Precise decay rates for $\eta_c \to \gamma\gamma$ and $\eta_b \to \gamma\gamma$ from lattice QCD

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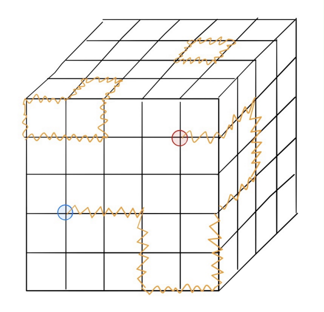
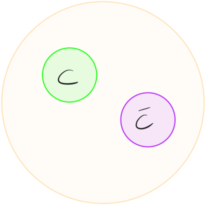


Heavionium decays

- Decays with photons can be used as tests of our understanding of internal structure of mesons from strong interaction physics
- $\eta_c \to \gamma\gamma$: experimental results give no clear consensus
  - Our result vastly improves picture from the lattice
- $\eta_b \to \gamma\gamma$: not yet seen; our result a prediction for Belle II



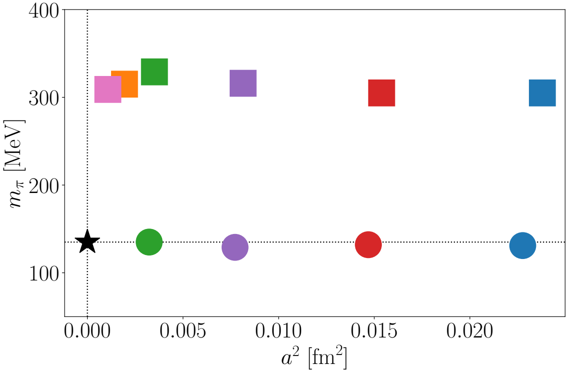
This work

- Precise calculation by using Highly Improved Staggered Quark (HISQ) action
  - Good action for heavy quarks, c.f. previous HPQCD work
- Calculate these decays with realistic sea
  - Effect of 2+1+1 quarks
- $< 1\%$ uncertainties for $\eta_c \to \gamma\gamma$, so more accurate now than experiment

Full details of $\eta_c \to \gamma\gamma$ process in [arXiv:2305.06231]; $\eta_b \to \gamma\gamma$ to appear soon.

Lattice details

- ★ $2+1+1$ HISQ gauge ensembles provided by MILC Collaboration
- ★ Lattice spacings from ≈ 0.15 fm down to ≈ 0.03 fm depending on process
- ★ Combination of $m_s/m_l = 5$ and physical m_l
- ★ Valence heavy quarks $m_c \leq m_h \leq m_b$ also use HISQ formalism
- ★ Quarks tuned so meson matches between lattice and experiment
  - Charm mass tuned to match J/ψ mesons, see (HPQCD '20 [2005.01845])
  - Tuned m_b to match η_b mesons



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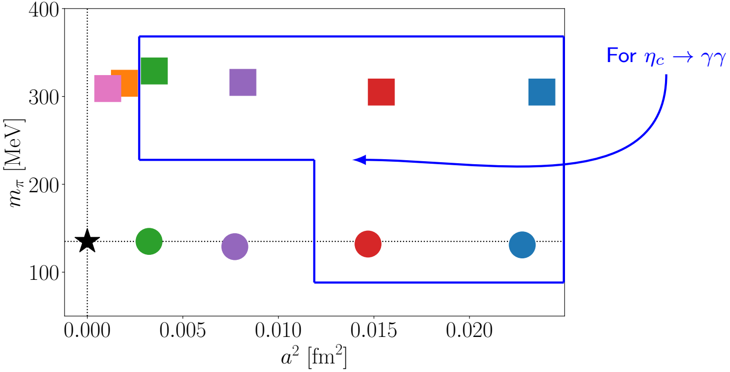

