KOLYA AND NEW RESULTS ON INCLUSIVE V_{CB}

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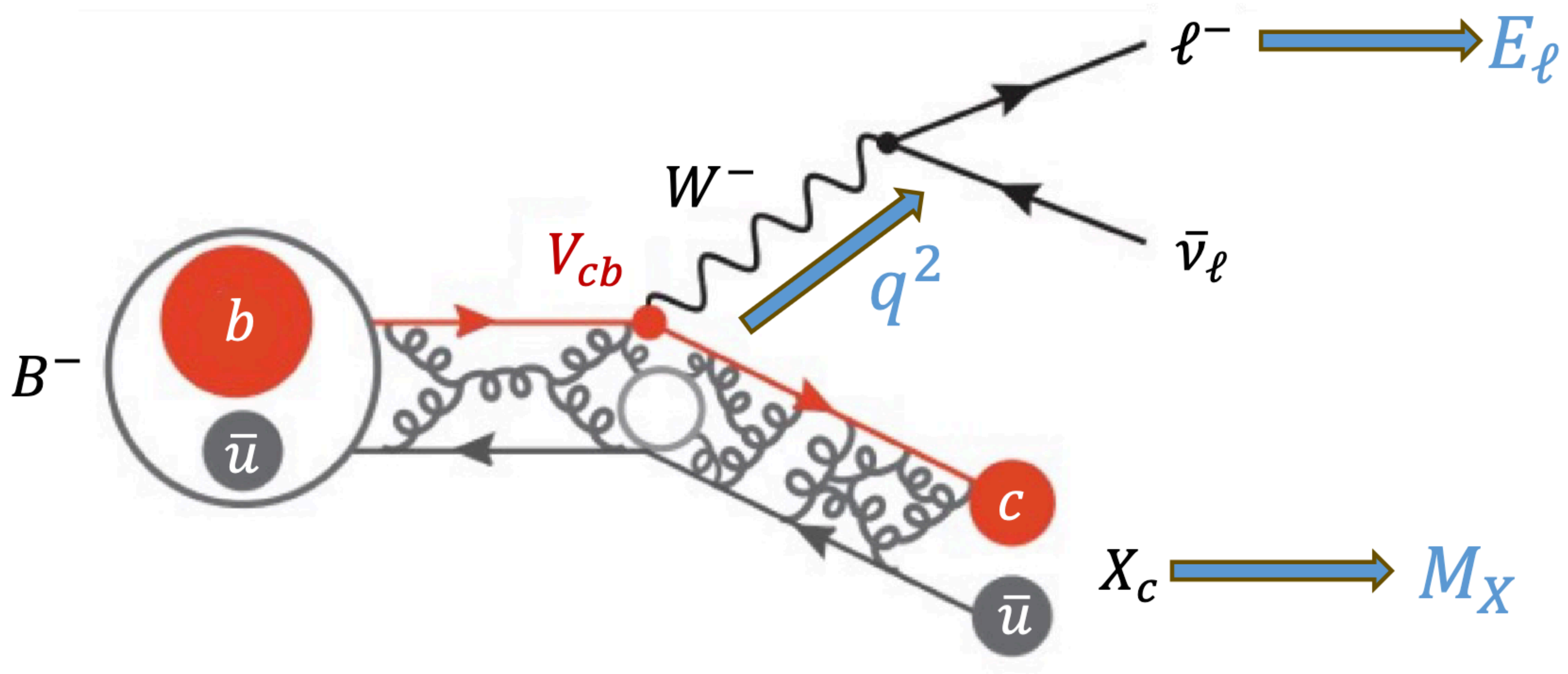
MARIE CURIE ACTIONS

Funded by the European Union

EXTRACTION OF $|V_{cb}|$ FROM INCLUSIVE $\overline{B} \to X_c l \bar{\nu}_l$



Observables

- Total rate $\Gamma_{\rm sl} = \Gamma(B \to X_c \ell \bar{\nu}_\ell)$
- Moments of the differential distribution of an observables O

- $O = E_\ell$: energy of the charged lepton in the B rest frame
- $O = M_X^2$: hadronic invariant mass
- $O = q^2$: leptonic invariant mass

$$\langle (O)^n \rangle_{\rm cut} = \int_{\rm cut} (O)^n \frac{\mathrm{d}\Gamma}{\mathrm{d}O} \mathrm{d}O \Big/ \int_{\rm cut} \frac{\mathrm{d}\Gamma}{\mathrm{d}O} \mathrm{d}O$$

HEAVY QUARK EXPANSION

Double series expansion in the strong coupling constant α_s and power suppressed terms Λ_{QCD}/m_b

- Total rate

$$\Gamma_{\text{sl}} = \frac{G_F^2 m_b^5 A_{\text{ew}}}{192\pi^3} |V_{cb}|^2 \left[\left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) \left(X_0(\rho) + \frac{\alpha_s}{\pi} X_1(\rho) + \left(\frac{\alpha_s}{\pi}\right)^2 X_2(\rho) + \left(\frac{\alpha_s}{\pi}\right)^3 X_3(\rho) + \ldots \right) + \left(\frac{\mu_G^2}{m_b^2} - \frac{\rho_D^3}{m_b^3} \right) \left(g_0(\rho) + \frac{\alpha_s}{\pi} g_1(\rho) + \ldots \right) + \frac{\rho_D^3}{m_b^3} \left(d_0(\rho) + \frac{\alpha_s}{\pi} d_1(\rho) + \ldots \right) + O\left(\frac{1}{m_b^4}\right) \right]$$

