Lattice QCD and flavor physics

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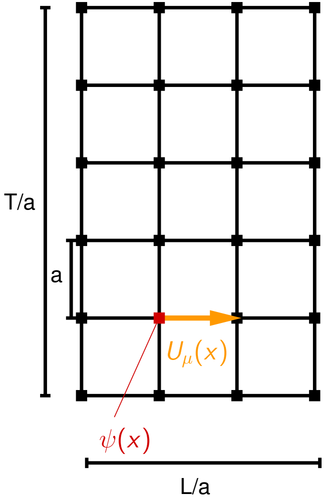
Lattice QCD

- Ab initio method to study nonperturbative phenomena of the strong interaction
- Systematically improvable uncertainties

- Discretize space-time and restrict to finite box $(L/a)^3 \times T/a$
  $\rightarrow$ Introduce finite value of the lattice spacing a
- Wick-rotate to Euclidean time $(t \rightarrow i\tau)$
- Implement discretized QCD Lagrangian
  $\rightarrow$ Numerical calculations based on Feynman's path integral formalism
- Stochastic procedure requiring stochastic data analysis

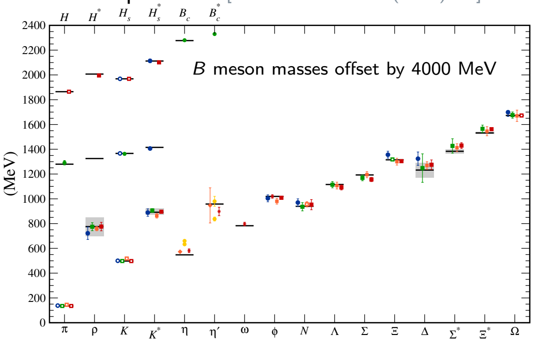
- Need experimental inputs to set lattice scale and quark masses
  $\rightarrow$ Simulate at different values of lattice spacing and quark masses
- Combine results to take continuum limit and inter-/extrapolate to physical quark masses



Lattice QCD

- Different discretizations
  - → Gauge action (Wilson, Symanzik, ...)
  - → Fermion action (Wilson, KS, DWF, ...)
- Results agree after continuum limit
- Few quantities needed as input
  e.g. M_π, f_π, f_K, M_{D_s}, M_{B_s}
- Numerous post- and predictions
- Well tested and established
- Subpercent level precision possible
  - → Simulate at physical pion mass
  - → Account for QED and isospin breaking
  - → Good control on all systematic effects
    (finite volume, discretization, etc.)

- Hadron spectrum [Kronfeld ARNPS62(2012)265]



- Data (inputs: open symbols)

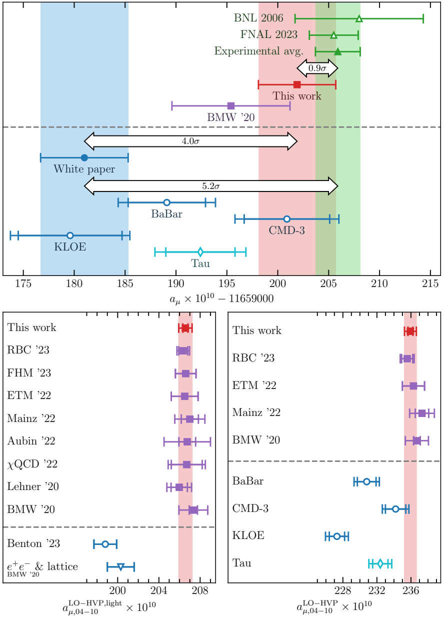
[MILC PRD70(2004)094505] [MILC RMP82(2010)1349]
[PACS-CS PRD79(2009)034503] [BMW Science 322(2008)1224]
[QCDSF-UKQCD PRD84(2011)054509]
[RBC-UKQCD PRL105(2010)241601]
[HadSpec PRD83(2011)111502] [UKQCD PRD86(2012)014504]
[FNAL-MILC PRD83(2011)034503] [HPQCD PRD83(2011)014506]
[Möhler, Woloshyn PRD84(2011)054505]

Highlights

$$g - 2$$

$g-2$

- Hadronic vacuum polarization (HVP) contribution to the anomalous magnetic moment $a_\mu = \frac{g_\mu - 2}{2}$
  - → Separate different contributions
  - → Define "windows" to focus on certain parts
- Update BMW [BMW Boccaletti et al. arXiv:2407.10913]
  - → New simulation at finer physical point ensemble
  - → Long distance tail complemented by data driven evaluation
  - → Excellent agreement for intermediate window with other lattice determinations
  - → SM confirmed to 0.37 ppm
  - → 40% improvement compared to BMW 2020
  - ⇝ Further details at Lattice 2024