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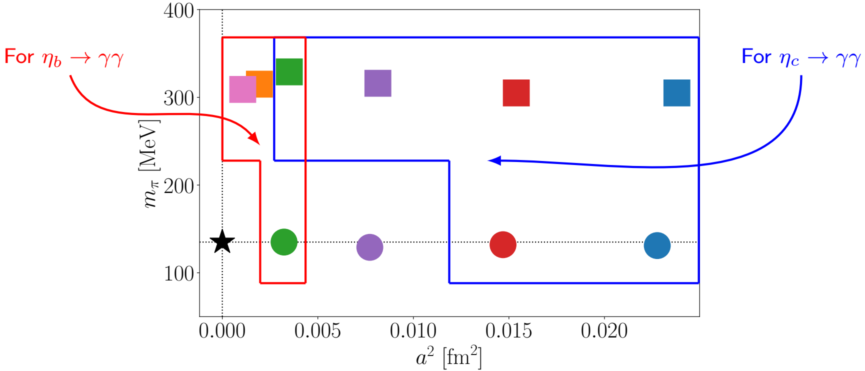
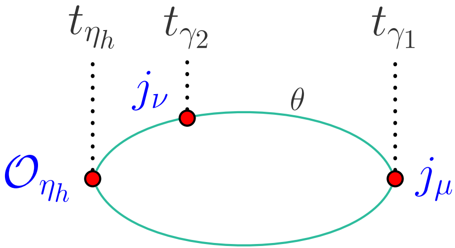




Ji & Jung [hep-lat/0101014] & [hep-lat/0103007]:

$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_h}) = a \sum_{t_{\gamma_1}} e^{-\omega_1 (t_{\gamma_1} - t_{\gamma_2})} C_{\mu\nu}(t_{\gamma_1}, t_{\gamma_2}, t_{\eta_h})$$

- For on-shell photons:

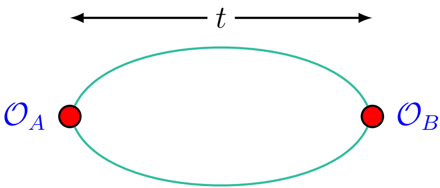
$$\omega_1 = |\vec{q}_1| = |\vec{q}_2| = \frac{M_{\eta_h}}{2}$$

- Impart momentum (θ in picture) to tune ω_1
- Currents require renormalisation; we use RI-SMOM scheme

Fitting correlators

Fit two sets of correlators:

$$C_{\eta_h}(t, t_{\eta_h}) = \sum_{n}^{N} a_n^2 \left(e^{-E_n t} + e^{-E_n (N_t - t)}\right)$$



and

$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_h}) = \sum_{n}^{N} a_n b_n \left(e^{-E_n (t_{\gamma_2} - t_{\eta_h})} + e^{-E_n (N_t - t_{\gamma_2} + t_{\eta_h})}\right)$$

Extract form factor $F_{\rm latt}(0, q_2^2)$ by:

Ground state

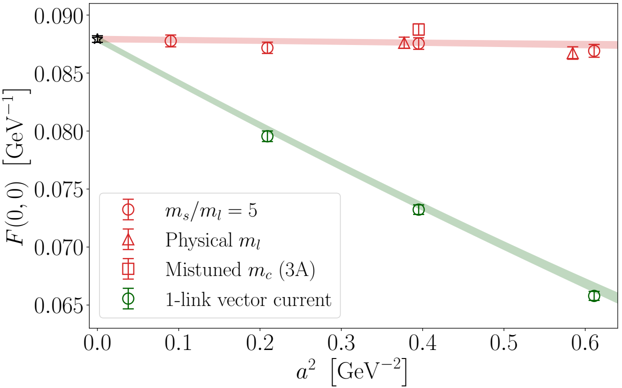
$$\frac{F_{\rm latt}(0, q_2^2)}{a} = b_0 Z_V^2 \frac{\sqrt{2aM_{\eta_h}^{\rm latt}}}{aM_{\eta_h}^{\rm latt} a q_1^y}$$