- Moments of differential distribution

$$\langle O^n \rangle_{\text{cut}} = (m_b)^{mn} \left[X_0^{(O,n)} + \frac{\alpha_s}{\pi} X_1^{(O,n)} + \left(\frac{\alpha_s}{\pi}\right)^2 X_2^{(O,n)} + \frac{\mu_\pi^2}{m_b^2} \left(p_0^{(O,n)} + \frac{\alpha_s}{\pi} p_1^{(O,n)} + \ldots \right) + \frac{\mu_G^2}{m_b^2} \left(g_0^{(O,n)} + \frac{\alpha_s}{\pi} g_1^{(O,n)} + \ldots \right) + \frac{\rho_D^3}{m_b^3} \left(d_0^{(O,n)} + \frac{\alpha_s}{\pi} d_1^{(O,n)} + \ldots \right) + \frac{\rho_{LS}}{m_b^2} \left(l_0^{(O,n)} + \frac{\alpha_s}{\pi} l_1^{(O,n)} + \ldots \right) + O\left(\frac{1}{m_b^4}\right) \right]$$

EXPERIMENTAL STATUS

Experiment	Hadron moments $<M^n_X>$	Lepton moments $<E^n_l>$	References
BaBar	n=2 c=0.9,1.1,1.3,1.5 n=4 c=0.8,1.0,1.2,1.4 n=6 c=0.9,1.3 [1]	n=0 c=0.6,1.2,1.5 n=1 c=0.6,0.8,1.0,1.2,1.5 n=2 c=0.6,1.0,1.5 n=3 c=0.8,1.2 [1,2]	[1] Phys.Rev. D81 (2010) 032003 [2] Phys.Rev. D69 (2004) 111104
Belle	n=2 c=0.7,1.1,1.3,1.5 n=4 c=0.7,0.9,1.3 [3]	n=0 c=0.6,1.4 n=1 c=1.0,1.4 n=2 c=0.6,1.4 n=3 c=0.8,1.2 [4]	[3] Phys.Rev. D75 (2007) 032005 [4] Phys.Rev. D75 (2007) 032001
CDF	n=2 c=0.7 n=4 c=0.7 [5]	.	[5] Phys.Rev. D71 (2005) 051103
CLEO	n=2 c=1.0,1.5 n=4 c=1.0,1.5 [6]	.	[6] Phys.Rev. D70 (2004) 032002
DELPHI	n=2 c=0.0 n=4 c=0.0 n=6 c=0.0 [7]	n=1 c=0.0 n=2 c=0.0 n=3 c=0.0 [7]	[7] Eur.Phys.J. C45 (2006) 35-59

- new Belle II measurements of q^2 moments

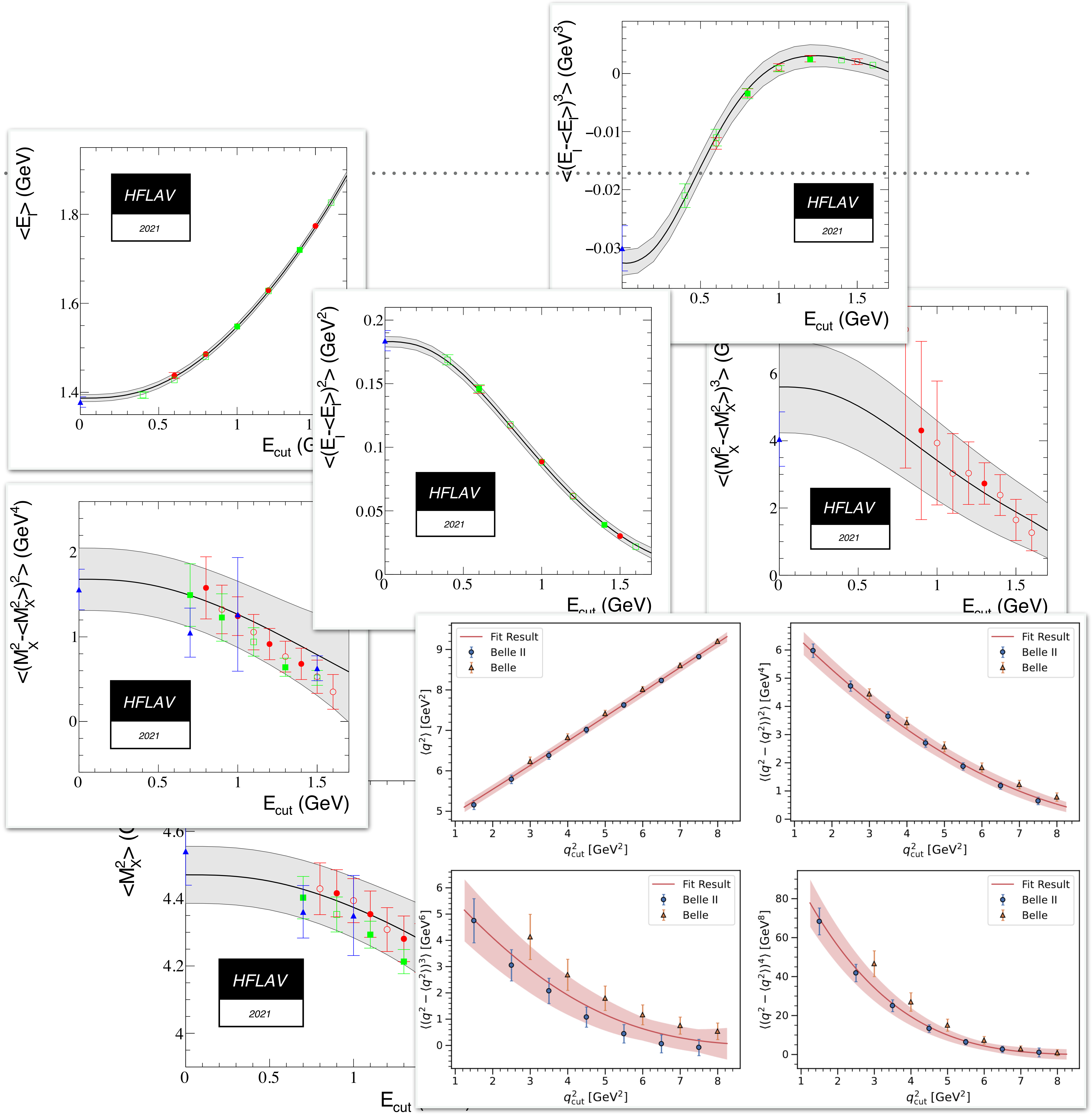
  Belle, Phys. Rev. D 104, 112011 (2022)
  Belle II, Phys. Rev. D 107, 072002 (2023)

