Universität Zürich[UZH]

Short- vs Long-Distance Dynamics in $b \to s\bar{\ell}\ell$ decays

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In collaboration with:

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$b \to s\bar{\ell}\ell$ decays

- Flavor-changing neutral transitions are prime candidates in the search for BSM physics.
- Long-standing tension with the SM in the exclusive $b \to s\bar{\ell}\ell$ in rates and angular distributions, especially in the low-q^2 region (q^2 is the invariant mass of the lepton pair).

[LHCb on $B \to K^*\bar{\mu}\mu$ (2405.17347), see talk by M. Andersson, and CMS on P5' (BPH-21-002)]

- **On the theoretical side**: difficulty of estimating non-perturbative contributions (*i.e.* local form factors and non-local hadronic matrix elements of the four-quark operators)
- **Goal**: disentangle possible short-distance physics from long-distance dynamics.
- It's necessary to look at **complementary observables** (different sensitivity to SD/LD physics and different uncertainties): **inclusive/exclusive level, low/high q^2**

Theoretical Challenges in the Inclusive $B \to X_s \bar{\ell} \ell$

Inclusive $B \to X_s \bar{\ell} \ell$:

- Treated with an Operator Product Expansion (OPE) in $1/m_b$
- In the **high-q^2 region**:
  - It is affected by large hadronic uncertainties as it is very sensitive to power corrections in the OPE
  - Breakdown of the OPE → becomes an expansion in $\Lambda_{QCD}/(m_b - \sqrt{q^2})$
- Normalizing $B \to X_s \bar{\ell} \ell$ to $B \to X_u \ell \bar{\nu}$ reduces these uncertainties

[Z. Ligeti and F. J. Tackmann, 0707.1694]

→ 1 inclusive rate

Theoretical Challenges in the Exclusive $B \to K^{(*)}\bar{\ell}\ell$

Exclusive $B \to K^{(*)}\bar{\ell}\ell$

$$\mathscr{L} = \mathscr{L}_{\text{QCD+QED}}^{[N_f=5]} + \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i \mathcal{O}_i$$

$$\mathcal{O}_1 = (\bar{s}_L^\alpha \gamma_\mu c_L^\beta)(\bar{c}_L^\beta \gamma^\mu b_L^\alpha) \qquad \mathcal{O}_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a$$

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell) \qquad \mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

To leading order in QED:

$$\mathscr{A}(B \to M\ell^+\ell^-) = \frac{G_F \alpha V_{ts}^* V_{tb}}{\sqrt{2}\pi}\Big[(C_9\ \ell\gamma^\mu\ell + C_{10}\ \ell\gamma^\mu\gamma_5\ell)\langle M|\bar{s}\gamma_\mu P_L b|\bar{B}\rangle\ - \frac{1}{q^2}\ell\gamma^\mu\ell\ \left(2im_b C_7 \langle M|\bar{s}\sigma_{\mu\nu}q^\nu P_R b|B\rangle + \mathscr{H}_\mu\right)\Big]$$

Local form factors

Non-local form factors

→ 2 parameterization of long-distance effects & 3 charm-rescattering effects

Inclusive decay rate $\Gamma(B \to X_s \bar{\ell} \ell)$

[G.Isidori, Z. Polonsky, AT, 2305.03076]

SM prediction for the **inclusive rate:**

[Z. Ligeti and F. J. Tackmann, 0707.1694]

$$R^{(\ell)}_{\text{incl}}(q_0^2) = \frac{\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_s \bar{\ell}\ell)}{dq^2}}{\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_u \bar{\ell}\nu)}{dq^2}} = \frac{|V_{tb}V_{ts}^*|^2}{|V_{ub}|^2} \left[\mathcal{R}_L + \Delta\mathcal{R}_{[q_0^2]}\right]$$

$q_0^2 = 15 \text{ GeV}^2$

from Belle, arXiv:2107.13855

$$\mathcal{R}_L = \frac{\alpha_e^2 C_L^2}{16\pi^2}$$

$$\Delta\mathcal{R}_{[15]} = \frac{\alpha_e^2}{8\pi^2}\Big[C_V^2 + C_V C_L + 0.485 C_L + 0.97 C_V + 0.93 + \Delta_{\text{n.p.}} + C_7(1.91 + 2.05 C_L + 4.27 C_7 + 4.1 C_V)\Big]$$

Significant cancellation of non-perturbative uncertainties since **the hadronic structure is very similar** ($b \to q_{light}$, left-handed current)

Change of basis: $\{\mathcal{O}_9, \mathcal{O}_{10}\} \to \{\mathcal{O}_V, \mathcal{O}_L\}$

$$\mathcal{O}_V = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell) \qquad C_L = -2C_{10}$$

$$\mathcal{O}_L = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}_L \gamma^\mu \ell_L) \qquad C_V = C_9 + C_{10}$$

Comparison with data in the inclusive rate

- Agreement in the SM between the inclusive rate and the sum over the leading exclusive modes $B \to K\bar{\ell}\ell, B \to K^*\bar{\ell}\ell, B \to K\pi\bar{\ell}\ell$ (via HHChPT)

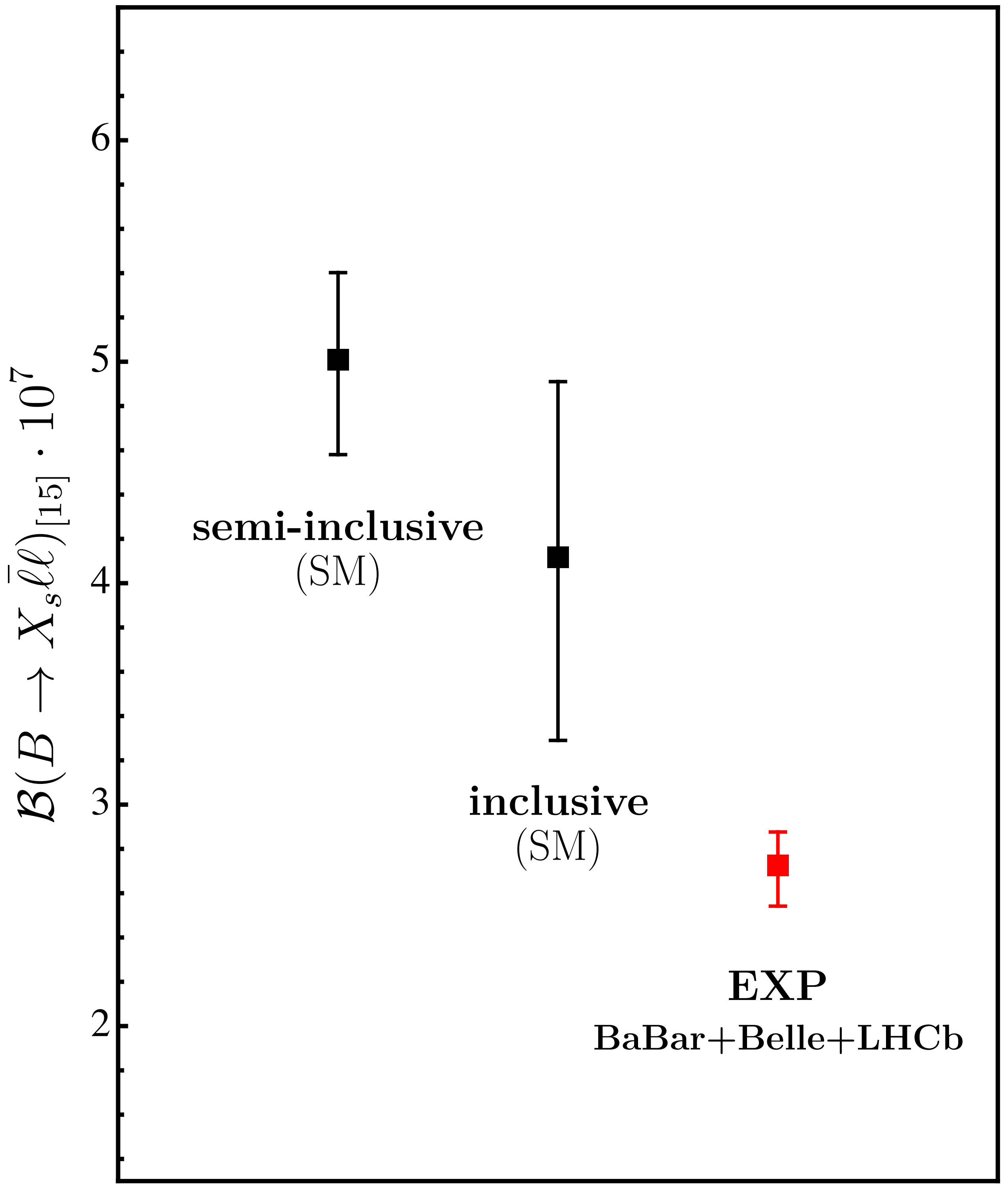
$$\sum_i \mathcal{B}(B \to X_s^i \bar{\ell}\ell)^{SM}_{[15]} = (5.07 \pm 0.42) \times 10^{-7}$$

$$\mathcal{B}(B \to X_s \bar{\ell}\ell)^{SM}_{[15]} = (4.10 \pm 0.81) \times 10^{-7}$$

- This compatibility opens up the possibility of comparing the inclusive SM prediction and a sum-over-exclusive experimental result (from LHCb):

$$B \to K\bar{\ell}\ell = (0.85 \pm 0.05) \times 10^{-7}$$
$$B \to K^*\bar{\ell}\ell = (1.58 \pm 0.35) \times 10^{-7}$$
$$B \to K\pi\bar{\ell}\ell = (0.05 \pm 0.09) \times 10^{-7}$$
$$B \to K\pi\pi\bar{\ell}\ell = (0.06 \pm 0.05) \times 10^{-7}$$
$$B \to K\pi\pi\pi\bar{\ell}\ell = (0.00 \pm 0.04) \times 10^{-7}$$

$$\rightarrow \boxed{\mathcal{B}(B \to X_s \bar{\ell}\ell)^{exp}_{[15]} = (2.65 \pm 0.17) \times 10^{-7}}$$



Modification of C_9 of $O(25\%)$ to explain tension

- **Confirmation of sizable suppression** on the $b \to s\bar{\mu}\mu$ rates at low q^2 compared to SM predictions
- Independent verification **not sensitive to uncertainties on the form factors**
- Sizable uncertainty but mainly **experimental** on $B \to X_u \ell \bar{\nu}$

Exclusive modes $B \to K^{(*)}\bar{\ell}\ell$

- The non-local form factors contain the matrix elements of the **four-quark operators** $\mathcal{O}_{1-6}$.

- Our goal is to extract information on these matrix elements from data.