which, when $q_2^2 = 0$, relates to the width for two on-shell photons:

$$\Gamma(\eta_h \to \gamma\gamma) = \pi \alpha_{\rm em}^2 Q_h^4 M_{\eta_h}^3 F(0,0)^2.$$

$$\eta_c \to \gamma\gamma$$

$\eta_c \to \gamma\gamma$ in the continuum



$$\frac{F^{(t)}_{\mathrm{latt}}(0,q_2^2)}{a} = \frac{F(0,0)}{(1-\frac{q_2^2}{M^2_{\mathrm{pole}}})}\left[1+\sum_{i=1}^{i_{\max}}\kappa^{(i,t)}_{a\Lambda}\left(a\Lambda^{(t)}\right)^{2i}+\kappa_{\mathrm{val},c}\delta^{\mathrm{val},c}+\kappa_{\mathrm{sea},c}\delta^{\mathrm{sea},c}\right.$$

$$\left.+\kappa^{(0)}_{\mathrm{sea},uds}\delta^{\mathrm{sea},uds}\left\{1+\kappa^{(1,t)}_{\mathrm{sea},uds}(a\tilde{\Lambda})^2+\kappa^{(2,t)}_{\mathrm{sea},uds}(a\tilde{\Lambda})^4\right\}\right]$$

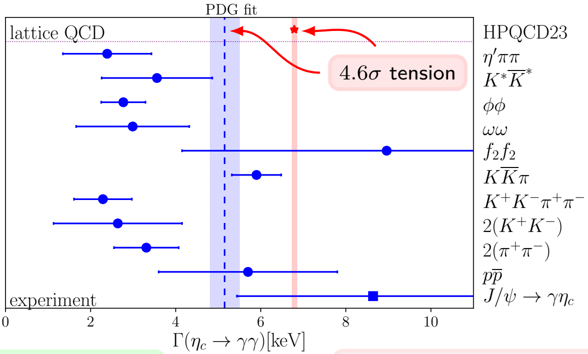
η_c Results

Continuum result gives

$$F(0,0) = 0.08793(29)_{\text{fit}}(26)_{\text{syst}} \text{ GeV}^{-1}$$

From which we can determine the width:

$$\Gamma(\eta_c \to \gamma\gamma) = 6.788(45)_{\text{fit}}(41)_{\text{syst}} \text{ keV}$$



PDG fit: 5.15(35) keV ← $\chi^2 = 118$ for 81 d.o.f.; $p = 0.005$

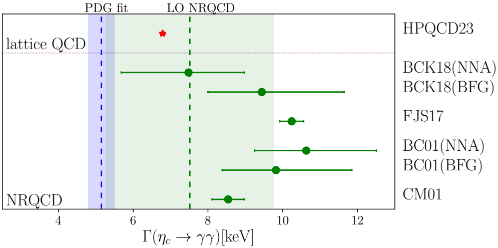
Nonrelativistic relations

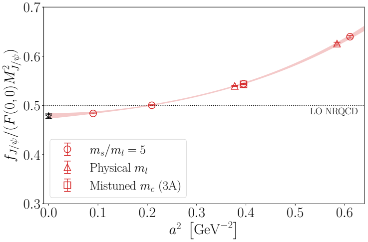
(Czarnecki & Melnikov '01 [hep-ph/0109054]):

Expectation in nonrelativistic limit:

$$\frac{\Gamma(J/\Psi \to e^+e^-)}{\Gamma(\eta_c \to \gamma\gamma)} \approx \frac{3}{4}\left(1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(v^2/c^2\right)\right)$$

$$\frac{f_{J/\psi}}{F(0,0)M_{J/\psi}^2} = \frac{1}{2}\left(1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(v^2/c^2\right)\right)$$



