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Femtoscopy with Lévy-stable sources from SPS through RHIC to LHC

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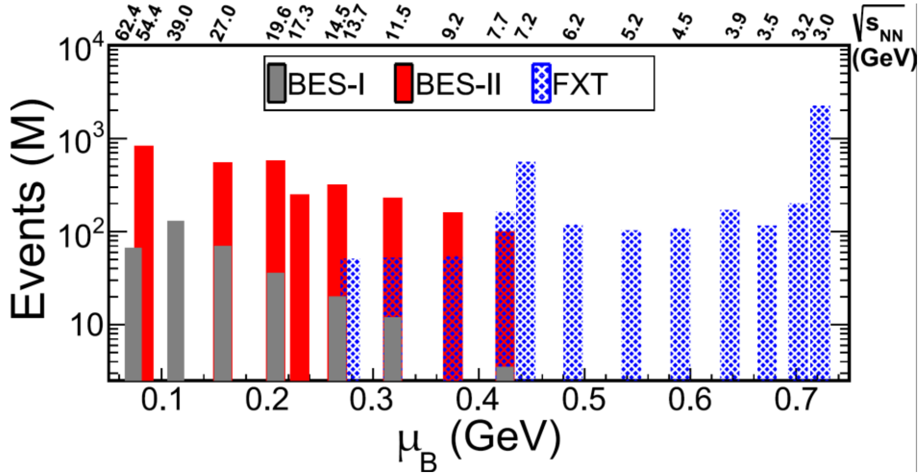
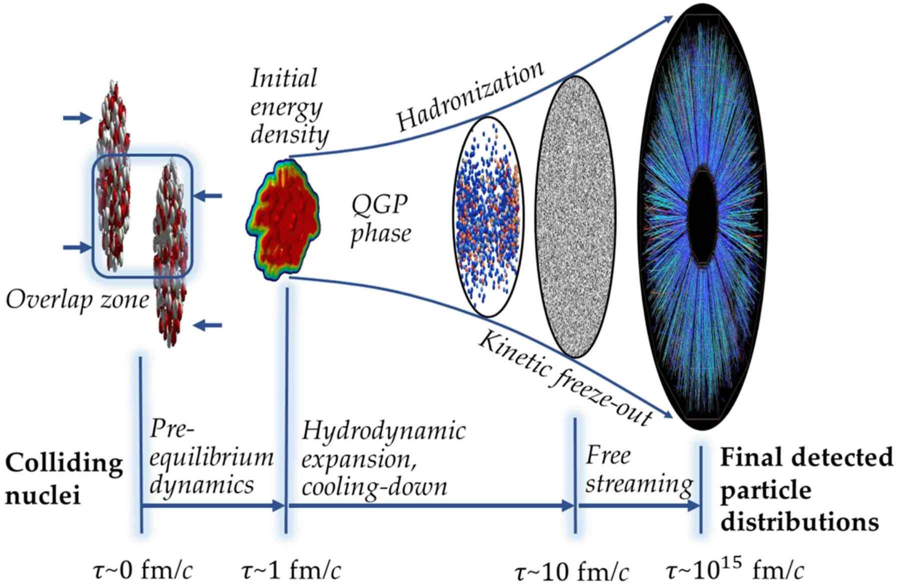
ICHEP 2024
18-24 July 2024, Prague

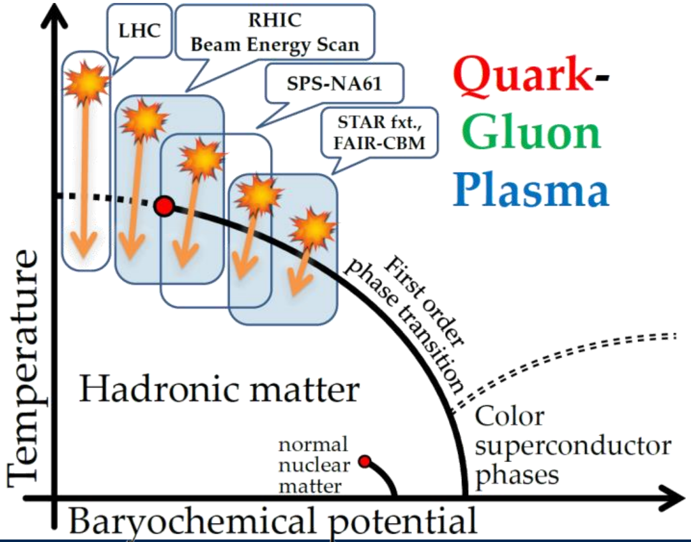
Exploring the properties of quark matter

- Strongly interacting **Quark-Gluon-Plasma (QGP)** discovery: early 2000s at RHIC, confirmed later by LHC
- Since then: exploring the **QCD phase diagram** with ongoing extensive research programs
- Many interesting measurements; focus of this talk: **femtoscopic correlations**

RHIC Beam Energy Scan

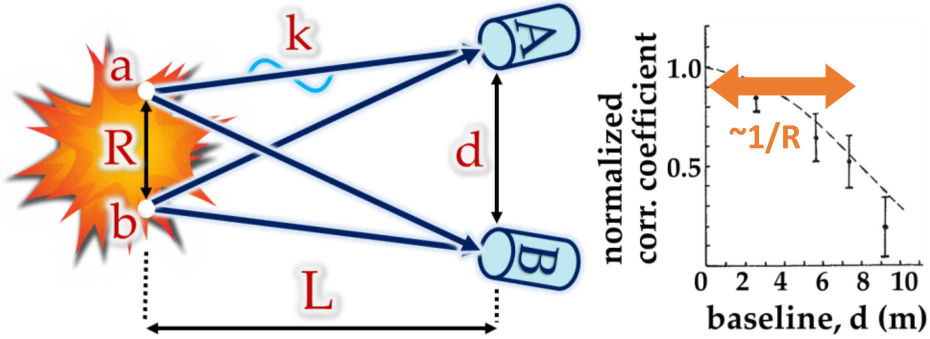




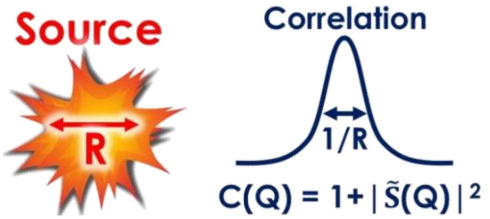


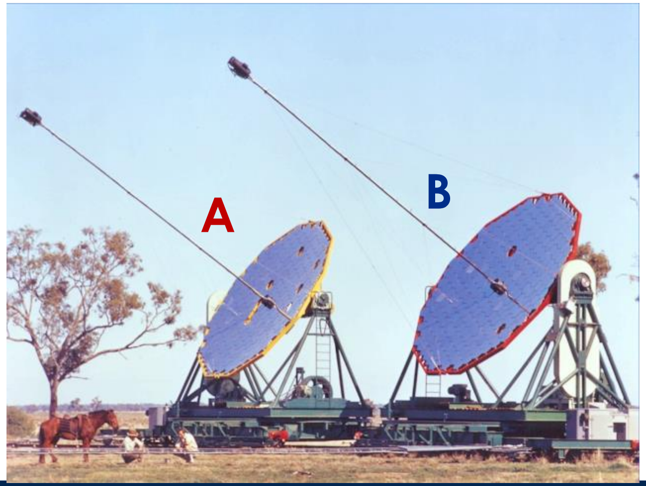
Intensity correlations on cosmic and nuclear scales

- **R. Hanbury Brown & R. Q. Twiss (Radio-astronomy)** *Nature 178 (1956), pp. 1046–1048.*
  - **Intensity correlation** vs. detector distance ⇒ source size



- **Goldhaber et al: discovery in high energy physics**
  Phys. Rev. 120 (1960), pp. 300–312.
  - Distant star ⇔ Quark-Gluon Plasma
  - Light ⇔ particles from freeze-out
  - Intensity correlation of light ⇔ **Momentum correlation** of identical (bosonic) particles
  - Measuring source shape on the fm scale!
  - Sub-field of heavy-ion physics encompassing such measurements: **femtoscopy**





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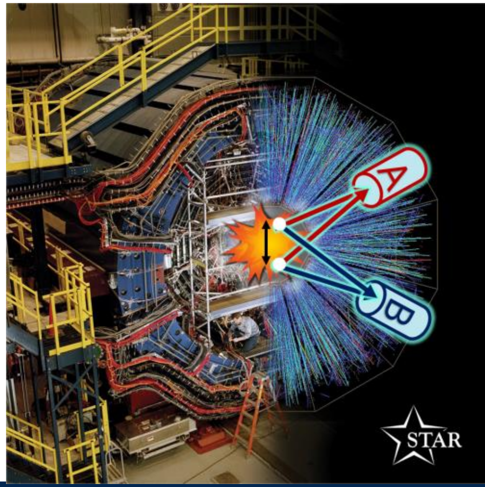


Basic definitions of femtoscopic correlation funtions

- Single particle momentum distribution: $N_1(p) = \int d^4x\, S(x,p)$

s: Single particle phase-space density (emission func.)
x: particle coordinate
p: particle momentum

- Pair momentum distribution: $N_2(p_a,p_b) = \int d^4x_a d^4x_b s(x_a,p_a)s(x_b,p_b)\left|\psi_{p_a,p_b}(x_a,x_b)\right|^2$

- **Correlation function:** $C(p_a,p_b) = \dfrac{N_2(p_a,p_b)}{N_1(p_a)N_1(p_b)}$
- **Pair source/spatial correlation:** $D_{\boldsymbol{K}}(\boldsymbol{r}) = \int d^4\rho\, s\left(\rho+\frac{r}{2},K\right) s\left(\rho-\frac{r}{2},K\right)$

pair separation: $r = x_a - x_b$
pair avg. mom.: $K = (p_a+p_b)/2$
pair center-of-mass: $\rho = (x_a+x_b)/2$

relative pair momentum | *average pair momentum**

*Pair wave function, **containing FSI!***

$$C(Q,K) = \int d^4r D(r,K)\left|\psi_Q(\mathrm{r})\right|^2$$

**Instead of K, m_T is often used:*

$$m_T = \sqrt{k_T^2 + m_\pi^2},\ k_T = \sqrt{K_x^2+K_y^2}$$

Ann. Rev. Nucl. Part. Sci. 55 (2005), pp. 357–402

- **Experiments: measuring C(Q) → information about D(r) and FSI**
- Experimental (and phenomenological) indications: power-law tail for pions, **non-Gaussianity?**

What is the shape of the source? Gaussian vs. Lévy distributions in heavy-ion physics

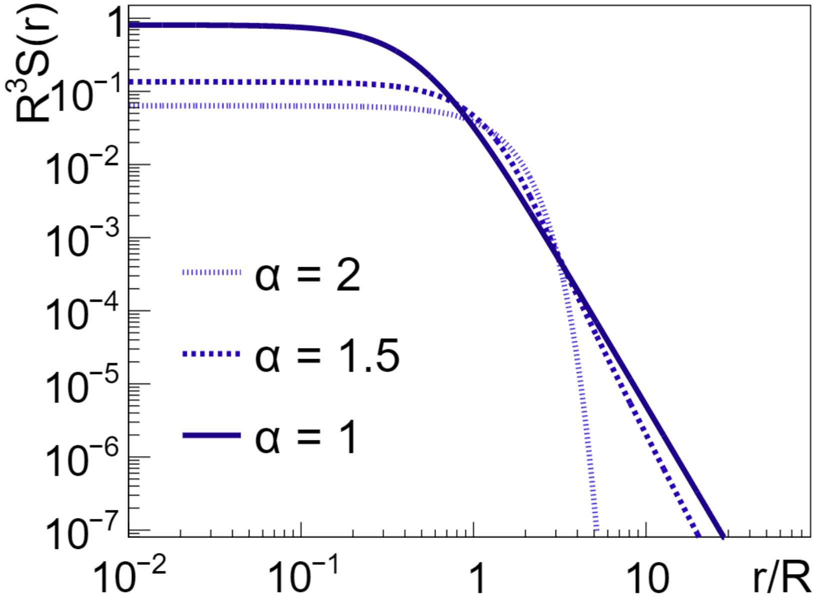
$$s(\boldsymbol{x},\boldsymbol{p}) = \mathcal{L}(\alpha,R;\boldsymbol{x}) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\boldsymbol{x}} e^{-\frac{1}{2}\left|\boldsymbol{q}^T\boldsymbol{R}^2\boldsymbol{q}\right|^{\alpha/2}}$$

$\boldsymbol{p}$ dependence through α, R

spherical symmetry: R^2 diagonal

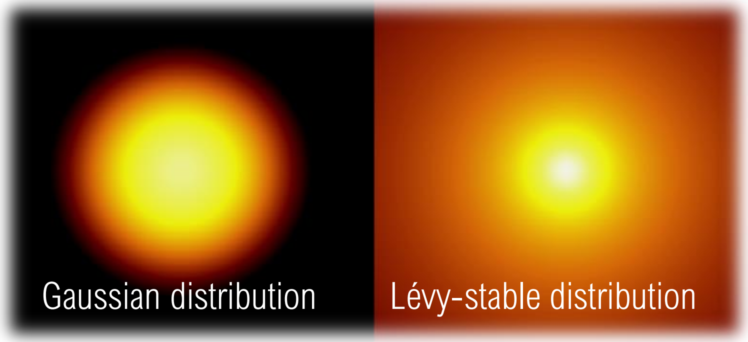
- **Symmetric Lévy-stable distribution:**
  - From generalized central limit theorem, **power-law tail (if $\alpha < 2$)** $\sim r^{-(1+\alpha)}$
  - $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy
  - Retains the same α under convolution

$$s(\boldsymbol{x},\boldsymbol{p}) = \mathcal{L}(\alpha,R;\boldsymbol{x})$$
$$\Downarrow$$
$$D_{\boldsymbol{K}}(\boldsymbol{r}) = \mathcal{L}\left(\alpha, 2^{1/\alpha}R;\boldsymbol{r}\right)$$



- **Experimental indications – Lévy source for pion pairs?**
  - **SPS (NA61/SHINE), RHIC (PHENIX, STAR), LHC (CMS)**
    Phys.Rev.C 97 (2018) no.6, 064911; Universe 10 (2024) 3, 102
    Phys.Rev.C 109 (2024) 2, 024914; Eur.Phys.J.C 83 (2024) 10, 919

- **Possible reasons for the $\alpha < 2$ Lévy exponent?**
  - Angle averaging of an elliptically contoured 3D Gaussian?
  - Averaging over events of many different shapes?



