D^0-$\bar{D}^0$ MIXING FROM NONLOCAL CONDENSATE CONTRIBUTIONS

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work in progress in collaboration with L. Dulibic (RBI, Zagreb) and A. Petrov (U. South Carolina)

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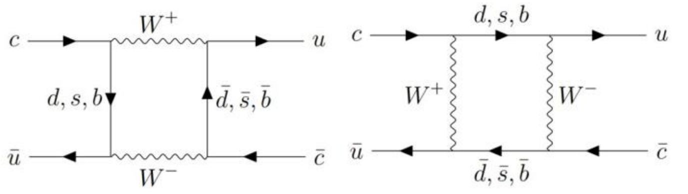
BASICS

neutral mesons mix:

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = \left(\begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11} \end{pmatrix}\right)\begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix}$$

off-shell states contribute   on -shell states contribute



parameters:

$$x = \frac{\Delta M_D}{\Gamma_D} \qquad y = \frac{\Delta \Gamma_D}{2\Gamma_D}$$

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$$

$$\Delta M \equiv M_1 - M_2, \quad \Delta\Gamma \equiv \Gamma_1 - \Gamma_2.$$

$$\Delta M_D = 2|M_{12}^D| \cdot (1 + O((\phi_{12}^D)^2))$$
$$\Delta \Gamma_D = 2|\Gamma_{12}^D| \cdot (1 + O((\phi_{12}^D)^2))$$

$$x \approx x_{12} = 2\frac{|M_{12}|}{\Gamma_D} \qquad y \approx y_{12} = \frac{|\Gamma_{12}|}{\Gamma_D}$$

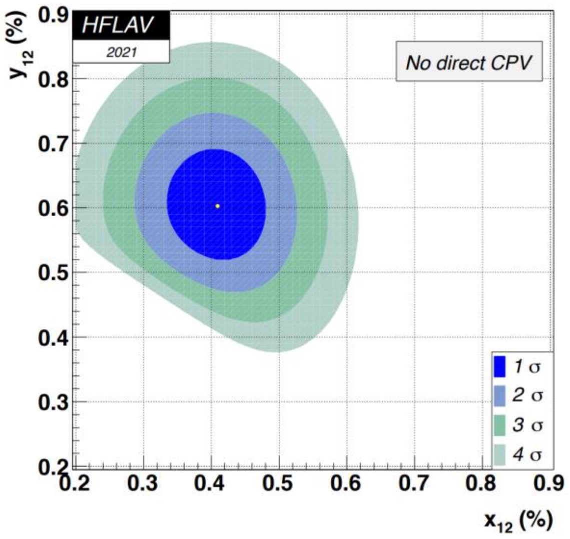
$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}$$

+ possible (indirect) CPV : $\phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right)$

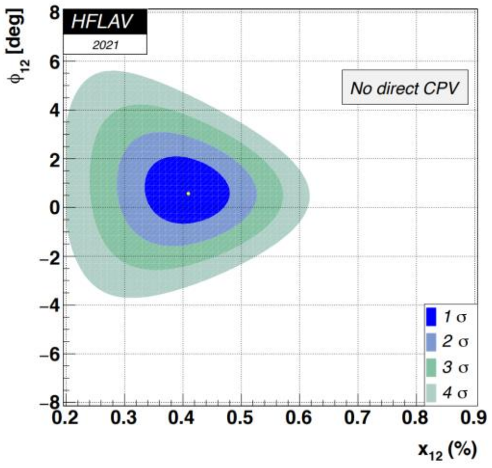
- more general approach with two phases Kagan, Silvestrini, 2001.07207 :

$$\phi_{12} \equiv \arg\left(\frac{M_{12}}{\Gamma_{12}}\right) = \phi^M - \phi^\Gamma$$

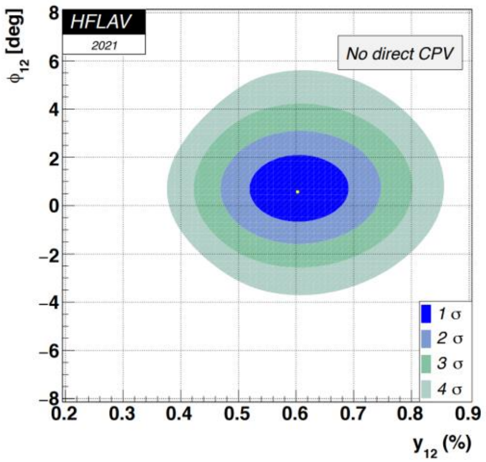
HFLAV fits, 2206.07501 - clear evidence for D^0-$\bar{D}^0$ mixing - no-mixing point x=y= 0 is excluded at >11.5 σ

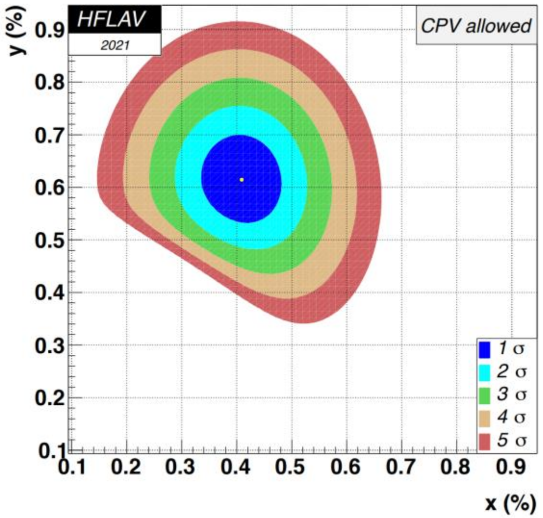


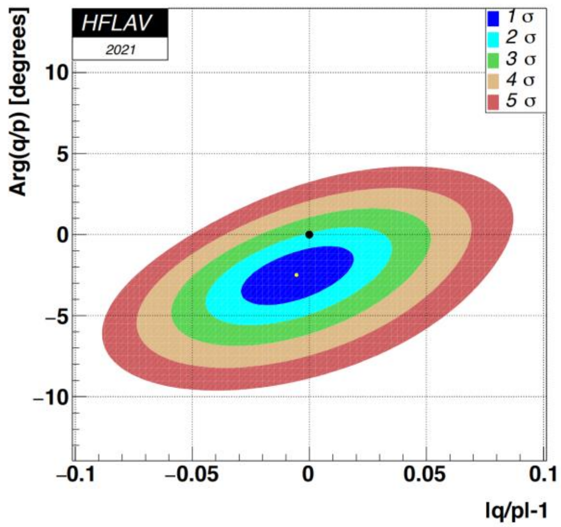
no direct evidence for CPV :