- Note that to all orders in α_s, and to first order in α_{em}, **these matrix elements have the same structure as the matrix elements of $\mathcal{O}_7$ and $\mathcal{O}_9$:**

$$\mathcal{M}(B \to H_\lambda \ell\ell)|_{C_{1-6}} = -i\frac{32\pi^2\mathcal{N}}{q^2}\bar{\ell}\gamma^\mu\ell\int d^4x e^{iqx}\langle H_\lambda | T\{j_\mu^{\text{em}}(x), \sum_{i=1,6} C_i\mathcal{O}_i(0)\} | B\rangle = \left(\Delta_9^\lambda(q^2) + \frac{m_B^2}{q^2}\Delta_7^\lambda\right)\langle H_\lambda\, \ell^+\ell^- | \mathcal{O}_9 | B\rangle$$

- The (regular for $q^2 \to 0$) contributions of the non-local matrix elements of the four-quark operators can be effectively taken into account by a **shift in C_9**:

$$C_9 \to C_9 + Y^\lambda(q^2) \qquad \lambda = K, \perp, //, 0$$

Effective shift in C_9

- **More precisely, the shift includes:**

$$C_9 \to C_9^\lambda(q^2) + Y_{q\bar{q}}^{[0]}(q^2) + Y_{b\bar{b}}^{[0]}(q^2) + Y_{c\bar{c}}^\lambda(q^2)$$

encodes (factorizable) perturbative contributions from 4-quark operators

encodes the perturbative charm-loop contributions and $c\bar{c}$ resonances

To estimate the non-perturbative contributions generated by the $c\bar{c}$ resonances, we use dispersive relations in combination with data:

$$Y_{c\bar{c}}^\lambda(q^2) = Y_{c\bar{c}}^\lambda(q_0^2) + \frac{16\pi^2}{\mathcal{F}_\lambda(q^2)} \Delta\mathcal{H}_{c\bar{c}}^\lambda(q^2),\ q_0^2 = 0$$

$$\Delta\mathcal{H}_{c\bar{c}}^{\lambda,1P} = \sum_V \eta_V^\lambda e^{i\delta_V^\lambda} \frac{q^2}{m_V^2} A_V^{\text{res}}(q^2) \qquad A_V^{\text{res}}(q^2) = \frac{m_V\Gamma_V}{m_V^2 - q^2 - i m_V \Gamma_V}$$

- We extract the residual contribution to C_9 from data:

$$C_9^\lambda(q^2) = C_9^{\text{SM}} + C_9^{\text{LD},\lambda}(q^2) + C_9^{\text{SD}}$$

Long-distance, no reason to assume it is independent of λ or q^2

Short-distance, independent of λ and q^2

Can we find this contribution from data?

Fit from data for every bin in q^2 and every polarization from LHCb+CMS data

2014 LHCb, 2023 CMS

2016 and 2020 LHCb

[M. Bordone, G.Isidori, S. Mächler, AT, 2401.18007]

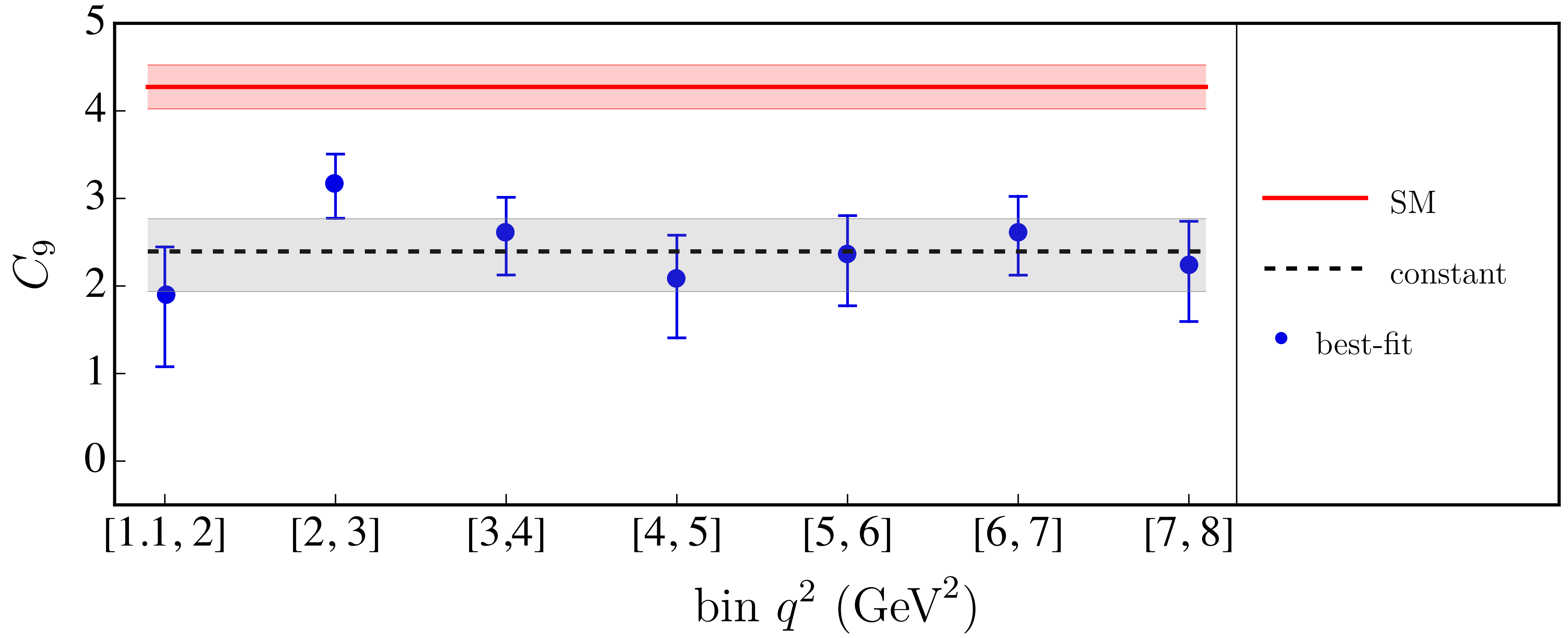
Results of the fit for C_9

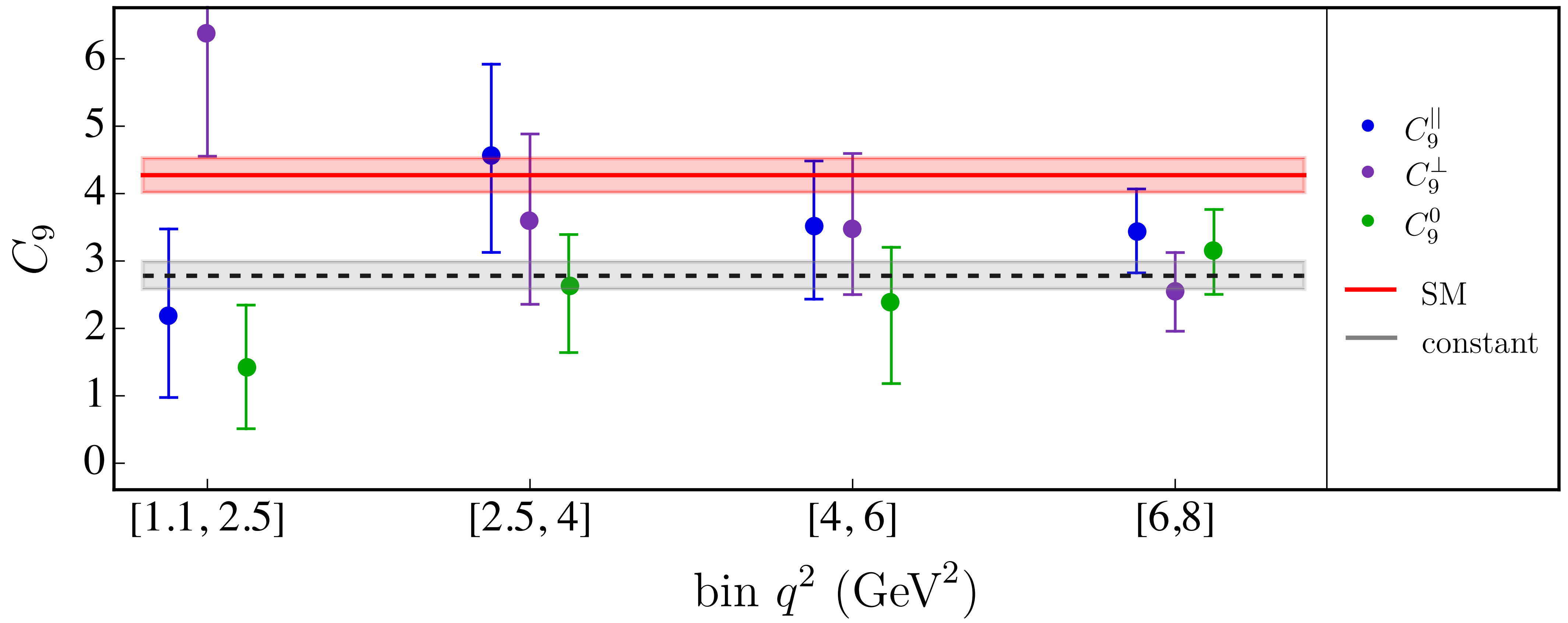
Compatible with recent LHCb analysis!