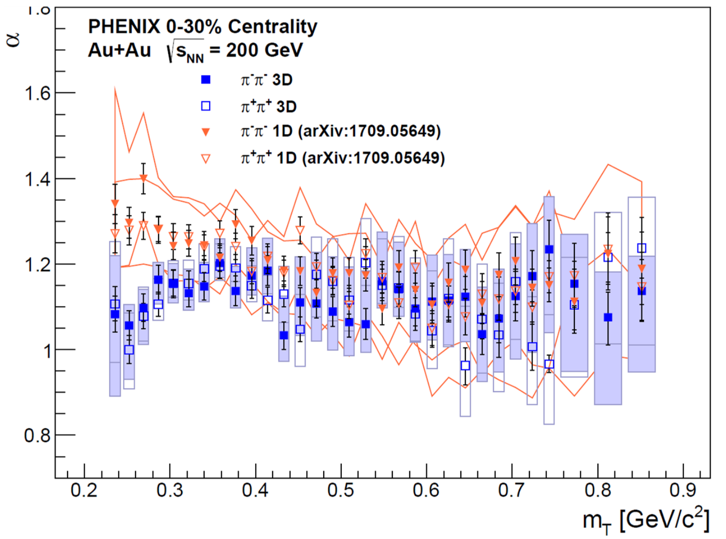
Lévy source not because of event averaging and 3D→1D conversion!

$$s(\boldsymbol{x},\boldsymbol{p}) = \mathcal{L}(\alpha, R; \boldsymbol{x}) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\boldsymbol{x}} e^{-\frac{1}{2}\left|\boldsymbol{q}^T \boldsymbol{R}^2 \boldsymbol{q}\right|^{\alpha/2}}$$

p *dependence through* α, R *spherical symmetry:* R^2 *diagonal*

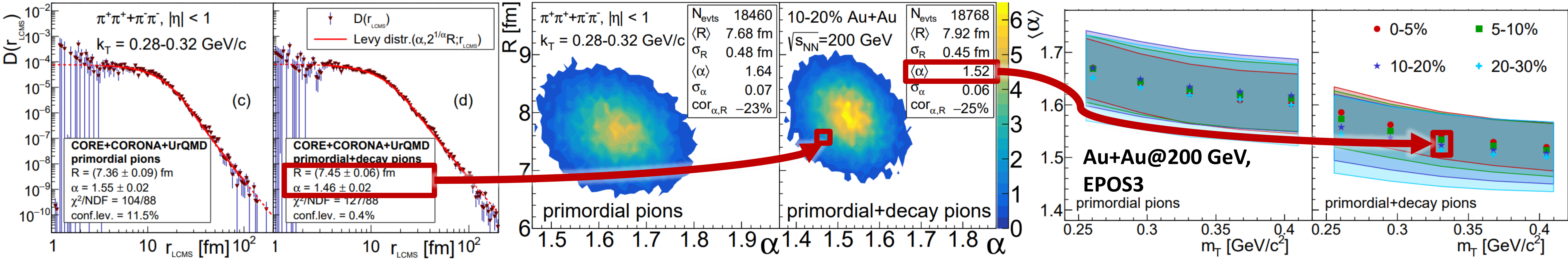
- **Not spherically sym. source: 3D vs. 1D α compatible! $\alpha < 2$ in 1D analyses not because of angle averaging!**

$$\boldsymbol{R^2} = \begin{pmatrix} R^2_{out} & 0 & 0 \\ 0 & R^2_{side} & 0 \\ 0 & 0 & R^2_{long} \end{pmatrix} \qquad \boldsymbol{q} = \begin{pmatrix} q_{out} \\ q_{side} \\ q_{long} \end{pmatrix}$$



Kurgyis, Acta Phys. Pol. B Proc. Suppl. vol. 12 (2), 477 (2019)
Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308

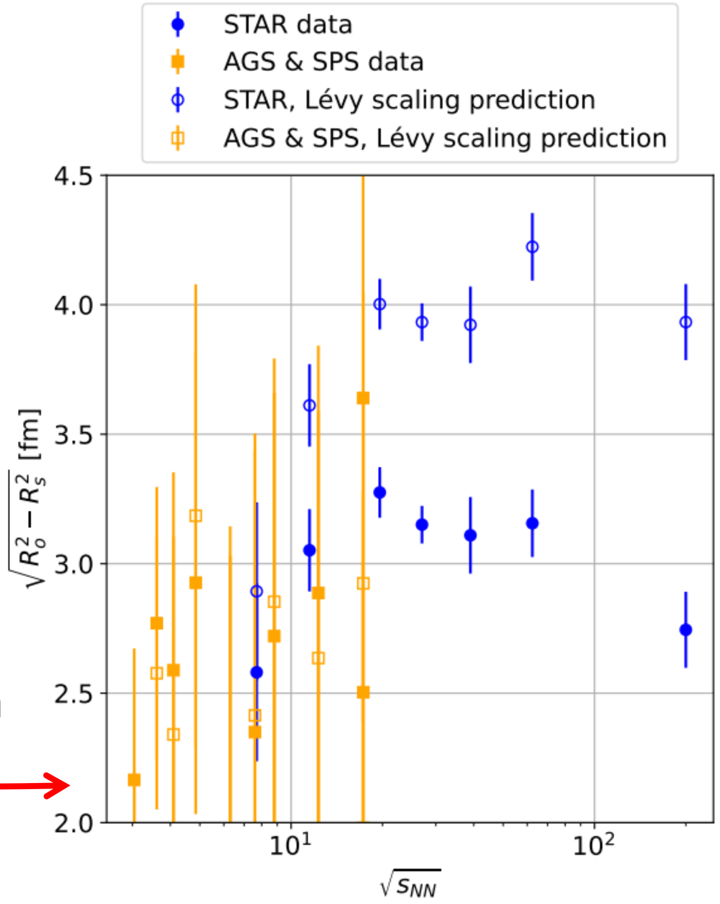
- **EPOS event-by-event analysis: decays and rescattering (UrQMD) – important role!**



Why do Lévy shapes appear, and why is it important?

- **A more comprehensive list of possible reasons:**
  - **Jet fragmentation**
    Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337
  - **Critical phenomena**
    Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532
  - **Resonance decays and hadronic rescattering**
    Kincses, Stefaniak, Csanád, Entropy 24 (2022) 3, 308
    Kórodi, Kincses Csanád, Phys. Lett. B 847 (2023) 138295
    Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002;

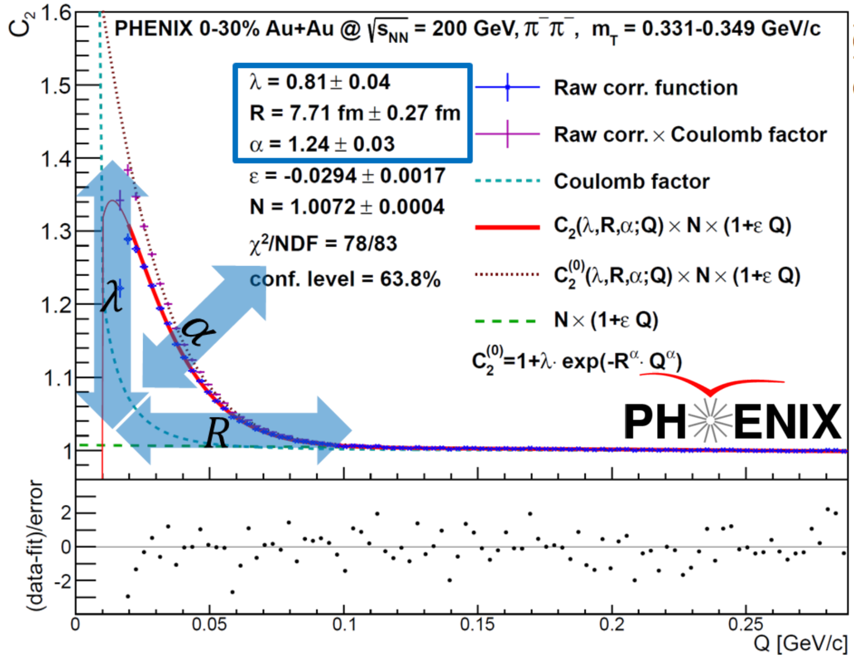
- **Importance of utilizing Lévy sources:**
  - Shape and size entangled, extracting radii with a Gaussian assumption might lead to wrong conclusions
    https://arxiv.org/pdf/2406.11435
  - Measuring α and R together:
    Order of quark-hadron transition, critical point search, understanding source dynamics

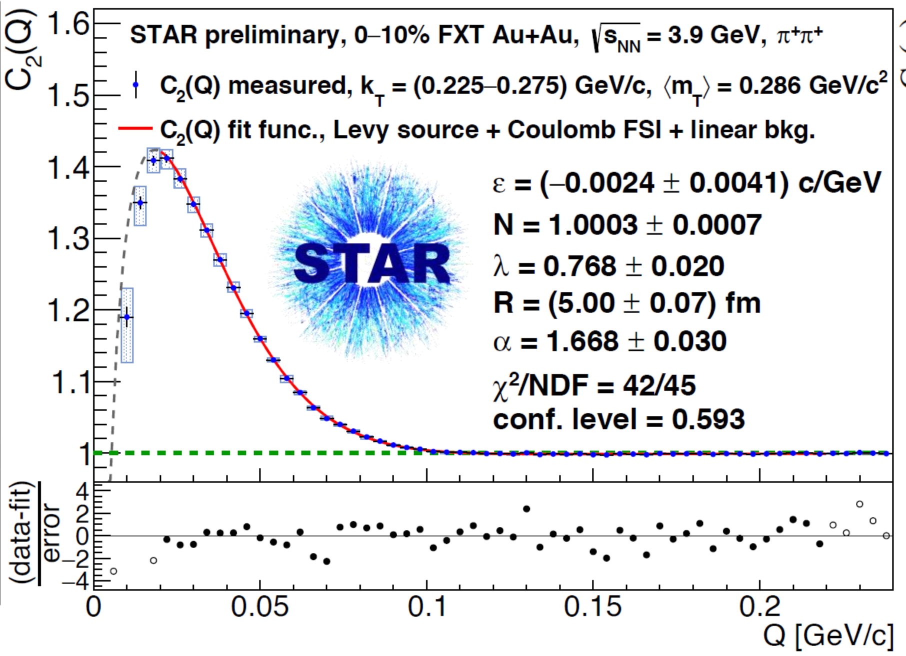


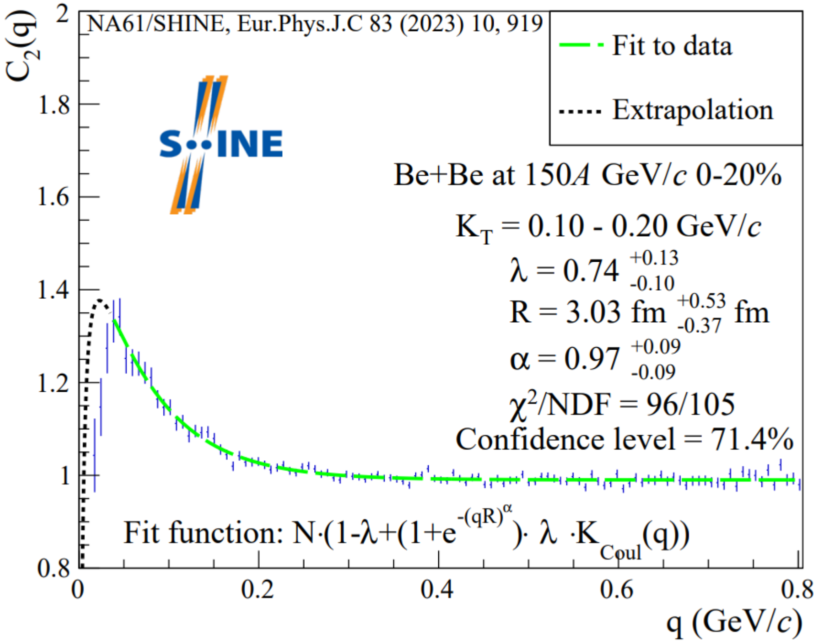
Example correlation function fits at different experiments

Correlation strength — *Lévy scale parameter (length of homogeneity)* — *Lévy exponent*

- 1D two-particle corr.func. with Lévy source and no FSI: $$C_2^{(0)}(Q) = 1 + \lambda \cdot e^{-(RQ)^{\alpha}}$$
- Fits in many different average transverse mass (m_T) bins, **incorporating Coulomb-interaction**



