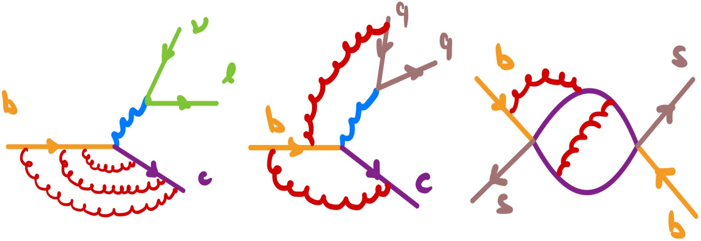
Karlsruhe Institute of Technology

Institute for Theoretical Particle Physics

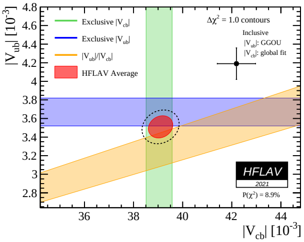
Precision Calculations in *B* physics

ICHEP 2024, Prague, 19 July 2024

Matthias Steinhauser | TTP KIT

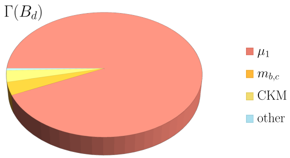


$V_{ub} \longleftrightarrow V_{cb}$

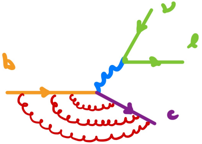


Lifetimes

$$\Gamma = \Gamma_3 + \Gamma_5 \frac{\langle B|O_5|B\rangle}{m_b^2} + \cdots$$



[from: Albrecht,Bernlochner,Lenz,Rusov'24]



$B - \overline{B}$ mixing

$$\Delta\Gamma_s^{\rm exp} = (8.3 \pm 0.5) \times 10^{-2} \text{ps}^{-1}$$
$$\Delta\Gamma_s^{\rm SM} = (7.6 \pm 1.7) \times 10^{-2} \text{ps}^{-1}$$

$$\Gamma(B \to X_c \ell\bar{\nu}) = \Gamma_0 + \Gamma_{\mu_\pi}\frac{\mu_\pi^2}{m_b^2} + \ldots$$

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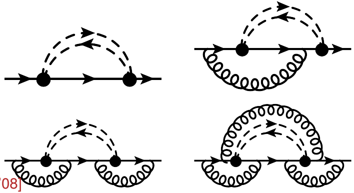
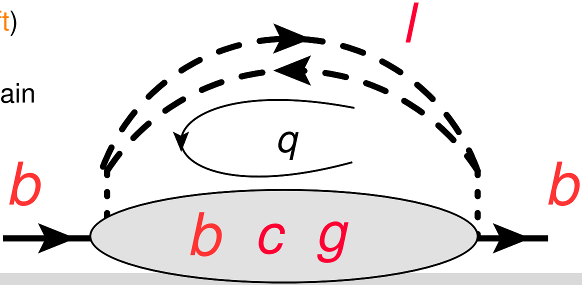
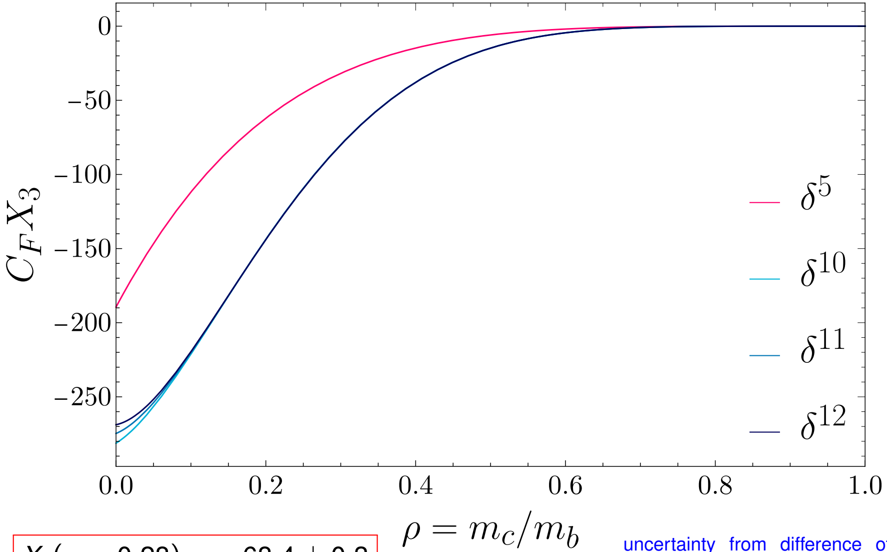
[Jezabek,Kühn'89; Nir'89 . . . ; Gambino et al.'05; Melnikov'08; Biswas,Melnikov'08; Pak,Czarnecki'08; Dowling,Piclum,Czarnecki'08; Fael,Schönwald,Steinhauser'20; Czakon,Czarnecki,Dowling'21]; [Becher,Boos,Lunghi'07; Alberti,Gambino,Nandi'14; Mannel,Pivovarov,Rosenthal'15; Mannel,Pivovarov'19; Dassinger,Mannel,Turczyk'07; Mannel,Turczyk,Uraltsev'10; Mannel,Vos'18; Fael,Mannel,Vos'19]