$B \to K\bar{\ell}\ell$

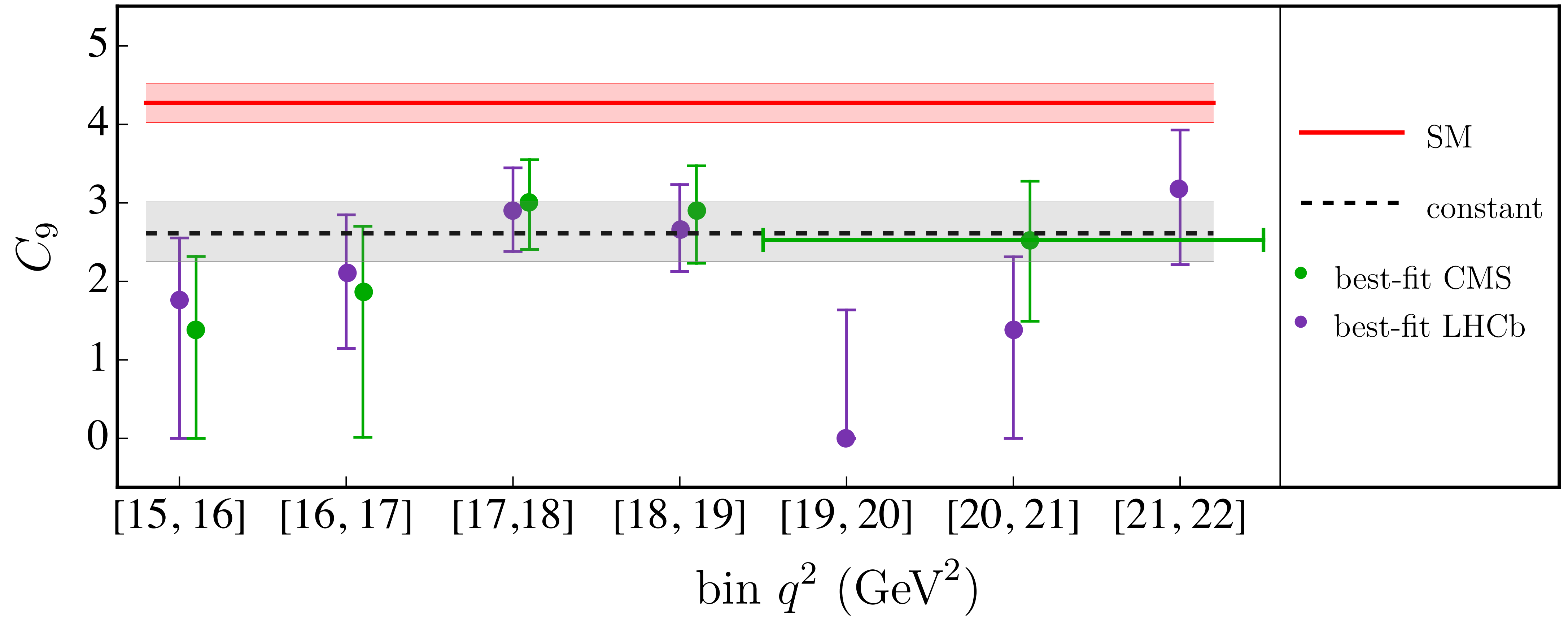
$B \to K^*\bar{\ell}\ell$

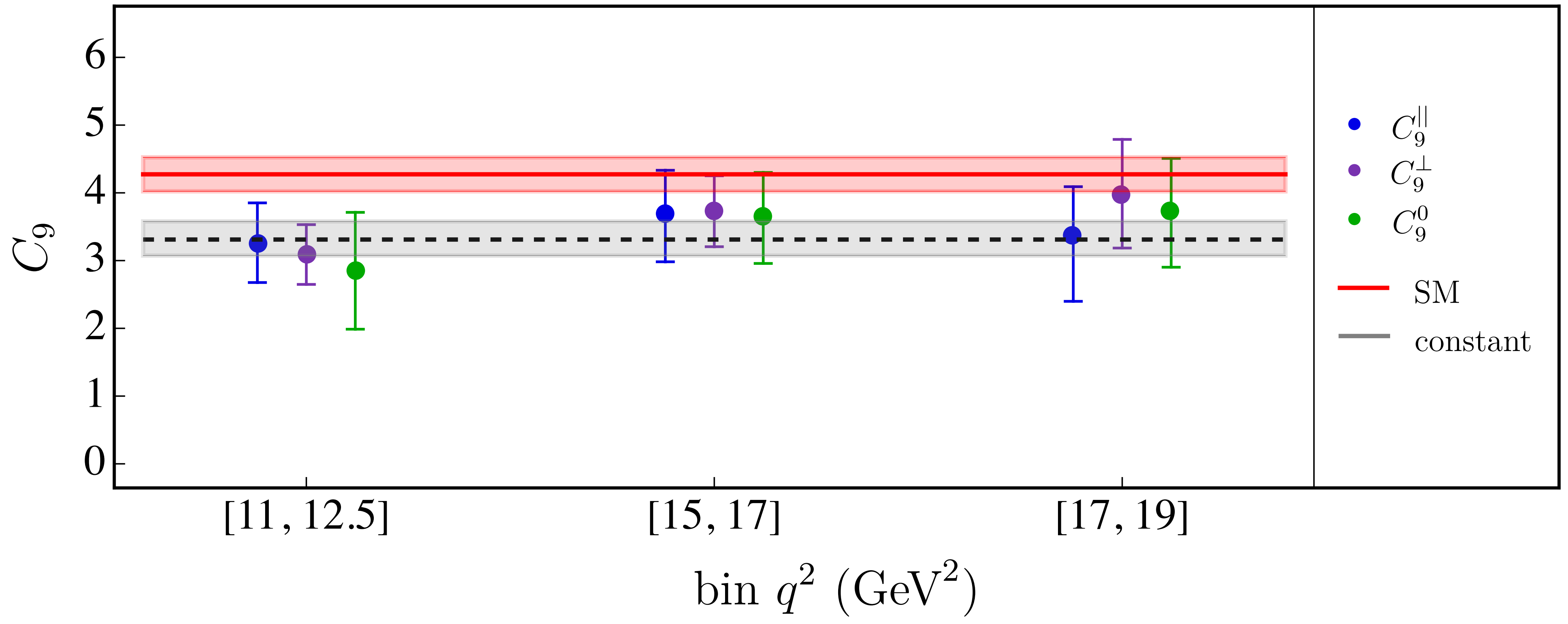
Low q^2



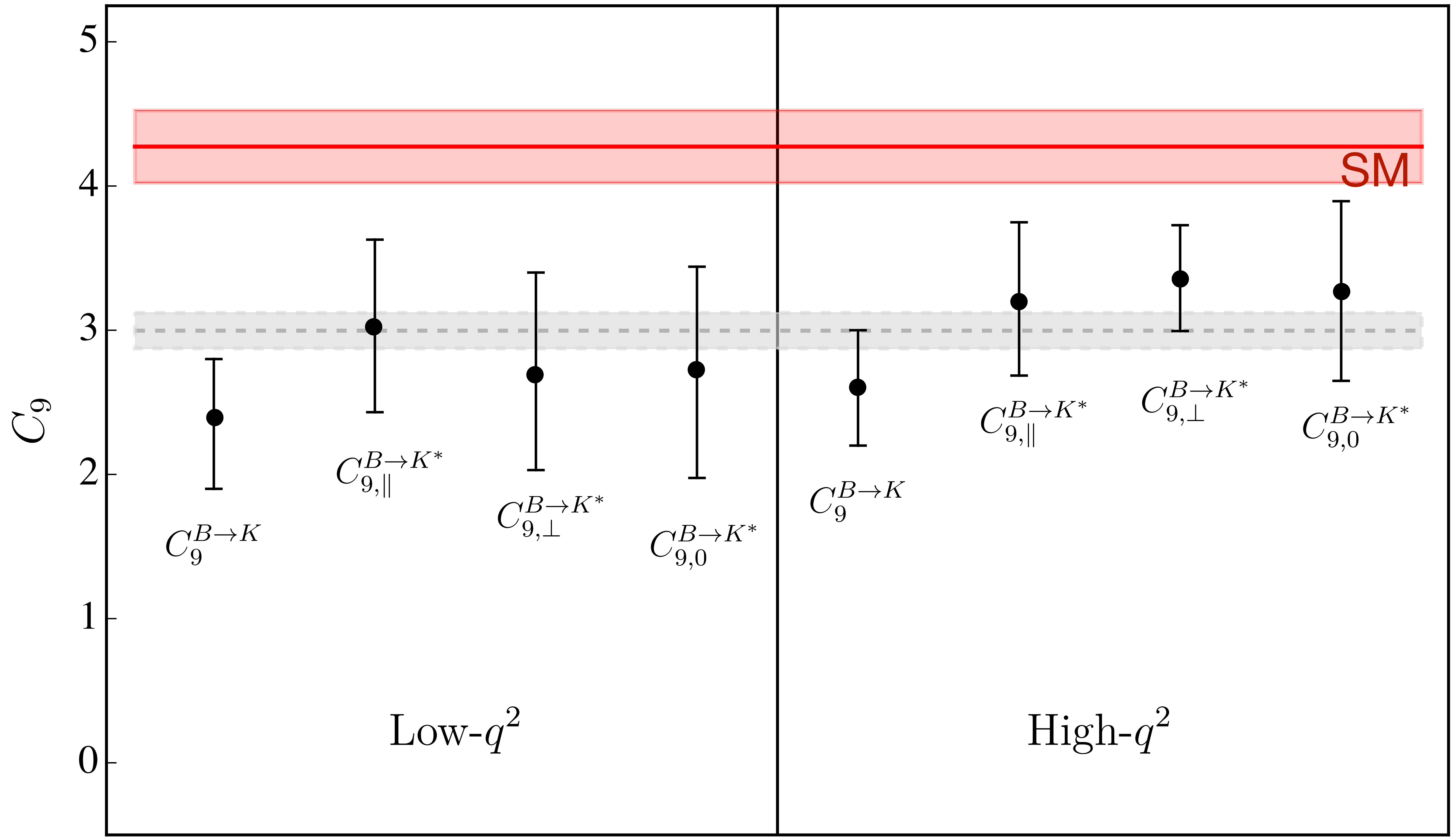


High q^2





Independent determinations of C_9

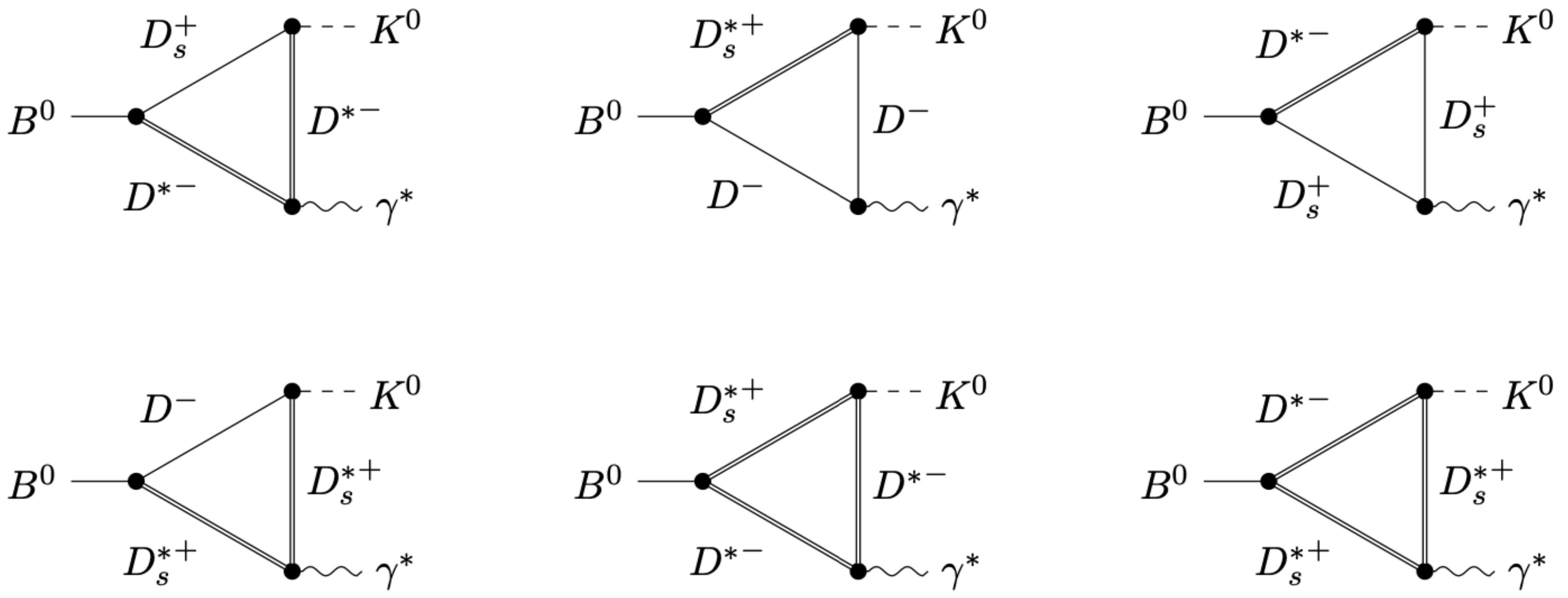


- We find that the values of C_9 are **consistent** throughout the different modes and polarizations, and that there is no significant q^2-dependence.
- Opposition to the expected behavior in the case of long-distance contributions beyond those already included.