x







$$y_D = \frac{\Delta\Gamma}{2\Gamma_D} = 0.615^{+0.056}_{-0.055}\ \%$$

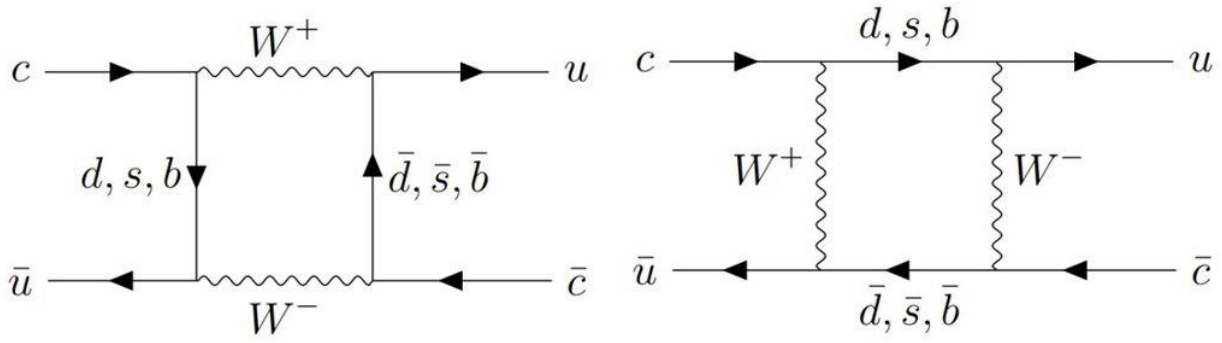
$$x_D = \frac{\Delta M}{2\Gamma_D} = 0.409^{+0.048}_{-0.049}\ \%$$

$$\phi(^\circ) = -2.5 \pm 1.2$$

$$2y_{CP} = (|q/p| + |p/q|)\, y \cos\phi - (|q/p| - |p/q|)\, x \sin\phi$$

$$\Rightarrow \quad y_{CP} = (0.719 \pm 0.113)\%$$

NAIVE HQE APPLICATION:



$\lambda_q = V_{cq} V_{uq}^*$

$$M_{12}^D = \lambda_s^2 \left[M_{ss}^D - 2M_{sd}^D + M_{dd}^D \right] + 2\lambda_s \lambda_b \left[M_{bs}^D - M_{bd}^D - M_{sd}^D + M_{dd}^D \right] + \lambda_b^2 \left[M_{bb}^D - 2M_{bd}^D + M_{dd}^D \right]$$

dispersive part

$$\Gamma_{12}^D = -\lambda_s^2 \left(\Gamma_{ss}^D - 2\Gamma_{sd}^D + \Gamma_{dd}^D \right) + 2\lambda_s \lambda_b \left(\Gamma_{sd}^D - \Gamma_{dd}^D \right) - \lambda_b^2 \Gamma_{dd}^D$$

absorptive part „Im part”

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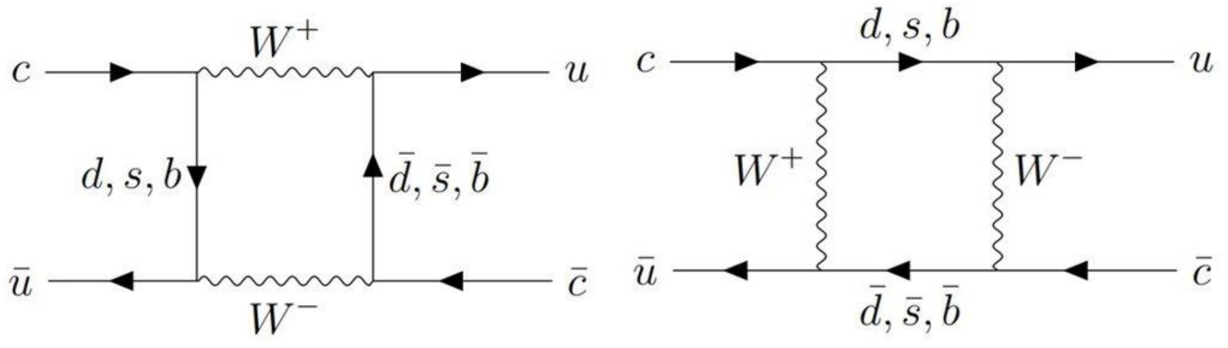

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CKM dominant <-> doubly GIM suppressed

CKM suppressed <-> GIM suppressed

doubly CKM suppressed <-> GIM dominant

all three contributions are of the same size and SMALL (equal zero in the chiral limit)
(although separate amplitudes are large: $\lambda_s^2 \Gamma_{ss}^D \tau_D \simeq 5.6\, y^{exp}$

EXTREME GIM suppression !

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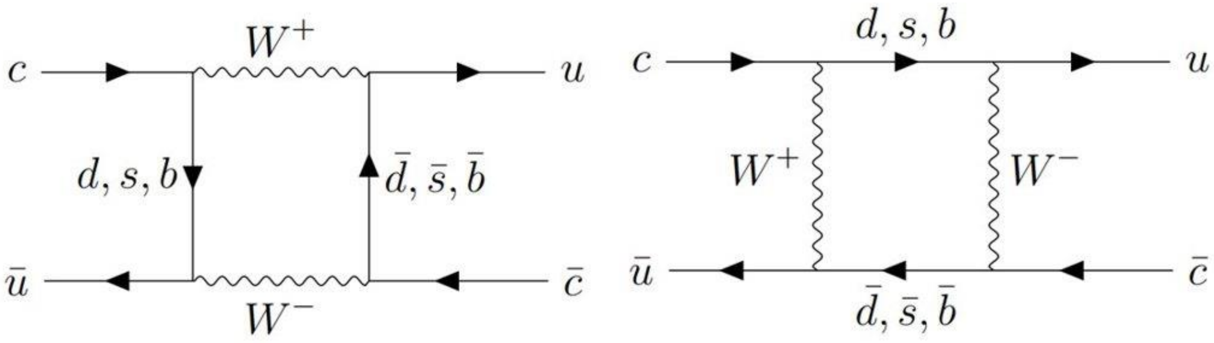

