Pushing the Heavy Quark Expansion for $b \to c\ell\bar{\nu}$ to Higher Order in $1/m_b$

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in collaboration with
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based on: JHEP 02 (2024) 226, [2311.12002]

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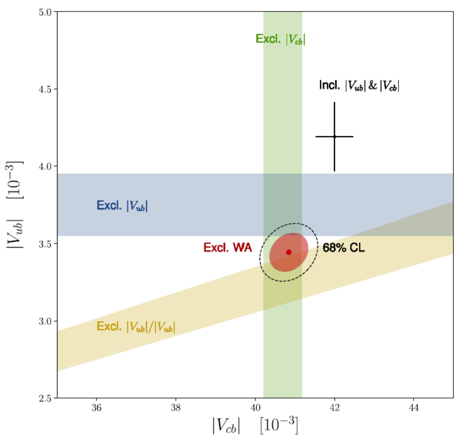
Motivation

- $b \to c\ell\bar{\nu}$ decays allow for studies of physics beyond the Standard Model and extraction of CKM parameters
- V_{cb} is an important input for many Standard Model predictions
- Long-standing puzzle between different determinations
  - exclusive: known-final state, e.g. $B \to D\ell\bar{\nu}$
  - inclusive: summed over all possible final states, e.g. $B \to X_c\ell\bar{\nu}$



Bernlochner, Prim, Vos (*Eur. Phys. J. Spec. Top.* (2024))

Setting up the Heavy Quark Expansion

- m_b is large compared to $\Lambda_{\rm QCD}$ $\to$ power series in $1/m_b$
  Chay, Georgi, bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel,...

$$
\begin{aligned}
\mathrm{d}\Gamma &\propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X|\mathcal{H}_{\rm eff}|B(v)\rangle|^2 \\
&= \int \mathrm{d}^4x \, \langle B(v)|\mathcal{H}^\dagger_{\rm eff}(x)\mathcal{H}_{\rm eff}(0)|B(v)\rangle \\
&= 2\,\mathrm{Im} \int \mathrm{d}^4x \, \langle B(v)|T\{\mathcal{H}^\dagger_{\rm eff}(x)\mathcal{H}_{\rm eff}(0)\}|B(v)\rangle \\
&= 2\,\mathrm{Im} \int \mathrm{d}^4x \, e^{-im_b v\cdot x}\langle B(v)|T\{\tilde{\mathcal{H}}^\dagger_{\rm eff}(x)\tilde{\mathcal{H}}_{\rm eff}(0)\}|B(v)\rangle
\end{aligned}
$$

- Split the momentum of the heavy quark as $p_b = m_b v + k$ where $v^2 = 1$
  $\to$ field redefinition of heavy quark field $b(x) = e^{-im_b v\cdot x} b_v(x)$
- Expand in residual momentum $k \sim iD$

HQE continued

- Perform Operator Product Expansion (OPE)
  Chay, Georgi, bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel,...

$$d\Gamma = d\Gamma^{(3)} + \frac{1}{m_b^2} d\Gamma^{(5)} + \frac{1}{m_b^3} d\Gamma^{(6)} + \frac{1}{m_b^4} d\Gamma^{(7)} + \ldots \,, \qquad d\Gamma^{(n)} = \sum_i \mathcal{C}_i^{(n)} \langle B | \mathcal{O}_i^{(n)} | B \rangle$$

  - $d\Gamma^{(n)}$ are power series in $\mathcal{O}(\alpha_s)$
  - $\mathcal{C}_i^{(n)}$ perturbative Wilson coefficients
  - $\mathcal{O}^{(n)}$ local operators of dimension $n \to$ chains of covariant derivatives iD
  - $\langle B | \mathcal{O}_i^{(n)} | B \rangle$ non-perturbative matrix elements $\to$ are extracted from data