Charm rescattering in $B^0 \to K^0 \bar{\ell}\ell$

[G.Isidori, Z. Polonsky, AT, 2405.17551]

- We cannot exclude a sizable long-distance contribution with a **reduced** q^2**- or** λ**-dependence** which would mimic a short-distance effect.
- For this reason, we try to estimate the simplest rescattering contribution from the leading two-body intermediate state $D_s D^*$ and $D_s^* D$.



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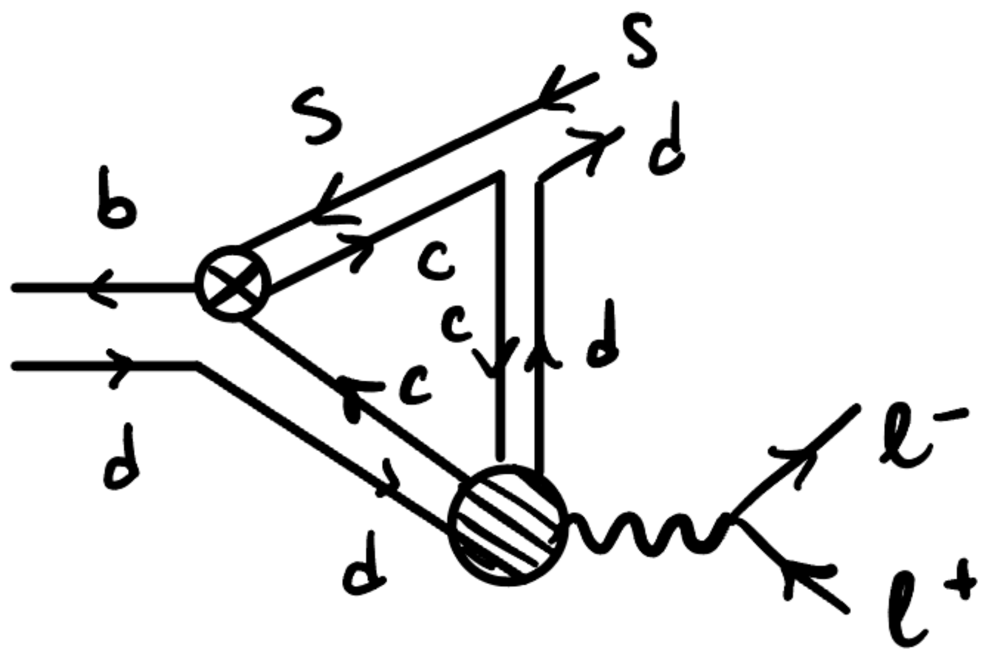
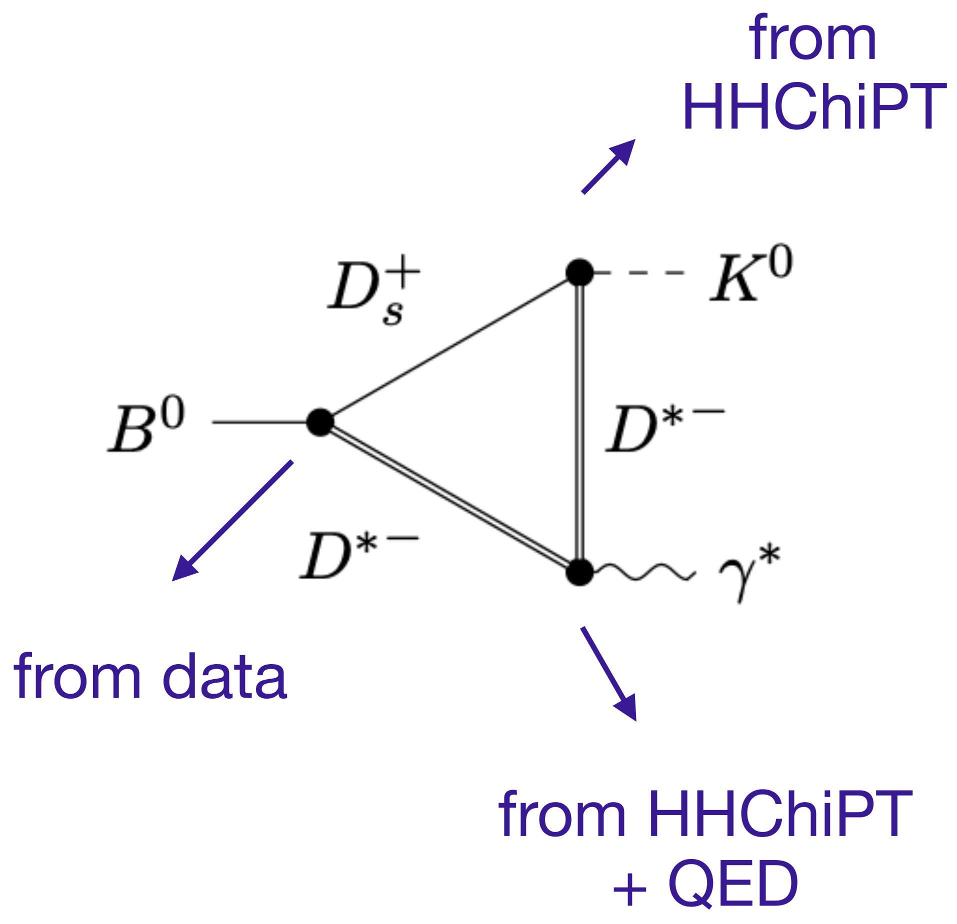
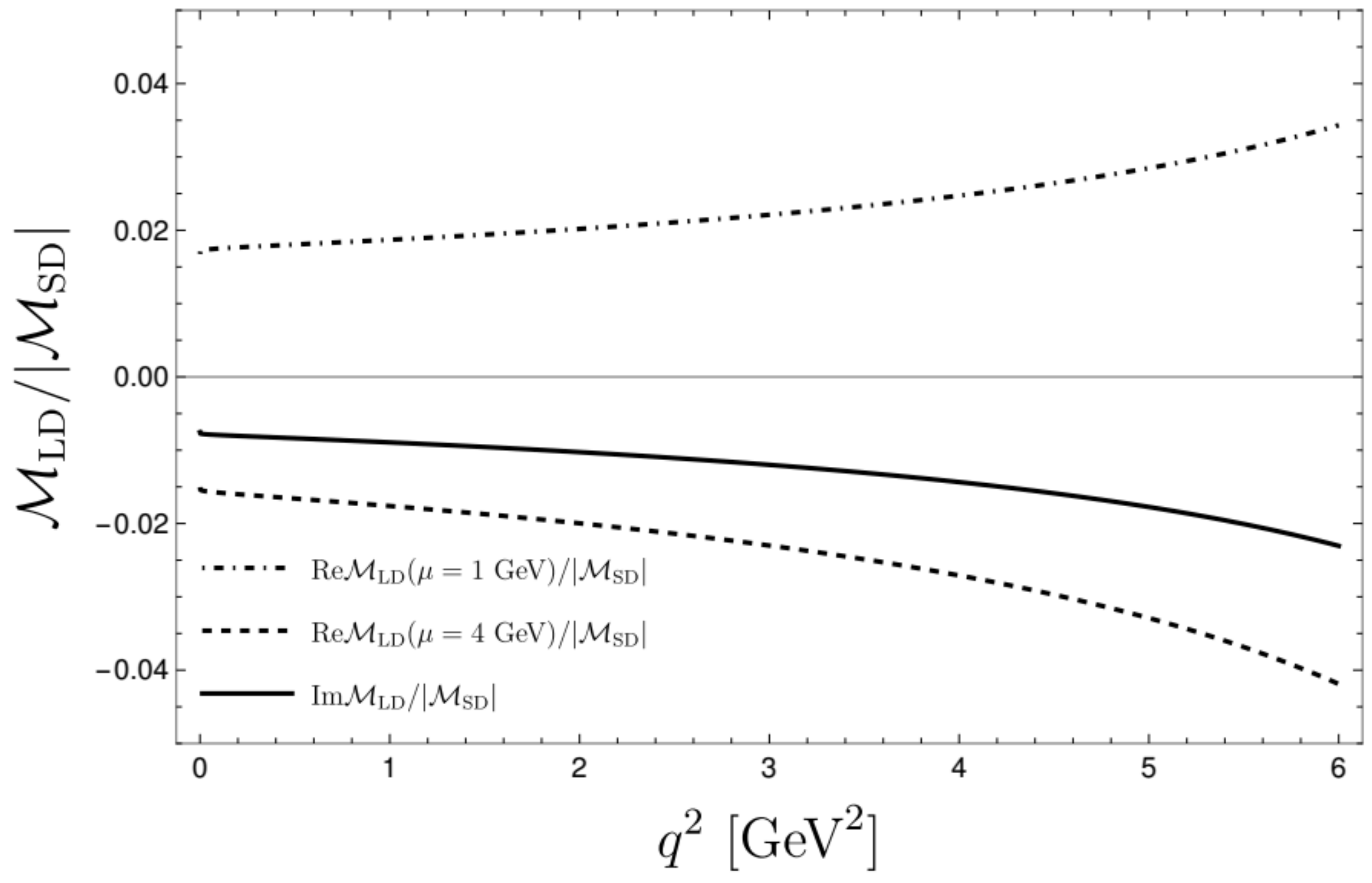
Charm rescattering in $B^0 \to K^0 \bar{\ell}\ell$