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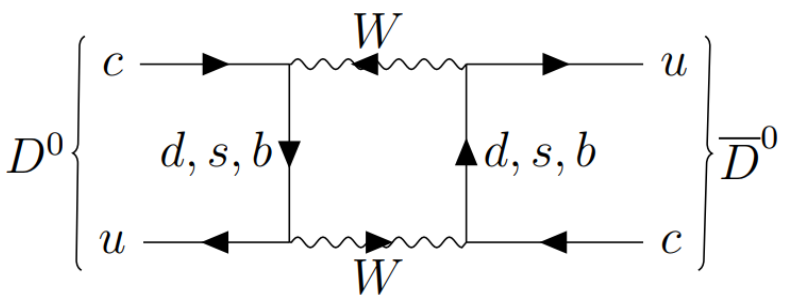
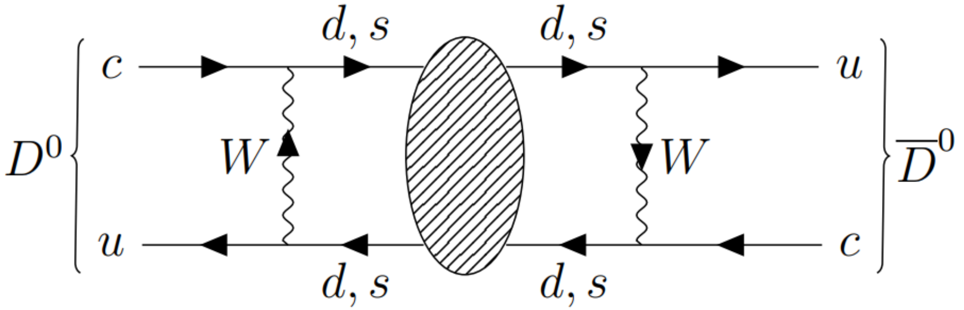
$$\Gamma_{12} = (2.08 \cdot 10^{-7} - 1.34 \cdot 10^{-11} I)\ (\text{1st term}) - (3.74 \cdot 10^{-7} + 8.31 \cdot 10^{-7} I)\ (\text{2nd term}) + (2.22 \cdot 10^{-8} - 2.5 \cdot 10^{-8} I)\ (\text{3rd term}).$$

$$y^{\text{naive HQE}} \sim (10^{-4}, 10^{-5})\ y^{\text{exp}}$$

SM results are 4 orders of magnitude smaller than experimental results ?!





the matrix element :

$$2M_D\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)=\langle D^0|\mathcal{H}^{\Delta C=2}|\bar{D}^0\rangle+\sum_n\frac{\langle D^0|\mathcal{H}^{\Delta C=1}|n\rangle\langle n|\mathcal{H}^{\Delta C=1}|\bar{D}^0\rangle}{M_D-E_n+i0^+}$$

M_{12}, local contribution at $\mu \sim M_D$

DOMINANT PART

M_{12} Γ_{12} intermediate states ($\pi\pi, \pi K, KK$ ...) contribution at $\mu \ll M_D$

INCLUSIVE (perturbative, HQE) APPROACH

EXCLUSIVE (nonperturbative) APPROACH

DISPERSIVE APPROACH – x and y are connected

LATTICE /HQET sum rules $\Delta C = 2$ operators only

lattice - Bazavov et al (Fermilab Lattice and MILC) 1706.04622
HQET sum rules - Kirk, Lenz , Rauch, 1711.02100

General solution to the problem in the HQE approach -> LIFTING THE GIM SUPPRESSION

INCLUSIVE HQE APPROACH

- NLO and mass corrections
- inclusion of new, higher operators
- quark-hadron duality violation
- different renormalization scales in the process

Golowich, Petrov - NLO corrections
Bobrowski, Lenz, Riedl, Rohrwild, 0904.3971 - alphaS and mass corrections
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Ohl, Ricciardi, 9301212 - higher dim operators in HQET – full matching
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- SU(3) breaking from final state phase space differences
- inclusion of multi-body states
- quark-hadron duality violation
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DISPERSIVE APPROACH

- x_D by modelling y_D; SU(3) breaking through physical thresholds of different D meson decay channels for y(s)

Falk, Grossmann, Ligeti, Nir, Petrov, 0402204 - from dispersion relation in HQET limit
H-n. Li, Umeeda, Xu, Yu, 2001.04079 - as an inverse problem
H-n. Li, 2208.14798

ADDITIONAL PROBLEMS IN D0-D0 MIXING IN ALL APPROCHES:

- charm mass m_c is close to the hadronic scale Λ_{QCD} $\longrightarrow$ expansion in Λ_{QCD} / m_c is slowly converging

although the expansion <O>/ m_c seems to work for c-hadron lifetimes

King, Lenz, Piscopo, Rauh, Rusov, Vlahos, Revisiting inclusive decay widths of charmed mesons, 2109.13219
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- the strong coupling at m_c is large ~ 0.3

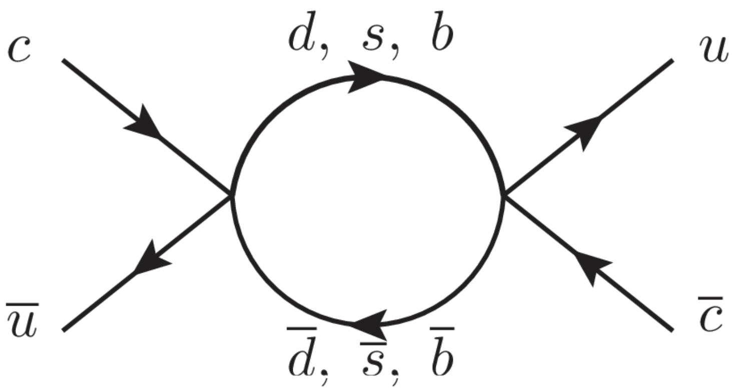
POSSIBLE EXISTENCE OF LARGE NONPERTURBATIVE EFFECTS

LIFTING GIM SUPPRESSION BY HIGHER DIMENSIONAL OPERATORS
- m_s EFFECTS IN PRACTICAL VERSION OF OPE -> CONDENSATES

Bigi, Uraltsev, 0005089

BOX DIAGRAM

$\propto (m_s/m_c)^4$



$$S_q = \frac{\not p + m_q}{p^2 - m_q^2} = \frac{\not p + m_q}{p^2}\left(1 + \frac{m_q^2}{p^2} + \cdots\right)$$