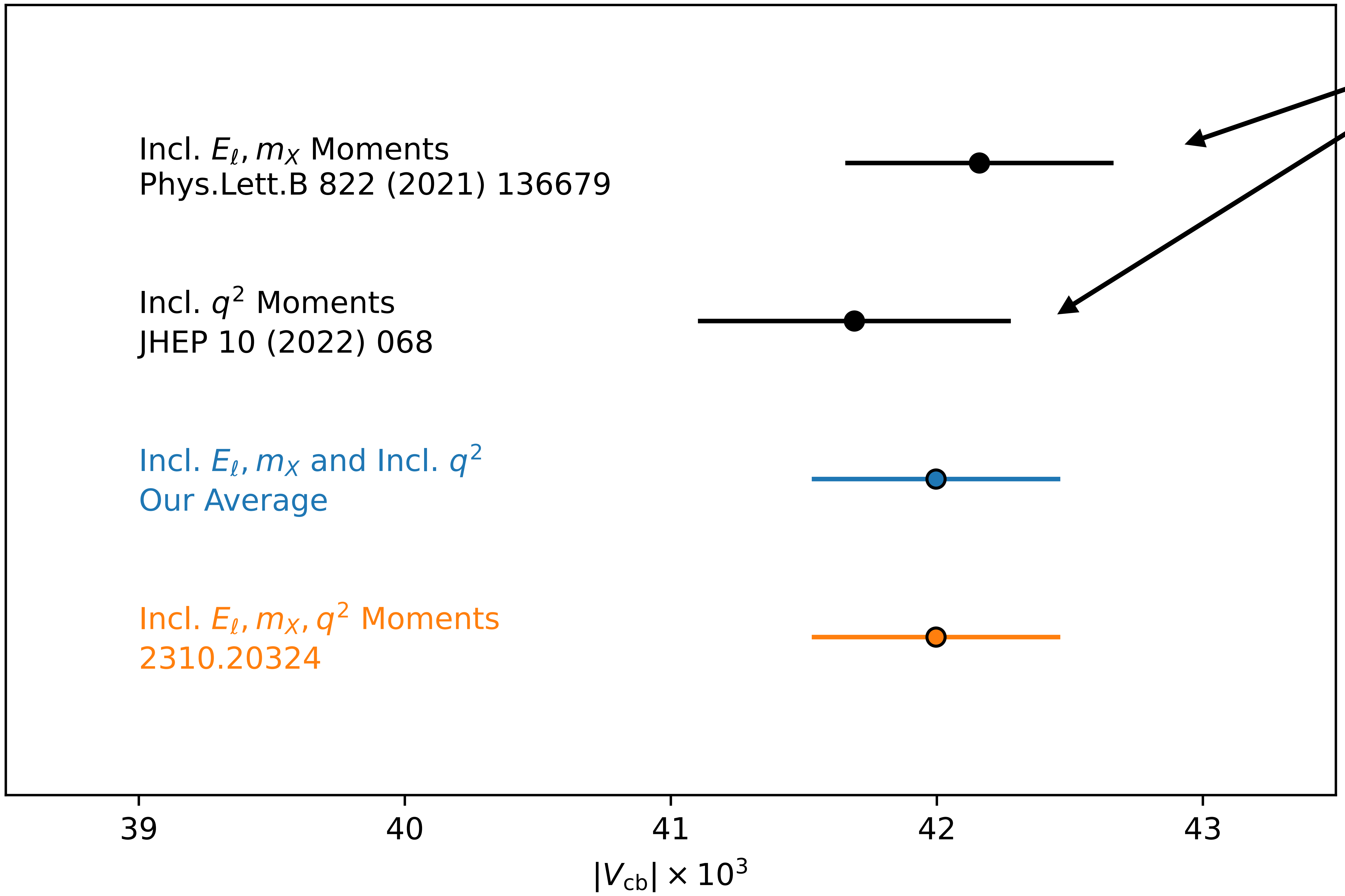
$$|V_{cb}|^{E_l, M_X, q^2} = (41.97 \pm 0.49) \times 10^{-3}$$

Gambino, Finauri, JHEP 02 (2024) 206

$$|V_{cb}|^{q^2} = (41.69 \pm 0.63) \times 10^{-3}$$

Bernlochner, **MF**, Olschewsky, Persson
van Tonder, Vos, Welsch *JHEP* 10 (2022) 068





Independent sets of data

MF, Prim, Vos, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w

- Difference mainly driven by the $\mathrm{Br}(B \to X_c l \bar{\nu}_l)$ average
- We need new $\mathrm{Br}(B \to X_c l \bar{\nu}_l)$ measurements to improve.