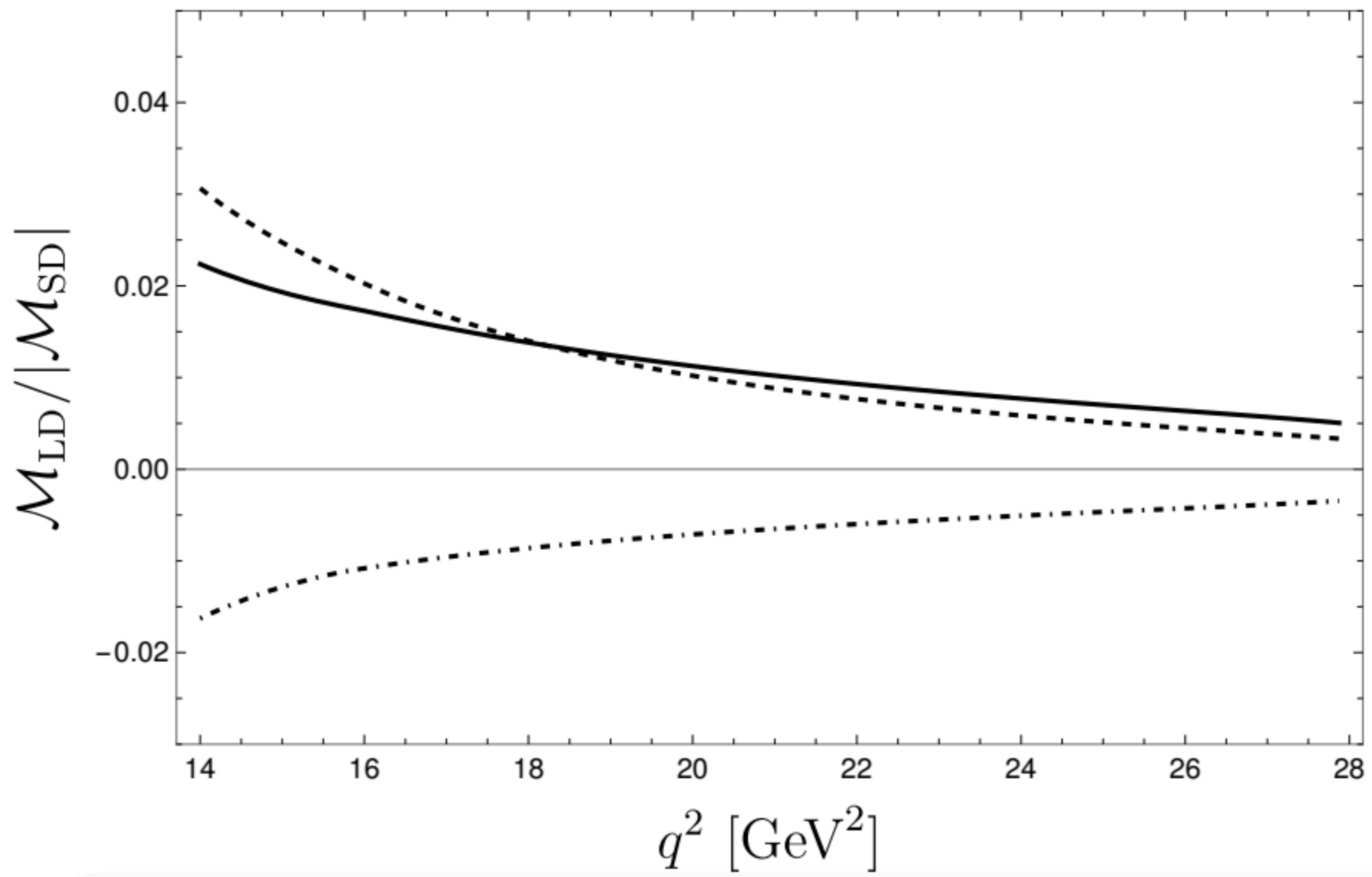




- We estimate this diagram using data on $B \to DD^*$ and Heavy Hadron Chiral Perturbation Theory (valid for soft kaons).
- Impose $SU(3)$-light flavor symmetry + heavy quark spin symmetry.
- Our result is **most reliable close to the q^2 end-point** (small kaon momentum), and satisfies constraints from gauge invariance.
- The absorptive part is finite and "exact" (no approximations) at the end-point.
- Add form factors for the $DD\gamma$ vertex and for the DDK vertex to extrapolate to the whole kinematical region

Charm rescattering in $B \to K\bar{\ell}\ell$





- Relatively **flat** in q^2 (far from narrow resonances)
- We find that these contributions are **not large enough** to explain the bulk of the tension on the value of C_9.

$$\left|\frac{\Delta C_9}{C_9}\right| \leq 3\,\%$$

$$\mathcal{N} = \frac{\sum_X \mathcal{B}(B^0 \to X)}{\mathcal{B}(B^0 \to D^* D_s) + \mathcal{B}(B^0 \to D D_s^*)} \approx 3$$

B^0 Decay	$\mathcal{B}(B^0 \to X) \times 10^3$
$D^* D_s$	8.0 ± 1.1
$D D_s^*$	7.4 ± 1.6
$D^* D_s^*$	17.7 ± 1.4
$D D_{s0}(2317)$	1.06 ± 1.6
$D^* D_{s1}(2457)$	9.3 ± 2.2
$D^* D_{s1}(2536)$	0.50 ± 0.14
$D D_{s2}(2573)$	$(3.4 \pm 1.8) \times 10^{-2}$
$D^* D_{s2}(2573)$	< 0.2
$D D_{s1}(2700)$	0.71 ± 0.12

- Unaccounted-for LD contributions are unlikely to exceed $8 - 10\,\%$ of the SD contribution to C_9

- Not enough to explain the tension with the SM value (the shift needed is of order $\approx 25\,\%$)

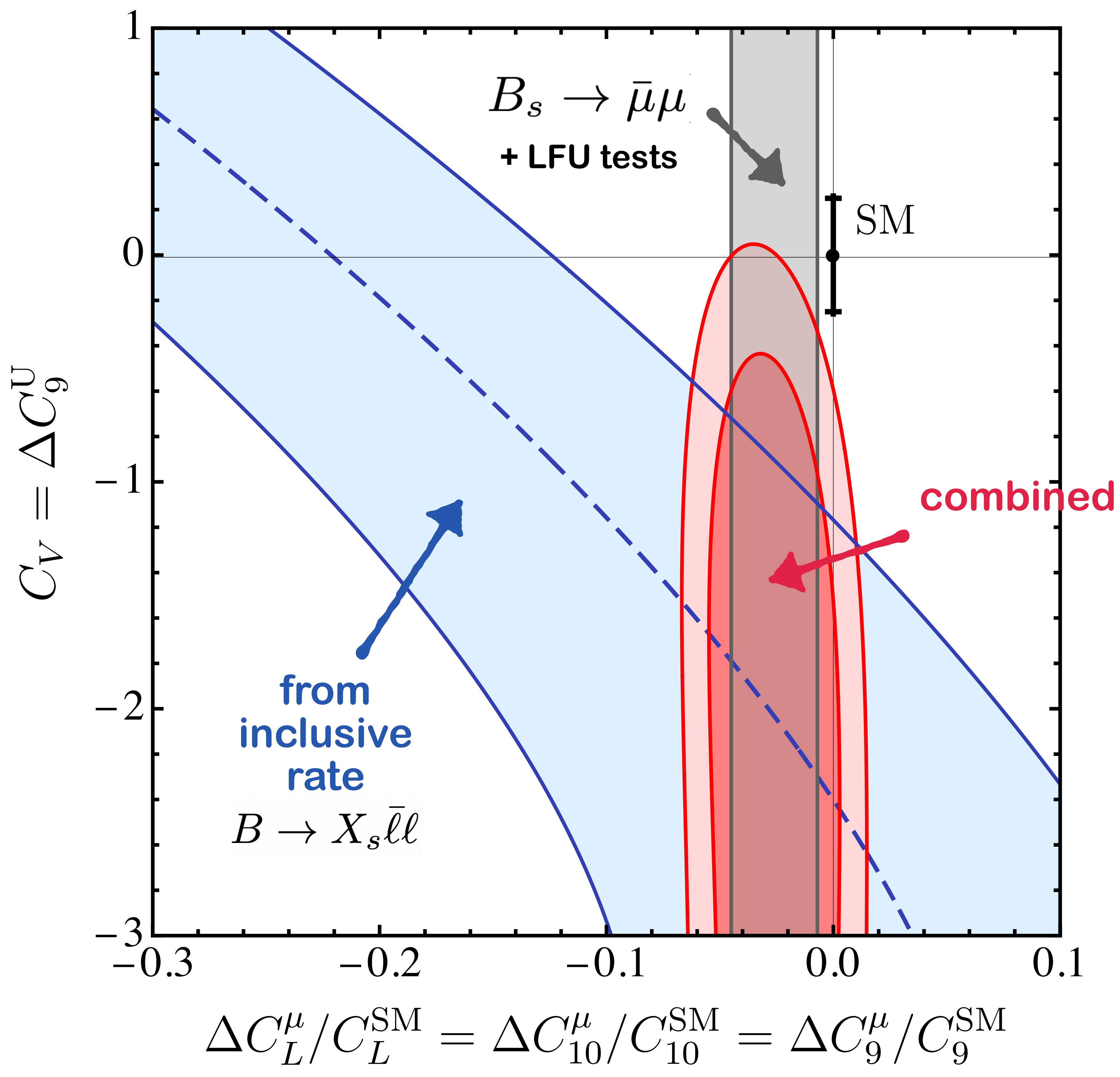
Conclusions

- Hard to explain the bulk of the tension in $b \to s\bar{\ell}\ell$ observables with **only long-distance** QCD effects.
- **<u>At the exclusive level</u>**, data provide no evidence of sizable unaccounted-for long-distance contributions (as we observe no dependence on mode, polarization or q^2), and our estimate of charm-rescattering contributions that mimic short-distance effects cannot explain all the tension.
- **<u>At the inclusive level at high-q^2,</u>** which has a different sensitivity to non-perturbative effects associated with charm-rescattering and is insensitive to local form factors uncertainties, we still observe this discrepancy (compatible with results from exclusive)

Thanks for your attention!