Matrix elements

- The number of matrix elements increases very fast with the dimension:

$$d\Gamma = d\Gamma^{(3)} + \frac{1}{m_b^2} d\Gamma^{(5)} + \frac{1}{m_b^3} d\Gamma^{(6)} + \frac{1}{m_b^4} d\Gamma^{(7)} + \frac{1}{m_b^5} d\Gamma^{(8)} + \ldots$$

  - $d\Gamma^{(3)}$: Partonic result ($d\Gamma^{(4)} = 0$ due to Heavy Quark Symmetries)
  - $d\Gamma^{(5)}$: 2 parameters

$$2m_B \mu_\pi^2 = -\langle B|\bar{b}_v (iD)^2 b_v|B\rangle$$
$$2m_B \mu_G^2 = \langle B|\bar{b}_v (-i\sigma^{\mu\nu})(iD_\mu)(iD_\nu) b_v|B\rangle$$

  - $d\Gamma^{(6)}$: 2 parameters

$$2m_B \rho_D^3 = \frac{1}{2}\langle B|\bar{b}_v [iD_\mu, [ivD, iD^\mu]] b_v|B\rangle$$
$$2m_B \rho_{LS}^3 = \frac{1}{2}\langle B|\bar{b}_v \{iD_\mu, [ivD, iD_\nu]\}(-i\sigma^{\mu\nu}) b_v|B\rangle$$

  - $d\Gamma^{(7)}$: 9 parameters (at tree level) Mannel, Turczyk, Uraltsev [1009.4622]; Kobach, Pal [1704.00008]
  - $d\Gamma^{(8)}$: 18 parameters (at tree level) Mannel, Turczyk, Uraltsev [1009.4622]; Kobach, Pal [1704.00008]

Reparametrization Invariance

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- Lorentz invariance of QCD $\to$ Reparametrization Invariance (RPI) imposed under infinitesimal change $v_\mu \to v_\mu + \delta v_\mu$, leading to

$$\delta_{\mathrm{RP}} v_\mu = \delta v_\mu \ , \qquad \delta_{\mathrm{RP}} iD_\mu = -m_b \delta v_\mu \ , \qquad \delta_{\mathrm{RP}} b_v(x) = i m_b (x \cdot \delta v) b_v(x)$$

- Lorentz invariant quantity $R(v) = \sum_{n=0}^{\infty} C^{(n)}_{\mu_1 \ldots \mu_n}(v) \otimes \bar{b}_v (iD^{\mu_1} \ldots iD^{\mu_n}) b_v$ Mannel, Vos [1802.09409]

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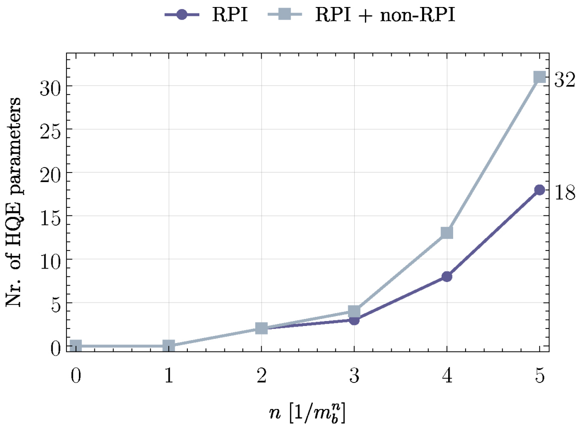
- RPI relates different orders n in $1/m_b$ expansion

$$\delta_{\mathrm{RP}} C^{(n)}_{\mu_1 \dots \mu_n}(v) = m_b \delta v^\alpha \left(C^{(n+1)}_{\alpha\mu_1 \dots \mu_n}(v) + C^{(n+1)}_{\mu_1 \alpha \dots \mu_n}(v) + \dots + C^{(n+1)}_{\mu_1 \dots \mu_n \alpha}(v) \right)$$

