$\Lambda_b \rightarrow \Lambda_c^* \text{ AT } O(1/m_c^2)$ IN HEAVY QUARK EXPANSION

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DAVIDE IACOBACCI





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HEAVY QUARK EFFECTIVE THEORY

- The presence of heavy quarks simplify the study of hadronic systems
- $m_Q \gg \Lambda_{QCD}$
- Λ_{QCD} separates the perturbative and non-pertubative regimes.
- In the Standard Model: *u*, *d*, *s* are light; *c*, *b*, *t* are heavy.
- The heavy quark, surrounded by the strongly interacting «brown muck» is almost on-shell $P^\mu_Q=m_Qv^\mu+k^\mu\ ,\quad k\sim\Lambda_{QCD}$
- "Large" and "Small" component of the field Q(x):

$$Q_{+}^{v}(x) = e^{im_{Q}v \cdot x} \Pi_{+}Q(x), \quad Q_{-}^{v}(x) = e^{im_{Q}v \cdot x} \Pi_{-}Q(x) ,$$

•
$$\Pi_{\pm}=rac{1\pm \psi}{2}$$
 are the projector operators and $\psi Q^v_+=Q^v_+$, $\psi Q^v_-=-Q^v_-$

M. Neubert, Phys. Rept., 245, 259 (1994)

HQET LAGRANGIAN

- The QCD Lagrangian becomes: $\mathcal{L}_{\text{QCD}} = \overline{Q}^v_+ iv \cdot DQ^v_+ + \overline{Q}^v_+ i \not\!\!D_\perp Q^v_- + \overline{Q}^v_- i \not\!\!D_\perp Q^v_+ \overline{Q}^v_- (iv \cdot D + 2m_Q)Q^v_-$
- $D^{\mu}_{\perp} = D^{\mu} (v \cdot D)v^{\mu}$ is transverse covariant derivative of QCD.
- $Q_{-}^{\nu}(x)$ can be integrated-out as it appears as a heavy degree of freedom with twice the heavy quark mass:

• The Lagrangian is now suitable for an expansion in $1/m_Q$:

$$\mathcal{L}_{\mathrm{HQET}} = \sum_{n=0} \mathcal{L}_n / (2m_Q)^n$$

• To second order: $\mathcal{L}_0 = \overline{Q}^v_+ i v \cdot D Q^v_+ \equiv \mathcal{L}^\infty_{\text{eff}}$,

$$\mathcal{L}_{1} = -\overline{Q}_{+}^{v} \not\!\!\!D_{\perp} \not\!\!\!D_{\perp} \not\!\!\!D_{\perp} \not\!\!\!Q_{+}^{v} = -\overline{Q}_{+}^{v} \left[D^{2} + a_{Q}(\mu) \frac{g}{2} \sigma_{\alpha\beta} G^{\alpha\beta} \right] Q_{+}^{v} \equiv O_{\mathrm{kin}} + O_{\mathrm{mag}} ,$$

$$\mathcal{L}_{2} = \overline{Q}_{+}^{v} [\not\!\!\!D_{\perp} iv \cdot D \not\!\!\!D_{\perp}] Q_{+}^{v} = g \overline{Q}_{+}^{v} \left[v_{\beta} D_{\alpha} G^{\alpha\beta} - iv_{\alpha} \sigma_{\beta\gamma} D^{\gamma} G^{\alpha\beta} \right] Q_{+}^{v} ,$$

• $ig G^{\alpha\beta} = [D^{\alpha}, D^{\beta}]$ is the field-strength.

MATCHING ONTO QCD

• In terms of the large field components, the generic source term becomes:

$$\overline{J}Q \equiv \overline{J}^{v}\mathcal{J}_{\mathrm{HQET}}Q^{v}_{+} = \overline{J}^{v} \left[1 + \frac{1}{iv \cdot D + 2m_{Q}} i \not{\!\!\!D}_{\perp} \right] Q^{v}_{+} , \quad J^{v} = e^{im_{Q}v \cdot x} J .$$

• The expansion in $1/m_Q$ gives

$$\mathcal{J}_{\text{HQET}} = 1 + \Pi_{-} \sum_{n=1} \mathcal{J}_{n} / (2m_{Q})^{n}$$
, $\mathcal{J}_{1} = i \not D$, and $\mathcal{J}_{2} = - \not D \not D$

• We are interested in $H_b \rightarrow H_c$ transitions. The matching onto QCD is:

$$\frac{\langle H_c | \bar{c} \, \Gamma \, b | H_b \rangle}{\sqrt{m_{H_c} m_{H_b}}} = \langle H_c^{v'} | \frac{1}{\mathcal{Z}} \int \mathcal{D} \bar{c}_+^{v'} \mathcal{D} \bar{b}_+^v \mathcal{D} \bar{b}_+^v \mathcal{D} \bar{b}_+^v$$
$$\exp \left\{ i \int d^4 x [\mathcal{L}'_{\text{HQET}} + \mathcal{L}_{\text{HQET}}] \right\} \bar{c}_+^{v'} \bar{\mathcal{J}}'_{\text{HQET}} \, \Gamma \, \mathcal{J}_{\text{HQET}} b_+^v | H_b^v \rangle \,, \qquad \overline{\mathcal{J}}_n \equiv \gamma^0 \overleftarrow{\mathcal{J}}_n^\dagger \gamma^0$$

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BARYON DECAYS $\Lambda_b \rightarrow \Lambda_c^*(2595, 2625)$

- Indipendent determination of V_{cb}
- Lepton Flavor Universality ratios R_{Λ^*}
- Comparison with Lattice QCD results