ρ and K^* at the physical point

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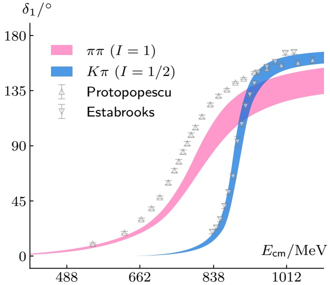
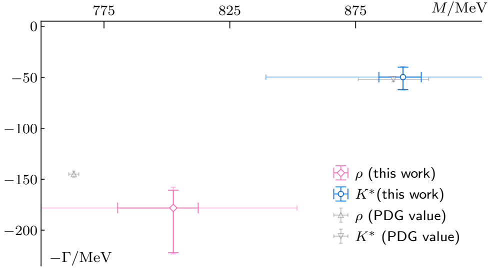




- Phase shift calculation of the $\pi\pi$ and $K\pi$ scattering amplitudes
  [RBC/UKQCD Boyle et al. arXiv:2406.19194] [arXiv:2406.19193]
- Mass and width determination of the vector channel resonance
- Calculation at physical pion mass at one lattice spacing
  → Large uncertainty due to estimating discretization effects
  ⇝ Further details at Lattice 2024

- $M_\rho = 796(5)(50)$ MeV
  $\Gamma_\rho = 192(10)(31)$ MeV
- $M_{K^*} = 893(2)(54)$ MeV
  $\Gamma_{K^*} = 51(2)(11)$ MeV

V_{cd}

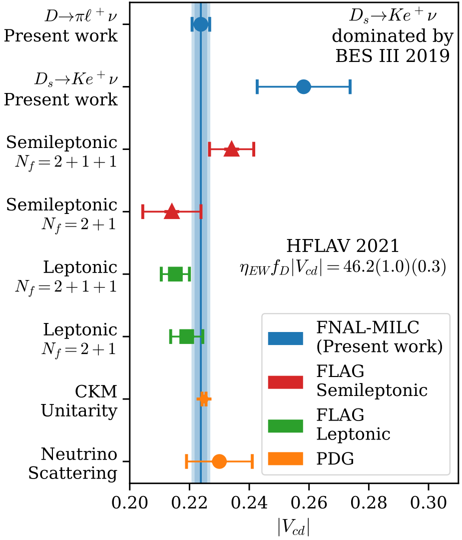
V_{cd}: PDG reports 1.8% uncertainty

- PDG averages three different determinations [PDG, Workman et al. PTEP (2022) 083C01]
  - → Earlier determination based on neutrino scattering data
    $|V_{cd}|^{\nu}_{PDG} = 0.230 \pm 0.011$
  - → Leptonic $D^+ \to \{\mu^+\nu_\mu, \tau^+\nu_\tau\}$ decays: LQCD (FNAL/MILC, ETMC) + experiment (BESIII, CLEO)
    $|V_{cd}|^{f_D}_{PDG} = 0.2181 \pm 0.0050$
  - → Semileptonic $D \to \pi\ell\nu$: LQCD (ETMC) + experiment (BaBar, BESIII, CLEO-c, Belle)
    $|V_{cd}|^{D\pi(0)}_{PDG} = 0.2330 \pm 0.014$

- $|V_{cd}|_{PDG} = 0.221 \pm 0.004$

- 2023: $D \to \pi\ell\nu$ and $D_s \to K\ell\nu$ determinations by FNAL/MILC exploiting full q^2 dependence
  $|V_{cd}|^{D\pi}_{FNAL/MILC} = 0.2238 \pm 0.0029$ (with BaBar, BESIII, CLEO-c, Belle data)
  $|V_{cd}|^{D_sK}_{FNAL/MILC} = 0.258 \pm 0.015$ (with BESIII data)
  [FNAL/MILC PRD107(2023)094516]

V_{cd}: new semileptonic determination exploiting full q^2 dependence



1.3% precision

[FNAL/MILC PRD107(2023)094516]

$$B \to D^* \ell \nu$$

How to determine V_{cb}?