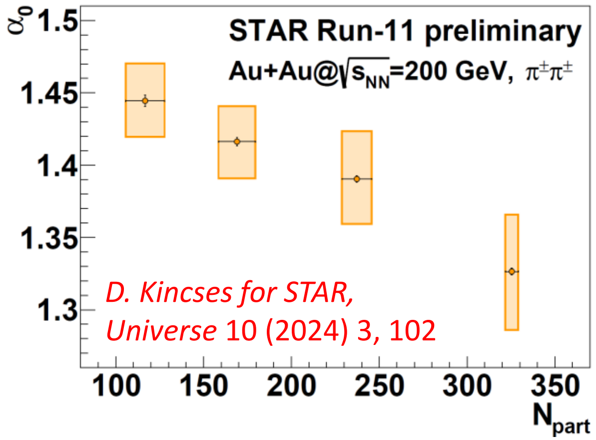


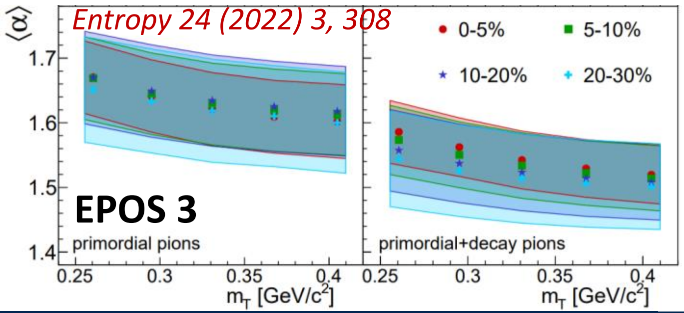


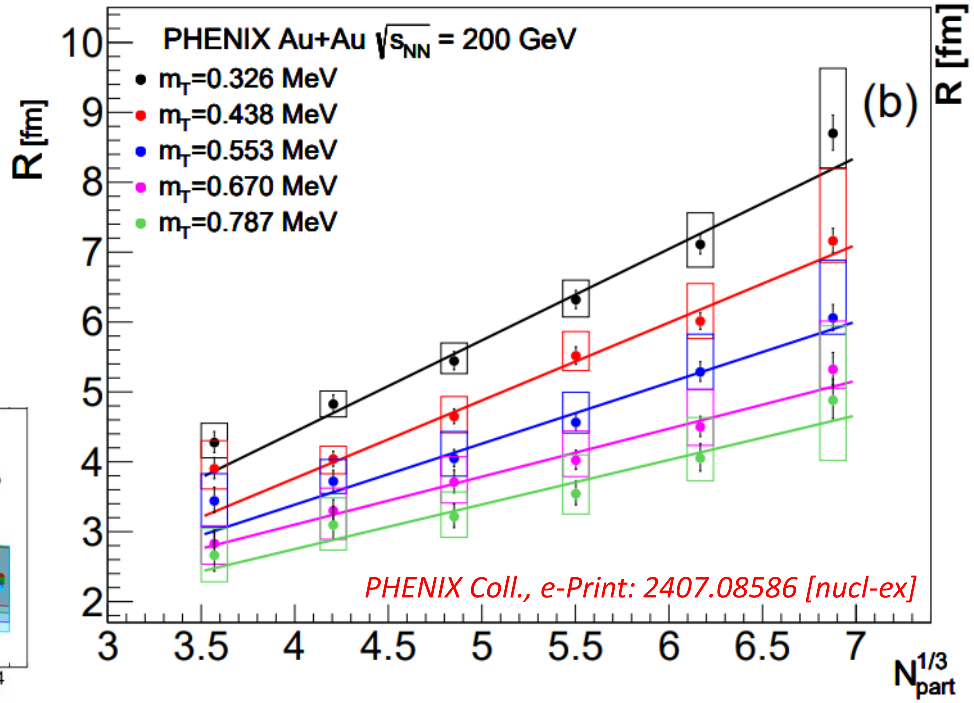
- **Lévy fits provide good description**, exponent far from the Gaussian (α = 2) case

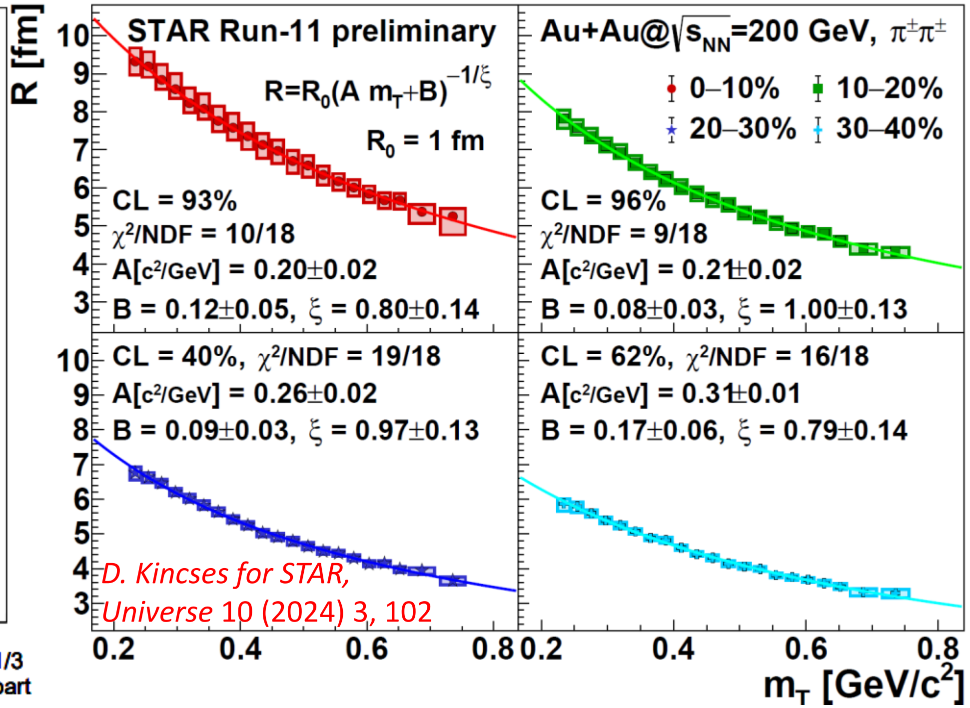
Selection of results from PHENIX, STAR
Lévy analyses at top RHIC energy (Au+Au @ 200 GeV)

- **Lévy exponent α** far from the Gaussian (α = 2) case, also lower than EPOS results
- **Lévy scale R** shows hydro-like scaling behavior, geometric centrality dependence
- **Systematic uncertainties estimated** from single-track- and pair-cuts, fit limits, handling of Coulomb effect





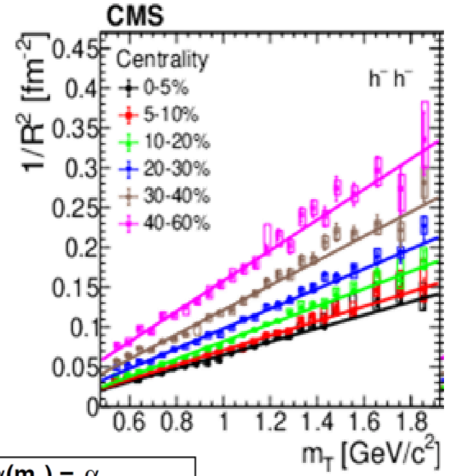


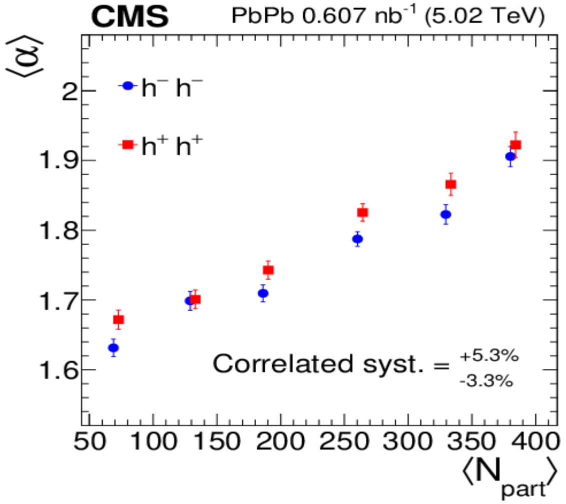


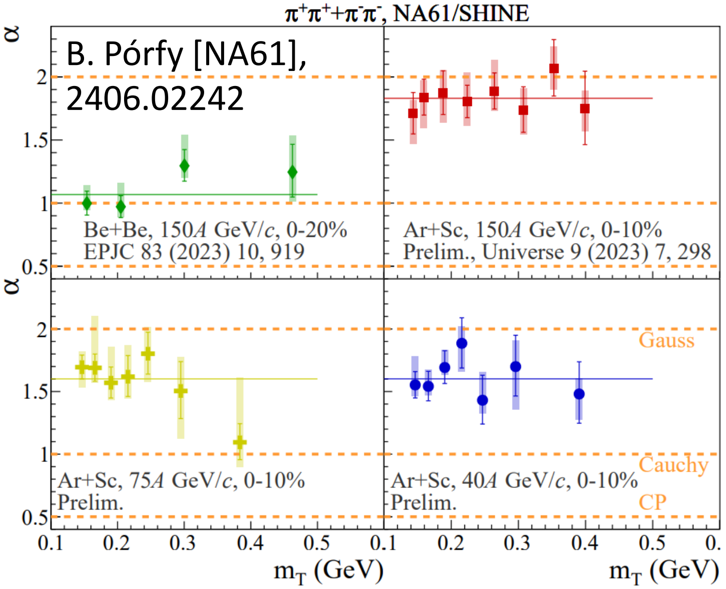
Example selection of recent measurements from PHENIX, STAR, CMS, NA61/SHINE

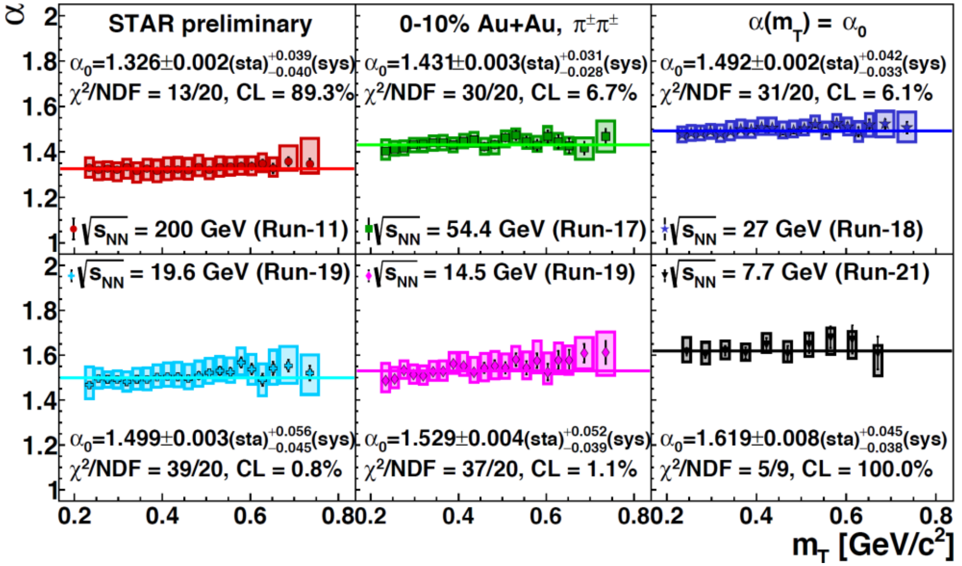
- Comprehensive measurements are ongoing
  - **Transverse mass dependence**
  - **Centrality dependence**
  - **Collision energy dependence**
  - **Particle species dependence**

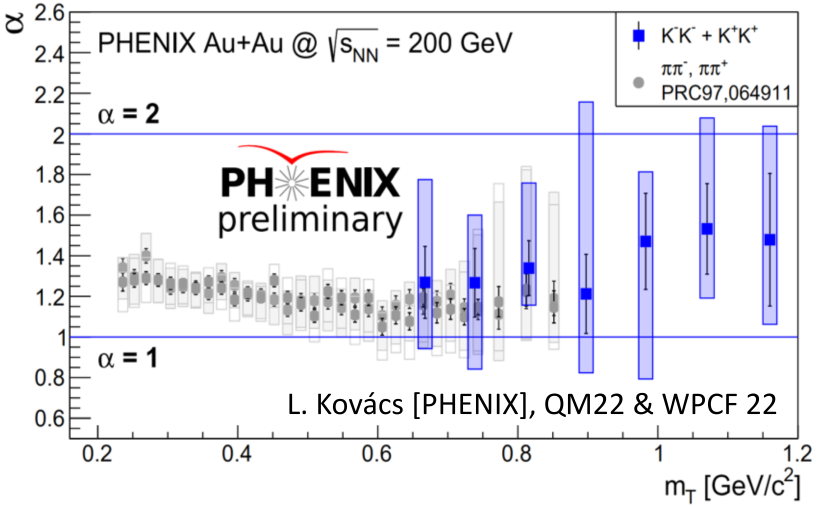
e-Print: 2407.08586 [nucl-ex]
Universe 10 (2024) 3, 102
Phys.Rev.C 109 (2024) 2, 024914
EPJ Web Conf. 296 (2024) 06004
Universe 10 (2024) 2, 54
Universe 9 (2023) 7, 336