SECOND-ORDER MATCHING

$$\begin{split} \frac{\langle H_c | \bar{c} \Gamma b | H_b \rangle}{\sqrt{m_{H_c} m_{H_b}}} &\simeq \langle H_c^{v'} | \bar{c}_+^{v'} \Gamma b_+^v | H_b^v \rangle \\ &+ \frac{1}{2m_c} \langle H_c^{v'} | (\bar{c}_+^{v'} \overline{\mathcal{J}}_1' + \mathcal{L}_1' \circ c_+^{v'}) \Gamma b_+^v | H_b^v \rangle \\ &+ \frac{1}{2m_b} \langle H_c^{v'} | \bar{c}_+^{v'} \Gamma (\mathcal{J}_1 b_+^v + b_+^v \circ \mathcal{L}_1) | H_b^v \rangle \\ &+ \frac{1}{4m_c^2} \langle H_c^{v'} | (\bar{c}_+^{v'} \overline{\mathcal{J}}_2' \Pi_-' + \mathcal{L}_2' \circ \bar{c}_+^{v'} + \mathcal{L}_1' \circ \bar{c}_+^{v'} \overline{\mathcal{J}}_1' \Pi_-' + \frac{1}{2} \mathcal{L}_1' \circ \mathcal{L}_1' \circ \bar{c}_+^{v'}) \Gamma b_+ | H_b^v \rangle \\ &+ \frac{1}{4m_b^2} \langle H_c^{v'} | \bar{c}_+^{v'} \Gamma (\Pi_- \mathcal{J}_2 b_+^v + b_+^v \circ \mathcal{L}_2 + \Pi_- \mathcal{J}_1 b_+^v \circ \mathcal{L}_1 + \frac{1}{2} b_+^v \circ \mathcal{L}_1 \circ \mathcal{L}_1) | H_b^v \rangle \\ &+ \frac{1}{4m_c m_b} \langle H_c^{v'} | (\bar{c}_+^{v'} \overline{\mathcal{J}}_1' + \mathcal{L}_1' \circ \bar{c}_+^{v'}) \Gamma (\mathcal{J}_1 b_+^v + b_+^v \circ \mathcal{L}_1) | H_b^v \rangle \end{split}$$

The symbol "o" denotes an operator product:

$$\mathcal{L}_1' \circ \bar{c}_+^{v'}(z) = i \int d^4x \, \mathcal{L}_1'(x) \bar{c}_+^{v'}(z)$$

PLUS SHORT DISTANCE CORRECTIONS $O(\alpha_s)$

RESIDUAL CHIRAL EXPANSION

 $\mathcal{L}_{\text{QCD}} = \overline{Q}^v_+ iv \cdot DQ^v_+ + \overline{Q}^v_+ i \not\!\!D_\perp Q^v_- + \overline{Q}^v_- i \not\!\!D_\perp Q^v_+ - \overline{Q}^v_- (iv \cdot D + 2m_Q)Q^v_-.$

- The mixed term breaks the $U(1)_+ \times U(1)_-$ chiral symmetry of the kinetic Lagrangian.
- New power counting: $i D\!\!\!/_\perp o i \theta D\!\!\!/_\perp$.
- At $\mathcal{O}(1/m_c^2, \theta^2)$:

$$\begin{split} \frac{\langle H_c | \bar{c} \, \Gamma \, b | H_b \rangle}{\sqrt{m_{H_c} m_{H_b}}} &\simeq \left\langle H_c^{v'} | \bar{c}_+^{v'} \, \Gamma \, b_+^v | H_b^v \right\rangle \\ &+ \frac{1}{2m_c} \left\langle H_c^{v'} | \left(\bar{c}_+^{v'} \overline{\mathcal{J}}_1' + \mathcal{L}_1' \circ c_+^{v'} \right) \Gamma \, b_+^v | H_b^v \right\rangle + \frac{1}{2m_b} \left\langle H_c^{v'} | \bar{c}_+ \, \Gamma \left(\mathcal{J}_1 b_+^v + b_+^v \circ \mathcal{L}_1 \right) | H_b^v \right\rangle \\ &+ \frac{1}{4m_c^2} \left\langle H_c^{v'} | \left(\bar{c}_+^{v'} \overline{\mathcal{J}}_2' \Pi_-' + \mathcal{L}_2' \circ \bar{c}_+^{v'} \right) \Gamma \, b_+^v | H_b^v \right\rangle. \end{split}$$

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VANISHING CHROMOMAGNETIC LIMIT

- In the VC limit $G_{\alpha\beta} \to 0 \longrightarrow \mathcal{L}_1 = -\overline{Q}^v_+ D^2 Q^v_+; \quad \mathcal{L}_2 = 0$.
- At $\mathcal{O}(1/m_c^2)$:

$$\begin{split} \frac{\langle H_c | \bar{c} \, \Gamma \, b | H_b \rangle}{\sqrt{m_{H_c} m_{H_b}}} &\simeq \left\langle H_c^{v'} | \bar{c}_+^{v'} \, \Gamma \, b_+^{v} | H_b^{v} \right\rangle + \\ &+ \frac{1}{2m_c} \left\langle H_c^{v'} | \left(\bar{c}_+^{v'} \overline{\mathcal{J}}_1' - \bar{c}_+^{v'} D^2 c_+^{v'} \circ \bar{c}_+^{v'} \right) \Gamma \, b_+^{v} | H_b^{v} \right\rangle + \frac{1}{2m_b} \left\langle H_c^{v'} | \bar{c}_+^{v'} \, \Gamma \left(\mathcal{J}_1 b_+^{v} - b_+^{v} \circ \bar{b}_+^{v} D^2 b_+^{v} \right) | H_b^{v} \right\rangle + \\ &+ \frac{1}{4m_c^2} \left\langle H_c^{v'} | \left(- \bar{c}_+^{v'} D^2 c_+^{v'} \circ \bar{c}_+^{v'} \overline{\mathcal{J}}_1' \Pi_-' + \frac{1}{2} \, \bar{c}_+^{v'} D^2 c_+^{v'} \circ \bar{c}_+^{v'} D^2 c_+^{v'} \circ \bar{c}_+^{v'} \right) \Gamma \, b_+^{v} | H_b^{v} \right\rangle \,. \end{split}$$