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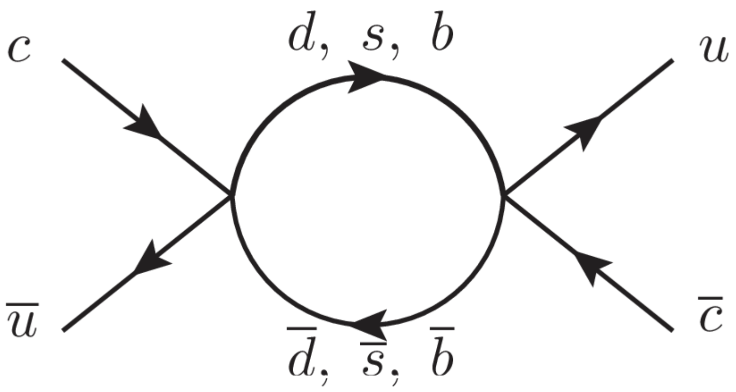

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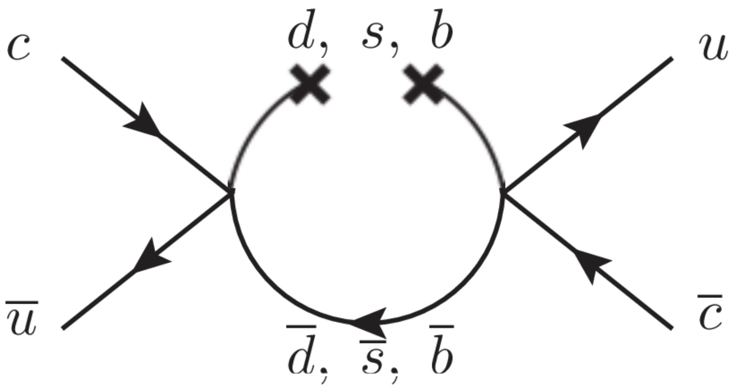
OPE -> dim 9 operator – **QCD CONDENSATES**



$$\langle \bar{q}(x) q(0) \rangle \propto 1 + i\not{x}\frac{m_q}{4} + \cdots$$

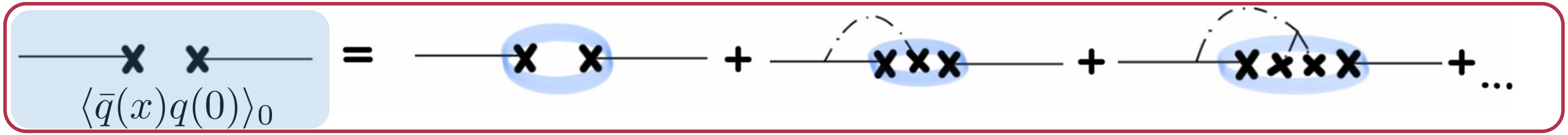
$\propto (m_s/m_c)^3$

- we trade a power of m_s/m_c suppression for a suppression of the higher dimensional operator.
- don't forget - there is also $16\pi^2$ relative enhancement since **this is not a loop calculation**

NONLOCAL CONDENSATE CONTRIBUTION

Local expansion of propagators in the soft background field generates condensates:

M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, QCD and resonance physics. theoretical foundations, Nucl. Phys. B147, Issue 5, 1979, 385-447



$$\langle \bar{q} q \rangle_0 = (-243\,\mathrm{MeV})^3$$

$$\frac{\langle \bar{q} i\sigma G q \rangle_0}{2\langle \bar{q} q \rangle_0} = 0.4 \pm 0.1\,\mathrm{GeV}^2 \approx \frac{\langle \bar{q} D^2 q \rangle}{\langle \bar{q} q \rangle} = \lambda_q^2$$

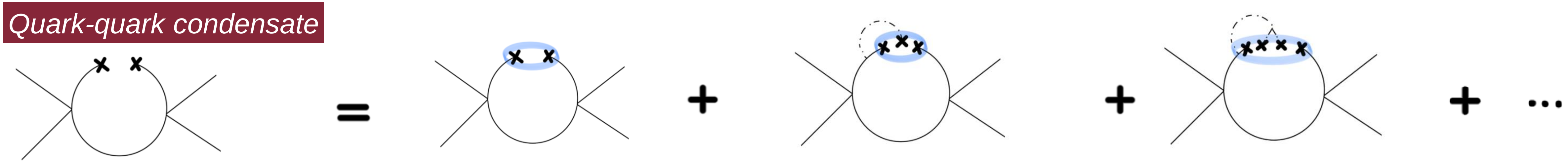
quark virtuality

$$\langle \bar{q} q \bar{q} q \rangle_0 \simeq \langle \bar{q} q \rangle_0^2$$

$$\langle \bar{q}(x)^a_\alpha q(0)^b_\beta \rangle = \frac{\langle \bar{q} q \rangle_0}{4N_C} \delta^{ab} [\delta_{\alpha\beta} \left(1 - \frac{x^2}{4}\left(\frac{m^2}{2} - \frac{\langle \bar{q}\sigma G q \rangle_0}{2\langle \bar{q} q \rangle_0}\right) \ldots\right) + i(x)_{\beta\alpha} \left(\frac{m}{4} - \frac{x^2}{4}\left(\frac{m^3}{12} - \frac{m}{12}\frac{\langle \bar{q}\sigma G q \rangle_0}{\langle \bar{q} q \rangle_0} + \frac{2}{81}\pi\alpha_s^{NP}\frac{\langle \bar{q} q \rangle_0^2}{\langle \bar{q} q \rangle_0}\right) \ldots\right)]$$