Backup

Comparison with data in the inclusive rate



$C_V = C_9 + C_{10}$ $\quad C_L = -2C_{10}$

- Fit of C_V, C_L from SM prediction on inclusive rate to experimental semi-inclusive determination
- Perturbative and non-perturbative corrections due to charm-rescattering can be accounted for via a modification of C_V
- If $C_L = C_L^{SM}$, C_V needs a large correction ($\sim 25\,\%$) to explain the data, and it is unlikely that charm re-scattering effects are so large in the high-q^2 region
- Modification of both C_V and C_L could explain well the data ➞ possible small LFU-violating amplitude (assuming LF non-universal modification to C_L)
- SM point not included within 2σ

Back-up

$$\mathcal{M}\left(B \to K\ell^+\ell^-\right)\big|_{C_{7,9}} = 2\mathcal{N}\left[C_9\langle K|\bar{s}_L\gamma_\mu b_L|B\rangle - \frac{2m_b}{q^2}C_7\langle K|\bar{s}_L i\sigma_{\mu\nu}q^\nu b_R|B\rangle\right]\ell\gamma^\mu\ell$$
$$= \mathcal{N}\,C_9\left[f_+(q^2)(p_B+p_K)^\mu + f_-(q^2)q^\mu\right]\ell\gamma^\mu\ell$$
$$+\mathcal{N}\,C_7\,\frac{f_T(q^2)}{(m_B+m_K)}\left[q^2(p_B+p_K)^\mu - (m_B^2-m_K^2)q^\mu\right]\left(\frac{2m_b}{q^2}\right)\ell\gamma^\mu\ell \tag{2.5}$$

and

$$\mathcal{M}\left(B \to K^*\ell^+\ell^-\right)\big|_{C_{7,9}} = \mathcal{N}\,C_9\Bigg[-2i\epsilon_{\mu\nu\rho\sigma}(\epsilon^*)^\nu p_B^\rho p_{K^*}^\sigma\frac{V(q^2)}{m_B+m_{K^*}}$$
$$+q_\mu\left(\epsilon^*\cdot q\right)\frac{2m_{K^*}}{q^2}A_0(q^2) + \left(\epsilon^*_\mu - q_\mu\frac{\epsilon^*\cdot q}{q^2}\right)\left(m_B+m_{K^*}\right)A_1(q^2)$$
$$-\left((p_B+p_{K^*})_\mu - q_\mu\frac{m_B^2-m_{K^*}^2}{q^2}\right)\frac{\epsilon^*\cdot q}{m_B+m_{K^*}}A_2(q^2)\Bigg]\bar{\ell}\gamma^\mu\ell$$
$$+\mathcal{N}\,C_7\Bigg[-2i\epsilon_{\mu\nu\rho\sigma}(\epsilon^*)^\nu p_B^\rho p_{K^*}^\sigma T_1(q^2) + (\epsilon^*\cdot q)\left(q_\mu - \frac{q^2}{m_B^2-m_{K^*}^2}\left(p_B+p_{K^*}\right)_\mu\right)T_3(q^2)$$
$$+\left(\epsilon^*_\mu(m_B^2-m_{K^*}^2) - (\epsilon^*\cdot q)\left(p_B+p_{K^*}\right)_\mu\right)T_2(q^2)\Bigg]\left(\frac{2m_b}{q^2}\right)\ell\gamma^\mu\ell\,, \tag{2.6}$$

where

$$q^\mu = p_B^\mu - p_{K^{(*)}}^\mu\,, \qquad \mathcal{N} = \sqrt{2}G_{\rm F}\alpha_{\rm em}V_{tb}V_{ts}^*/(4\pi)\,, \tag{2.7}$$

Back-up

$$\mathcal{M}\left(B \to K \ell^+ \ell^-\right)\big|_{C_{7,9}} = \mathcal{N}\left[C_9 + \frac{2m_b}{m_B + m_K}\frac{f_T(q^2)}{f_+(q^2)}C_7\right] f_+(q^2)(p_B + p_K)_\mu\, \bar{\ell}\gamma^\mu \ell$$

$$\begin{aligned}\mathcal{M}\left(B \to K^* \ell^+ \ell^-\right)\big|_{C_{7,9}} = \mathcal{N}\Bigg\{ \\ & - \left[C_9 + \frac{2m_b(m_B + m_{K^*})}{q^2}\frac{T_1(q^2)}{V(q^2)}C_7\right] \frac{2V(q^2)}{m_B + m_{K^*}} i\epsilon_{\mu\nu\rho\sigma}(\epsilon^*)^\nu p_B^\rho p_{K^*}^\sigma \\ & - \left[C_9 + \frac{2m_b(m_B + m_{K^*})}{q^2}\frac{T_2(q^2)}{A_2(q^2)}C_7\left(1 + O\left(\frac{q^2}{m_B^2}\right)\right)\right] \frac{A_2(q^2)}{m_B + m_{K^*}}(\epsilon^* \cdot q)(p_B + p_{K^*})_\mu \\ & + \left[C_9 + \frac{2m_b(m_B^2 - m_{K^*})}{q^2}\frac{T_2(q^2)}{A_1(q^2)}C_7\right] A_1(q^2)\,(m_B + m_{K^*})\,\epsilon_\mu^* \Bigg\} \bar{\ell}\gamma^\mu \ell\,, \qquad (2.8)\end{aligned}$$

Back-up

<table>
<tr><th>q^2 region</th><th>Amplitude</th><th colspan="4">C_9 values</th></tr>
<tr><td rowspan="4">Low q^2</td><td>$B \to K$</td><td colspan="2">$2.4^{+0.4}_{-0.5}$</td><td rowspan="4">$2.7^{+0.2}_{-0.2}$
(χ^2/dof=1.27 (0.10))</td><td rowspan="8">$3.1^{+0.1}_{-0.1}$
(χ^2/dof = 1.33 (0.02))</td></tr>
<tr><td>$B \to K^*(\epsilon_\parallel)$</td><td>$3.1^{+0.6}_{-0.6}$</td><td rowspan="3">$2.8^{+0.2}_{-0.2}$</td></tr>
<tr><td>$B \to K^*(\epsilon_\perp)$</td><td>$2.8^{+0.7}_{-0.7}$</td></tr>
<tr><td>$B \to K^*(\epsilon_0)$</td><td>$2.7^{+0.7}_{-0.8}$</td></tr>
<tr><td rowspan="4">High q^2</td><td>$B \to K$</td><td colspan="2">$2.6^{+0.4}_{-0.4}$</td><td rowspan="4">$3.1^{+0.2}_{-0.2}$
(χ^2/dof=1.04 (0.40))</td></tr>
<tr><td>$B \to K^*(\epsilon_\parallel)$</td><td>$3.3^{+0.5}_{-0.5}$</td><td rowspan="3">$3.4^{+0.3}_{-0.3}$</td></tr>
<tr><td>$B \to K^*(\epsilon_\perp)$</td><td>$3.5^{+0.4}_{-0.4}$</td></tr>
<tr><td>$B \to K^*(\epsilon_0)$</td><td>$3.5^{+0.6}_{-0.6}$</td></tr>
</table>

Table 4.1: Best-fit points assuming constant C_9 values in the low- and high-q^2 regions, separating or combining the different decay amplitudes, or considering the same value over the full q^2 spectrum for all the decay amplitudes (last column).

Back-up

$$Y^{\lambda}(q^2)\big|_{\alpha_s^0} = Y^{[0]}_{\mathrm{q\bar{q}}}(q^2) + Y^{[0]}_{\mathrm{c\bar{c}}}(q^2) + Y^{[0]}_{\mathrm{b\bar{b}}}(q^2)\,, \tag{2.14}$$

where

$$\begin{aligned}
Y^{[0]}_{\mathrm{q\bar{q}}}(q^2) &= \frac{4}{3}C_3 + \frac{64}{9}C_5 + \frac{64}{27}C_6 - \frac{1}{2}h(q^2,0)\left(C_3 + \frac{4}{3}C_4 + 16C_5 + \frac{64}{3}C_6\right)\,,\\
Y^{[0]}_{\mathrm{c\bar{c}}}(q^2) &= h(q^2,m_c)\left(\frac{4}{3}C_1 + C_2 + 6C_3 + 60C_5\right)\,,\\
Y^{[0]}_{\mathrm{b\bar{b}}}(q^2) &= -\frac{1}{2}h(q^2,m_b)\left(7C_3 + \frac{4}{3}C_4 + 76C_5 + \frac{64}{3}C_6\right)\,,
\end{aligned}$$

with

$$h(q^2,m) = -\frac{4}{9}\left(\ln\frac{m^2}{\mu^2} - \frac{2}{3} - x\right) - \frac{4}{9}(2+x)\begin{cases}\sqrt{x-1}\,\arctan\frac{1}{\sqrt{x-1}}\,, & x = \frac{4m^2}{q^2} > 1\,,\\ \sqrt{1-x}\left(\ln\frac{1+\sqrt{1-x}}{\sqrt{x}} - \frac{i\pi}{2}\right), & x = \frac{4m^2}{q^2} \le 1\,.\end{cases}$$

Back-up

$$\mathcal{O}_1 = \frac{4\pi}{\alpha_e}(\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L) \qquad \mathcal{O}_2 = \frac{4\pi}{\alpha_e}(\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)$$

$$\mathcal{O}_3 = \frac{4\pi}{\alpha_e}(\bar{s}_L\gamma_\mu b_L)\sum_q(\bar{q}_L\gamma^\mu q_L) \qquad \mathcal{O}_4 = \frac{4\pi}{\alpha_e}(\bar{s}^a_L\gamma^\mu T^a b^b_L)\sum_q(\bar{q}_L{}^b\gamma^\mu T^a q^a_L)$$

$$\mathcal{O}_5 = \frac{4\pi}{\alpha_e}(\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho b_L)\sum_q(\bar{q}_L\gamma^\mu\gamma^\nu\gamma^\rho q_R) \qquad \mathcal{O}_6 = \frac{4\pi}{\alpha_e}(\bar{s}^a_L\gamma_\mu\gamma_\nu\gamma_\rho T^a b^b_L)\sum_q(\bar{q}_L{}^b\gamma^\mu\gamma^\nu\gamma^\rho T^a q^a_R)$$

$$\mathcal{O}_7 = \frac{m_b}{e}(\bar{s}_L\sigma^{\mu\nu}b_R)F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{e^2}m_b(\bar{s}_L\sigma^{\mu\nu}T^a b_R)G^a_{\mu\nu}$$

$$\mathcal{O}_9 = (\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\ell) \qquad \mathcal{O}_{10} = (\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\gamma_5\ell)$$

$$\mathrm{Re}[C_V^{\mathrm{eff}}(q^2 = 15\ \mathrm{GeV}^2)] = 0.43 \pm 0.26\ ,$$
$$\mathrm{Re}[C_V^{\mathrm{eff}}(q^2 = 1\ \mathrm{GeV}^2)] = 0.13 \pm 0.13\ .$$

$$\mathcal{C}_L = \frac{4G_F}{\sqrt{2}}\frac{\alpha_e}{4\pi}C_L\ .$$

The corresponding one-loop expression is

$$\mathcal{C}_L^{(0)} = \frac{2G_F^2 m_W^2}{\pi^2}Y_0(x_t) = \frac{y_t^2}{16\pi^2 v^2}\left[1 + O(g^2/y_t^2)\right]$$

$$C_V^{(0)}(\mu) = -4Z_0(x_t) + \frac{4}{9} - \frac{4}{9}\ln\left(\frac{\mu^2}{M_W^2}\right) \approx 0.22 - 0.89 \times \ln\left(\frac{\mu^2}{m_b^2}\right)\ ,$$