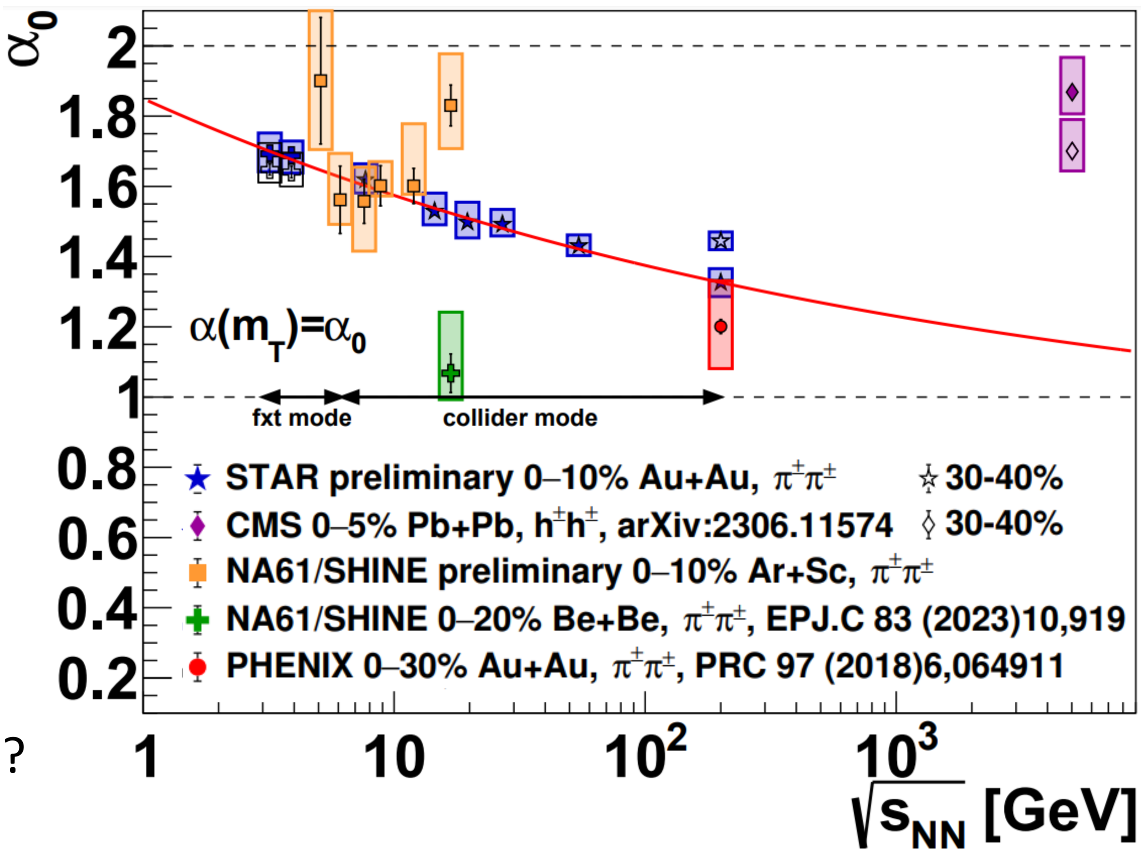




Lévy exponent α from SPS through RHIC to LHC

- Different values for **small (Be+Be)** & **medium (Ar+Sc)** systems at SPS
- System size ordering: **BeBe** < **AuAu** < **ArSc** ??
- **STAR beam energy scan**: monotonic decrease
- **RHIC** vs. **LHC (CMS)**: **non-monotonic trend**
  - Minimum around top RHIC energy?
- **No signs of critical behavior (very small α, ~0.5)**
- **Opposite centrality trend at RHIC & LHC (?)**
- Many new and ongoing experimental measurements, **<u>many open questions for phenomenology!</u>**
  - Can there be anomalous diffusion in the quark stage?
  - What is the role of finite size and finite time?
  - If decays & rescattering are not enough to reproduce the data, what else needed?

Csanád, Kincses, Universe 10 (2024) 54. arXiv:2401.01249



Thank you for your attention!

Phenomenology of femtoscopy with Lévy-stable sources

T. Csörgő et al. In: ***Acta Phys. Polon. B 36 (2005), pp. 329–337.***
T. Csörgő et al. In: ***AIP Conf. Proc. 828.1 (2006), pp. 525–532.***
M. Csanád et al. In: ***Braz. J. Phys. 37 (2007), pp. 1002–1013***
D. Kincses, M. I. Nagy, and M. Csanád, ***Phys. Rev. C 102.6 (2020), p. 064912.***
D. Kincses, M. Stefaniak, and M. Csanád, ***Entropy 24 (2022), p. 308.***

2023-2024

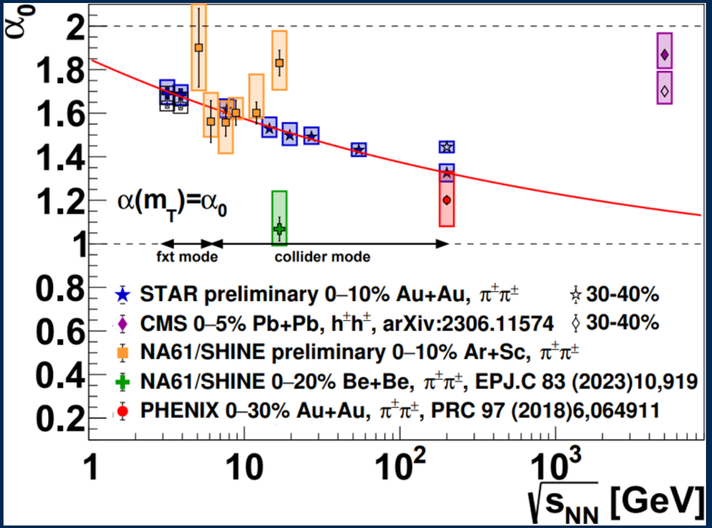
Kórodi, Kincses, Csanád, ***Phys. Lett. B 847 (2023) 138295***
Kurgyis, Kincses, Nagy, Csanád, ***Universe 9 (2023) 7, 328***
Nagy, Purzsa, Csanád, Kincses, ***Eur. Phys. J. C 83, 1015 (2023)***
Csanád, Kincses, ***Universe 10 (2024) 2, 54***
Csanád, Kincses, ***e-Print: 2406.11435 [nucl-th]***

Experimental measurements of Lévy-type correlations

PHENIX Coll., ***Phys. Rev. C 97.6 (2018), p. 064911.***
L. Kovács for the PHENIX Coll., ***Universe 9 (2023) 7, 336***

2024

PHENIX Coll., ***e-Print: 2407.08586 [nucl-ex]***
CMS Coll., ***Phys.Rev.C 109 (2024) 2, 024914***
D. Kincses for the STAR Coll., ***Universe 10 (2024) 3, 102***
NA61/SHINE Coll., ***Eur.Phys.J.C 83 (2024) 10, 919***
B. Pórfy for the NA61/SHINE Coll, ***EPJ Web Conf. 296 (2024) 06004***



If interested in a follow-up, come to the Zimányi School in December!

http://zimanyischool.kfki.hu/24/

Further details, backup slides

Event-by-event investigation of the source function

(Entropy 24 (2022), p. 308.)

- **Experiments – no direct access to $D(r)$ pair-source**

$$C_2(Q) = \int D(r)\left|\psi_Q(\mathrm{r})\right|^2 dr$$

- **Event generator models (like EPOS) – direct access to freeze-out coordinates!**
  - **Phenomenological investigations of $\boldsymbol{D(r)}$ possible**
- **EPOS: E**nergy conserving quantum-mechanical multiple scattering approach, based on **P**artons (parton ladders), **O**ff-shell remnants, and **S**plitting of parton ladders
  - **Monte Carlo based heavy-ion event generator model**, reproduces basic observables (spectra, flow) in high-energy collisions, femtoscopy was not investigated before
  - **Main parts of the EPOS model:**
    - **Core-Corona division** (based on dE/dx of string segments)
    - **Hydrodynamical evolution** (vHLLE 3D+1 viscous hydro)
    - **Hadronic cascades** (UrQMD afterburner)

Event-by-event investigation of the source function

(Entropy 24 (2022), p. 308.)

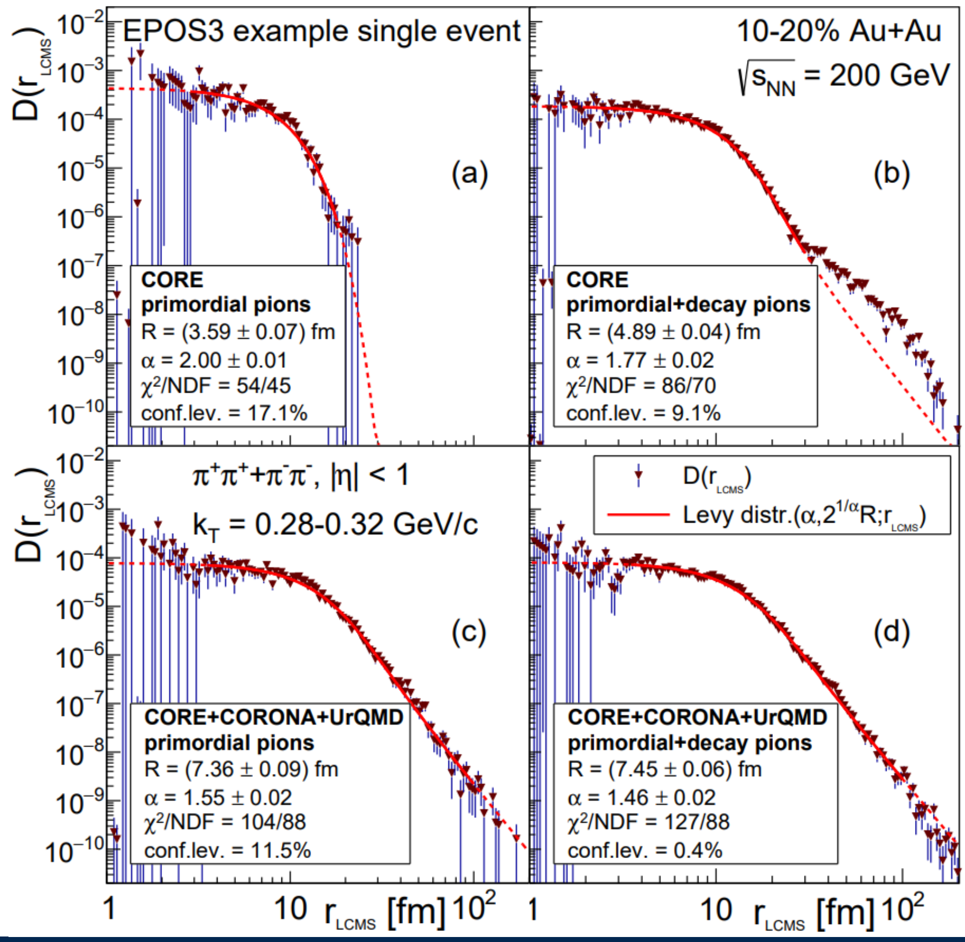
- $\sqrt{s_{NN}}$ = 200 GeV Au+Au collisions generated by EPOS359
- **Angle-avgeraged $\pi\pi$ radial source distribution**

$$D\left(r_{1,2}^{LCMS}\right) = \int d\Omega dt D(r)$$

- **Event-by-event investigation:**

a) CORE, primordial pions – **Gaussian source shape**
b) CORE, decay products incl. – **power-law structures appear**
c) CORE+CORONA+UrQMD, primordial pions – **Lévy-shape**
d) CORE+CORONA+UrQMD, decay products incl. – **Lévy-shape**

- **Recall possible reasons for Lévy-type sources:**
  event averaging, resonance decays, rescattering
- **Event-by-event non-Gaussianity observed!**

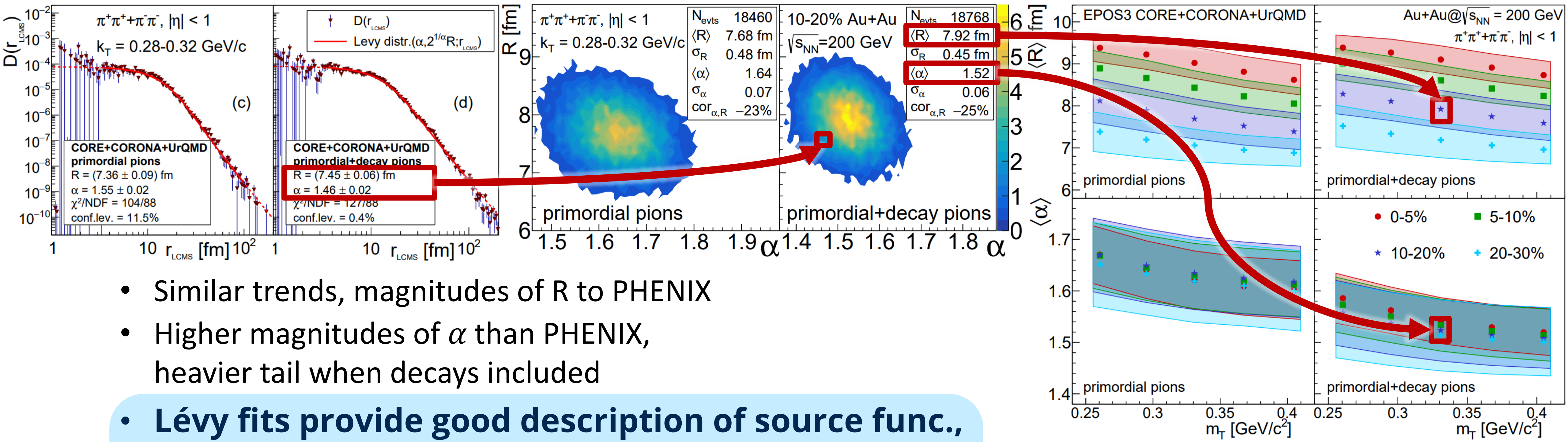


$$r_{1,2}^{LCMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{LCMS})^2};\ \Delta z_{LCMS} = \Delta z - \beta(\Delta t)/\sqrt{1-\beta^2};\ \beta = (p_{z,1}+p_{z,2})/(E_1+E_2)$$

Event-by-event investigation of the source function

(Entropy 24 (2022), p. 308.)