QCD CONDENSATES The relevant contributions in the local expansion

Quark-quark condensate



$$\langle \overline{q}(x)^a_\alpha q(0)^b_\beta \rangle = \frac{\langle \overline{q}q\rangle_0}{4N_C}\delta^{ab}\left[\delta_{\alpha\beta}\left(1-\frac{x^2}{4}\left(\frac{m^2}{2}-\frac{\lambda_q^2}{2}\right)\ldots\right)++i(\not{x})_{\beta\alpha}\left(\frac{m}{4}-\frac{x^2}{4}\left(\frac{m^3}{12}-\frac{m}{12}\lambda_q^2+\frac{2}{81}\pi\alpha_s\frac{\langle \overline{q}q\rangle_0^2}{\langle \overline{q}q\rangle_0}\right)\ldots\right)\right]$$

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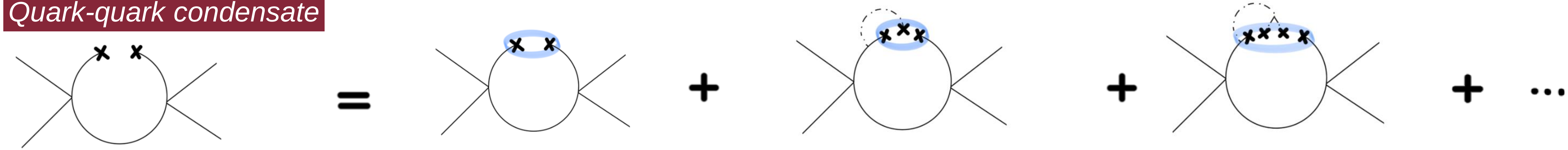

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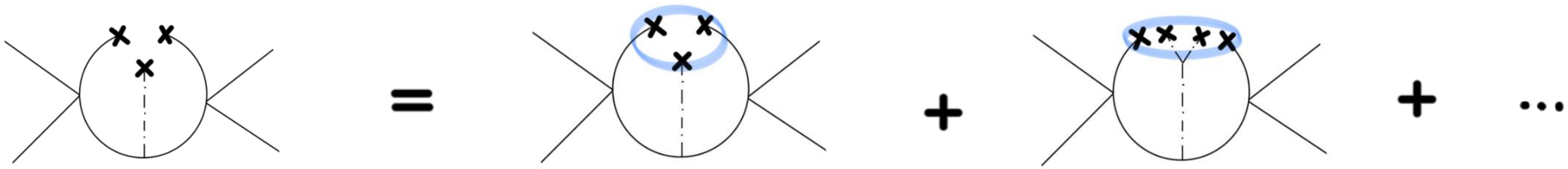
Mixed condensate



$$\langle\overline{q}^a_\alpha(x)G^{cd}_{\mu\nu}(0)q^b_\beta(0)\rangle = \langle\bar{q}q\rangle_0\frac{8}{1536}\left(\delta^{bd}\delta^{ac}-\frac{1}{3}\delta^{cd}\delta^{ab}\right)\left[\lambda_q^2\left(\sigma_{\mu\nu}+\frac{m}{2}\left(i\sigma_{\mu\nu}\not{x}+\gamma_\mu x_\nu-\gamma_\nu x_\mu\right)\right)_{\beta\alpha}+\frac{i}{4}\left(\not{x}\sigma_{\mu\nu}\right)_{\beta\alpha}\frac{16\pi}{9}\alpha_s^{NP}\frac{\langle\overline{q}q\rangle_0^2}{\langle\overline{q}q\rangle_0}\right]$$

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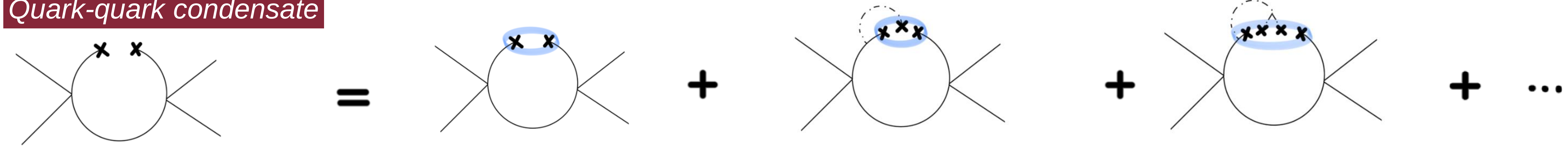

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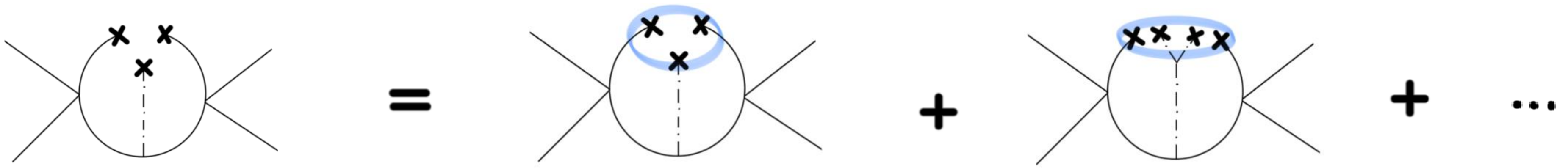

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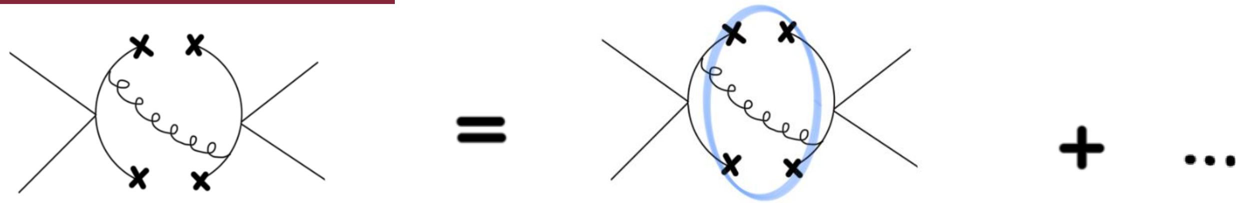
Four quark condensate



$$\alpha_s\langle \overline{q}^a_\alpha(x) q(0)^b_\beta \overline{q}^c_\gamma(0) q^d_\delta(x)\rangle \sim \alpha_s\langle\overline{q}q\rangle_0^2 \quad !$$