Back-up

SM prediction for the inclusive rate

$$\mathcal{B}(B \to X_s \bar{\ell}\ell)^{\rm SM}_{[15]} = (4.5 \pm 1.0) \times 10^{-7}$$
$$= 4.5 \times 10^{-7} \left[1 \pm 0.16_{\rm exp} \pm 0.11_{\rm CKM} \pm 0.09_{\Delta\mathcal{R}}\right]$$

Sum of the SM predictions for the leading exclusive modes

$$\mathcal{B}(B \to K \bar{\ell}\ell)^{\rm SM}_{[15]} = \left(1.31 \pm 0.08_{\rm lat} \pm 0.09_{\rm par}\right) \times 10^{-7}$$
$$\mathcal{B}(B \to K^* \bar{\ell}\ell)^{\rm SM}_{[15]} = \left(3.19 \pm 0.21_{\rm lat} \pm 0.22_{\rm par}\right) \times 10^{-7}$$
$$\mathcal{B}\big(B \to (K\pi)_s \bar{\ell}\ell\big)^{\rm SM}_{[15]} = (5.8 \pm 2.5) \times 10^{-8}$$

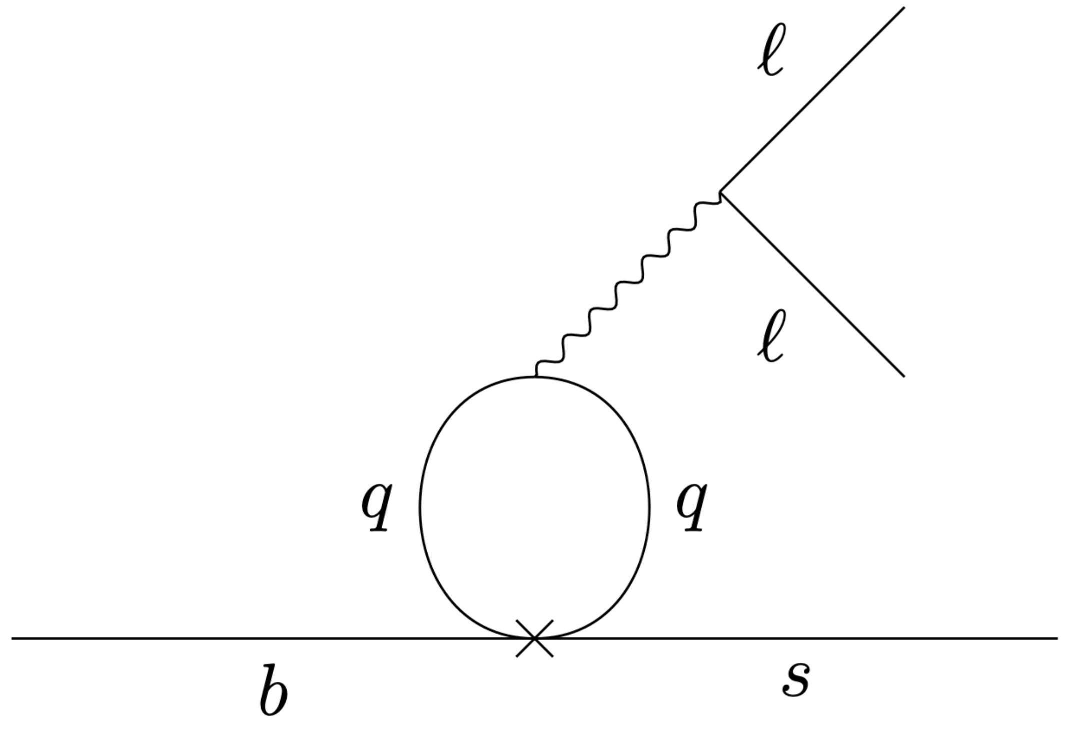
$$\sum_i \mathcal{B}(B \to X_s^i \bar{\ell}\ell)^{\rm SM}_{[15]} = \left(5.07 \pm 0.42\right) \times 10^{-7}$$

Correction factor due to $B \to K\pi\bar{\ell}\ell$

$$\Delta^{[15]}_{K\pi} = \frac{\mathcal{B}(B \to (K\pi)_s \bar{\ell}\ell)_{[15]}}{\mathcal{B}(B \to K\ell)_{[15]} + \mathcal{B}(B \to K^*\ell\ell)_{[15]}} = 0.13 \pm 0.06$$

Back-up

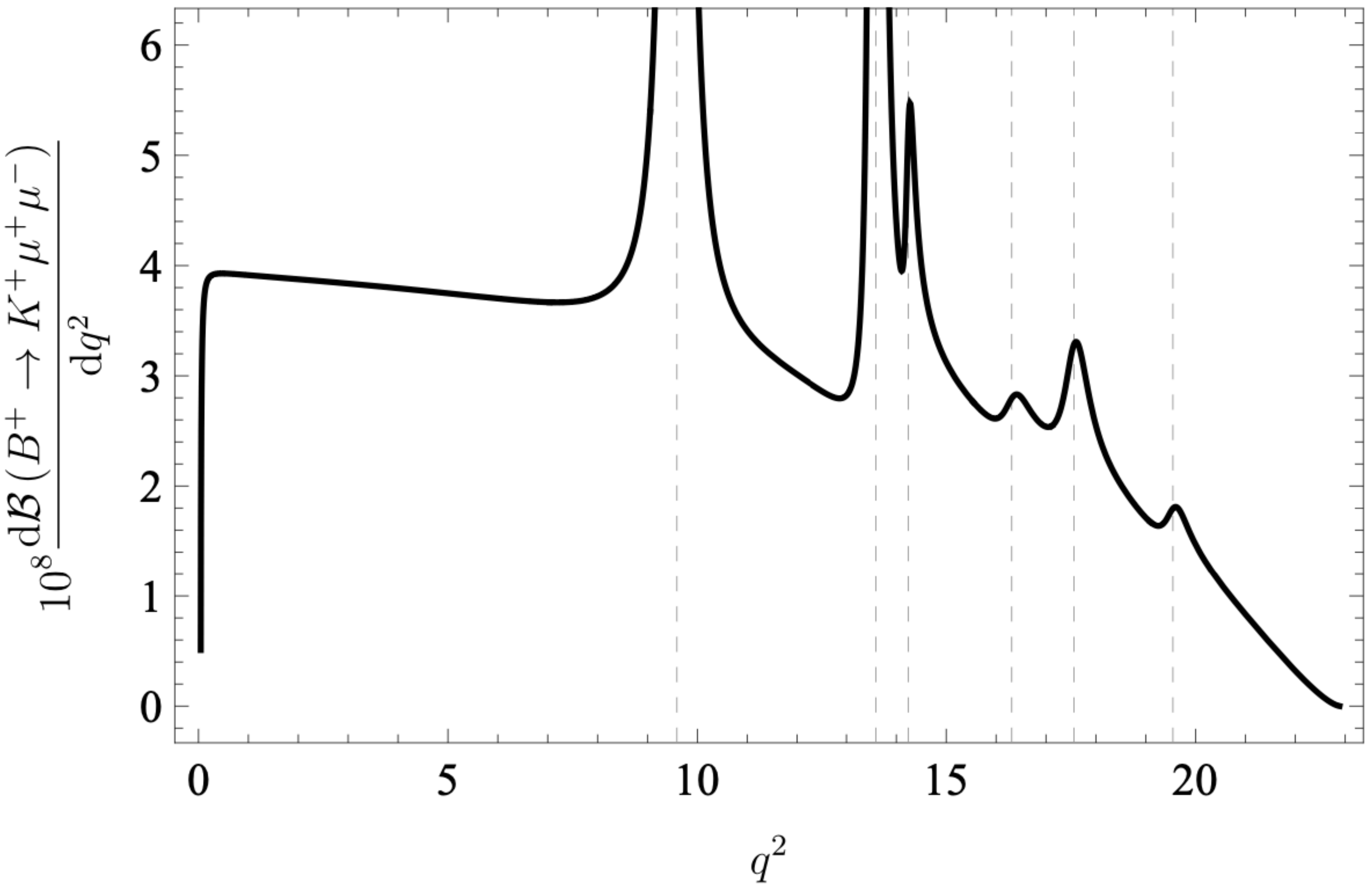
V	η_V	δ_V
J/ψ	32.3 ± 0.6	-1.50 ± 0.05
$\psi(2s)$	7.12 ± 0.32	2.08 ± 0.11
$\psi(3770)$	$(1.3 \pm 0.1) \times 10^{-2}$	-2.89 ± 0.19
$\psi(4040)$	$(4.8 \pm 0.8) \times 10^{-3}$	-2.69 ± 0.52
$\psi(4160)$	$(1.5 \pm 0.1) \times 10^{-2}$	-2.13 ± 0.33
$\psi(4415)$	$(1.1 \pm 0.2) \times 10^{-2}$	-2.43 ± 0.43