- This allows us to find combinations of operators which are RPI

Counting RPI operators



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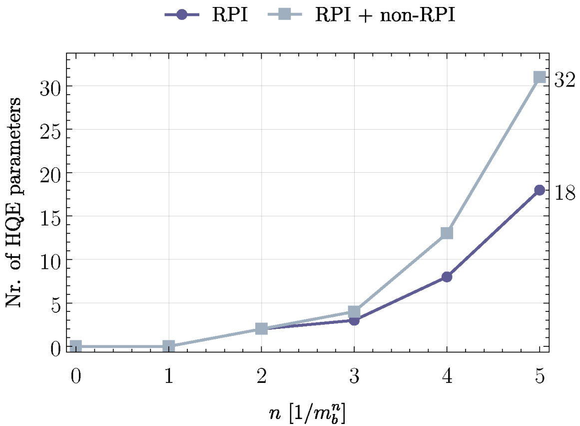

- Up to $1/m_b^4$: total of 8 independent parameters Mannel, Vos [1802.09409]
- At $1/m_b^5$, we find 4 spin-indep. and 6 spin-dep. RPI operators:
  Only 10 RPI operators at dimension 8, instead of 18 in full basis Mannel, ISM, Vos [2311.12002]

New inclusive V_{cb} determination

- Need RPI observable: dilepton invariant mass (q^2) moments used to extract $|V_{cb}^{\mathrm{incl}}|$
  (q^2-cut needed due to experimental setup)
  Bernlochner, Welsch, Fael, Olschewsky, Persson, von Tonder, Vos [2205.10274]

$$\langle (q^2)^n \rangle_{\mathrm{cut}} = \frac{\int_{q^2 > q^2_{\mathrm{cut}}} \mathrm{d}q^2 \, (q^2)^n \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}}{\int_{q^2 > q^2_{\mathrm{cut}}} \mathrm{d}q^2 \, \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}}$$

- RPI $\to$ only depend on reduced set of RPI operators

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- RPI $\rightarrow$ only depend on reduced set of RPI operators
- Data $\rightarrow$ values for reduced set of RPI parameters up to $1/m_b^4 \rightarrow Br(\bar{B} \rightarrow X_c \ell \bar{\nu}) \rightarrow$

$$|V_{cb}^{\text{incl}}| = (41.69 \pm 0.63) \times 10^{-3}$$ Bernlochner, Vos, et al. [2205.10274]

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- First determination of V_{cb} up to $\mathcal{O}(1/m_b^4)$ and first extraction of $1/m_b^4$ matrix elements from data
- Agreement at $1 - 2\sigma$ level with previous $\mathcal{O}(1/m_b^3)$ determinations
  Finauri, Gambino [2310.20324]; Bordone, Capdevila, Gambino [2107.00604]; Alberti, Gambino, Healey, Nandi[1411.6560]; Gambino, Schwanda [1307.4551]

Where do we currently stand?

Γ	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓	✓	✓
$1/m_b^2$	✓	✓		
$1/m_b^3$	✓	✓		
$1/m_b^4$	✓			
$m_b^{\text{kin}}/\bar{m}_c$		✓	✓	✓

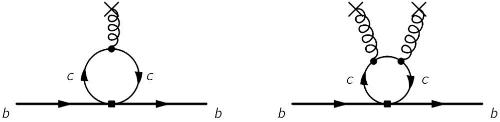
$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓	✓	
$1/m_b^2$	✓	✓		
$1/m_b^3$	✓	✓		
$1/m_b^4$	✓			

- Green: known perturbative corrections Jezabek, Kuhn (1989); Melnikov (2008); Pak, Czarnecki (2008); Becher, Boos, Lunghi (2007); Alberti, Gambino, Nandi (2014); Mannel, Pivovarov, Rosenthal (2015); Gambino, Healey, Turczyk (2016); Mannel, Pivovarov (2020); Fael, Schonwald, Steinhauser (2020, 2021); Fael, Herren (2024)
- Next step in HQE is dimension 8: $1/m_b^5$