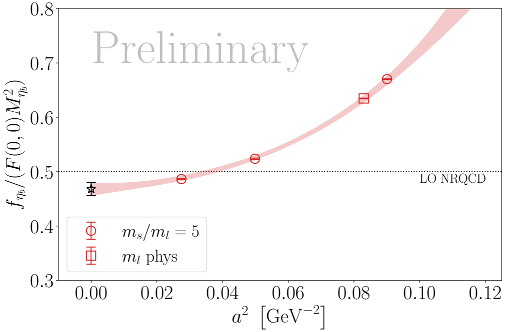


$$\text{Our result}: \frac{f_{J/\psi}}{F(0,0)M_{J/\psi}^2} = 0.4786(57)_{\text{fit}}(14)_{\text{syst}}$$

$M_{J/\psi}$, $f_{J/\psi}$ & $\Gamma(J/\psi \to e^+e^-)$ (for LO NRQCD central value) from HPQCD '20 [2005.01845]

$$\eta_b \to \gamma\gamma$$

$\eta_b \to \gamma\gamma$ from ratio with decay constant



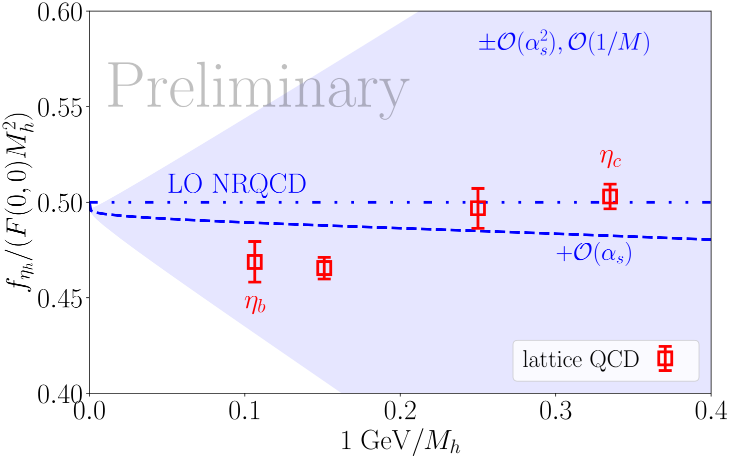
$$R^{\text{latt}}_{\eta_b} = R^{\text{phys}}_{\eta_b}\Bigg[1 + \sum^{i_{\max}} \kappa^{(i)}_{a\Lambda}(a\Lambda)^{2i} + \kappa_{\text{val},b}\delta^{\text{val},b} + \kappa_{\text{sea},c}\delta^{\text{sea},c} + \kappa^{(0)}_{\text{sea},uds}\delta^{\text{sea},uds}\{1 + \kappa^{(1)}_{\text{sea},uds}(a\tilde{\Lambda})^2 + \kappa^{(2)}_{\text{sea},uds}(a\tilde{\Lambda})^4\}\Bigg]$$

$$R^{\text{phys}}_{\eta_b} = 0.468(12); \quad F(0,0) = 0.01751(53)\ \text{GeV}^{-1}$$

$$\Gamma(\eta_b \to \gamma\gamma) = 0.526(32)\ \text{keV}$$

f_{η_b} result for conversion from (HPQCD '21 [2101.08103])

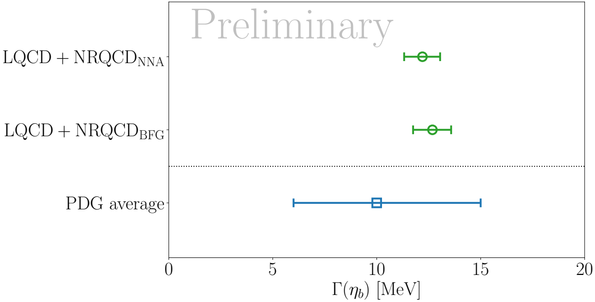
Comparison with LO NRQCD



$$R_{\eta_h} \equiv \frac{f_{\eta_h}}{F_{\eta_h}(0,0)M_{\eta_h}^2} = \frac{1}{2}\left(1 + \mathcal{O}\left(\alpha_s\right) + \mathcal{O}\left(v^2/c^2\right)\right)$$