	$\mathcal{B}(B \to X \ell \bar{\nu}_\ell)$ (%)	$\mathcal{B}(B \to X_c \ell \bar{\nu}_\ell)$ (%)	In Average
Belle [63] $E_\ell > 0.6$ GeV	-	10.54 ± 0.31	✓
Belle [63] $E_\ell > 0.4$ GeV	-	10.58 ± 0.32	
CLEO [65] incl.	10.91 ± 0.26	10.72 ± 0.26	
CLEO [65] $E_\ell > 0.6$	10.69 ± 0.25	10.50 ± 0.25	✓
BaBar [62] incl.	10.34 ± 0.26	10.15 ± 0.26	✓
BaBar SL [64] $E_\ell > 0.6$ GeV	-	10.68 ± 0.24	✓
Our Average	-	10.48 ± 0.13	
Average Belle [63] & BaBar [64] ($E_\ell > 0.6$ GeV)	-	10.63 ± 0.19	

EXPERIMENTAL PERSPECTIVES

Proposed analysis @ Belle II

2023 Belle II physics week
https://indico.belle2.org/event/9402/overview

- Redo spectral moment measurements of q^2, M_X^2, and E_l moments in a single analysis.
- Very valuable to capture the full experimental correlations.
- A_{FB} & other differential measurements (q^2, M_X^2, and E_l moments for forward and backward events)
- Measurements w/ and w/o QED FSR corrections

comprehensive open-source framework where all available corrections in the HQE are implemented and validated.

INCLUSIVE DECAYS: OPEN-SOURCE LIBRARY

MF, Milutin, Vos, hep-ph/2407.XXXX

Nikolai Uraltsev 1957 - 2013

Open-source library in python: **KOLYA**

https://gitlab.com/vcb-inclusive/kolya

- Prediction in the HQE for
  - $\Gamma_{\rm sl}$ and $\Delta\Gamma_{\rm sl}(E_{\rm cut})$
  - Centralised moments $\langle E_\ell \rangle_{E_{\rm cut}}$, $\langle M_X^2 \rangle_{E_{\rm cut}}$
  - Centralised moments $\langle q^2 \rangle_{q^2_{\rm cut}}$

- Use the kinetic scheme

Bigi, Shifman, Uraltsev, Vainshtein, *Phys.Rev.D* 56 (1997) 4017
Czarnecki, Melnikov, Uraltsev, Phys.Rev.Lett. 80 (1998) 3189
MF, Schönwald, Steinhauser, *Phys.Rev.Lett.* 125 (2020) 052003

- Interface to CRunDec for automatic α_s, $m_b^{\rm kin}$ and $\overline{m}_c$ RGE evolution

Chetyrkin, Kuhn, Steinhauser, Comput. Phys. Commun. 133 (2000) 43
Schmidt, Steinhauser, Comput. Phys. Commun. 183 (2012) 1845
Herren, Steinhauser, Comput. Phys. Commun. 224 (2018) 333

HEAVY QUARK EXPANSION

Double series expansion in the strong coupling constant α_s and power suppressed terms $\Lambda_{\mathrm{QCD}}/m_b$

- Total rate

$$\Gamma_{\mathrm{sl}} = \frac{G_F^2 m_b^5 A_{\mathrm{ew}}}{192\pi^3} |V_{cb}|^2 \left[\left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) \left(X_0(\rho) + \frac{\alpha_s}{\pi} X_1(\rho) + \left(\frac{\alpha_s}{\pi}\right)^2 X_2(\rho) + \left(\frac{\alpha_s}{\pi}\right)^3 X_3(\rho) + \ldots \right) \right.$$
$$\left. + \left(\frac{\mu_G^2}{m_b^2} - \frac{\rho_D^3}{m_b^3}\right) \left(g_0(\rho) + \frac{\alpha_s}{\pi} g_1(\rho) + \ldots \right) + \frac{\rho_D^3}{m_b^3} \left(d_0(\rho) + \frac{\alpha_s}{\pi} d_1(\rho) + \ldots \right) + O\left(\frac{1}{m_b^4}\right) \right]$$

- Moments of differential distribution

$$\langle O^n \rangle_{\mathrm{cut}} = (m_b)^{mn} \left[X_0^{(O,n)} + \frac{\alpha_s}{\pi} X_1^{(O,n)} + \left(\frac{\alpha_s}{\pi}\right)^2 X_2^{(O,n)} + \frac{\mu_\pi^2}{m_b^2} \left(p_0^{(O,n)} + \frac{\alpha_s}{\pi} p_1^{(O,n)} + \ldots \right) \right.$$
$$\left. + \frac{\mu_G^2}{m_b^2} \left(g_0^{(O,n)} + \frac{\alpha_s}{\pi} g_1^{(O,n)} + \ldots \right) + \frac{\rho_D^3}{m_b^3} \left(d_0^{(O,n)} + \frac{\alpha_s}{\pi} d_1^{(O,n)} + \ldots \right) + \frac{\rho_{LS}}{m_b^2} \left(l_0^{(O,n)} + \frac{\alpha_s}{\pi} l_1^{(O,n)} + \ldots \right) + O\left(\frac{1}{m_b^4}\right) \right]$$