- $\mathcal{O}(\alpha_s^3)$ [Fael,Schönwald,Steinhauser'20]
- optical theorem
- integrate out $(\ell\bar{\nu})$ loop
- asymptotic expansion [Beneke,Smirnov'97]
  $m_b \approx m_c$: $\delta = 1 - m_c/m_b$ [Dowling,Piclum,Czarnecki'08]
  $|k^\mu| \sim m_b$ (hard)
  $|k^\mu| \sim \delta \cdot m_b$ (ultra-soft)
- $\int \mathrm{d}^D q$
  ➪ at most 3 loops remain
- expansion up to δ^{12}
- analytic calculation





N³LO: $\Gamma(B \to X_c \ell \bar{\nu}) = \Gamma_0 \left[X_0 + C_F \sum_{n \geq 1} \left(\frac{\alpha_s}{\pi}\right)^n X_n \right] + \ldots$



$X_3(\rho = 0.28) = -68.4 \pm 0.3$

uncertainty from difference of δ^{11} and δ^{12} expansion $\times 5$

Numerical results

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$$\Gamma(B \to X_c \ell \bar{\nu}) = \Gamma_0 X_0 \left[1 + \sum_{n\geq 1} \left(\frac{\alpha_s}{\pi}\right)^n Y_n\right] + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}\right)$$

$$\alpha_s \equiv \alpha_s^{(4)}$$

	Y_1	Y_2	Y_3
$m_b^{\rm OS}, m_c^{\rm OS}$	-1.72	-13.09	-163.3
$m_b^{\rm kin}, m_c^{\rm kin}$	-0.94	-3.75	-20.8
$m_b^{\rm kin}, \overline{m}_c(3\ \text{GeV})$	-1.67	-7.24	-28.6
$m_b^{\rm kin}, \overline{m}_c(2\ \text{GeV})$	-1.25	-3.64	-0.9
$\overline{m}_b(\overline{m}_b), \overline{m}_c(3\ \text{GeV})$	3.07	-13.36	62.7

New fit

[Bordone,Capdevila,Gambino'21]

[Bernlochner,Fael,Olschewsky,Persson,van Tonder,Vos,Welsch'22]

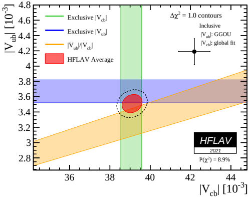
[Hayashi,Mishima,Sumino,Takaura'23]

$$|V_{cb}| = 42.16(30)_{\rm th}(32)_{\rm exp}(25)_{\Gamma} \times 10^{-3}$$

- $\Gamma(B \to X_c \ell \bar{\nu})_{\mathcal{O}(\alpha_s^3)}$:

  shift $|V_{cb}|$ by $+0.6\%$

  reduce uncertainty: $(50)_\Gamma$ ➪ $(25)_\Gamma$



[see talk by Matteo Fael]

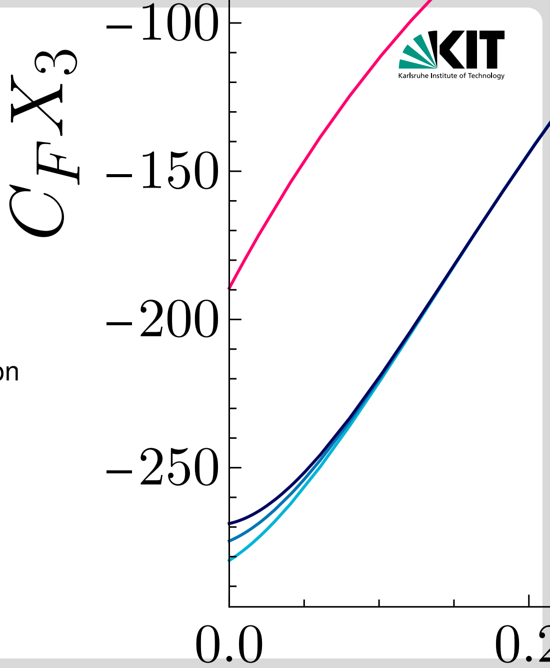
$b \to u \ell \bar{\nu}$

$X_3^u \approx -202 \pm 20$

Dedicated calculations:

- complete fermionic contribution [Fael,Usovitsch'23]
- large-N_c [Chen,Li,Li,Wang,Wang,Wu'23]

⇨ $X_3^u \approx -200.9 \pm 2.0$



QED: $\mu^- \to e^- \nu_\mu \bar{\nu}_e$

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$$\frac{1}{\tau_\mu} \equiv \Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192\pi^3} (1 + \Delta q)$$

$\Delta q^{(1)} \approx \frac{\alpha(m_\mu)}{\pi} \left(\frac{25}{8} - 3\zeta_2\right)$ [Kinoshita,Sirlin'59; Berman'58]

$\Delta q^{(2)}$: [van Ritbergen,Stuart'98; Seidensticker,Steinhauser'99]

$\Delta q^{(3)} \approx \left(\frac{\alpha(m_\mu)}{\pi}\right)^3 (-15.3 \pm 2.3)$ [Fael,Schönwald,Steinhauser'20]

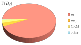
$$\tau_\mu^{\rm exp} = (2.1969811 \pm 2.2 \times 10^{-6})\ \mu s$$

$$\delta\tau_\mu\Big|_{\alpha^2} = 41 \times 10^{-6}\ \mu s$$

$$\delta\tau_\mu\Big|_{\alpha^3} = (0.09 \pm 0.01) \times 10^{-6}\ \mu s$$

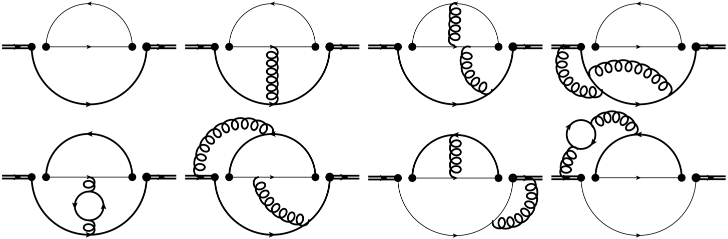
Nonleptonic B meson decays

[Lenz,Piscopo,Rusov'22] $\Gamma(B^+) = 0.58^{+0.11}_{-0.07}\,\text{ps}^{-1}, \Gamma(B_d) = 0.63^{+0.11}_{-0.07}\,\text{ps}^{-1}$



Experiment: $\Gamma(B^+) = 0.6105 \pm 0.0015\,\text{ps}^{-1}, \Gamma(B_d) = 0.6583 \pm 0.0017\,\text{ps}^{-1}$

- NLO: [Altarelli,Petrarca,'91; Buchalla'93; Bagan,Ball,Braun,Gosdzinsky'94'95; Krinner,Lenz,Rauh'13]
- NNLO: [Czarnecki,Slusarczyk,Tkachov'06]
  BUT: no resummation of large logarithms; massless final-state quarks



Nonleptonic B meson decays

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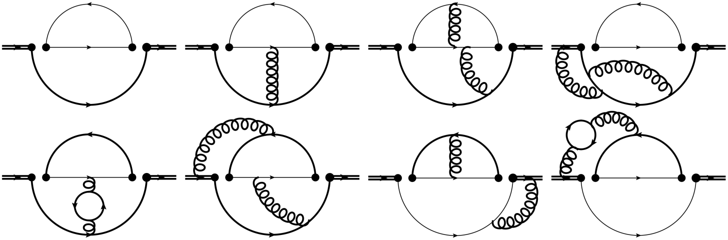
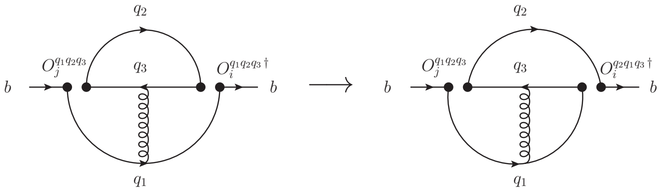