HADRONIC MATRIX ELEMENTS IN QCD

 $\Lambda_b^0
ightarrow \Lambda_c^*[2595] \ (J^P = 1/2^-)$ matrix element in QCD

$$\begin{split} \left\langle \Lambda_{c}(2595)^{+}(k,\eta(\lambda_{c}),s_{c}) \left| \, \bar{c}\gamma^{\mu}b \left| \Lambda_{b}^{0}(p,s_{b}) \right\rangle &= +\bar{u}_{\alpha}^{(1/2)}(k,\eta(\lambda_{c}),s_{c}) \left[\sum_{i} f_{i}(q^{2})\Gamma_{V,i}^{\alpha\mu} \right] u(p,s_{b}) , \\ \left\langle \Lambda_{c}(2595)^{+}(k,\eta(\lambda_{c}),s_{c}) \left| \, \bar{c}\gamma^{\mu}\gamma_{5}b \left| \Lambda_{b}^{0}(p,s_{b}) \right\rangle &= -\bar{u}_{\alpha}^{(1/2)}(k,\eta(\lambda_{c}),s_{c}) \left[\sum_{i} g_{i}(q^{2})\gamma_{5}\Gamma_{A,i}^{\alpha\mu} \right] u(p,s_{b}) , \\ \left\langle \Lambda_{c}(2595)^{+}(k,\eta(\lambda_{c}),s_{c}) \left| \, \bar{c}\,i\sigma^{\mu\nu}q_{\nu}b \left| \Lambda_{b}^{0}(p,s_{b}) \right\rangle &= -\bar{u}_{\alpha}^{(1/2)}(k,\eta(\lambda_{c}),s_{c}) \left[\sum_{i} t_{i}(q^{2})\Gamma_{T,i}^{\alpha\mu} \right] u(p,s_{b}) , \\ \left\langle \Lambda_{c}(2595)^{+}(k,\eta(\lambda_{c}),s_{c}) \left| \, \bar{c}\,i\sigma^{\mu\nu}q_{\nu}\gamma_{5}b \left| \Lambda_{b}^{0}(p,s_{b}) \right\rangle &= -\bar{u}_{\alpha}^{(1/2)}(k,\eta(\lambda_{c}),s_{c}) \left[\sum_{i} t_{i}^{5}(q^{2})\gamma_{5}\Gamma_{T,i}^{\alpha\mu} \right] u(p,s_{b}) . \end{split}$$

$\Lambda_b^0 \to \Lambda_c^*[2625](J^P=3/2^-)$ matrix element in QCD

$$\begin{split} \left\langle \Lambda_{c}(2625)^{+}(k,\eta(\lambda_{c}),s_{c}) \left| \, \bar{c}\gamma^{\mu}b \left| \Lambda_{b}^{0}(p,s_{b}) \right\rangle &= +\bar{u}_{\alpha}^{(3/2)}(k,\eta(\lambda_{c}),s_{c}) \left[\sum_{i} F_{i}(q^{2})\Gamma_{V,i}^{\alpha\mu} \right] u(p,s_{b}) , \\ \left\langle \Lambda_{c}(2625)^{+}(k,\eta(\lambda_{c}),s_{c}) \left| \, \bar{c}\gamma^{\mu}\gamma_{5}b \left| \Lambda_{b}^{0}(p,s_{b}) \right\rangle &= -\bar{u}_{\alpha}^{(3/2)}(k,\eta(\lambda_{c}),s_{c}) \left[\sum_{i} G_{i}(q^{2})\gamma_{5}\Gamma_{A,i}^{\alpha\mu} \right] u(p,s_{b}) , \\ \left\langle \Lambda_{c}(2595)^{+}(k,\eta(\lambda_{c}),s_{c}) \left| \, \bar{c}\,i\sigma^{\mu\nu}q_{\nu}b \left| \Lambda_{b}^{0}(p,s_{b}) \right\rangle &= -\bar{u}_{\alpha}^{(3/2)}(k,\eta(\lambda_{c}),s_{c}) \left[\sum_{i} T_{i}(q^{2})\Gamma_{T,i}^{\alpha\mu} \right] u(p,s_{b}) , \\ \left\langle \Lambda_{c}(2595)^{+}(k,\eta(\lambda_{c}),s_{c}) \left| \, \bar{c}\,i\sigma^{\mu\nu}q_{\nu}\gamma_{5}b \left| \Lambda_{b}^{0}(p,s_{b}) \right\rangle &= -\bar{u}_{\alpha}^{(3/2)}(k,\eta(\lambda_{c}),s_{c}) \left[\sum_{i} T_{i}^{5}(q^{2})\gamma_{5}\Gamma_{T5,i}^{\alpha\mu} \right] u(p,s_{b}) , \end{split}$$

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HQET MATRIX ELEMENTS

MATRIX ELEMENT PARAMETERISATION FOR A GENERIC CURRENT

$$\Gamma^{\mu} \mapsto J^{\mu}_{\Gamma} = \underbrace{\sum_{\Gamma} C_{\Gamma}(w) J_{\Gamma}}_{\mathcal{O}(\alpha_{s})} + \underbrace{\Delta J^{\mu}_{\Gamma} + \Delta \overline{J}^{\mu}_{\Gamma}}_{\mathcal{O}(1/m_{b,c})} + \underbrace{\delta \overline{J}^{\mu}_{\Gamma}}_{\mathcal{O}(1/m_{c}^{2})} + \mathcal{O}(1/m_{b}^{2}, 1/m_{b}m_{c}, \alpha_{s}/m) , \quad \Gamma = V, A, T, T5 .$$