BUILDING BLOCKS IN THE HQE

$\Gamma_{\rm sl}$	tree	α_s	α_s^2	α_s^3
Partonic		[1]	[2–5]	[6]
μ_π^2, μ_G^2	[7, 8]	[9–12]		
ρ_D^3, ρ_{LS}^3	[13]	[14]		
$1/m_b^4, 1/m_b^5$	[15–18]			
$q_n(q^2_{\rm cut})$	tree	α_s	α_s^2	
Partonic		[14, 19]	[20]	
μ_G^2, μ_π^2	[7, 8]	[10, 11]		
ρ_D^3, ρ_{LS}	[13]	[14]		
$1/m_b^4, 1/m_b^5$	[17, 18]			
$\ell_n(E_{\rm cut}), h_n(E_{\rm cut})$	tree	α_s	$\alpha_s^2\beta_0$	α_s^2
Partonic		[19, 21, 22]	[19, 22]	[23]
μ_G^2	[7, 8]	[10, 11]		
ρ_D^3	[13]			
$1/m_b^4, 1/m_b^5$	[15, 16, 18]			

complete references in backup slides

talk by I. Milutin

- Power up to $1/m_b^5$
- Perturbative corrections to $\Gamma_{\rm sl}$ up to $O(\alpha_s^3)$

MF, Schönwald, Steinhauser, Phys.Rev.D 104 (2021) 016003, JHEP 08 (2022) 039

- NLO corrections to power suppressed terms for q^2 moments

Mannel, Moreno, Pivovarov, JHEP 08 (2020) 089

- NNLO corrections to q^2 moments

MF, Herren, JHEP 05 (2024) 287

IMPLEMENTATION

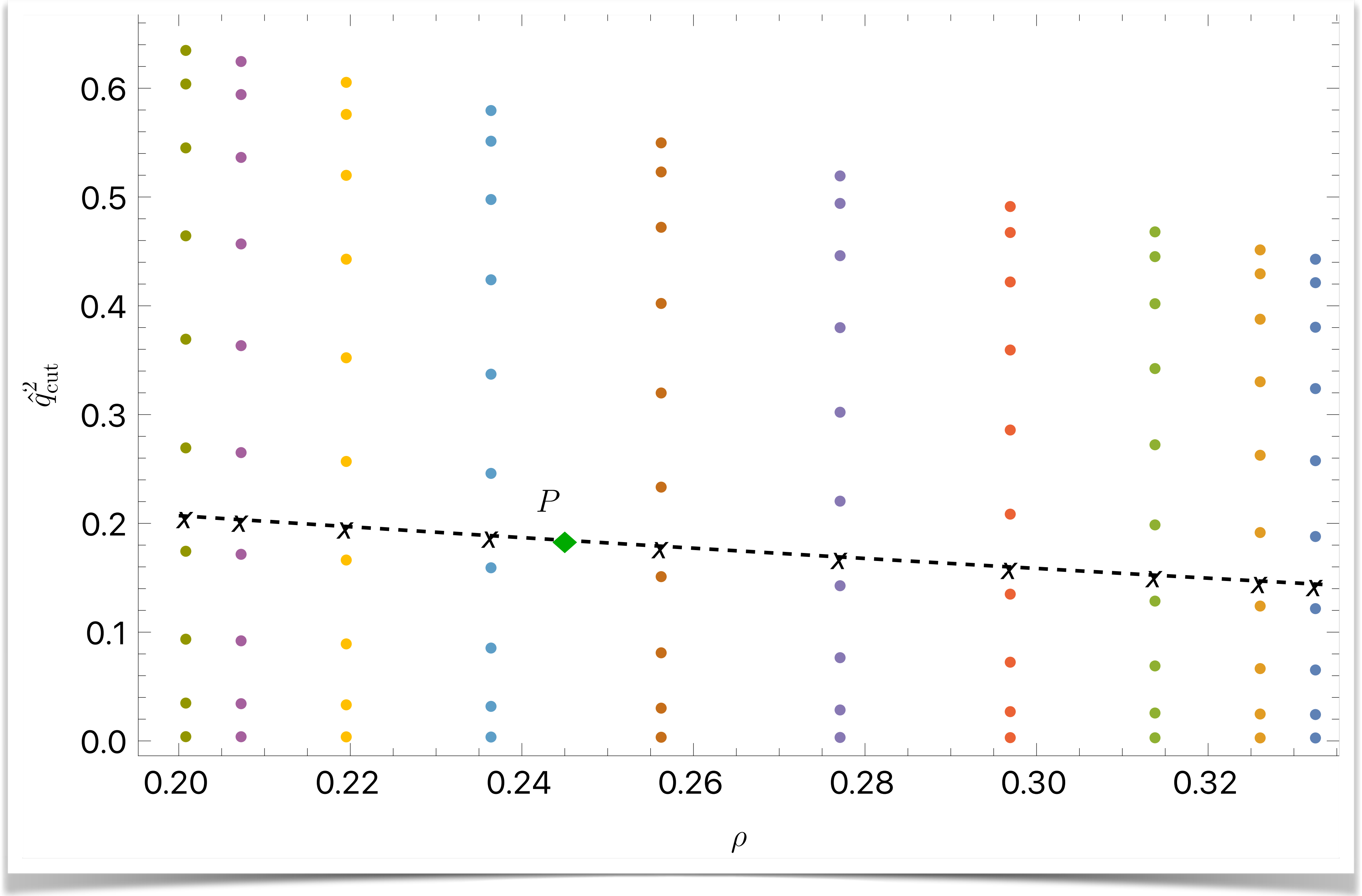
$\rho = m_c/m_b \quad \hat{q}^2 = q^2/m_b^2$