Inclusive width

- Can get full width $\Gamma(\eta_b)$ by combining our decay width with $\dfrac{\Gamma(\eta_b)}{\Gamma(\eta_b \to \gamma\gamma)}$ from NRQCD



$$\Gamma(\eta_b)_{\mathrm{NNA}} = 12.20 \left({}^{+42}_{-47}\right)_{\mathrm{NRQCD}} (74)_{\mathrm{LQCD}} \text{ MeV}$$

$$\Gamma(\eta_b)_{\mathrm{BFG}} = 12.68 \left({}^{+47}_{-53}\right)_{\mathrm{NRQCD}} (77)_{\mathrm{LQCD}} \text{ MeV}$$

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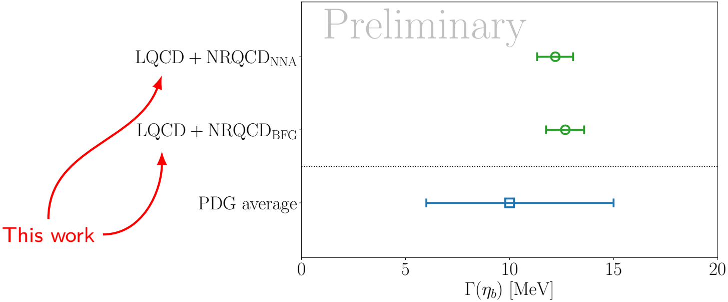

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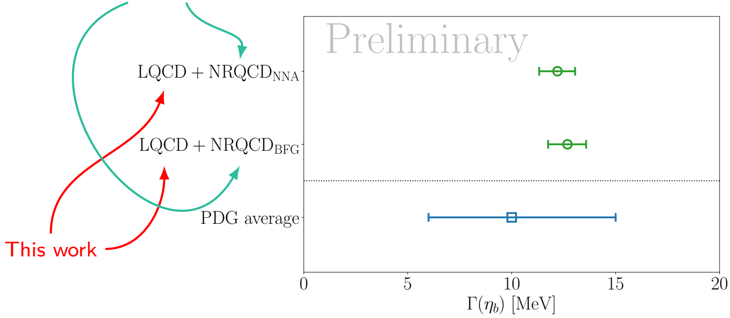
(Brambilla, Chung & Komijani [1810.02586])



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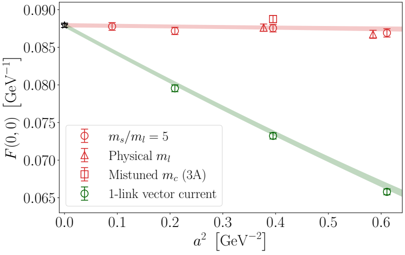