• We work out the vector current example

$$\gamma^{\mu} \mapsto J_{V}^{\mu} = \underbrace{C_{1}(w)\gamma^{\mu} + C_{2}(w)v^{\mu} + C_{3}(w)v'^{\mu}}_{\mathcal{O}(\alpha_{s})} + \underbrace{\Delta J_{V}^{\mu} + \Delta \overline{J}_{V}^{\mu}}_{\mathcal{O}(1/m_{b,c})} + \underbrace{\delta \overline{J}_{V}^{\mu}}_{\mathcal{O}(1/m_{c}^{2})} + \underbrace{\mathcal{O}(1/m_{b}^{2}, 1/m_{b}m_{c}, \alpha_{s}/m)}_{\mathcal{O}(1/m_{c}^{2})}$$

• At $\mathcal{O}(1/m)$ order the local terms are:

$$\begin{split} &\langle \Lambda_c^*(k,\eta,s_c) | \, \bar{c}_+^{v'} \gamma^\mu \, b_+^v \, \left| \bar{\Lambda}(p,s_b) \right\rangle = \sqrt{4} \bar{u}_\alpha(m_{\Lambda_c^*} v',\eta,s_c) \gamma^\mu u(m_{\Lambda_b} v,s_b) \zeta^\alpha(w) \,, \\ &\langle \Lambda_c^*(k,\eta,s_c) | \, \bar{c}_+^{v'} \gamma^\mu \mathcal{J}_1 b_+^v \, \left| \bar{\Lambda}(p,s_b) \right\rangle = \sqrt{4} \bar{u}_\alpha(m_{\Lambda_c^*} v',\eta,s_c) \gamma^\mu \gamma^\beta u(m_{\Lambda_b} v,s_b) \zeta_b^{\alpha\beta}(w) \,, \\ &\langle \Lambda_c^*(k,\eta,s_c) | \, \bar{c}_+^{v'} \overline{\mathcal{J}}_1 \gamma^\mu b_+^v \, \left| \bar{\Lambda}(p,s_b) \right\rangle = \sqrt{4} \bar{u}_\alpha(m_{\Lambda_c^*} v',\eta,s_c) \gamma^\beta \gamma^\mu u(m_{\Lambda_b} v,s_b) \zeta_c^{\alpha\beta}(w) \,. \end{split}$$

$$w = v \cdot v' = \frac{m_{\Lambda_b}^2 + m_{\Lambda_c^*}^2 - q^2}{2m_{\Lambda_b}m_{\Lambda_c^*}} , \quad q = p - k .$$

$$\zeta^{\alpha}(w) = \zeta(w)(v - v')^{\alpha} ,$$

$$\zeta^{\alpha\beta}_{(q)}(w) = (v - v')^{\alpha} \left[\zeta_1^{(q)}(w)v^{\beta} + \zeta_2^{(q)}(w)v'^{\beta}\right] + g^{\alpha\beta}\zeta_3^{(q)}(w)$$

- $\zeta(w)$, $\zeta_i^{(q)}$ are the leading and sub-leading Isgur-Wise functions.
- Only one independent sub-leading IW function: $\zeta_3^{(b)} \equiv \zeta_{SL}$.

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• At $\mathcal{O}(1/m)$ the non-local terms $\mathcal{L}_1^{(\prime)} = \mathcal{L}_{kin}^{(\prime)} + \mathcal{L}_{mag}^{(\prime)}$:

$$\begin{split} &\langle \Lambda_c^*(k,\eta,s_c) | \mathcal{L}_{\rm kin} \circ \bar{c}_+^{v'} \gamma^{\mu} b_+^v | \Lambda_b(p,s_b) \rangle = \sqrt{4} \eta_{\rm kin}^{(b)}(w) \, v^{\alpha} \bar{u}_{\alpha}(m_{\Lambda_c^*}v',\eta,s_c) \gamma^{\mu} u(m_{\Lambda_b}v,s_b) \,, \\ &\langle \Lambda_c^*(k,\eta,s_c) | \mathcal{L}_{\rm kin}' \circ \bar{c}_+^{v'} \gamma^{\mu} b_+^v | \Lambda_b(p,s_b) \rangle = \sqrt{4} \eta_{\rm kin}^{(c)}(w) \, v^{\alpha} \bar{u}_{\alpha}(m_{\Lambda_c^*}v',\eta,s_c) \gamma^{\mu} u(m_{\Lambda_b}v,s_b) \,, \\ &\langle \Lambda_c^*(k,\eta,s_c) | \mathcal{L}_{\rm mag} \circ \bar{c}_+^{v'} \gamma^{\mu} b_+^v | \Lambda_b(p,s_b) \rangle = \eta_{\rm mag}^{(b)}(w) \, g_{\mu\alpha} v_{\nu}' \bar{u}^{\alpha}(m_{\Lambda_c^*}v',\eta,s_c) \, \gamma^{\mu} \Pi_+ \, i \sigma^{\mu\nu} u(m_{\Lambda_b}v,s_b) \,, \\ &\langle \Lambda_c^*(k,\eta,s_c) | \mathcal{L}_{\rm mag}' \circ \bar{c}_+^{v'} \gamma^{\mu} b_+^v | \Lambda_b(p,s_b) \rangle = \eta_{\rm mag}^{(c)}(w) \, g_{\mu\alpha} v_{\nu} \bar{u}^{\alpha}(m_{\Lambda_c^*}v',\eta,s_c) \, i \sigma^{\mu\nu} \Pi_+' \gamma^{\mu} u(m_{\Lambda_b}v,s_b) \,. \end{split}$$