- Tree level implemented in exact form
- We implement analytic results for higher QCD corrections for Γ_{sl}
  - Exact results at NLO
  - Asymptotic expansions at NNLO and N3LO
- Use Numba for fast numerical evaluation

https://numba.pydata.org

- Chebyshev interpolation grids for QCD corrections to the moments

$$f(\rho, \hat{q}^2_{\text{cut}}) = \int_{q^2 > q^2_{\text{cut}}} (q^2)^i \, (q_0)^j \, \frac{d^3\Gamma^{\text{NLO}}}{dq^2\, dq_0\, dE_l} dq^2\, dq_0\, dE_l$$



SEMILEPTONIC INCLUSIVE DECAYS: NP EFFECTS

MF, Rahimi, Vos, JHEP 02 (2023) 086

- We include also effects of heavy NP parametrised in terms of dimension-six operators:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F V_{cb}}{\sqrt{2}} \left[\left(1 + C_{V_L}\right) \mathcal{O}_{V_L} + \sum_{i=V_R,S_L,S_R,T} C_i \, \mathcal{O}_i \right]$$

$$\mathcal{O}_{V_{L(R)}} = \left(\bar{c}\gamma_\mu P_{L(R)} b\right) \left(\bar{\ell}\gamma^\mu P_L \nu_\ell\right)$$

$$\mathcal{O}_{S_{L(R)}} = \left(\bar{c} P_{L(R)} b\right) \left(\bar{\ell} P_L \nu_\ell\right)$$

$$\mathcal{O}_T = \left(\bar{c}\, \sigma_{\mu\nu} P_L b\right) \left(\bar{\ell}\, \sigma^{\mu\nu} P_L \nu_\ell\right)$$

- Contribution to the moments of $B \to X_c l \bar{\nu}_l$

$$\langle O \rangle = \xi_{\text{SM}} + |C_{V_R}|^2 \, \xi_{\text{NP}}^{\langle V_R,V_R \rangle} + |C_{S_L}|^2 \, \xi_{\text{NP}}^{\langle S_L,S_L \rangle} + |C_{S_R}|^2 \, \xi_{\text{NP}}^{\langle S_R,S_R \rangle} + |C_T|^2 \, \xi_{\text{NP}}^{\langle T,T \rangle}$$
$$+ \text{Re}((C_{V_L} - 1) C_{V_R}^*) \, \xi_{\text{NP}}^{\langle V_L,V_R \rangle} + \text{Re}(C_{S_L} C_{S_R}^*) \, \xi_{\text{NP}}^{\langle S_L,S_R \rangle} + \text{Re}(C_{S_L} C_T^*) \, \xi_{\text{NP}}^{\langle S_L,T \rangle}$$
$$+ \text{Re}(C_{S_R} C_T^*) \, \xi_{\text{NP}}^{\langle S_R,T \rangle}$$

INSTALLATION

```
$: git clone https://gitlab.com/vcb-inclusive/kolya.git
$: cd kolya
$: pip3 install .
```

[1]:
```python
import kolya
import numpy as np
```

Physical parameters

Physical parameters like quark masses like $m_b^{\rm kin}(\mu_{WC})$, $\overline{m}_c(\mu_c)$ and $\alpha_s(\mu_s)$ are declared in the class `parameters.physical_parameters`. Initialization set default values

[2]:
```python
par = kolya.parameters.physical_parameters()
par.show()
```

```
bottom mass:      mbkin( 1.0  GeV)     =  4.563  GeV
charm mass:       mcMS( 3.0  GeV)      =  0.989  GeV
coupling constant: alpha_s( 4.563  GeV) =  0.2182
```

In order to set the quark masses at scales different from the default ones in a consistent way, we include the method FLAG2023 which internally use CRunDec. For instance, we set the quark masses at a scale $\mu_{WC} = \mu_c = 2$ GeV in the following way:

[3]:
```python
par = kolya.parameters.physical_parameters()
par.FLAG2023(scale_mcMS=2.0, scale_mbkin=2.0)
par.show()
```

```
bottom mass:      mbkin( 2.0  GeV)     =  4.295730717092438  GeV
charm mass:       mcMS( 2.0  GeV)      =  1.0940623249384822  GeV
coupling constant: alpha_s( 4.563  GeV) =  0.21815198098622618
```

HQE parameters

Non-perturbative matrix elements in the HQE are declared in the class `parameters.HQE_parameters`. This class is defined in the historical basis of hep-ph/1307.4551. By default they are initalized to zero. We can set their values in the following way