NONLOCAL QCD CONDENSATES

- Assumption - long distances play a crucial role in $D^0\overline{D}^0$ mixing
- Questions have been raised in the literature as to whether this expansion is well behaved

Mikhailov, Radyushkin, *Nonlocal condensates and QCD sum rules for the pion wave function*, Phys.Rev.D 45 (1992) 1754

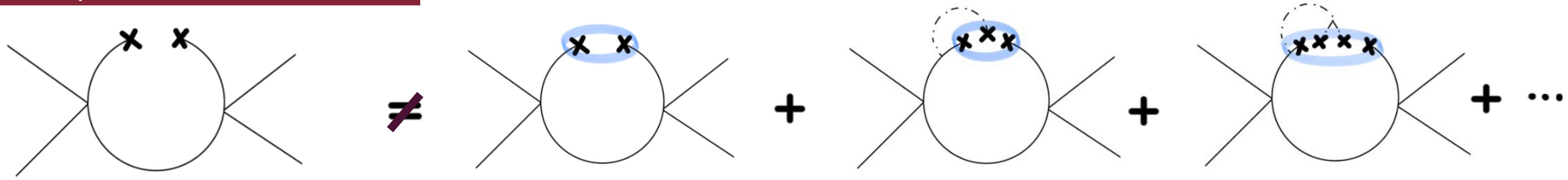
$$\langle\bar{q}(x)^a_\alpha q(0)^b_\beta\rangle = \frac{\langle\bar{q}q\rangle_0}{4N_C}\delta^{ab}[\delta_{\alpha\beta}\left(1-\frac{x^2}{4}\left(\frac{m^2}{2}-\frac{\langle\bar{q}\sigma Gq\rangle_0}{2\langle\bar{q}q\rangle_0}\right)\ldots\right)+i(x)_{\beta\alpha}\left(\frac{m}{4}-\frac{x^2}{4}\left(\frac{m^3}{12}-\frac{m}{12}\frac{\langle\bar{q}\sigma Gq\rangle_0}{\langle\bar{q}q\rangle_0}+\frac{2}{81}\pi\alpha_s^{NP}\frac{\langle\bar{q}q\rangle_0^2}{\langle\bar{q}q\rangle_0}\right)\ldots\right)]$$

$$\langle\bar{q}(x)^a_\alpha q(0)^b_\beta\rangle = \frac{\langle\bar{q}q\rangle}{4N_C}\delta^{ab}[\delta_{\alpha\beta}F_S(x)+i(x)_{\beta\alpha}F_V(x)]$$

partial resummation of the OPE to all orders
– F_S (x) and F_V (x) as vacuum distribution functions

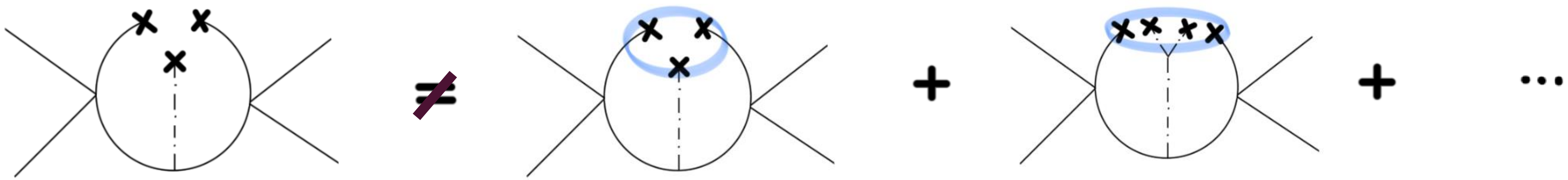
NONLOCAL QCD CONDENSATES — Nonlocal generalization

Quark-quark nonlocal condensate



$$\langle \bar{q}(x)^a_\alpha q(0)^b_\beta \rangle = \frac{\langle \bar{q}q \rangle}{4N_C} \delta^{ab} [\delta_{\alpha\beta} F_S(x) + i(\not{x})_{\beta\alpha} F_V(x)]$$

Mixed nonlocal condensate



$$\langle \bar{q}^a_\alpha(x) G^{cd}_{\mu\nu}(0) q^b_\beta(0) \rangle$$
$$= \frac{4}{1536}\left(\delta^{ac}\delta^{bd} - \frac{1}{3}\delta^{ab}\delta^{cd}\right)\left(\left(\sigma_{\mu\nu} + \frac{m}{2}(i\sigma_{\mu\nu}\not{x} + \gamma_\mu x_\nu - \gamma_\nu x_\mu)\right)\langle \bar{q} i\sigma G q \rangle + \frac{i}{2}\not{x}\sigma_{\mu\nu}\frac{16}{9}\pi\alpha_s^{NP}\langle \bar{q}q \rangle^2\right)_{\beta\alpha} F_G(x)$$

How is the nonlocality modeled?

- *condition 1:* must reproduce the expansion in small-x limit
- *condition 2:* must decay in the large-x limit

fixed by the first moments of the expansion

$$F_{S,V,G}(x) = \int_0^\infty d\alpha \left(B_{S,V,G} f(\alpha) + A_{S,V,G} f'(\alpha)\right) e^{-\alpha \frac{x^2}{4}}$$