RESIDUAL CHIRAL

• At $\mathcal{O}(1/m_c^2, \theta^2)$ in the RC expansion:

 $\left\langle \Lambda_c^*(k,\eta,s_c) \right| \bar{c}_+^{v'} \overline{\mathcal{J}}_2' \Pi_-' \Gamma b_+^v \left| \Lambda_b(p,s_b) \right\rangle = \sqrt{4} \bar{u}_\mu \gamma_\alpha v_\beta' \Gamma u \, \psi^{\alpha\beta\mu}(v,v') \; ,$

 $\psi^{\alpha\beta\mu}(v,v') = \psi_1(w)v^{\mu}(v^{\alpha}v'^{\beta} - v^{\beta}v'^{\alpha}) + \psi_2(w)(g^{\alpha\mu}v^{\beta} - g^{\beta\mu}v^{\alpha}) + \psi_3(w)(g^{\alpha\mu}v'^{\beta} - g^{\beta\mu}v'^{\alpha}) \,.$

Non-local terms are reabsorbed:

$$\begin{split} \eta_{\rm kin}^{(c)}(w) &+ \frac{1}{2m_c}\lambda(w) \to \eta_{\rm kin}^{(c)}(w) \ ,\\ \eta_{\rm mag}^{(c)}(w) &+ \frac{1}{2m_c}\eta(w) \to \eta_{\rm mag}^{(c)}(w). \end{split}$$

VANISHING CHROMOMAGNETIC

• At $\mathcal{O}(1/m_c^2)$ in the VC limit:

 $\langle \Lambda_{c}^{*}(k,\eta,s_{c})| - \bar{c}_{+}^{v'} D^{2} c_{+}^{v'} \circ \bar{c}_{+}^{v'} \overline{\mathcal{J}}_{1}^{\prime} \Pi_{-}^{\prime} \Gamma b_{+}^{v} |\Lambda_{b}(p,s_{b})\rangle = \sqrt{4} \bar{u}_{\mu} \gamma_{\alpha} \Pi_{-}^{\prime} \Gamma u \beta^{\mu\alpha}(v,v') ,$ $\langle \Lambda_{c}^{*}(k,\eta,s_{c})| \bar{c}_{+}^{v'} D^{2} c_{+}^{v'} \circ \bar{c}_{+}^{v'} D^{2} c_{+}^{v'} \circ \bar{c}_{+}^{v'} \Gamma b_{+}^{v} |\Lambda_{b}(p,s_{b})\rangle = \sqrt{4} \bar{u}_{\mu} \Gamma u \alpha^{\mu}(v,v') ,$

 $\beta^{\mu\alpha}(v,v') = v^{\mu}(\beta_1(w)v^{\alpha} + \beta_2(w)v'^{\alpha}) + \beta_3(w) g^{\mu\alpha} ,$ $\alpha^{\mu}(v,v') = \alpha(w)(v-v')^{\mu} .$

FIT TO THE LATTICE RESULTS

The Form Factors depend on 7 and 5 Isgur-Wise functions in the RC expansion and VC limit respectively.

- Isgur-Wise functions are not known from first principles.
- We use the lattice results given in the form:

 $f_i = F_i + A_i(w - 1).$

• We expand the IW functions to the first order in (w - 1):

 $\zeta = \zeta^{(0)} + \zeta^{(1)}(w - 1).$

- We fit our Form Factors to the lattice results.
- We use a χ^2 minimisation and find remarkably good fits.
- Neglecting NNLO terms, we achieve a poor fit.
- NNLO corrections are necessary to reconcile HQET with LQCD results.

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RC EXPANSION		VC LIMIT	
Parameter	Best fit	Parameter	Best fit
$\zeta^{(0)}$	0.52 ± 0.16	$\zeta^{(0)}$	0.52 ± 0.16
$\zeta^{(1)}$	-6.11 ± 1.27	$\dot{\zeta}^{(1)}$	-5.97 ± 1.24
$\zeta^{(0)}_{SL}$	0.15 ± 0.01	$\zeta^{(0)}_{SL}$	0.15 ± 0.01
$\zeta_{SL}^{(1)}$	-0.38 ± 0.11	$\zeta^{(1)}_{SL}$	-0.31 ± 0.10
$\eta_{kin,c}^{(0)}$	-0.26 ± 0.44	$\eta_{kin,c}^{(0)}$	-0.24 ± 0.42
$\eta_{kin,c}^{(1)}$	9.77 ± 3.08	$\eta_{kin,c}^{(1)}$	9.48 ± 3.06
$\eta_{mag,c}^{(0)}$	0.01 ± 0.10	$eta_1^{(0)}$	1.58 ± 0.71
$\eta^{(1)}_{mag,c}$	-0.15 ± 1.44	$eta_3^{(0)}$	0.81 ± 0.05
$\eta_{mag,b}^{(0)}$	-0.08 ± 0.06	$eta_3^{(1)}$	-0.96 ± 0.50
$\eta_{maq,b}^{(1)}$	0.25 ± 0.70		
$\psi_1^{(0)}$	1.58 ± 0.72	χ^2_{Va}	
$\psi^{'(0)}$	0.82 ± 0.05	$\frac{1}{d o}$	$\frac{1}{f} = 0.84$
$\psi^{'(1)}$	-1.13 ± 0.52	u. U.	J•