[4]:
```
hqe = kolya.parameters.HQE_parameters(
    muG = 0.306,
    rhoD = 0.185,
    rhoLS = -0.13,
    mupi = 0.477,
)
hqe.show()
```

```
mupi  =  0.477  GeV^2
muG   =  0.306  GeV^2
rhoD  =  0.185  GeV^3
rhoLS =  -0.13  GeV^3
```

[5]:
```
hqe.show(flagmb4=1)
```

```
mupi  =  0.477  GeV^2
muG   =  0.306  GeV^2
rhoD  =  0.185  GeV^3
rhoLS =  -0.13  GeV^3

m1 =  0  GeV^4
m2 =  0  GeV^4
m3 =  0  GeV^4
m4 =  0  GeV^4
m5 =  0  GeV^4
m6 =  0  GeV^4
m7 =  0  GeV^4
m8 =  0  GeV^4
m9 =  0  GeV^4
```

Wilson coefficients

The Wilson coefficients in the effective Hamiltonian are declared in the class `parameters.WCoefficients`. They are initialized to zero and can be set in the following way

[6]:
```
wc = kolya.parameters.WCoefficients(
    VL = 0,
    VR = 0,
    SL = 0.1,
    SR = 0.1,
    T = 0,
)
wc.show()
```

```
C_{V_L} =  0
C_{V_R} =  0
C_{S_L} =  0.1
C_{S_R} =  0.1
C_{T} =  0
```

Total Rate

We define the total rate as

$$\Gamma_{\rm sl} = \frac{G_F^2 (m_b^{\rm kin})^5}{192\pi^3} |V_{cb}|^2 X$$

The coefficients X is a function of the quark masses, α_s, the HQE parameters and the Wilson coefficients. It is evaluated by the function `X_Gamma_KIN_MS(par, hqe, wc)`

[5]:
```
hqe = kolya.parameters.HQE_parameters(
    muG = 0.306,
    rhoD = 0.185,
    rhoLS = -0.13,
    mupi = 0.477,
)
wc  = kolya.parameters.WCoefficients()
kolya.TotalRate.X_Gamma_KIN_MS(par,hqe,wc)
```

[5]:
```
0.539225163728085
```

The branching ratio is given by the function `BranchingRatio_KIN_MS(Vcb,par,hqe,wc)`

[6]:
```
Vcb = 42.2e-2
kolya.TotalRate.BranchingRatio_KIN_MS(Vcb,par,hqe,wc)
```

[6]:
```
10.555834162102016
```

Centralized q^2 moments

Q2 moments are evaluated with `Q2moments.moment_n_KIN_MS(q2cut, par, hqe, wc)`, where $q^2_{\rm cut}$ must be provided in GeV^2. The first centralized moment is calculated as follows:

[6]:
```
q2cut = 8.0 # GeV^2
kolya.Q2moments.moment_1_KIN_MS(q2cut,par,hqe,wc)
```

[6]: 8.996406491856465

The result for the moment $\langle q^{2n} \rangle$ is in GeV^{2n}

Centralized electron energy moments

E_l moments are evaluated with `Elmoments.moment_n_KIN_MS(Elcut, par, hqe, wc)`, where $E_{\rm cut}$ must be provided in GeV. The first centralized moment is calculated as follows:

[9]:
```
elcut = 0.5 # GeV
kolya.Elmoments.moment_1_KIN_MS(elcut,par,hqe,wc)
```

[9]: 1.4192938891883413

The result for $\langle E_l^n \rangle$ is in GeV^n

Centralized M_X^2 moments

M_X^2 moments are evaluated with `MXmoments.moment_n_KIN_MS(El_cut, par, hqe, wc)`, where $E_{\rm cut}$ must be provided in GeV. The first centralized moment is calculated as follows:

[13]:
```
elcut = 0.5 #GeV
kolya.MXmoments.moment_1_KIN_MS(elcut,par,hqe,wc)
```

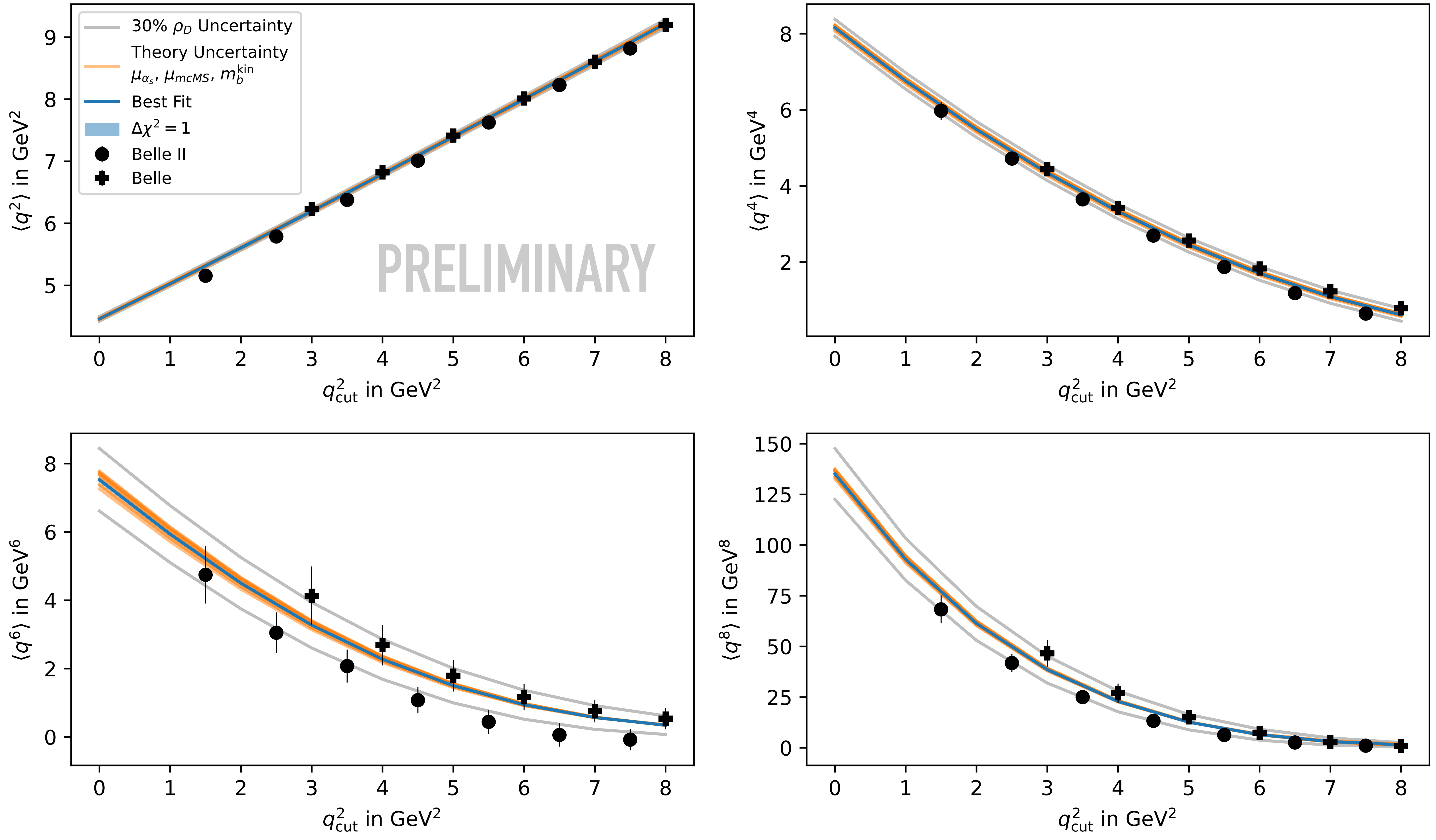
[13]: 4.492408891792521

The result for $\langle M_X^{2n} \rangle$ is in GeV^{2n}

FIT UP TO $1/m_b^5$

Bernlochner, **MF**, Milutin, Prim, Vos, in preparation

- ➤ Uncertainties from perturbative QCD estimated via scale variation.
- ➤ Very low p-Value in the fit.
- ➤ Current recipe: assign uncertainty on μ_G and ρ_D to cover the theory uncertainty from truncation.
- ➤ Can we use new expressions up to $O(1/m_b^5)$ to better assess theory uncertainty in the $1/m_b$ expansion?



$\chi^2/\mathrm{d.o.f.} = 801/51$

LSSA APPROXIMATION

- Too many parameters at $O(1/m_b^4)$ and $O(1/m_b^5)$
- Reparametrization invariance reduce the number

  Mannel, Vos, JHEP 1806 (2018) 115; **MF**, Mannel, Vos, JHEP 02 (2019) 177, Mannel, Milutin, Vos hep-ph/2311.1200

  - $1/m_b^4$: 5 instead of 8
  - $1/m_b^5$: 10 instead of 19
- Idea: catch "higher order" effects” by estimating $O(1/m_b^4)$ and $O(1/m_b^5)$ and remove ρ_D uncertainty

- Lower state saturation ansatz (LSSA)

  Mannel, S. Turczyk and N. Uraltsev, JHEP 1011, 109 (2010)
  Heinonen and T. Mannel, Nucl. Phys. B 889 (2014) 46

Approximate with the lightest state $|B\rangle\langle B|$

$$\langle B|\bar{b}_v iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_N} \Gamma b_v|B\rangle = \langle B|\bar{b}_v A_k B_{n-k} \Gamma b_v|B\rangle$$

$$= \frac{1}{2M_B} \sum_n \langle B|\bar{b}_v A_k b_v|H_n\rangle\langle H_n|\bar{b}_v B_k \Gamma b_v|B\rangle$$

CONCLUSIONS

- Kolya is an open-source code for inclusive decays written in python.
- Comprehensive framework where all available corrections are implemented and validated.
- The library is open source, so code contributions and improvements are very welcome.
- New higher order corrections can be implemented like
  - QED corrections
  - Exact NNLO corrections to E_l and M_X moments with cuts
  - NLO corrections to ρ_D for E_l and M_X moments with cuts
- Additional variable
  - Forward-backward asymmetries A_{FB}
  - Lepton universality ratio $R_X = \Gamma_{B\to X_c\tau\bar{\nu}_\tau}/\Gamma_{B\to X_c l\bar{\nu}_l}$
  - Decay to charmless final state $B \to X_u l\bar{\nu}_l$

References

[1] Y. Nir, *The Mass Ratio m_c/m_b in Semileptonic B Decays*, *Phys. Lett. B* **221** (1989) 184–190.

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[3] A. Pak and A. Czarnecki, *Heavy-to-heavy quark decays at NNLO*, *Phys. Rev. D* **78** (2008) 114015, [0808.3509].

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[6] M. Fael, K. Schönwald and M. Steinhauser, *Third order corrections to the semi-leptonic $b \to c$ and the muon decays*, 2011.13654.

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