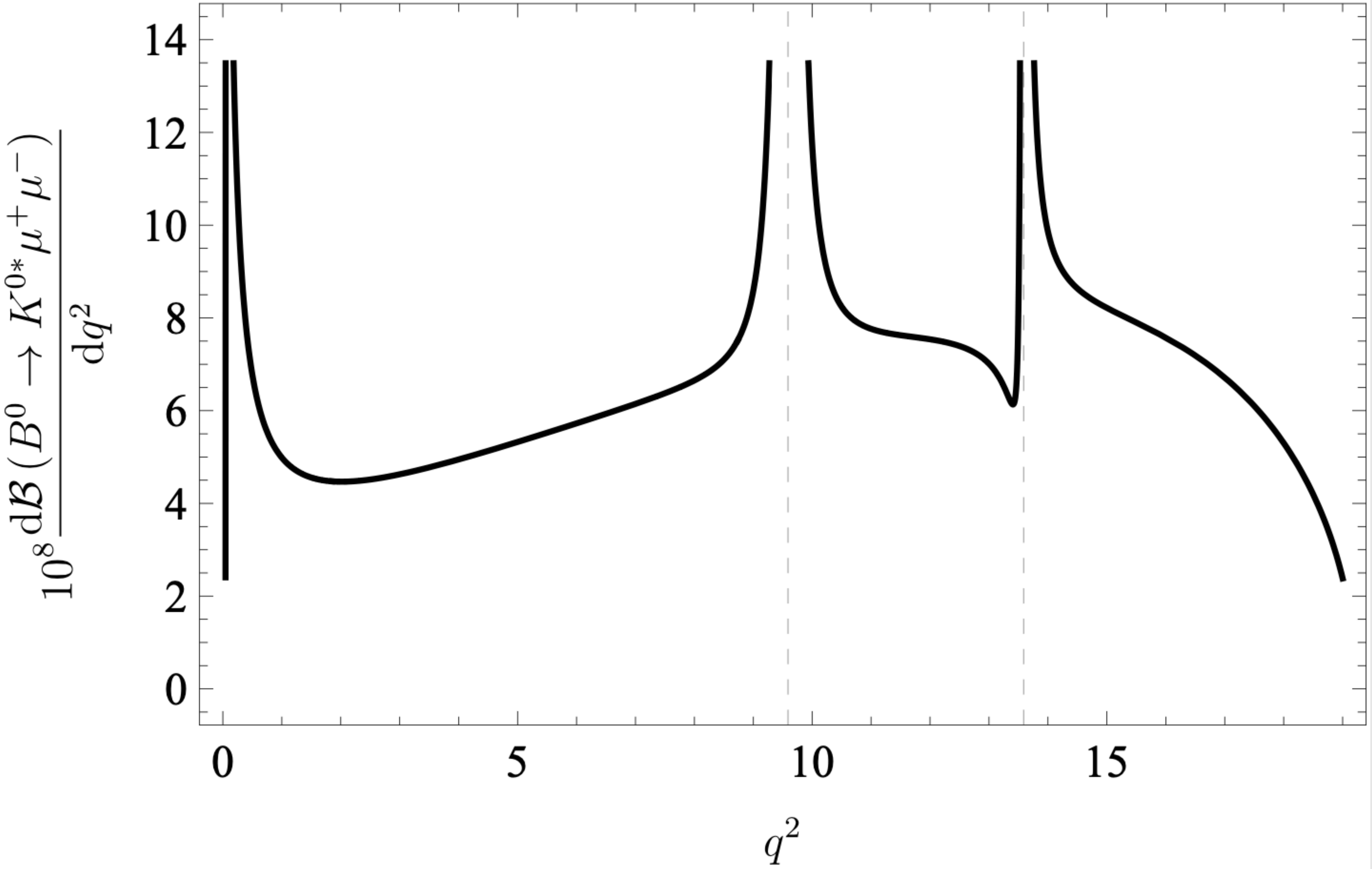
V	Polarization	η_V	δ_V
J/ψ	$\perp$	26.6 ± 1.1	1.46 ± 0.06
	$\parallel$	12.3 ± 0.5	-4.42 ± 0.06
	longitudinal	13.9 ± 0.5	-1.48 ± 0.05
$\psi(2s)$	$\perp$	3.0 ± 0.9	3.2 ± 0.4
	$\parallel$	1.11 ± 0.30	-3.32 ± 0.22
	longitudinal	1.14 ± 0.06	2.10 ± 0.11

$$\frac{\mathrm{d}^4\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\phi} = \frac{9}{32\pi}\Big[I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K + I_2^s \sin^2\theta_K \cos 2\theta_\ell + I_2^c \cos^2\theta_K \cos 2\theta_\ell + I_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + I_5 \sin 2\theta_K \sin\theta_\ell \cos\phi + I_6 \sin^2\theta_K \cos\theta_\ell + I_7 \sin 2\theta_K \sin\theta_\ell \sin\phi + I_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + I_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi \Big] ,$$

Back-up



Back-up



Back-up

q^2 (GeV^2)	C_9^K
[1.1, 2]	$1.9^{+0.5}_{-0.8}$
[2, 3]	$3.2^{+0.3}_{-0.4}$
[3, 4]	$2.6^{+0.4}_{-0.5}$
[4, 5]	$2.1^{+0.5}_{-0.7}$
[5, 6]	$2.4^{+0.4}_{-0.6}$
[6, 7]	$2.6^{+0.4}_{-0.5}$
[7, 8]	$2.3^{+0.5}_{-0.7}$
constant	$2.4^{+0.4}_{-0.5}$ ($\chi^2/\text{dof} = 1.35$)

q^2 (GeV^2)	C_9^K (LHCb)	C_9^K (CMS)
[15, 16]	$1.8^{+0.8}_{-1.8}$	$1.4^{+0.9}_{-1.4}$
[16, 17]	$2.1^{+0.7}_{-1.0}$	$1.9^{+0.8}_{-1.9}$
[17, 18]	$2.9^{+0.5}_{-0.5}$	$3.0^{+0.5}_{-0.6}$
[18, 19]	$2.7^{+0.6}_{-0.5}$	
[18, 19.24]		$2.9^{+0.6}_{-0.7}$
[19, 20]	$0^{+1.6}_{-0}$	
[20, 21]	$1.4^{+0.9}_{-1.4}$	
[21, 22]	$3.2^{+0.8}_{-0.9}$	
[19.24, 22.9]		$2.5^{+0.7}_{-1.0}$
constant	2.6 ± 0.4 ($\chi^2/\text{dof} = 1.06$)	

Table 3.3: Determinations of C_9 from $B \to K\mu^+\mu^-$ in the low-q^2 (left) and high-q^2 (right) regions. The p-values for the constant fits are 0.17 (low-q^2) and 0.39 (high-q^2).

Back-up

q^2 region	Amplitude	C_9 values			
Low q^2	$B \to K$	$2.4^{+0.4}_{-0.5}$		$2.7^{+0.2}_{-0.2}$ (χ^2/dof=1.28, p-value=0.09)	$3.0^{+0.1}_{-0.1}$ (χ^2/dof = 1.33, p-value=0.01)
	$B \to K^*(\epsilon_\parallel)$	$3.0^{+0.6}_{-0.6}$	$2.8^{+0.2}_{-0.2}$		
	$B \to K^*(\epsilon_\perp)$	$2.7^{+0.7}_{-0.7}$			
	$B \to K^*(\epsilon_0)$	$2.7^{+0.7}_{-0.8}$			
High q^2	$B \to K$	$2.6^{+0.4}_{-0.4}$		$3.0^{+0.2}_{-0.2}$ (χ^2/dof=1.06, p-value=0.37)	
	$B \to K^*(\epsilon_\parallel)$	$3.2^{+0.5}_{-0.5}$	$3.3^{+0.3}_{-0.2}$		
	$B \to K^*(\epsilon_\perp)$	$3.4^{+0.4}_{-0.4}$			
	$B \to K^*(\epsilon_0)$	$3.3^{+0.6}_{-0.6}$			

Table 4.1: Best-fit points assuming constant C_9 values in the low- and high-q^2 regions, separating or combining the different decay amplitudes, or considering the same value over the full q^2 spectrum for all the decay amplitudes (last column).

Back-up

q^2 (GeV^2)	$C_9^{\parallel}$	$C_9^{\perp}$	C_9^{0}
$[1.1, 2.5]$	$2.2^{+1.3}_{-1.2}$	$6.4^{+1.7}_{-1.8}$	$1.4^{+0.9}_{-0.9}$
$[2.5, 4]$	$4.6^{+1.4}_{-1.4}$	$3.6^{+1.3}_{-1.2}$	$2.6^{+0.8}_{-1.0}$
$[4, 6]$	$3.5^{+1.0}_{-1.1}$	$3.5^{+1.1}_{-1.0}$	$2.4^{+0.8}_{-1.2}$
$[6, 8]$	$3.4^{+0.6}_{-0.6}$	$2.5^{+0.6}_{-0.6}$	$3.1^{+0.6}_{-0.6}$
constant	$2.8^{+0.2}_{-0.2}$ $(\chi^2/\mathrm{dof} = 1.26)$		

$$\mathcal{L}_{BD} = g_{DD^*}\left(\Phi^{\mu\dagger}_{D_s^*}\,\Phi_D\partial_\mu\Phi_B + \Phi^\dagger_{D_s}\Phi^\mu_{D^*}\partial_\mu\Phi_B\right) + \text{h.c.}$$

$$\mathcal{L}_{DK} = \frac{2ig_\pi m_D}{f_K}\left(\Phi^{\mu\dagger}_{D^*}\Phi_{D_s}\partial_\mu\Phi^\dagger_K - \Phi^\dagger_D\Phi^\mu_{D_s^*}\partial_\mu\Phi^\dagger_K\right) + \text{h.c.}$$

$$\Phi^{\mu\nu}_V = D^\mu\Phi^\nu_V - D^\nu\Phi^\mu_V\,,$$

$$D_\mu\Phi = \partial_\mu\Phi + i\,eA_\mu\Phi\,,$$

q^2 (GeV^2)	[illegible]	[illegible]	[illegible]
$[11, 12.5]$	$3.3^{+0.6}_{-0.6}$	$3.1^{+0.4}$ [illegible]	[illegible]$^{+0.8}_{-0.9}$
$[15, 17]$	$3.7^{+0.6}_{-0.7}$	[illegible]	[illegible]$^{+0.7}_{-0.7}$
$[17, 19]$	$3.4^{+0.7}_{-1.0}$	$4.0^{+0.8}_{-0.8}$	$3.7^{+0.8}_{-0.8}$
constant	$3.3^{+0.3}_{-0.2}$ $(\chi^2/\mathrm{dof} = 0.82)$		

Table 3.4: Determinations of C_9 in different q^2 bins from the different polarizations of the $B \to K^*\mu^+\mu^-$ decay. The p-values for the constant fits are 0.14 (low-q^2) and 0.73 (high-q^2).

Back-up

$$\mathcal{L}_{D,\text{free}} = -\frac{1}{2}\left(\Phi^{\mu\nu}_{D^*}\right)^\dagger \Phi_{D^*\,\mu\nu} - \frac{1}{2}\left(\Phi^{\mu\nu}_{D_s^*}\right)^\dagger \Phi_{D_s^*\,\mu\nu} + \left(D_\mu \Phi_D\right)^\dagger D^\mu \Phi_D + \left(D_\mu \Phi_{D_s}\right)^\dagger D^\mu \Phi_{D_s} + m_D^2\left[\left(\Phi^\mu_{D^*}\right)^\dagger \Phi_{D^*\,\mu} + \left(\Phi^\mu_{D_s^*}\right)^\dagger \Phi_{D_s^*\,\mu}\right] - m_D^2\left[\Phi^\dagger_D \Phi_D + \Phi^\dagger_{D_s}\Phi_{D_s}\right] + \text{h.c.}\,.$$

$$\mathcal{L}_{BD} = g_{DD^*}\left(\Phi^{\mu\dagger}_{D_s^*}\,\Phi_D \partial_\mu \Phi_B + \Phi^\dagger_{D_s}\Phi^\mu_{D^*}\partial_\mu \Phi_B\right) + \text{h.c.}$$

$$\mathcal{L}_{DK} = \frac{2 i g_\pi m_D}{f_K}\left(\Phi^{\mu\dagger}_{D^*}\Phi_{D_s}\partial_\mu \Phi^\dagger_K - \Phi^\dagger_D \Phi^\mu_{D_s^*}\partial_\mu \Phi^\dagger_K\right) + \text{h.c.}$$

$$\Phi^{\mu\nu}_V = D^\mu \Phi^\nu_V - D^\nu \Phi^\mu_V\,,$$

$$D_\mu \Phi = \partial_\mu \Phi + i\,e A_\mu \Phi\,,$$