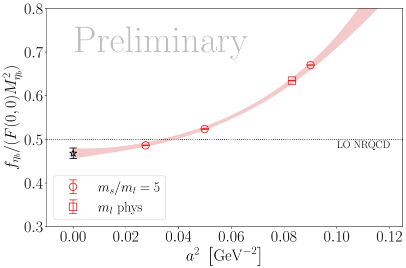

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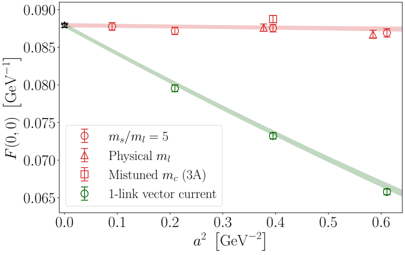
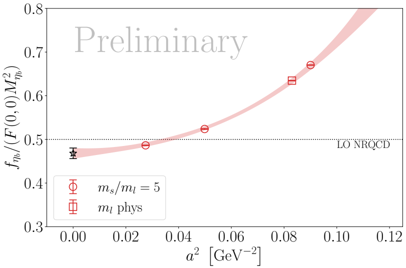
  - $F(0,0) = 0.08793(29)_{\text{fit}}(26)_{\text{syst}}$ GeV^{-1}
- $\Gamma(\eta_b \to \gamma\gamma) = 0.526(32)$ keV Preliminary!
  - $\Gamma(\eta_b) = 12.20(88)$ MeV (NNA); $\Gamma(\eta_b) = 12.68(93)$ MeV (BFG) Preliminary!
- New/updated information on $\eta_c \to \gamma\gamma$ to make picture more clear from experiment side would be welcome!
- We look forward to results from experiment for $\eta_b \to \gamma\gamma$

Belle II?





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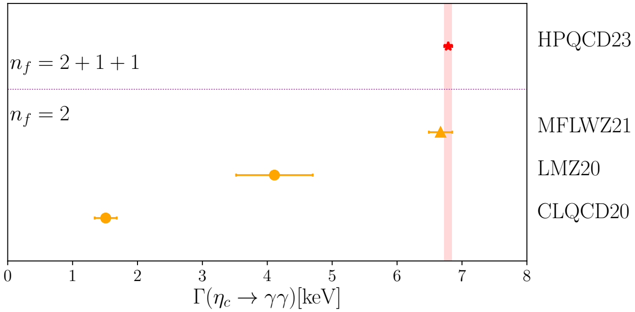
Thank you!

Belle II?

Extra stuff

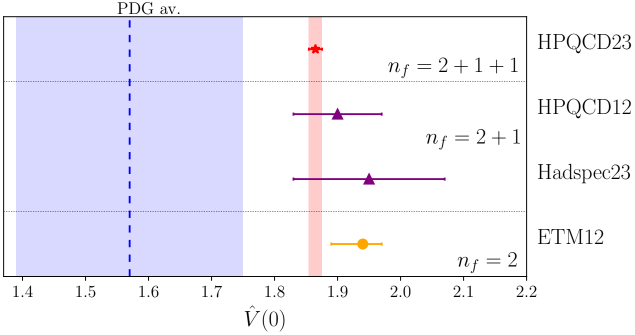
$\Gamma(\eta_c \to \gamma\gamma)$ lattice comparison

★ Comparison with other lattice calculations for $\Gamma(\eta_c \to \gamma\gamma)$



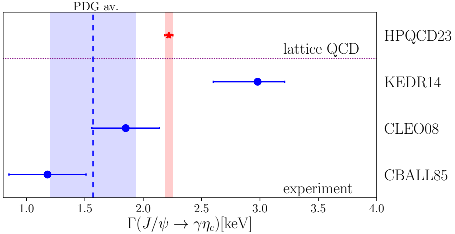
Summary plot $J/\psi \to \gamma\eta_c$: lattice results

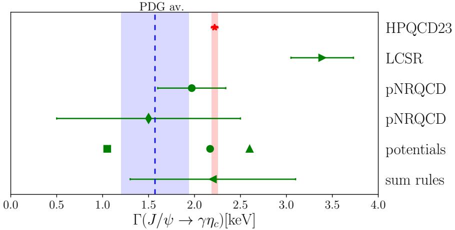
★ Comparison with other lattice calculations of $\hat{V}(0)$ for $J/\psi \to \gamma\eta_c$



Summary results: $J/\psi \to \gamma\eta_c$

★ Comparison with experiment (top) and other theory calculations (bottom) for $\Gamma(J/\psi \to \gamma\eta_c)$





Product of fractions

★ Product of $\mathrm{Br}(J/\psi \to \gamma\eta_c)$ and $\mathrm{Br}(\eta_c \to \gamma\gamma)$ compared to CLEO and BESIII measurements