 $\frac{\chi^2_{RC}}{\chi^2_{RC}} = 0.89$

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USING VECTOR AND AXIAL FORM FACTORS AS INPUT

WE PREDICT TENSOR AND PSEUDO-TENSOR FORM FACTORS

RC EXPANSION		VC LIMIT	
Parameter	Best fit	Parameter	Best fit
$\zeta^{(0)}$	0.43 ± 0.50	$\zeta^{(0)}$	0.52 ± 0.43
$\zeta^{(1)}$	-6.59 ± 1.62	$\zeta^{(1)}$	-6.49 ± 1.55
$\zeta_{SL}^{(0)}$	0.16 ± 0.01	$\zeta^{(0)}_{SL}$	0.16 ± 0.01
$\zeta_{SL}^{(1)}$	-0.58 ± 0.17	$\zeta_{SL}^{(1)}$	-0.57 ± 0.17
$\eta_{kin,c}^{(\widetilde{0})}$	0.01 ± 1.30	$\eta_{kin.c}^{(0)}$	-0.21 ± 1.10
$\eta_{kin,c}^{(1)}$	10.72 ± 4.01	$\eta_{kin,c}^{(1)}$	10.56 ± 3.94
$\eta_{mag,c}^{(0)}$	-0.04 ± 0.12	$eta_1^{(0)}$	1.42 ± 0.96
$\eta_{mag,c}^{(1)}$	-0.07 ± 1.61	$eta_3^{(0)}$	0.84 ± 0.09
$\eta_{mag,b}^{(0)}$	-0.01 ± 0.07	$eta_3^{(1)}$	-1.69 ± 0.64
$\eta_{mag,b}^{(1)}$	-0.75 ± 0.93		
$\psi_1^{(0)}$	1.48 ± 1.01	χ^2_{Va}	
$\psi^{'(0)}$	0.83 ± 0.10	$\frac{d_{10}}{d_{10}} = 0.72$	
$\psi^{'(1)}$	-1.76 ± 0.68	u. 0.	J •
$\psi^{\prime(1)}$ χ^2_{RC}	-1.76 ± 0.68	u. 0.	J·

d.*o*.*f*



CONCLUSIONS

- HQET IS A VALUABLE TOOL TO STUDY HADRONS CONTAINING A HEAVY QUARK
- $\succ \Lambda_b \rightarrow \Lambda_c^*(2595, 2625)$ represent relevant applications.
- > We went beyond previous analysis of this channel computing Form Factors at $O(1/m_c^2)$.
- > We showed the relevance of the novel $\mathcal{O}(1/m_c^2)$ terms to reconcile HQET with the lattice QCD results.

THANK YOU FOR YOUR ATTENTION!

BACKUP SLIDES

SPIN-FLAVOUR SYMMETRY & ISGUR-WISE FUNCTION

- $\mathcal{L}_{\text{eff}}^{\infty}$ is invariant under $SU(2)_{N_{h}}$ spin group.
- For N_h heavy quarks $\mathcal{L}_{eff}^{\infty} = \sum_{i=1}^{n} \overline{Q}_{+}^{v\,i} iv \cdot DQ_{+}^{v\,i}$, and $SU(2) \Rightarrow SU(2N_h)$

SPIN-FLAVOUR SYMMETRY

- Symmetry breaking terms are suppressed by powers of $1/m_Q$.
- Scattering of a pseudo-scalar meson $P(v) \rightarrow P'(v')$ involving the transition $Q \rightarrow Q'$.

What happens to the brown muck?

If v = v' nothing happens If $v \neq v'$ form factor suppression

The hadronic matrix element can be written as

$$\langle P'(v') | \bar{Q}_{+}'^{v'} \gamma^{\mu} Q_{+}^{v} | P(v) \rangle = \xi (v \cdot v') (v + v')^{\mu}$$

• The universal form factor $\xi(v \cdot v')$ is the "Isgur-Wise" function.

HELICITY AMPLITUDES

$$\mathcal{A}_{\Gamma}(s_b, s_c, \lambda_c, \lambda_q) \equiv \langle \Lambda_c^*(s_c, \eta(\lambda_c)) | \, \bar{c} \, \Gamma^{\mu} \varepsilon_{\mu}^*(\lambda_q) b \, | \Lambda_b(s_b) \rangle \, ,$$

Helicity Amplitudes for J = 1/2

$$s_{b,c} \in \left\{ +\frac{1}{2}, -\frac{1}{2} \right\}, \\ \lambda_c \in \{0, +1, -1\}, \\ \lambda_q \in \{t, 0, +1, -1\} .$$

$\begin{aligned} \mathcal{A}_{\Gamma}^{(1/2)}(+1/2,+1/2,0) &\equiv -\sqrt{\frac{1}{3}}\mathcal{A}_{\Gamma}(+1/2,+1/2,0,0) + \sqrt{\frac{2}{3}}\mathcal{A}_{\Gamma}(+1/2,-1/2,+1,0) \,, \\ \mathcal{A}_{\Gamma}^{(1/2)}(+1/2,+1/2,t) &\equiv -\sqrt{\frac{1}{3}}\mathcal{A}_{\Gamma}(+1/2,+1/2,0,t) + \sqrt{\frac{2}{3}}\mathcal{A}_{\Gamma}(+1/2,-1/2,+1,t) \,, \\ \mathcal{A}_{\Gamma}^{(1/2)}(+1/2,-1/2,-1) &\equiv \sqrt{\frac{1}{3}}\mathcal{A}_{\Gamma}(+1/2,-1/2,0,-1) - \sqrt{\frac{2}{3}}\mathcal{A}_{\Gamma}(+1/2,+1/2,-1,-1) \,. \end{aligned}$