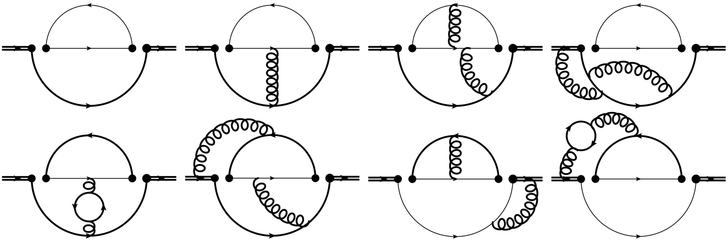
- $b \to c\bar{u}d$, $b \to c\bar{c}s$, ... [CKM-suppressed]
- γ_5 !?
- 4-loop 2-point functions with 2 mass scales (m_c, m_b)

Nonleptonic B meson decays



- $b \to c\bar{u}d$, $b \to c\bar{c}s$, ... [CKM-suppressed]
- γ_5 !?
  ➪ restore Fierz symmetry in d dimensions by proper choice of evanescent operators [Buras,Weisz'90; Herrlich,Nierste'94; Buras,Gorbahn,Haisch,Nierste'06]
- 4-loop 2-point functions with 2 mass scales (m_c, m_b)
  "expand and match" [Fael,Lange,Schönwald,Steinhauser'22'23]
  overlapping series expansions; precise-stable-fast

Nonleptonic B meson decays



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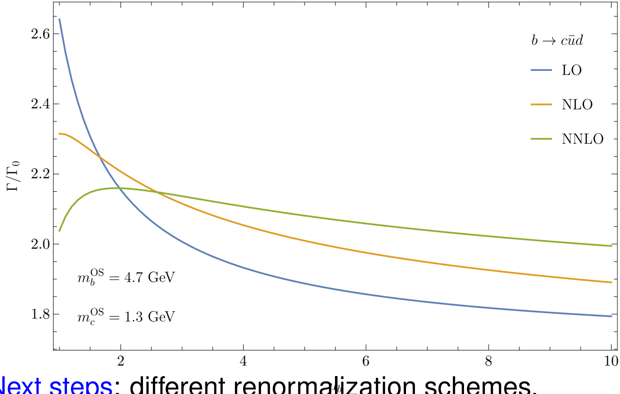
  overlapping series expansions; precise-stable-fast

Results: $b \to c\bar{u}d$

[Egner,Fael,Schönwald,Steinhauer'24]

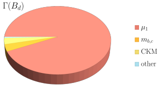


$\mu_b \in [m_b/2, 2m_b]$

⇨

NLO: $\pm 6.3\%$

NNLO: $\pm 3.5\%$

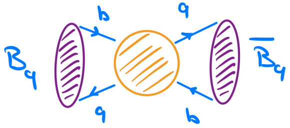


Next steps: different renormalization schemes,
include $1/m_b$ corrections, phenomenological analysis

Lifetime differences

- weak interaction
- $\Delta B = 2$:
  $B_q \sim (\bar{b}, q) \leftrightarrow (b, \bar{q}) \sim \overline{B}_q,\ q = d, s$

  mass matrix: M^q decay matrix: Γ^q
- M_{12}^q: dominated by top quarks
  Γ_{12}^q: internal u, c quarks



$$\frac{\Delta\Gamma_q}{\Delta M_q} = -\mathrm{Re}\,\frac{\Gamma_{12}^q}{M_{12}^q}$$

$$\Delta M_q = M_H^q - M_L^q \qquad \Delta\Gamma_q = \Gamma_L^q - \Gamma_H^q$$

$$|B_{q,L}\rangle = p|B_q\rangle + q|\bar{B}_q\rangle \qquad |B_{q,H}\rangle = p|B_q\rangle - q|\bar{B}_q\rangle$$

Experiment

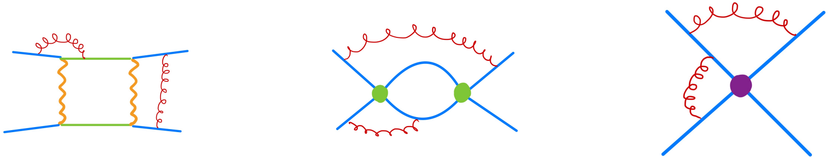
[. . . ,CLEO,BABAR,Belle,CDF,D0,ATLAS,CMS,LHCb] [HFLAV'22]

$$\Delta M_s^{\rm exp} = (17.765 \pm 0.006)\ \text{ps}^{-1}$$
$$\Delta \Gamma_s^{\rm exp} = (0.083 \pm 0.005)\ \text{ps}^{-1}$$

$$\Delta M_d^{\rm exp} = (0.5065 \pm 0.0019)\ \text{ps}^{-1}$$
$$\Delta \Gamma_d^{\rm exp} = (0.001 \pm 0.010)\ \text{ps}^{-1}$$