- **Such fits repeated for thousands of events** in case (c) and (d) (final stage of EPOS)
- Normal distribution of α, R for given centrality & k_T
- **Mean and std.dev. values of the source params. extracted**, for different centrality and m_T classes

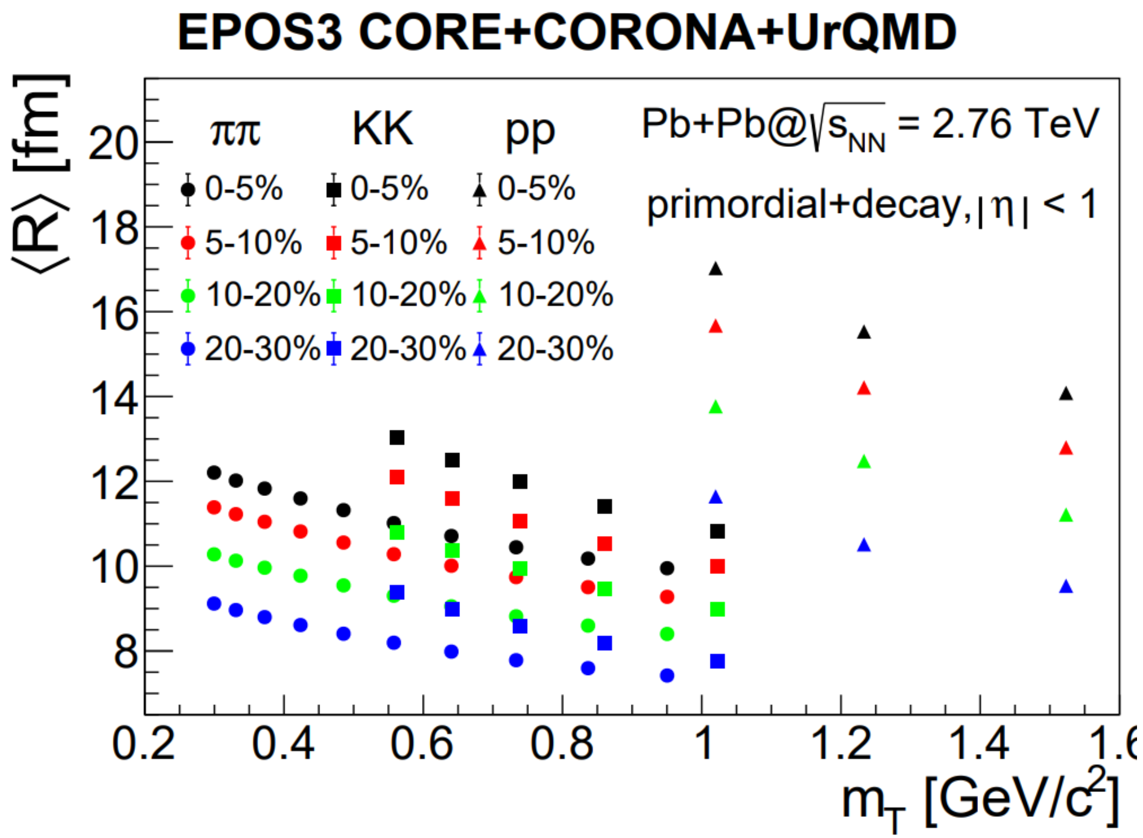


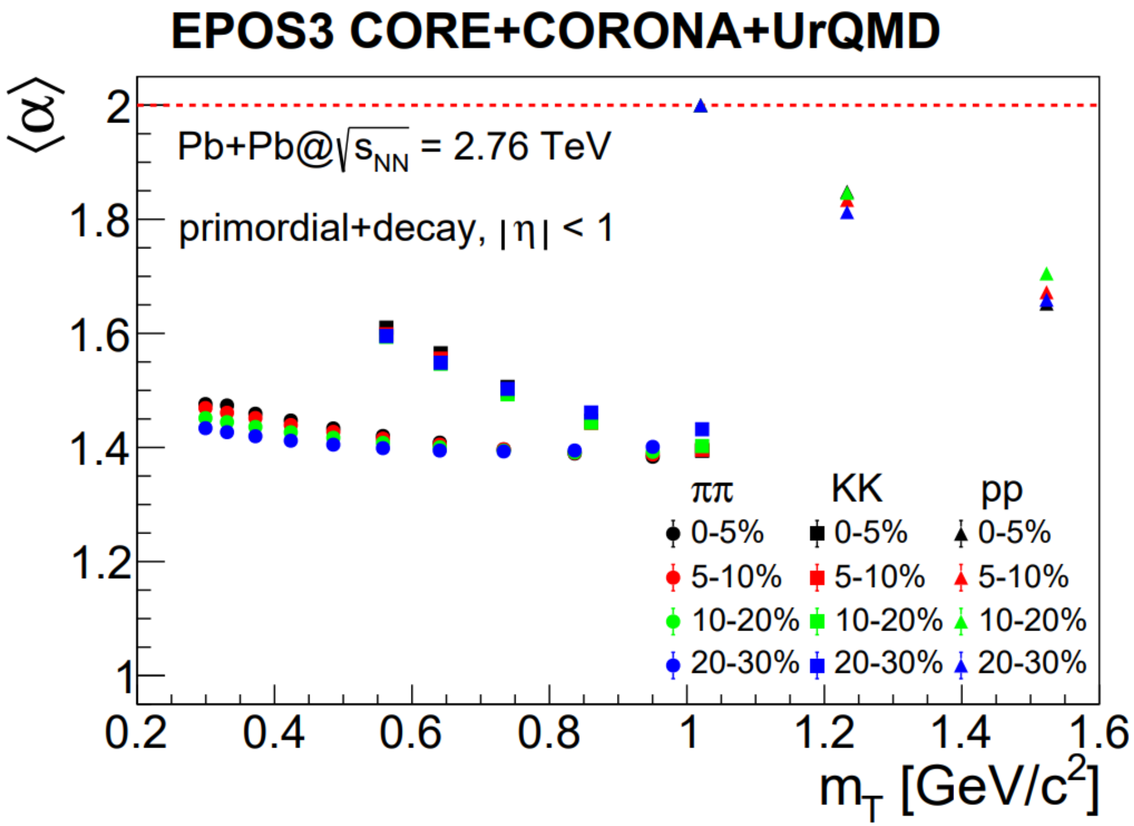
- Similar trends, magnitudes of R to PHENIX
- Higher magnitudes of α than PHENIX, heavier tail when decays included

- **Lévy fits provide good description of source func., tail strongly affected by rescattering, decays**

EPOS analysis at LHC energies

(Phys. Lett. B 847 (2023) 138295)





Stay tuned!

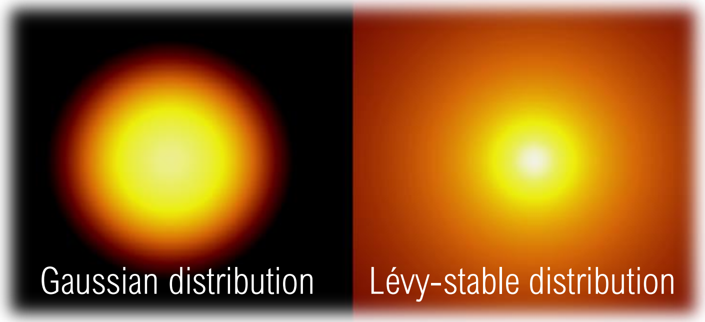
Properties of univariate stable distributions

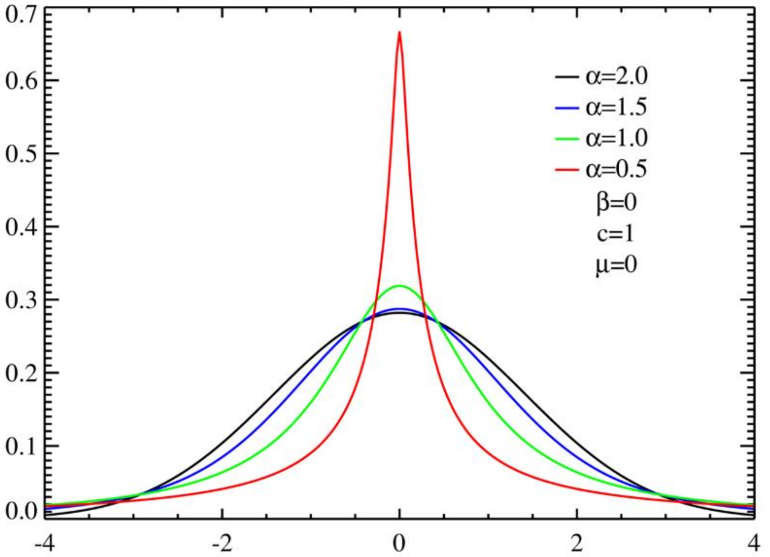
- **Univariate stable distribution:** $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi(q) e^{-ixq} dq$, where the characteristic function:
- $\varphi(q;\alpha,\beta,R,\mu) = \exp(iq\mu - |qR|^{\alpha}(1 - i\beta \mathrm{sgn}(q)\Phi))$

$$\Phi = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right), \alpha \neq 1 \\ -\frac{2}{\pi}\log|q|, \alpha = 1 \end{cases}$$

- α: index of stability
- β: skewness, symmetric if $\beta = 0$
- R: scale parameter
- μ: location, equals the median,

  if $\alpha > 1$: $\mu =$ mean





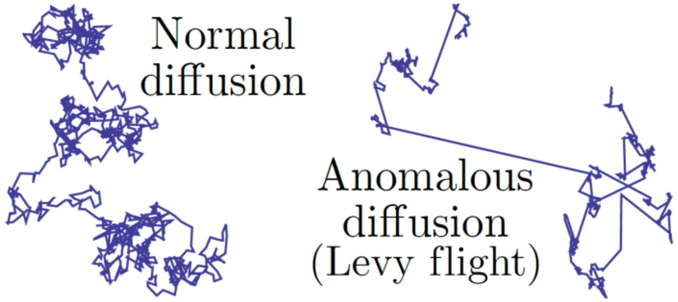
In 3D: $\mathcal{L}(\boldsymbol{r};\alpha,R) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{q} e^{i\boldsymbol{qr}} e^{-\frac{1}{2}|\boldsymbol{qRq}|^{\alpha/2}}$

$$R^2_{\sigma\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & R^2_{\mathrm{out}} & 0 & 0 \\ 0 & 0 & R^2_{\mathrm{side}} & 0 \\ 0 & 0 & 0 & R^2_{\mathrm{long}} \end{pmatrix}$$

- **Important characteristics of stable distributions:**
  - Retains same α and β under convolution of random variables
  - Any moment greater than α isn't defined

Lévy-type sources in heavy-ion collisions

- **Anomalous diffusion**
  - Elastic rescattering of hadrons
  - Expanding hadron gas → time dependent increasing mean free path
  - Hadronic Resonance Cascade (HRC) model
  - α depends on total inelastic cross-section
  - $\alpha_{\pi}^{HRC} > \alpha_{K}^{HRC}$ (smaller c.s. → larger m.f.p.)
  - **Kaon vs. pion measurements can test the anom.diff. Picture**
  - **Motivation for Lévy femtoscopy with kaons!**



Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002;

T. J. Humanic, Int. J. Mod. Phys. E 15, 197 (2006)

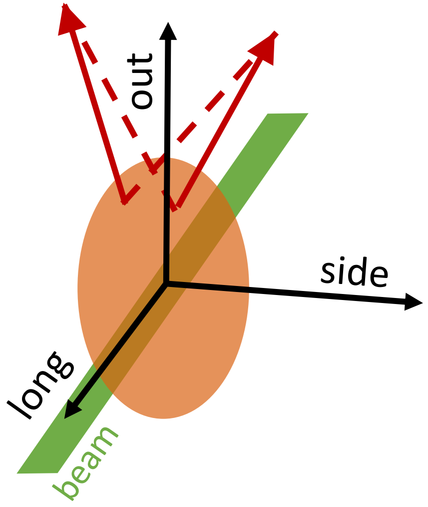
S. Chapman, P. Scotto, U. Heinz, Phys.Rev.Lett. 74 (1995) 4400;
T. Csörgő and B. Lörstad, Phys.Rev. C54 (1996) 1390;
S. Pratt, Nucl.Phys. A830 (2009) 51C

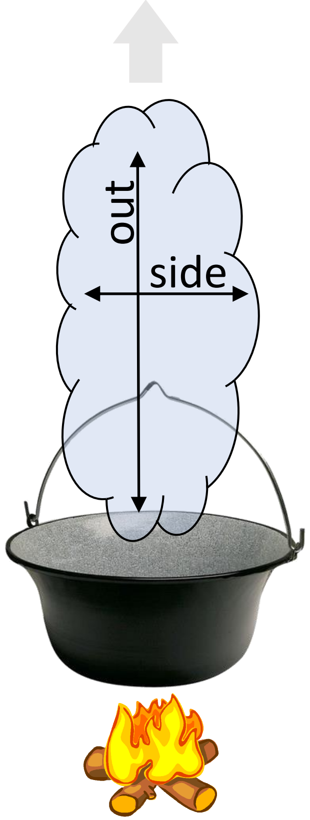
HBT and the phase transition

- C(q) usually measured in the Bertsch-Pratt pair coordinate-system
  - **out:** direction of the average transverse momentum
  - **long:** beam direction
  - **side:** orthogonal to the latter two
- $R_{out}, R_{side}, R_{long}$: HBT radii
- Δτ emission duration, i.e. $S(r,\tau) \sim e^{-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}}$
- From a simple hydro calculation:

$$R_{\text{out}}^2 = \frac{R^2}{1+u_T^2 m_T/T_0} + \beta_T^2 \Delta\tau^2, \quad R_{\text{side}}^2 = \frac{R^2}{1+u_T^2 m_T/T_0}$$

- RHIC, 200 GeV: $R_{out} \approx R_{side} \rightarrow$ no strong 1st order phase trans.
- Plus lots of other details: pre-equilibrium flow, initial state, EoS, ...