Helicity Amplitudes for J = 3/2

$$\begin{aligned} \mathcal{A}_{\Gamma}^{(3/2)}(+1/2,+3/2,+1) &\equiv \mathcal{A}_{\Gamma}(+1/2,+1/2,+1,+1) ,\\ \mathcal{A}_{\Gamma}^{(3/2)}(+1/2,+1/2,0) &\equiv \sqrt{\frac{2}{3}} \mathcal{A}_{\Gamma}(+1/2,+1/2,0,0) + \sqrt{\frac{1}{3}} \mathcal{A}_{\Gamma}^{(3/2)}(+1/2,-1/2,+1,0) ,\\ \mathcal{A}_{\Gamma}^{(3/2)}(+1/2,+1/2,t) &\equiv \sqrt{\frac{2}{3}} \mathcal{A}_{\Gamma}(+1/2,+1/2,0,t) + \sqrt{\frac{1}{3}} \mathcal{A}_{\Gamma}^{(3/2)}(+1/2,-1/2,+1,t) ,\\ \mathcal{A}_{\Gamma}^{(3/2)}(+1/2,-1/2,-1) &\equiv \sqrt{\frac{2}{3}} \mathcal{A}_{\Gamma}(+1/2,-1/2,0,-1) + \sqrt{\frac{1}{3}} \mathcal{A}_{\Gamma}^{(3/2)}(+1/2,+1/2,-1,-1) \end{aligned}$$

(RC) VECTOR FORM FACTORS

$$\begin{split} f_{1/2,0} &= \frac{\zeta s_- \sqrt{s_+}}{2(m_{\Lambda_b} m_{\Lambda_2^+})^{3/2}} \left(C_1 + \frac{C_2 s_+}{2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Lambda_2^+})} + \frac{C_3 s_+}{2m_{\Lambda_a^+}(m_{\Lambda_b} + m_{\Lambda_a^+})} \right) + \\ &+ \varepsilon_b \frac{\sqrt{s_+}(m_{\Lambda_b} - m_{\Lambda_a^+})}{(m_{\Lambda_b} + m_{\Lambda_a^+})\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \left(\zeta \bar{\Lambda}' - 2\zeta_{\rm SL} - \frac{\zeta \bar{\Lambda}' \left(m_{\Lambda_b}^2 + m_{\Lambda_a^+}^2 - q^2 \right)}{2m_{\Lambda_b}m_{\Lambda_a^+}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_+}(m_{\Lambda_b} - m_{\Lambda_a^+})}{(m_{\Lambda_b} + m_{\Lambda_a^+})\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \left(\zeta \bar{\Lambda} - 2\zeta_{\rm SL} - \frac{\zeta \bar{\Lambda}' \left(m_{\Lambda_b}^2 + m_{\Lambda_a^+}^2 - q^2 \right)}{2m_{\Lambda_b}m_{\Lambda_a^+}} \right) + \\ &+ \frac{s_-\sqrt{s_+}}{(m_{\Lambda_b} + m_{\Lambda_a^+})\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \left(3(\psi_3 + w\psi_2) - \frac{s_-s_+\psi_1}{4(m_{\Lambda_b}m_{\Lambda_a^+})^2} \right) \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_+}(m_{\Lambda_b} - m_{\Lambda_a^+})}{(m_{\Lambda_b} - m_{\Lambda_a^+})\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \left(\zeta \bar{\Lambda}' - 2\zeta_{\rm SL} - \frac{\zeta \bar{\Lambda} \left(m_{\Lambda_b}^2 + m_{\Lambda_a^+}^2 - q^2 \right)}{2m_{\Lambda_b}m_{\Lambda_a^+}} \right) + \\ &+ \varepsilon_b \frac{\sqrt{s_-}(m_{\Lambda_b} + m_{\Lambda_a^+})}{(m_{\Lambda_b} - m_{\Lambda_a^+})\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \left(\zeta \bar{\Lambda}' - 2\zeta_{\rm SL} - \frac{\zeta \bar{\Lambda} \left(m_{\Lambda_b}^2 + m_{\Lambda_a^+}^2 - q^2 \right)}{2m_{\Lambda_b}m_{\Lambda_a^+}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-}(m_{\Lambda_b} + m_{\Lambda_a^+})}{(m_{\Lambda_b} - m_{\Lambda_a^+})\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \left(\zeta \bar{\Lambda} - 2\zeta_{\rm SL} - \frac{\zeta \bar{\Lambda} \left(m_{\Lambda_b}^2 + m_{\Lambda_a^+}^2 - q^2 \right)}{2m_{\Lambda_b}m_{\Lambda_a^+}}} \right) + \\ &+ \frac{s_+\sqrt{s_-}}{(m_{\Lambda_b} - m_{\Lambda_a^+})\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \left(\zeta \bar{\Lambda} - 2\zeta_{\rm SL} - \frac{\zeta \bar{\Lambda} \left(m_{\Lambda_b}^2 + m_{\Lambda_a^+}^2 - q^2 \right)}{2m_{\Lambda_b}m_{\Lambda_a^+}}} \right) + \\ &+ \frac{s_+\sqrt{s_-}}{(m_{\Lambda_b} - m_{\Lambda_a^+})\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \left(3(\psi_3 + w\psi_2) - \frac{s_-s_+\psi_1}{4(m_{\Lambda_b}m_{\Lambda_a^+})^2} \right) , \\ f_{1/2,\perp} &= \frac{C_1\zeta s_-\sqrt{s_+}}{2(m_{\Lambda_b}m_{\Lambda_a^+})^{3/2}} + \varepsilon_b \frac{\sqrt{s_+}}{\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \left(\frac{\zeta \bar{\Lambda} \left(m_{\Lambda_b}^2 + m_{\Lambda_a^+}^2 - q^2 \right)}{2m_{\Lambda_b}m_{\Lambda_a^+}}} \right) + \\ &+ \frac{s_-\sqrt{s_+}}{\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \left(\bar{\Lambda} - 2\zeta_{\rm SL} - \frac{\zeta \bar{\Lambda}' \left(m_{\Lambda_b}^2 + m_{\Lambda_a^+}^2 - q^2 \right)}{2m_{\Lambda_b}m_{\Lambda_a^+}}} \right) + \\ &+ \frac{s_-\sqrt{s_+}}{2(m_{\Lambda_b}m_{\Lambda_a^+})^{3/2}} \left(\varepsilon_c \eta_{\rm km}^{(c)} - \varepsilon_c \eta_{\rm mag}^{(c)} + \varepsilon_c \frac{\sqrt{s_+}}{\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \right) + \\ &+ \frac{s_-\sqrt{s_+}}{2(m_{\Lambda_b}m_{\Lambda_a^+})^{3/2}} \left(\varepsilon_c \eta_{\rm km}^{(c)} - \varepsilon_c \eta_{\rm mag}^{(c)} + \varepsilon_c \frac{\sqrt{s_+}}{\sqrt{m_{\Lambda_b}m_{\Lambda_a^+}}} \right) \right) + \\ &+ \frac{s_-\sqrt{s_+}}{2(m_{\Lambda_b}m_{$$