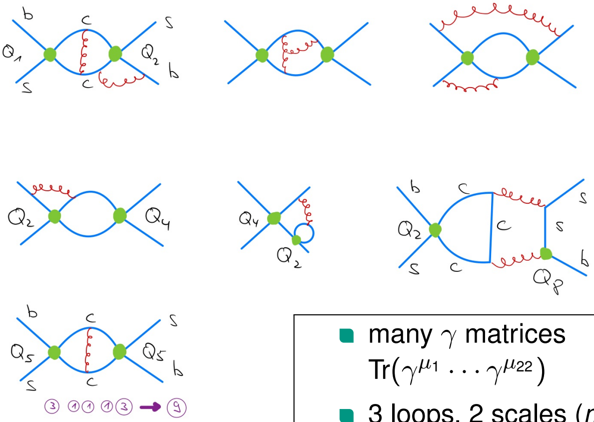
Effective theories

$$\text{SM} \quad \rightarrow \quad \mathcal{H}_{\text{eff}}^{|\Delta B|=1} \quad \rightarrow \quad \mathcal{H}_{\text{eff}}^{|\Delta B|=2}$$



$\Delta B = 1$

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}}\left[-\lambda_t^s\left(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8\right) - \lambda_u^s \sum_{i=1}^{2} C_i(Q_i - Q_i^u) + \ldots\right.$$

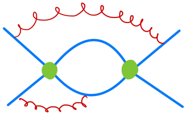


- many γ matrices
  $\mathrm{Tr}(\gamma^{\mu_1} \ldots \gamma^{\mu_{22}})$
- 3 loops, 2 scales (m_c, m_b)

$\Delta B = 2$

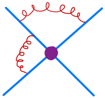
- Heavy Quark Expansion [Khoze,Shifman'83; ...; Manohar,Wise'94]

$$\Gamma_{12}^s = \frac{1}{2M_{B_s}} \text{Abs}\langle B_s | i \int \mathrm{d}^4x \; T \; \mathcal{H}_{\rm eff}^{\Delta B=1}(x)\mathcal{H}_{\rm eff}^{\Delta B=1}(0)|\bar{B}_s\rangle$$



- $\Delta\Gamma_s$ in terms of $|\Delta B| = 2$ operators [Beneke,Buchalla,Greub,Lenz,Nierste'99; ...]

$$\Gamma_{12}^s = -(\lambda_c^s)^2\Gamma_{12}^{cc} - 2\lambda_c^s\lambda_u^s\Gamma_{12}^{uc} - (\lambda_u^s)^2\Gamma_{12}^{uu}$$



$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}}\left[H^{ab}(z)\langle B_s|Q|\bar{B}_s\rangle + \widetilde{H}_S^{ab}(z)\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle\right] + \mathcal{O}(\Lambda_{\rm QCD}/m_b)$$

$$Q = \bar{s}_i\gamma^\mu\,(1-\gamma^5)\,b_i\;\bar{s}_j\gamma_\mu\,(1-\gamma^5)\,b_j \qquad \widetilde{Q}_S = \bar{s}_i\,(1+\gamma^5)\,b_j\;\bar{s}_j\,(1+\gamma^5)\,b_i$$

- Nonperturbative MEs from lattice or sum rules [...; Kirk,Lenz,Rauh'17; King,Lenz,Rauh'19'21; Bazavov et al.'16; Dowdall,Davies,Horgan,Lepage,Monahan,et al.'19; Di Luzio,Kirk,Lenz,Rauh'19]
- $H^{ab}(z)$, $\widetilde{H}_S^{ab}(z)$: Wilson coefficients from matching

Known results

- LO [. . . , Beneke,Buchalla,Greub,Lenz,Nierste'99; Beneke,Buchalla,Dunietz'96]

- NLO Beneke,Buchalla,Greub,Lenz,Nierste'99; Ciuchini,Franco,Lubicz,Mescia,Tarantino'03; Beneke,Buchalla,Lenz,Ulrich'03; Lenz,Nierste'06; Asatrian,Asatryan,Hovhannisyan,Nierste,Tumasyan,Yeghiazaryan'20; Gerlach,Nierste,Shtabovenko,Steinhauser'21'22]