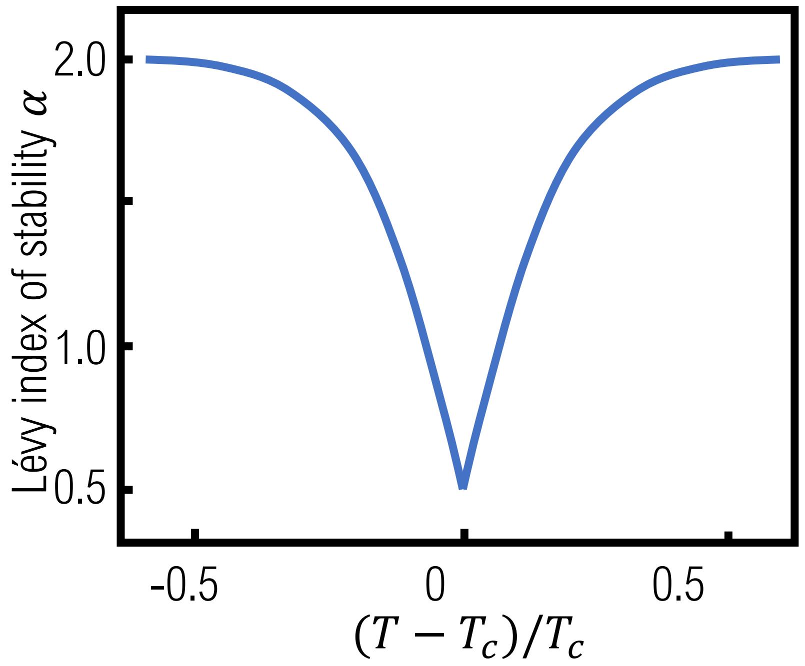


Second order phase transition?

- Second order phase transitions: **critical exponents**
  - **Near the critical point**
    - Specific heat ~ $((T-T_c)/T_c)^{-\alpha}$
    - Order parameter ~ $((T-T_c)/T_c)^{-\beta}$
    - Susceptibility/compressibility ~ $((T-T_c)/T_c)^{-\gamma}$
    - Correlation length ~ $((T-T_c)/T_c)^{-\nu}$
  - **At the critical point**
    - Order parameter ~ (source field)$^{1/\delta}$
    - **Spatial correlation function** ~ $r^{-d+2-\eta}$
  - Ginzburg-Landau: $\alpha=0, \beta=0.5, \gamma=1, \eta=0.5, \delta=3, \eta=0$
- QCD ↔ 3D Ising model
- Can we measure the η power-law exponent?
- Depends on spatial distribution: measurable with femtoscopy!
- **What distribution has a power-law exponent? Levy-stable distribution!**

Lévy index as critical exponent?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$;
  Lévy source: $\sim r^{-(1+\alpha)}$; $\boldsymbol{\alpha \Leftrightarrow \eta}$?
  Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67
- QCD universality class ↔ 3D Ising
  Halasz et al., Phys.Rev.D58 (1998) 096007
  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: $\eta = 0.50 \pm 0.05$
    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)
    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- **Change in α_{Levy} - proximity of CEP?**



- Modulo finite size/time and non-equilibrium effects
- Other possible reasons for Lévy distributions: anomalous diffusion, QCD jets, …

Kinematic variables of the correlation function I.

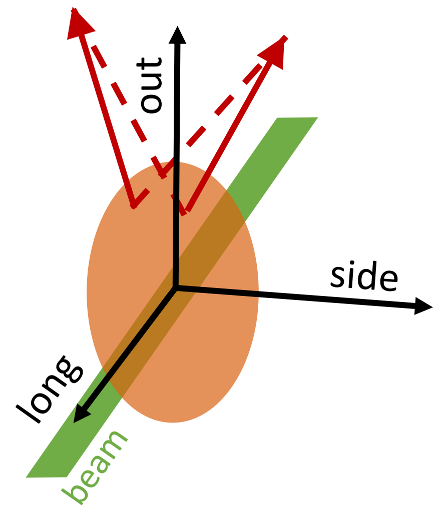
- Smoothness approximation ($p_1 \approx p_2 \approx K$): $S(x_1, K - q/2)\, S(x_2, K + q/2) \;\approx\; S(x_1, K)\, S(x_2, K)$
- $C_2(q,K) = \int d^4 r D(r,K) \left|\psi_q^{(2)}(\mathrm{r})\right|^2$
- Without any FSI $\left|\psi_q^{(2)}(\mathrm{r})\right|^2 = 1 + \cos(\mathrm{qr})$

$\Big\}$ $C_2^{(0)}(q,K) \simeq 1 + \frac{\widetilde{D}(q,K)}{\widetilde{D}(0,K)}$, where $\widetilde{D}(q,K) = \int D(x,K) e^{iqx} d^4 x$

- **HBT correlation function in direct connection with Fourier transform of the pair-source function**
- Important to determine the nature and dimensionality of the correlation function
- Lorentz-product of $q = (q_0, \boldsymbol{q})$ and $K = (K_0, \boldsymbol{K})$ is zero, i.e.: $qK = q_0 K_0 - \boldsymbol{qK} = 0$
- Energy component of q can be expressed as $q_0 = \boldsymbol{q}\frac{\boldsymbol{K}}{K_0}$
- If the energy of the particles are similar, K is approximately on shell
- **Correlation function can be measured as a function of three-momentum variables**

Kinematic variables of the correlation function II.

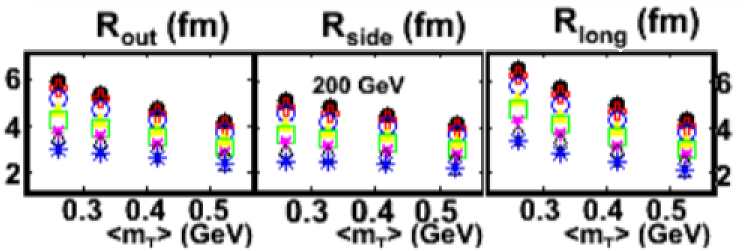
- $C_2(\boldsymbol{q}, \boldsymbol{K})$ as a function of three-momentum variables
- $\boldsymbol{K}$ dependence is smoother, $\boldsymbol{q}$ is the main kinematic variable
- Close to mid-rapidity one can use $k_T = \sqrt{K_x^2 + K_y^2}$, or $m_T = \sqrt{k_T^2 + m^2}$
- For any fixed value of m_T, the correlation function can be measured as a function of $\boldsymbol{q}$ only
- Usual decomposition: **out-side-long or Bertsch-Pratt (BP) coordinate-system**
  - $\boldsymbol{q} \equiv (q_{out}, q_{side}, q_{long})$
  - long: beam direction
  - out: k_T direction
  - side: orthogonal to the others
  - Essentially a rotation in the transverse plane



- Customary to use a Lorentz-boost in the long directon and change to the **Longitudnal Co-Moving System (LCMS)** where the average longitudinal momentum of the pair is zero

Kinematic variables of the correlation function III.

- Drawback of a 3D measurement: lack of statistics, difficulties of a precise shape analysis
- **What is the appropriate one-dimensional variable?**
- Lorentz-invariant relative momentum: $q_{inv} \equiv \sqrt{-q^\mu q_\mu} = \sqrt{q_x^2 + q_y^2 + q_z^2 - (E_1 - E_2)^2}$
- Equivalent to three-mom. diff. in Pair Co-Moving System (PCMS), where $E_1 = E_2$: $q_{inv} = |\boldsymbol{q}_{PCMS}|$
- In LCMS using BP variables: $q_{inv} = \sqrt{(1-\beta_T)^2 q_{out}^2 + q_{side}^2 + q_{long}^2}$ $\beta_T = 2k_T/(E_1 + E_2)$
- **Value of q_{inv} can be relatively small even when q_{out} is large!**
- Experimental indications: **in LCMS source is ≈ spherically symmetric**
- Correlation function boosted to PCMS will not be spherically symmetric
- Let us introduce the following variable invariant to Lorentz boosts in the beam direction:



$$Q \equiv |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{z,LCMS}^2},$$

$$\text{where } q_{z,LCMS}^2 = \frac{4(p_{1z}E_2 - p_{2z}E_1)^2}{(E_1+E_2)^2 - (p_{1z}+p_{2z})^2}.$$

Kinematic variables of the correlation function IV.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

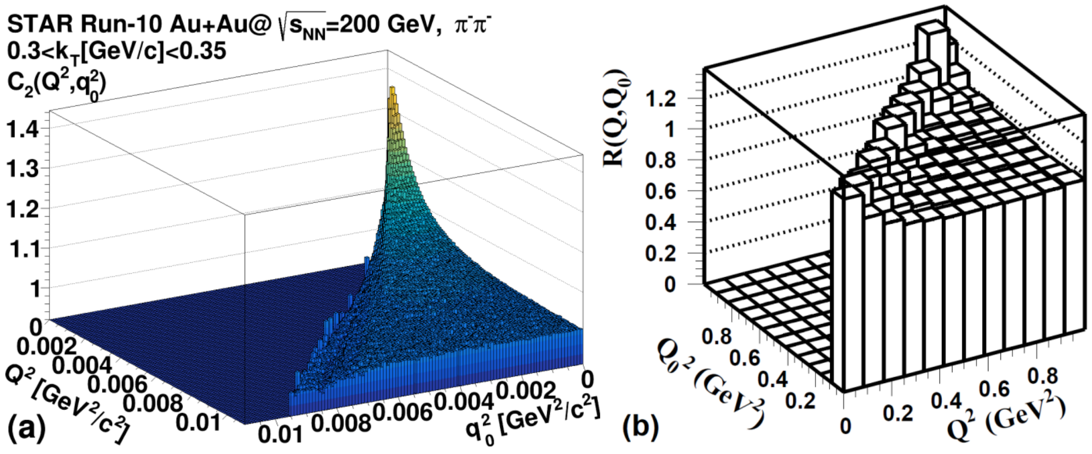


Figure 3.4: Example two-dimensional pion correlation functions for $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions (a) and $\sqrt{s} = 91$ GeV e^+e^- collisions (b). The latter figure is taken from the thesis of Tamás Novák [161].

Q dep. corr.func. $\boldsymbol{q_{inv}}$ dep. corr.func.

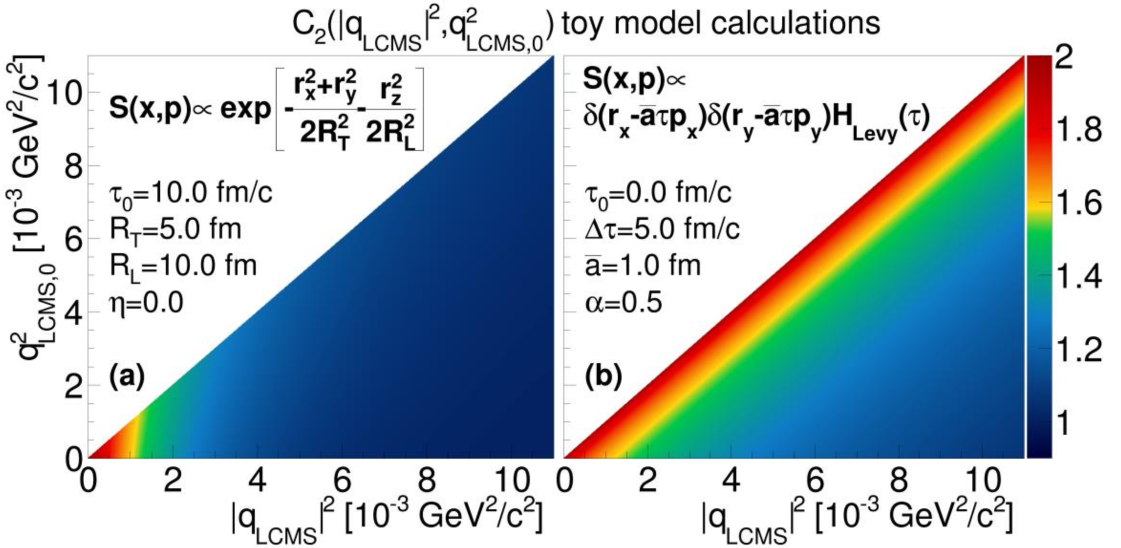
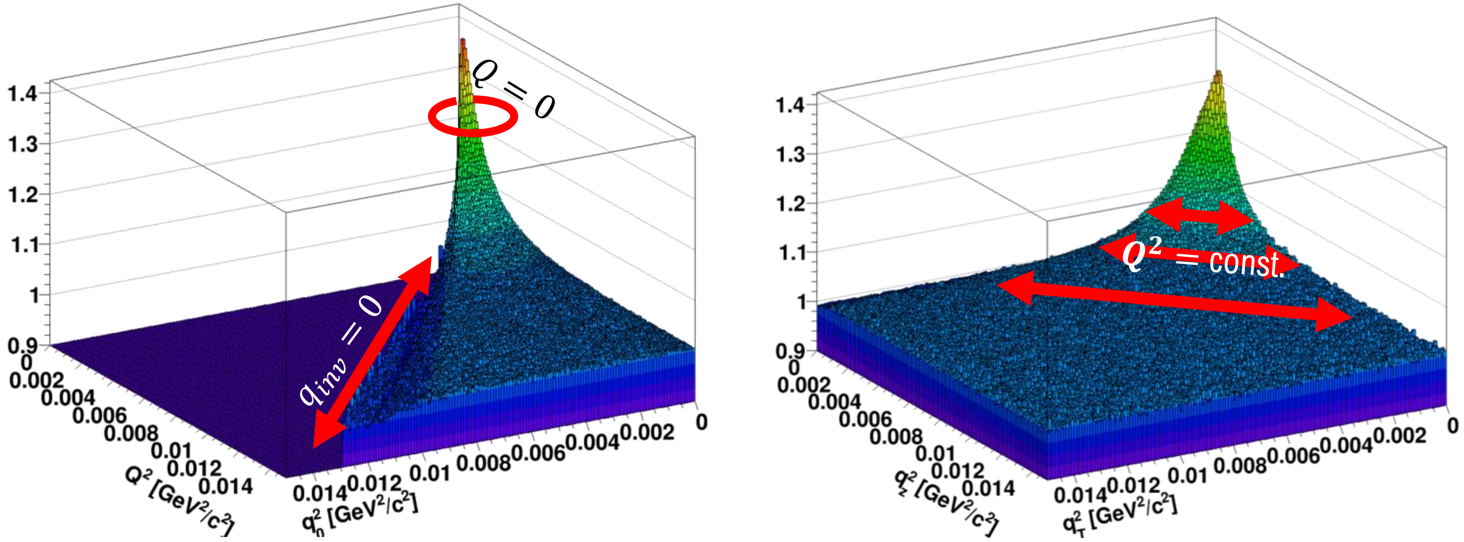