- Leptonic $B_c \to \tau\nu_\tau$ decays
  - → Experimentally very challenging
- Semileptonic decays
  - → B or B_s initial state
  - → Inclusive decays

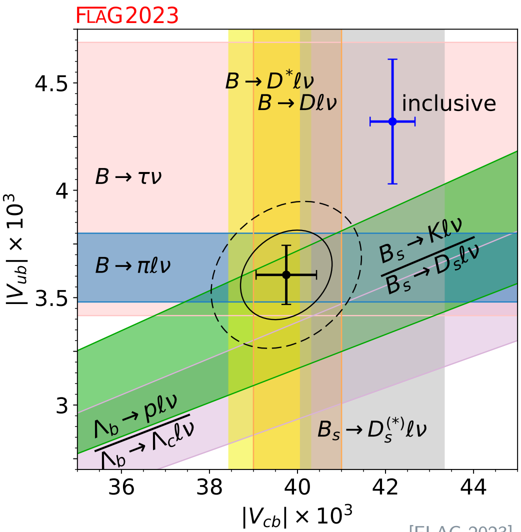
    Progress toward first lattice determination
  - → Exclusive decays

    hadronic pseudoscalar final state

    hadronic vector final state

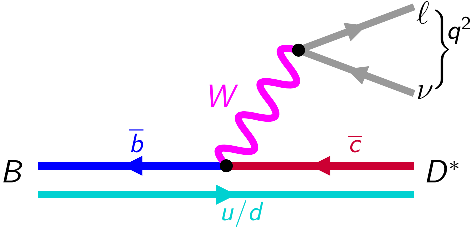
    $B \to D^*\ell\nu$ experimentally preferred

    (BaBar, Belle, Belle II, LHCb)
- Long standing $2-3\sigma$ discrepancy between inclusive and exclusive

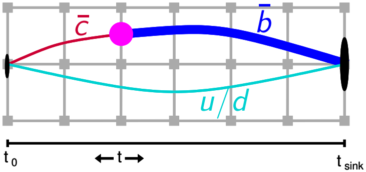


[FLAG 2023]

Exclusive semi-leptonic decays on the lattice



$$q^2 = M_B^2 + M_{D^*}^2 - 2 M_B E_{D^*}$$



- Treat D^* as QCD-stable particle (narrow-width approximation)
- Conventionally parametrized placing the B meson at rest in terms of

$$\frac{d\Gamma(B \to D^* \ell \nu)}{dq^2} = \mathcal{K}_{D^*}(q^2, m_\ell) \cdot |\mathcal{F}(q^2)|^2 \cdot |V_{cb}|^2$$

experiment known theory input (nonperturbative) CKM

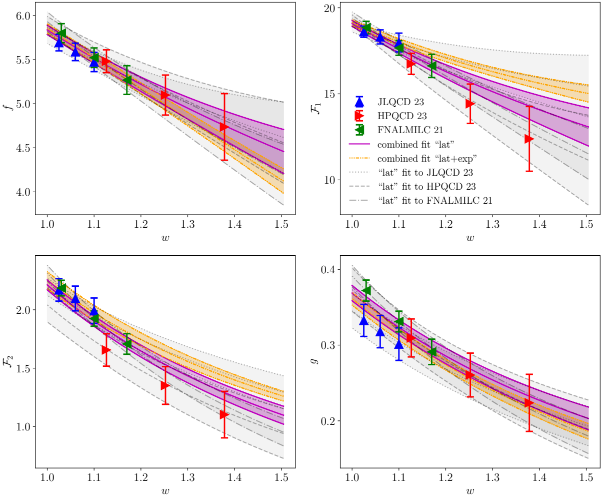
- Calculate hadronic matrix elements for form factors $f(w)$, $g(w)$, $\mathcal{F}_1(w)$, $\mathcal{F}_2(w)$ with $w = v_{D^*} \cdot v_B$

Three lattice calculations over the full q^2 range

- $B \to D^*\ell\nu$
  FNAL/MILC 2021 [Bazavov et al. EPJC 82 (2022) 1141]
  HPQCD 2023 [Harrison, Davies, PRD 109 (2024) 094515]
  JLQCD 2023 [Y. Aoki et al. PRD 109 (2024) 074503]

- Some tension in the shape of the form factors
  → Limited range in w (FNAL/MILC, JLQCD)
  → Slope not well enough constraint
  → HQET ratios to be scrutinized
  ⇝ Further details at Lattice 2024

- Combined analysis [Bordone, Jüttner, arXiv:2406.10074]
  Belle 2023 [Belle PRD 108 (2023) 012002]
  Belle II 2023 [Belle II PRD 108 (2023) 092013]