- NNLO n_f part: [Asatrian, Hovhannisyan,Nierste,Yeghiazaryan'17]
  full $Q_{1,2} \times Q_{1,2}$: [Gerlach,Nierste,Shtabovenko,Steinhauser'22] [up to $(m_c/m_b)^2$]
  NEW: NNLO MIs for arb. m_c/m_b: [Reeck,Shtabovenko,Steinhauser'24]

Numerical results for $\Delta\Gamma_s$

[Gerlach,Nierste,Shtabovenko,Steinhauser'22]

$$\frac{\Delta\Gamma_s}{\Delta M_s} = \left(4.33^{+0.23}_{-0.44}{}_{\rm scale}{}^{+0.09}_{-0.19}{}_{\rm scale,\ 1/m_b} \pm 0.12_{B\tilde{B}_s} \pm 0.78_{1/m_b} \pm 0.05_{\rm input}\right) \times 10^{-3}\ (\overline{\rm MS})$$

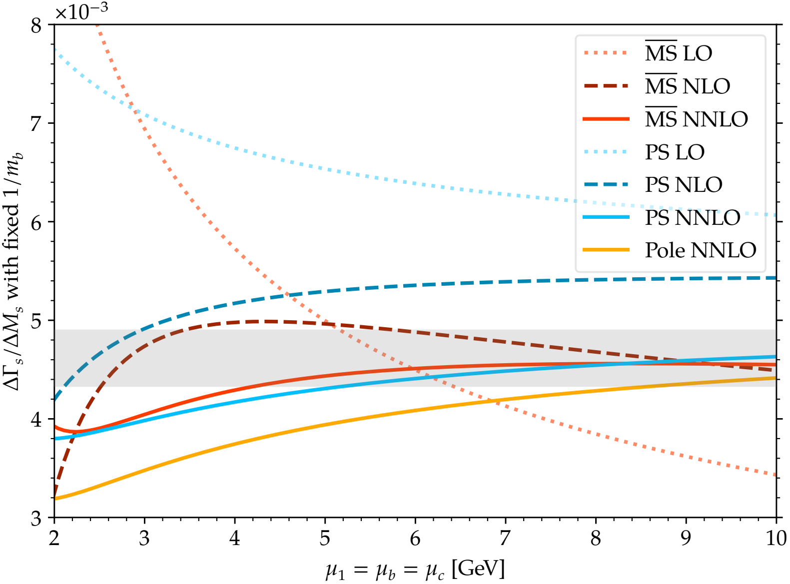
$$\frac{\Delta\Gamma_s}{\Delta M_s} = \left(4.20^{+0.36}_{-0.39}{}_{\rm scale}{}^{+0.09}_{-0.19}{}_{\rm scale,\ 1/m_b} \pm 0.12_{B\tilde{B}_s} \pm 0.78_{1/m_b} \pm 0.05_{\rm input}\right) \times 10^{-3}\ ({\rm PS})$$

$$\Delta\Gamma_s^{\rm SM} = (7.6 \pm 1.7) \times 10^{-2} {\rm ps}^{-1}$$
$$\Delta\Gamma_s^{\rm exp} = (8.3 \pm 0.5) \times 10^{-2} {\rm ps}^{-1}$$

- $\overline{\rm MS}$ + PS
- $\mu_1 = \mu_c = \mu_b \in \{2.1, 8.4\}$ GeV
- NLO$\to$ NNLO: scale dependence reduced by factor 2
- uncertainty dominated by $1/m_b$ correction
- pole scheme inadequate
- TODO: NNLO penguin contribution

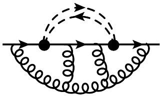
μ dependence

[Gerlach,Nierste,Shtabovenko,Steinhauser'22]

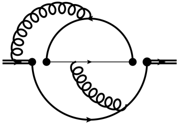


Summary

- $\Gamma(b \to c\ell\bar{\nu})$ to $\mathcal{O}(\alpha_s^3)$ ⇨ $|V_{cb}|_{\mathrm{incl.}}$
  $\Gamma(b \to u\ell\bar{\nu})$ to $\mathcal{O}(\alpha_s^3)$
  $\Gamma(\mu^- \to e^-\nu_\mu\bar{\nu}_e)$ to $\mathcal{O}(\alpha^3)$



- Non-leptonic B decays, $\mathcal{O}(\alpha_s^2)$



- $B - \overline{B}$ mixing
  NNLO corrections to $\Delta\Gamma_s$