Figure 3.5: Toy model calculation for two different types of source functions. Taking a Gaussian source in both space and time leads to a correlation function that depends mostly on $|q_{LCMS}|$ (a), while a source that shows strong space-time and momentum space correlation leads to a q_{inv} dependent correlation function (b).

Kinematic variables of the correlation function V.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

$$Q = |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + \left(p_{1y} - p_{2y}\right)^2 + q_{long,LCMS}^2}$$

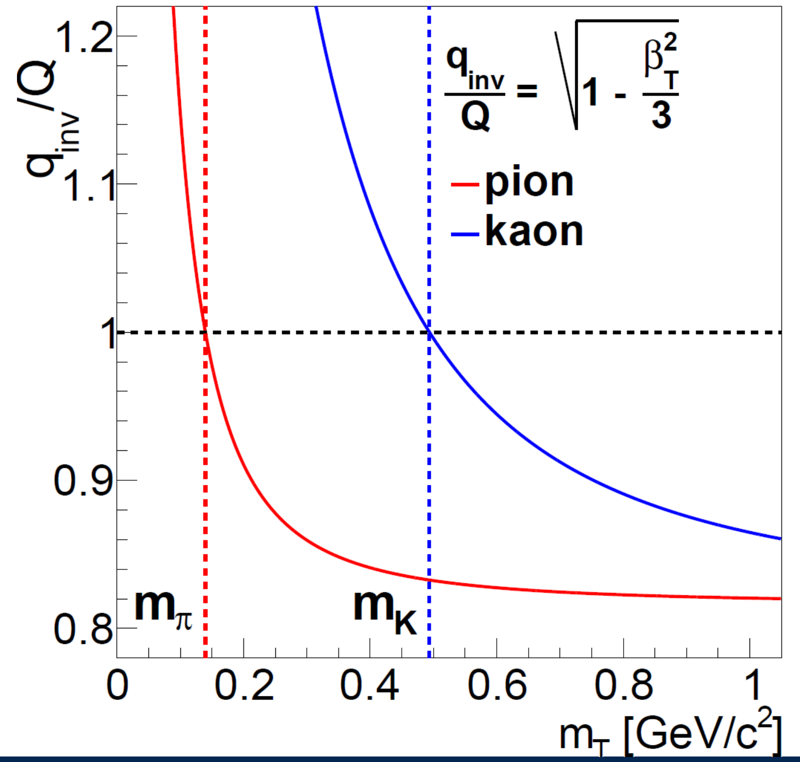


Kinematic variables of the correlation function V.

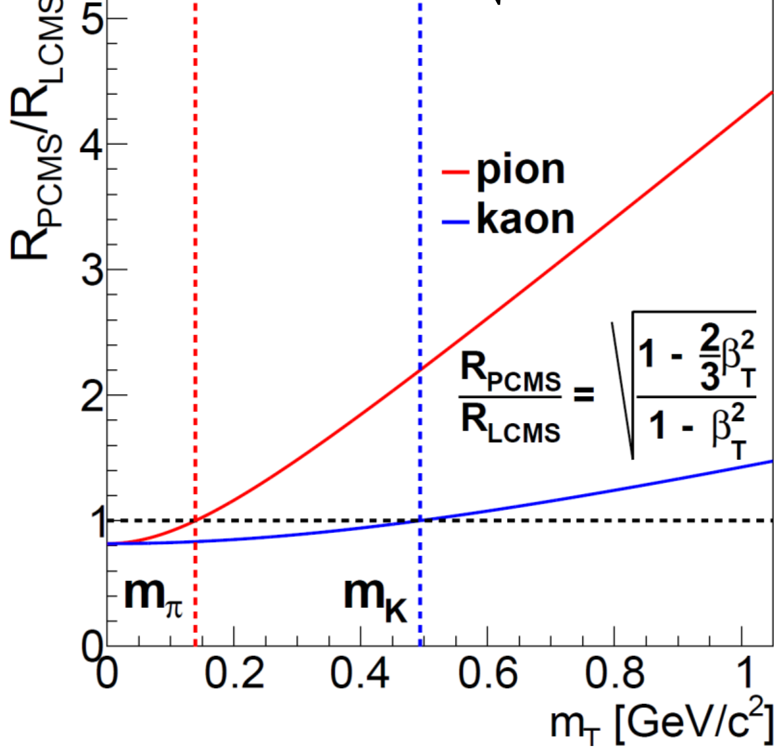
- Correlation function measured in LCMS, Coulomb effect calculated in PCMS
- Approximation:
- (Note $m_T < m$

  not physical of course)

$$q_{\mathrm{inv}} \equiv q_{\mathrm{PCMS}} \approx q_{\mathrm{LCMS}} \cdot \sqrt{1 - \beta_T^2/3}$$



$$R_{\mathrm{PCMS}} \approx R_{\mathrm{LCMS}} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}}$$



Coulomb correction and fitting of the corr. function

- Core-Halo model, Bowler-Sinyukov method: $C_2(Q,k_T) = 1 - \lambda + \lambda \int d^3\boldsymbol{r} D_{(c,c)}(\boldsymbol{r},k_T) \left|\Psi_Q^{(2)}(\boldsymbol{r})\right|^2$
- Neglecting FSI and using a Lévy-stable source function: $C_2^{(0)}(Q,k_T) = 1 + \lambda e^{-|RQ|^\alpha}$
- **Using numerical integral calculation as fit function results in numerically fluctuating χ^2 landscape**
- Treat FSI as correction factor: $K(Q,k_T) = \frac{C_2(Q,k_T)}{C_2^{(0)}(Q,k_T)}$
- An iterative method can be used: $C_2^{(fit)}(Q;\lambda,R,\alpha) = C_2^{(0)}(Q;\lambda,R,\alpha) \cdot K(Q;\lambda_0,R_0,\alpha_0)$
- Procedure continued until $\Delta_{\mathrm{iteration}} = \sqrt{\frac{(\lambda_{\mathrm{n}+1}-\lambda_{\mathrm{n}})^2}{\lambda_{\mathrm{n}}^2} + \frac{(R_{\mathrm{n}+1}-R_{\mathrm{n}})^2}{R_{\mathrm{n}}^2} + \frac{(\alpha_{\mathrm{n}+1}-\alpha_{\mathrm{n}})^2}{\alpha_{\mathrm{n}}^2}} < 0.01$
- **Iterations usually converge within 2-3 rounds, fit parameters can be reliably extracted**

Coulomb correction and fitting of the corr. function

- Lévy-type correlation function without final state effects: $C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^{\alpha}}$
- Sinyukov method:

$$C(Q_{\mathrm{LCMS}}; \lambda, R_{LCMS}, \alpha) = \left(1 - \lambda + \lambda \cdot K(q_{inv}; \alpha, R_{\mathrm{PCMS}}) \cdot \left(1 + e^{-|R_{\mathrm{LCMS}} Q_{\mathrm{LCMS}}|^{\alpha}}\right)\right) \cdot N \cdot (1 + \varepsilon Q_{\mathrm{LCMS}})$$

Intercept parameter (correlation strength): λ

Coulomb correction: K

Lévy scale parameter: R_{LCMS}

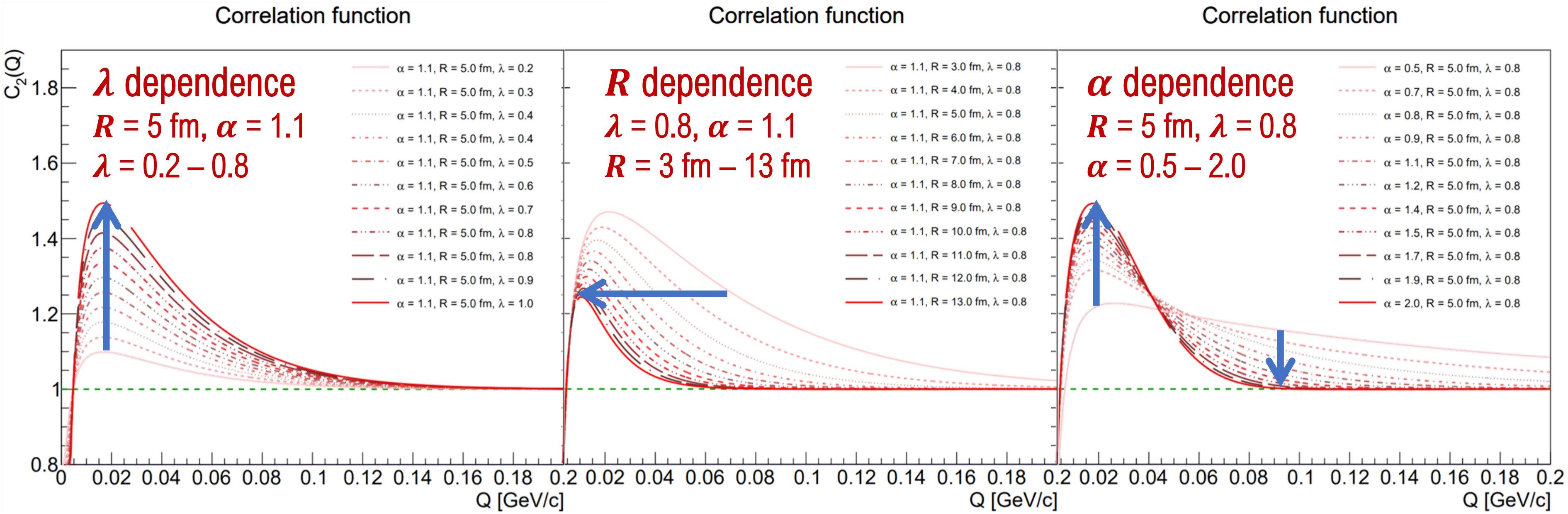
Lévy exponent: α

Possible linear background (usually negligible): $N \cdot (1 + \varepsilon Q_{\mathrm{LCMS}})$

- Coulomb-correction calculated numerically (in PCMS)

$$q_{\mathrm{inv}} \equiv q_{\mathrm{PCMS}} \approx q_{\mathrm{LCMS}} \cdot \sqrt{1 - \beta_T^2/3} \qquad R_{\mathrm{PCMS}} \approx R_{\mathrm{LCMS}} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}}$$

Shape of the correlation function

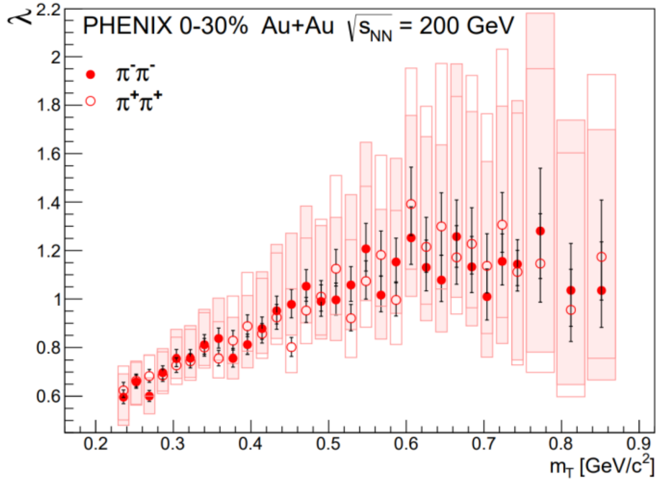


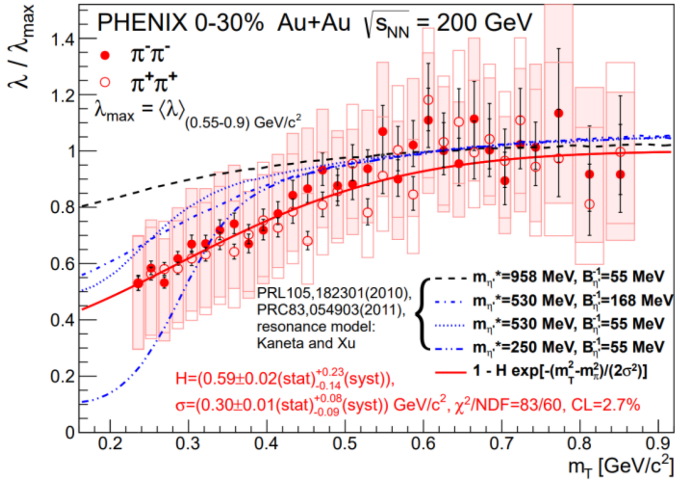
$$C_2(Q) = 1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)$$