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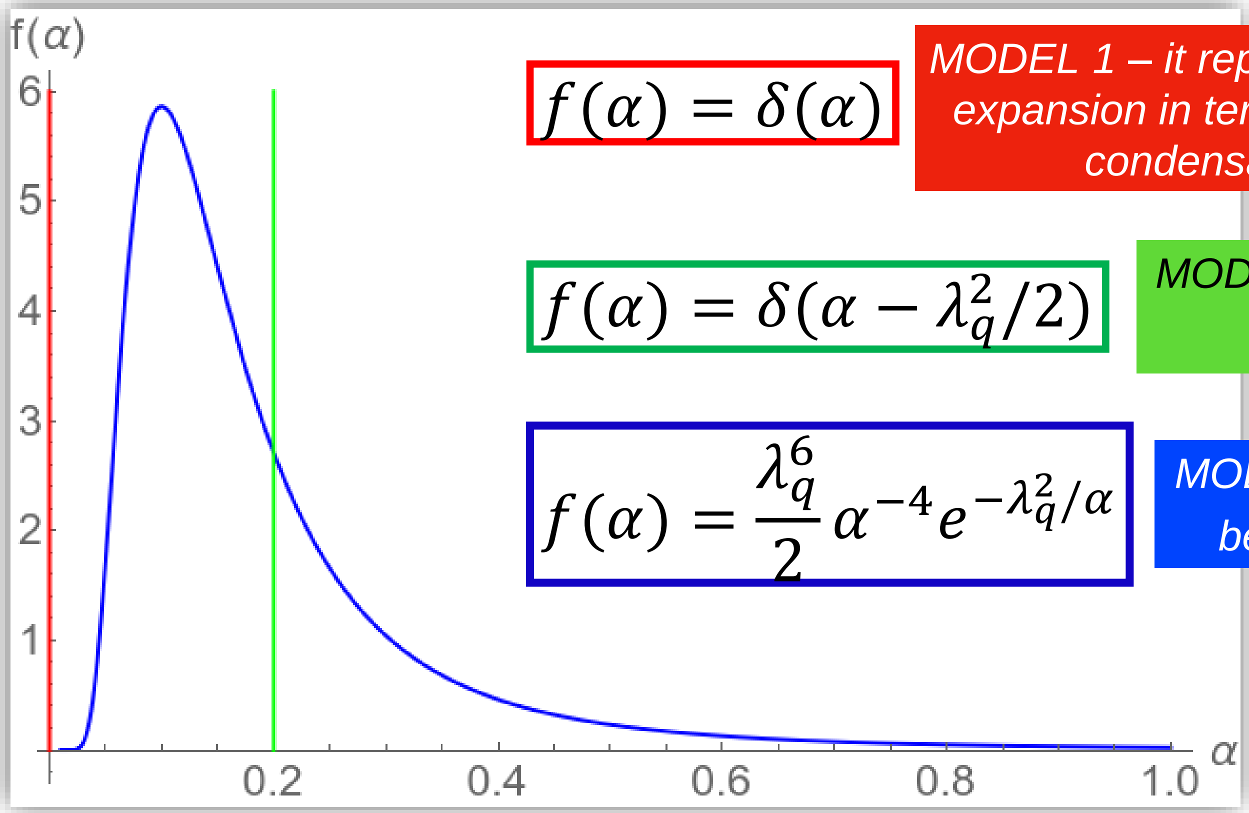


$$f(\alpha) = \delta(\alpha)$$

MODEL 1 – it reproduces the expansion in terms of local condensates

$$f(\alpha) = \delta(\alpha - \lambda_q^2/2)$$

MODEL 2 – it models large-x behavior to be Gaussian-like $e^{-\lambda^2 x^2}$

$$f(\alpha) = \frac{\lambda_q^6}{2} \alpha^{-4} e^{-\lambda_q^2/\alpha}$$

MODEL 3 – it models large-x behavior to be like $e^{-\lambda x}$

V. M. Braun, D. Y. Ivanov and G. P. Korchemsky, *The B meson distribution amplitude in QCD*, [0309330]

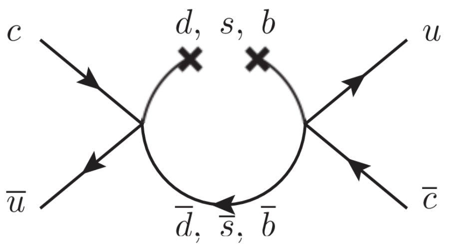
$$\lambda_q^2 = \frac{\langle \bar{q}(D)^2 q\rangle}{\langle \bar{q}q\rangle} \approx \frac{\langle \bar{q} i\sigma G q\rangle}{2\langle \bar{q}q\rangle}$$

average quark virtuality

(Preliminary) **Results** for D0-D0bar mixing (x-parameter) :