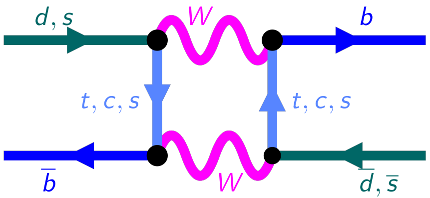
mixing and lifetimes

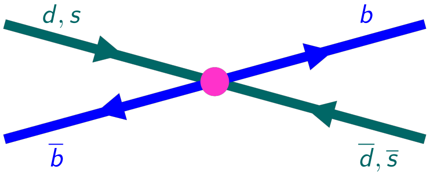
Neutral $B_{(s)}$ meson mixing

- Standard model process described by box diagrams
- Top quark contribution dominates
  $\Rightarrow$ short-distance process
  $\rightarrow$ Describe by point-like 4-quark operators
- Parameterize experimentally measured oscillation frequencies Δm_q for $q = d, s$ by

$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 M_{B_q} f_{B_q}^2 \hat{B}_{B_q} \left| V_{tq}^* V_{tb} \right|^2$$

  $\rightarrow$ Nonperturbative contribution:
  decay constant $f_{B_q}^2$ times bag parameter $\hat{B}_{B_q}$
  $\rightarrow$ SM: $\mathcal{O}_1^q = \bar{b}^\alpha \gamma^\mu (1 - \gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1 - \gamma_5) q^\beta$





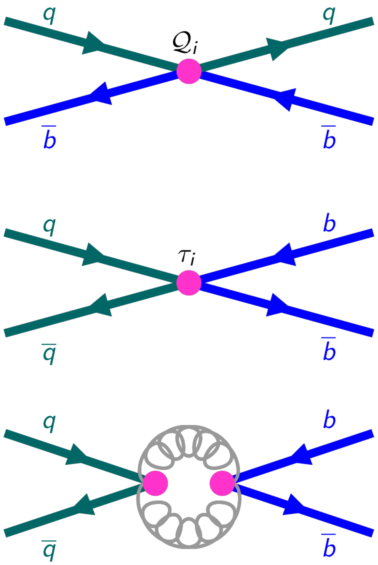
Heavy meson lifetimes ($\Delta B = 0$ operators)

- Using heavy quark expansion (HQE), lifetimes of heavy mesons are described by 4-quark operators with $\Delta B = 0$

- Operators $\mathcal{Q}_1$, $\mathcal{Q}_2$, $\mathcal{T}_1$, $\mathcal{T}_2$, contribute

- $\Delta B = 0$ operators mix under renormalization
  → To date no complete LQCD determination (only exploratory work 20+ years ago)

- Quark-line disconnected contributions
  → Notoriously noisy, hard to calculate on the lattice

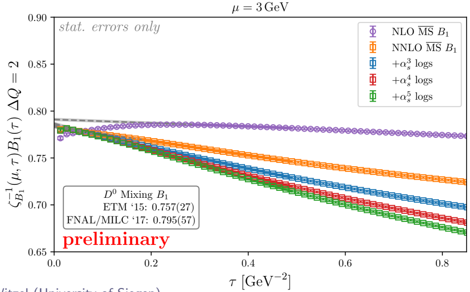


Pioneering calculation with new renormalization procedure

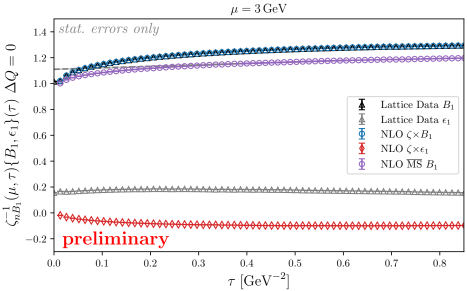
[Black, Harlander, Lange, Rago, Shindler, OW PoS Lattice 263] ⇝ update by M. Black at Lattice 2024

- Simplified calculation with some caveats for "neutral" charm-strange meson
- Use gradient flow in combination with short-flow-time expansion to renormalize operators
  - → Suppresses operator mixing on the lattice
  - → Take $a \to 0$ continuum limit as function of the gradient flow time τ
  - → Account for operator mixing as part of PT matching to $\overline{\text{MS}}$ in the continuum

- Validation: (short distance) meson mixing



- Pioneer lifetime determination



Highlights

- $g-2$ updated value for a_μ [BMW]
- Physical point calculation of ϱ and K^* [RBC/UKQCD]
- New determination of V_{cd} [Fermilab/MILC]
- Updates on $B \to D^*\ell\nu$ [Fermilab/MILC, HPQCD, JLQCD]
- First steps to determine heavy meson lifetimes on the lattice [Black et al.]