Going higher in the $1/m_b$ expansion

- Dimension 8 contains: Bigi, Mannel, Turczyk, Uraltsev [0911.3322]

"genuine" $\frac{1}{m_b^5}$ & Intrinsic Charm $\frac{1}{m_b^3 m_c^2}$



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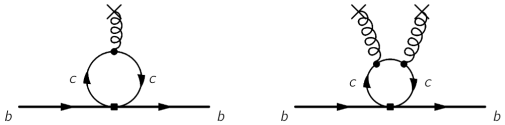

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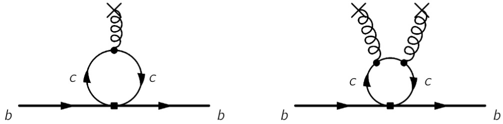

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- We need $1/m_b^3 \times 1/m_c^2$ contributions to complete the calculation at $1/m_b^4$
  - We calculate the full dimension 8 contributions
  - We extract the Intrinsic Charm contribution
  - We find that only 1 combination of parameters describes the IC in the q^2-moments

  Mannel, ISM, Vos [2311.12002]

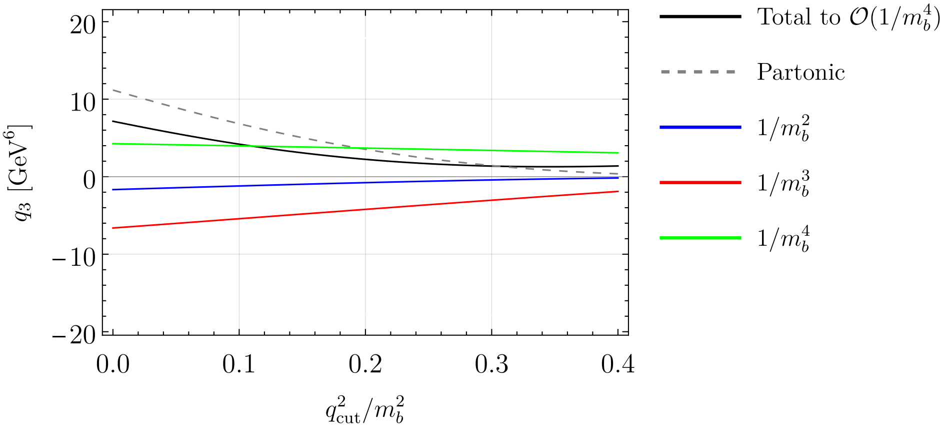
Lowest-Lying-State Approximation

- We can employ the LLSA: Lowest-Lying-State Approximation to estimate the higher order matrix elements by linking them to lower order ones
  Mannel, Turczyk, Uraltsev [1009.4622]

$$\langle B|\bar{b}\ AC\Gamma\ b(0)|B\rangle = \frac{1}{2m_B}\sum_n \langle B|\bar{b}\ A\ b(0)|n\rangle \cdot \langle n|\bar{b}\ C\Gamma\ b(0)|B\rangle$$

  - $A = iD_{\mu_1}...iD_{\mu_k},\ C = iD_{\mu_{k+1}}...iD_{\mu_n}$

q^2-moments



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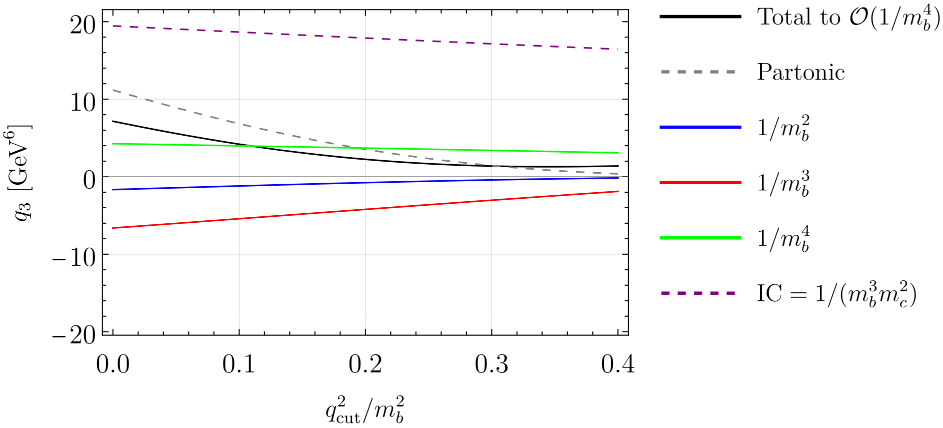