PHENIX correlation strength parameter

- Without FSI: $C_2^{(0)}(Q=0)=2$
- Experimental resolution limits measurement to Q ~ few MeV
- $\lambda \equiv \lim_{Q\to 0} C_2^{(0)}(Q)-1$, experimentally often $\lambda < 1$
- **Core-halo picture**: $S = S_{core} + S_{halo} \Rightarrow D = D_{(c,c)} + D_{(c,h)} + D_{(h,h)}$
- $C_2(Q) = 1-\lambda+\lambda\int d^4r D_{(c,c)}(r)\left|\psi_Q^{(2)}(r)\right|^2$
- $\lambda = N_{core}^2/(N_{core}+N_{halo})^2$

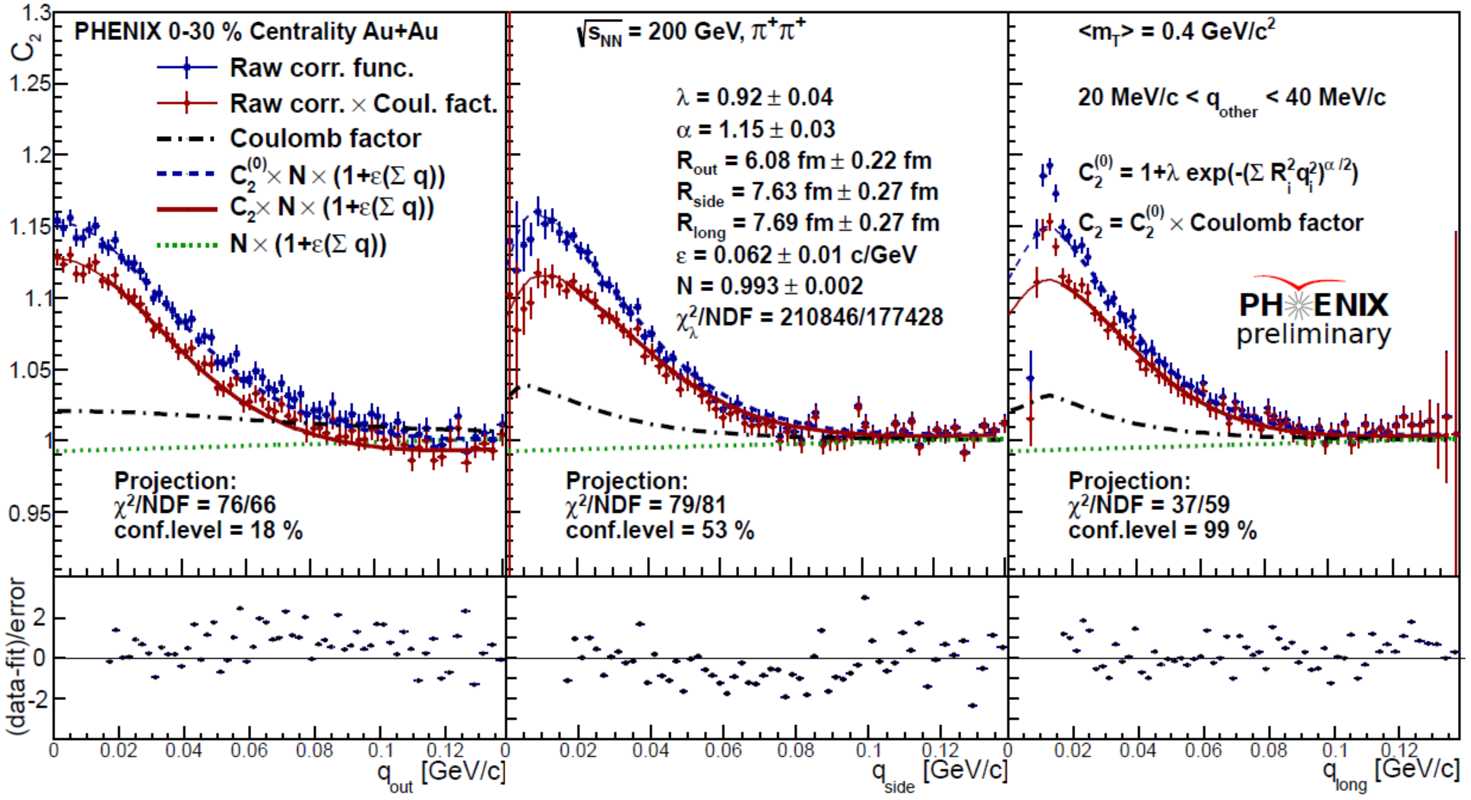
- Possible reasons behind $\lambda < 1$:
  - In-medium mass modification of η' meson
    Phys. Rev. Lett.81 (1998), pp. 2205–2208
  - Partially coherent particle emission
    Phys. Rev. D47 (1993), pp. 3860–3870





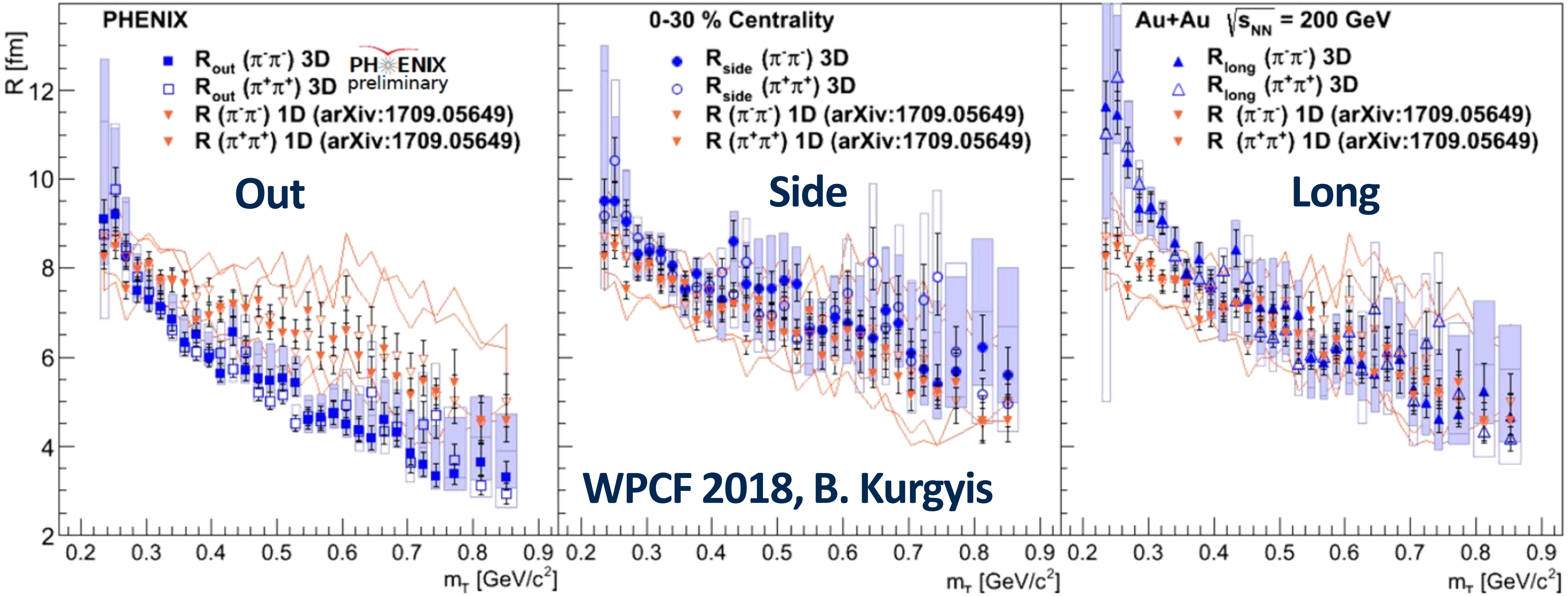
PHENIX 3D Lévy femtoscopy – example corr.func.

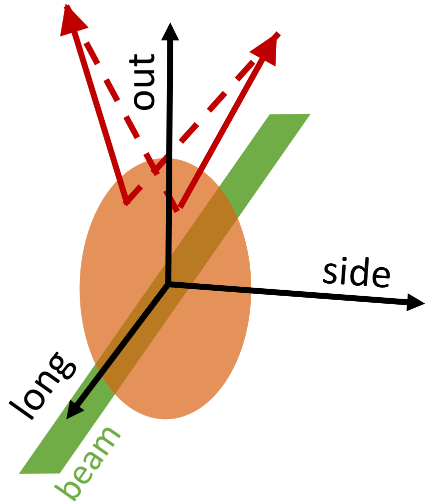
- Femtoscopy done in 3D: Bertsch-Pratt pair frame (out/side/long coordinates)
- Physical parameters: $R_{out,side,long}, \lambda, \alpha$ measured versus pair m_T
- Fit in this case: modified log-likelihood (small stat. in peak range)



PHENIX cross-check with 3D vs 1D

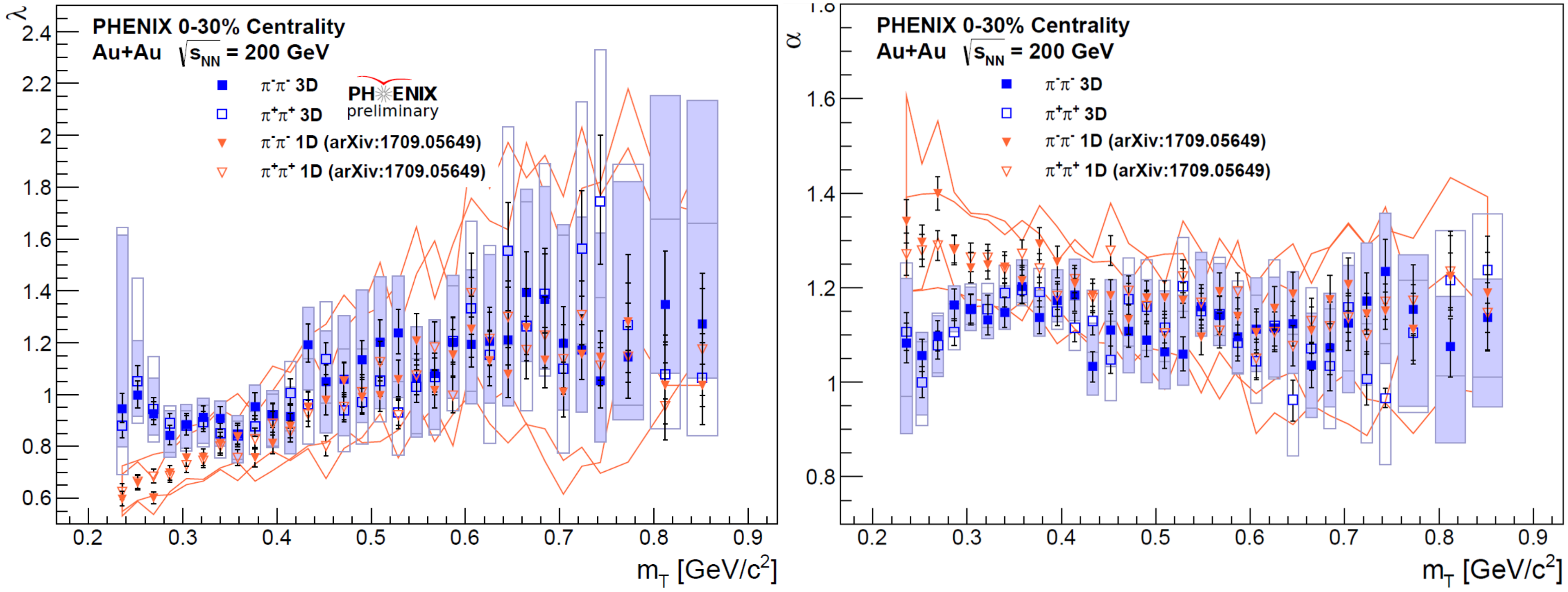
B. Kurgyis for the PHENIX Coll.,
Acta Phys. Pol. B Proc. Suppl. vol. 12 (2),
pp. 477 - 482 (2019)





- **Compatibility with 1D Lévy analysis**
- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS radius!
- There is no 2nd moment (variance or root mean square) for Lévy distributions with $\alpha < 2$!
- Asymmetric source for small m_T, validity of Coulomb-approximation?

PHENIX 3D versus 1D: strength λ and shape α



- **Compatible with 1D (Q_{LCMS}) measurement** of Phys. Rev. C 97, 064911 (2018)
- Small discrepancy at small m_T: due to large R_{long} at small m_T?