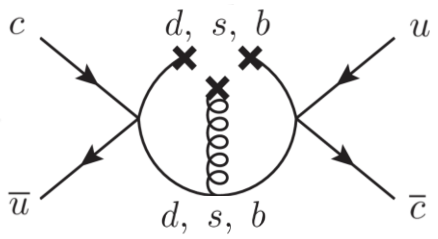
expanding in terms of local condensates

using the simplest nonlocal model M2

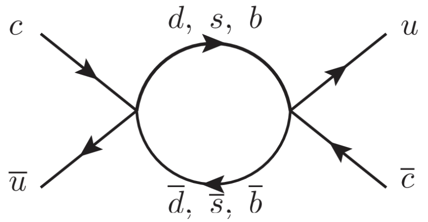


-4.6×10^{-6}

-5.8×10^{-6}



-1.9×10^{-6}



the perturbative contribution

-3.6×10^{-6}

total from nonlocal condensates

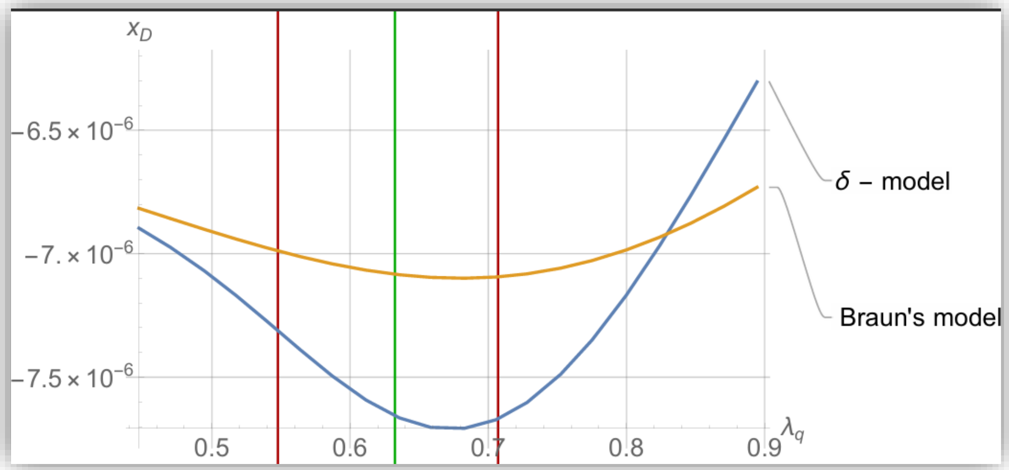
-7.7×10^{-6}

$x = -1.13 \times 10^{-5}$

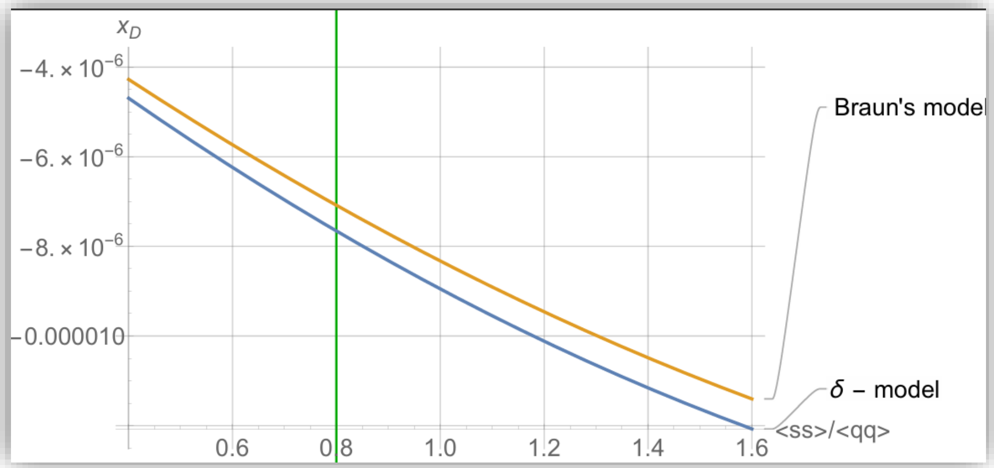
FINAL RESULT

(Preliminary) **Results** (model & parameter dependence) :

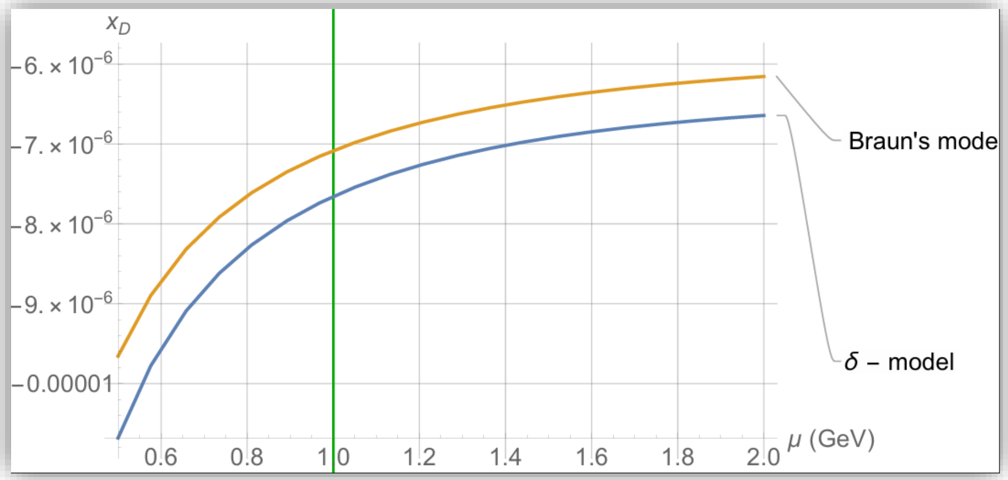
Dependence on the quark virtuality



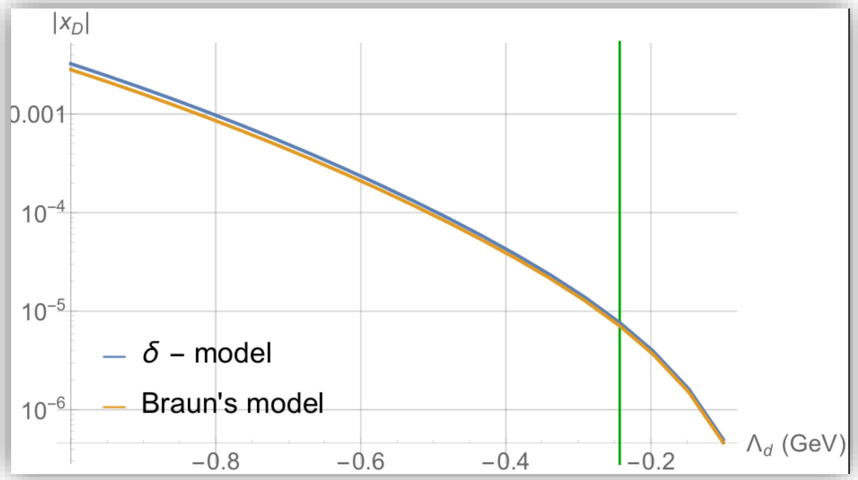
Dependence on the SU(3) breaking in quark condensate



Dependence on the mu-scale



Dependence on the value of the quark condensate - STRONG



Concluding remarks

- The result is sensitive to

P. Gubler and D. Satow, *Recent Progress in QCD Condensate Evaluations and Sum Rules*, [1812.00385]

  - **Condensate value** $\langle\bar{q}q\rangle$, as well as **the ratio** $\frac{\langle\bar{s}s\rangle}{\langle\bar{q}q\rangle}$ = 0.8 +/- 0.3
  - **Quark virtuality** λ^2_q = 0.4 +/- 0.1 GeV^2 , but much larger values are also reported
  - **Scale μ dependence**

$$x = -1.13 \times 10^{-5}$$

PRELIMINARY FINAL RESULT

Concluding remarks

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P. Gubler and D. Satow, *Recent Progress in QCD Condensate Evaluations and Sum Rules,* [1812.00385]

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  - **Scale μ dependence**

$$x = -1.13 \times 10^{-5}$$

PRELIMINARY FINAL RESULT

- **FUTURE RESEARCH** - calculation of the **four-quark condensate** contributions
  - both propagators in the box diagram are replaced by condensates - expected dependence on strange mass $\propto (m_s/m_c)^2$
  - this is supposed to be the (parametrically) **leading contribution!**

A. F. Falk, Y. Grossman, Z. Ligeti, and A. A. Petrov, *SU(3) breaking and D0−D0 mixing,* [hep-ph/0110317]