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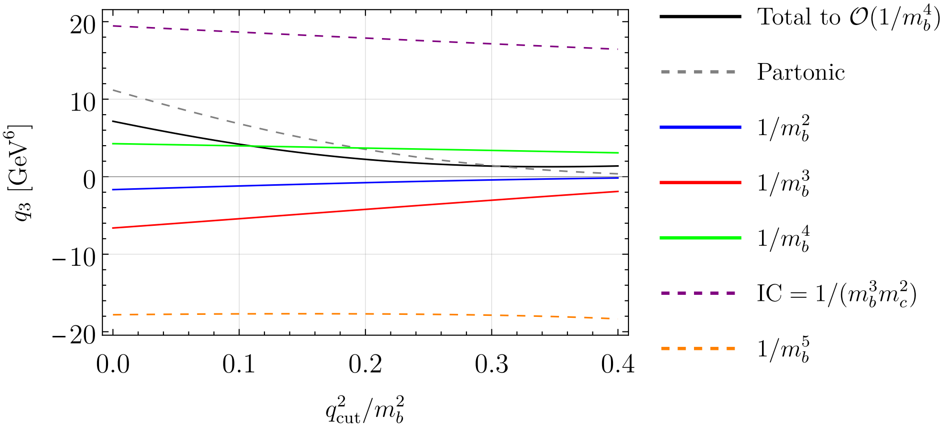

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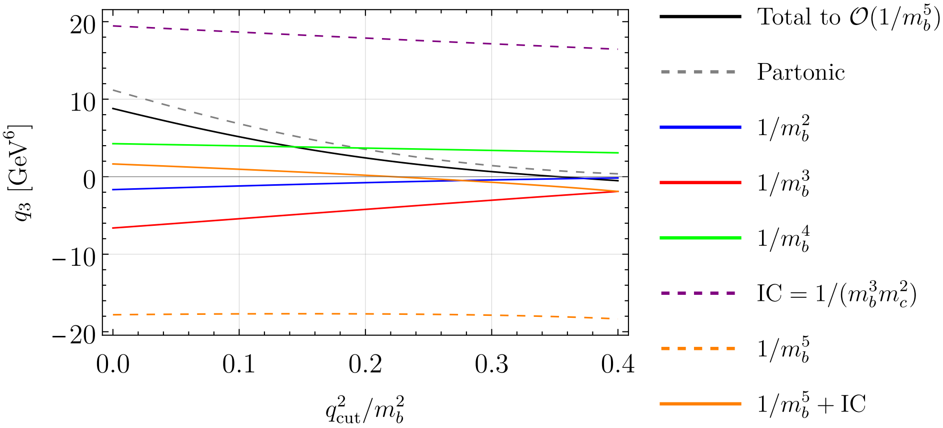

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Conclusion

- We identified the 10 RPI operators at $1/m_b^5$
- The $1/m_b^3 \times 1/m_c^2$ contributions are (partially) cancelled* by the strict $1/m_b^5$ contributions (*within the LLSA)
- We find an unexpectedly small overall contribution of the dimension-8 operators