$$\begin{split} & \Lambda_{b} \rightarrow \Lambda_{c}^{*}(2595) \\ & \Lambda_{b} \rightarrow \Lambda_{c}^{*}(2625) \\ & \Lambda_{c} \rightarrow \Lambda_{c}^{*}(2625) \\ & \Lambda_{b} \rightarrow \Lambda_{c}^{*}(26625) \\ & \Lambda_{b} \rightarrow \Lambda_{c}$$

(VC) VECTOR FORM FACTORS

$$\begin{split} f_{1/2,0} &= \frac{\zeta s_-\sqrt{s_+}}{2(m_{A_1}m_{A_2})^{3/2}} \left(C_1 + \frac{C_{2s_+}}{2m_{A_1}(m_{A_1} + m_{A_2})} + \frac{C_{3s_+}}{2m_{A_1}(m_{A_1} + m_{A_2})} \right) + \\ &+ \varepsilon_b \frac{\sqrt{s_-(m_{A_1} - m_{A_2})}}{(m_{A_1} + m_{A_2})\sqrt{m_{A_1}m_{A_2}}} \left(\zeta \lambda^2 - 2\zeta_{SL} - \frac{\zeta \lambda^2(m_{A_1}^2 + m_{A_2^2}^2 - q^2)}{2m_{A_1}m_{A_2^2}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2})}}{(m_{A_1} + m_{A_2})\sqrt{m_{A_1}m_{A_2^2}}} \left(\zeta \lambda - 2\zeta_{SL} - \frac{\zeta \lambda^2(m_{A_1}^2 + m_{A_2^2}^2 - q^2)}{2m_{A_1}m_{A_2^2}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2})}}{(m_{A_1} - m_{A_2})\sqrt{m_{A_1}m_{A_2^2}}} \left(\zeta \lambda^2 - 2\zeta_{SL} - \frac{\zeta \lambda^2(m_{A_1}^2 + m_{A_2^2}^2 - q^2)}{2m_{A_1}m_{A_2^2}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2})}}{(m_{A_1} - m_{A_2})\sqrt{m_{A_1}m_{A_2^2}}} \left(\zeta \lambda^2 - 2\zeta_{SL} - \frac{\zeta \lambda^2(m_{A_1}^2 + m_{A_2^2}^2 - q^2)}{2m_{A_1}m_{A_2^2}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2})}}{(m_{A_1} - m_{A_2^2})\sqrt{m_{A_1}m_{A_2^2}}}} \left(\zeta \lambda^2 - 2\zeta_{SL} - \frac{\zeta \lambda^2(m_{A_1}^2 + m_{A_2^2}^2 - q^2)}{2m_{A_1}m_{A_2^2}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2})}}{(m_{A_1} - m_{A_2^2})\sqrt{m_{A_1}m_{A_2^2}}}} \left(\zeta \lambda^2 - 2\zeta_{SL} - \frac{\zeta \lambda^2(m_{A_1}^2 + m_{A_2^2}^2 - q^2)}{2m_{A_1}m_{A_2^2}}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2^2})}}{(m_{A_1} - m_{A_2^2})\sqrt{m_{A_1}m_{A_2^2}}}} \left(\zeta \lambda^2 - 2\zeta_{SL} - \frac{\zeta \lambda^2(m_{A_1}^2 + m_{A_2^2}^2 - q^2)}{2m_{A_1}m_{A_2^2}}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2^2})}}{(m_{A_1} - m_{A_2^2})\sqrt{m_{A_1}m_{A_2^2}}}} \left(\zeta \lambda^2 - 2\zeta_{SL} - \frac{\zeta \lambda^2(m_{A_1}^2 + m_{A_2^2}^2 - q^2)}{2m_{A_1}m_{A_2^2}}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2^2})}}{(m_{A_1} - m_{A_2^2})\sqrt{m_{A_1}m_{A_2^2}}}} \left(\zeta \lambda^2 - 2\zeta_{SL} - \frac{\zeta \lambda^2(m_{A_1}^2 + m_{A_2^2}^2 - q^2)}{2m_{A_1}m_{A_2^2}}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2^2})}}{(m_{A_1} - m_{A_2^2})\sqrt{m_{A_1}m_{A_2^2}}}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2^2})}}{(m_{A_1} - m_{A_2^2})\sqrt{m_{A_1}m_{A_2^2}}}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2^2})}}{(m_{A_1} - m_{A_2^2})\sqrt{m_{A_1}m_{A_2^2}}}} \right) + \\ &+ \varepsilon_c \frac{\sqrt{s_-(m_{A_1} - m_{A_2^2})}}{(m_{A_1} - m_{A_2^2})\sqrt{m_{A_1}m_{A_2^2}}}} {(\delta \lambda - 2\zeta_{SL} - \frac{\zeta \lambda^2(m_{A_1} - m_{A_2^2} - q^2)}{2m$$