Outlook

- We want to improve the precision of $|V_{cb}|^{\mathrm{incl}}$ even further
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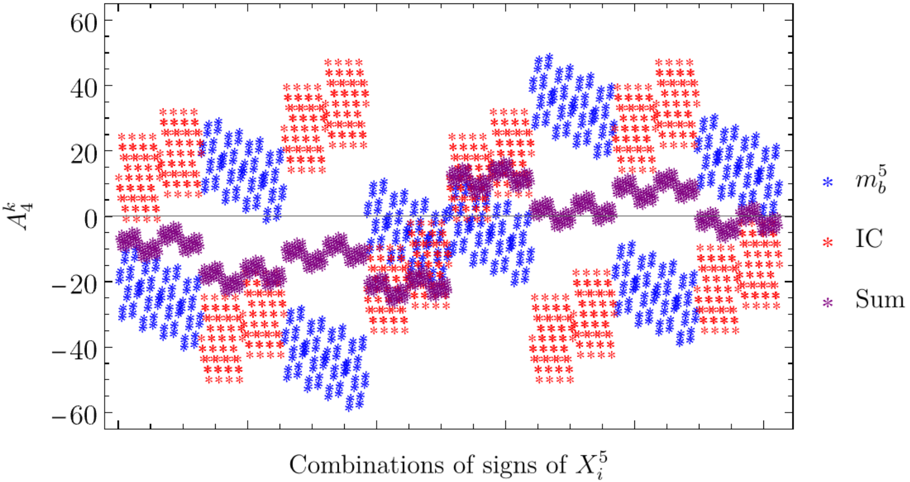
Thank you for your attention

Back-up: Cancellation without LLSA

- Assume for all dimension 8 operators that $X_i^5 \sim \Lambda^5_{\rm QCD}$ and vary signs:



Back-up: Calculation of forward matrix element

- **Step 1:** expand charm propagator ($Q^\mu = m_b v^\mu$)

$$
\begin{aligned}
-iS_{\mathrm{BGF}} &= \frac{1}{Q + iD - m_c} \\
&= \frac{1}{Q - m_c} - \frac{1}{Q - m_c}(iD)\frac{1}{Q - m_c} \\
&\quad + \frac{1}{Q - m_c}(iD)\frac{1}{Q - m_c}(iD)\frac{1}{Q - m_c} + \ldots
\end{aligned}
$$

Back-up: Calculation of forward matrix element

- **Step 2:** insert in forward matrix element

$$
\begin{aligned}
T = & \left[\Gamma \frac{1}{Q - m_c} \Gamma^\dagger\right]_{\alpha\beta} \langle \bar{b}_\alpha b_\beta \rangle \\
& - \left[\Gamma \frac{1}{Q - m_c} \gamma^\mu \frac{1}{Q - m_c} \Gamma^\dagger\right]_{\alpha\beta} \langle \bar{b}_\alpha (iD_\mu) b_\beta \rangle \\
& + \left[\Gamma \frac{1}{Q - m_c} \gamma^\mu \frac{1}{Q - m_c} \gamma^\nu \frac{1}{Q - m_c} \Gamma^\dagger\right]_{\alpha\beta} \langle \bar{b}_\alpha (iD_\mu)(iD_\nu) b_\beta \rangle \\
& + \dots
\end{aligned}
$$

Back-up: Calculation of forward matrix element

- **Step 3:** Determine Trace formula
  - Start at dimension 8 $\rightarrow$ lengthy, but systemically calculable
  - Compute dim-7, including $1/m_b$ correction through e.o.m.

$$(ivD)b_v = -\frac{1}{2m_b}(iD)(iD)b_v$$

  - ...
  - Compute dim-3, including corrections up to $1/m_b^5$

$$\langle \bar{b}_\alpha b_\beta \rangle = 2m_B \left(\frac{1+v}{4} + \frac{1}{8m_b^2}(\mu_G^2 - \mu_\pi^2) + \mathcal{O}(1/m_b^6) \right)_{\beta\alpha}$$

    - Need full (non-RPI) set of basic parameters up to $1/m_b^5$
- **Step 4:** Compute the trace with the geometric series