Probing Hydrodynamics in PbPb Collisions at 5.02 TeV using Higher-order Cumulants

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Outline

- Introduction
  - Anisotropic flow
  - Asymmetry of v_2 distribution

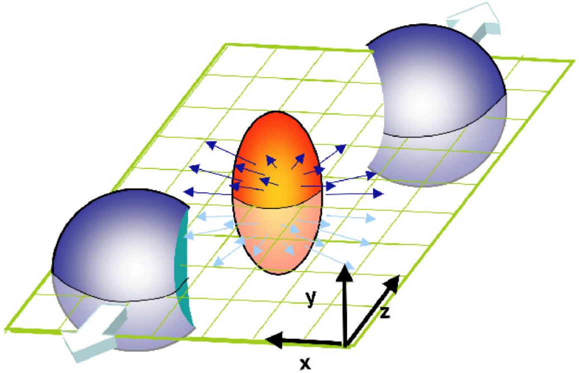
- Motivation
  - Hydrodynamic probes

- Analysis Technique
  - Collectivity - $v_2\{2k\}$ in terms of Q-cumulants in PbPb collisions

- Results
  - Measurement of two hydrodynamic probes vs centrality
  - High-precision measurement of central moments of v_2 distribution

- Summary

Anisotropic Flow

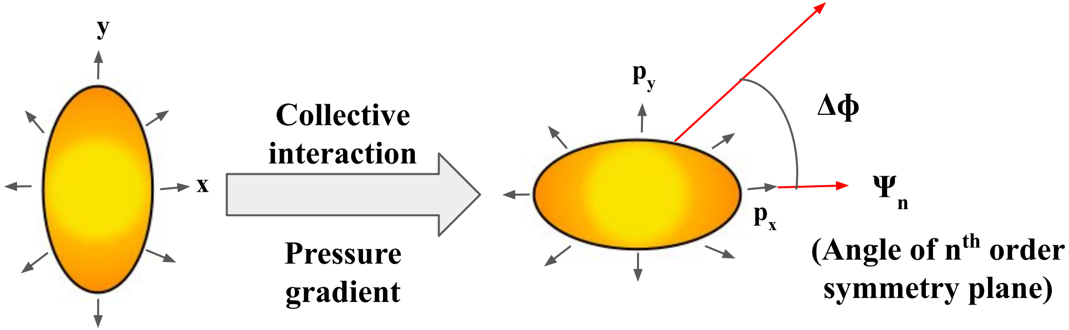


Azimuthal Anisotropy

$$\frac{2\pi}{N}\frac{dN}{d\phi} = 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\Delta\Phi)$$

$$\Delta\phi = \phi - \psi_n$$

$$\boxed{v_n \equiv < \cos[n(\phi - \psi_n)] >}$$



Collective interaction

Pressure gradient

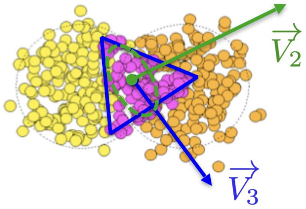
(Angle of nth order symmetry plane)

Initial state spatial anisotropy

Final state momentum anisotropy

v_n - Fourier flow harmonics depend on :

- initial state geometry
- initial state fluctuations
- medium transport properties (η/s)



$$\overrightarrow{V_m} = v_m e^{-im\Psi_m}$$

$$\overrightarrow{V_n} = v_n\, e^{-in\Psi_n}$$

Motivation - Asymmetry of the v_2 Distribution

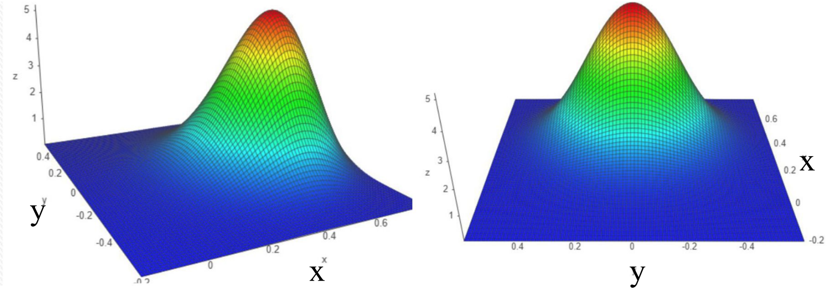
- Hydrodynamics : azimuthally anisotropic expansion of QGP formed in AA collisions
- Event-by-event fluctuations : the early stage dynamics
- Non-Gaussianities in e-by-e v_2 distribution
- Hydrodynamic expansion - $\mathbf{v_2 \propto \varepsilon_2}$
- **Fluctuations** present in initial state = **non-gaussianities** in v_2 distribution in final state
- Precise measurements of the Non-Gaussian flow fluctuations : test hydrodynamics and constrain IS models

$$\boldsymbol{v_2} = v_x \boldsymbol{e_x} + v_y \boldsymbol{e_y}$$

$$v_2 = \sqrt{v_x^2 + v_y^2}$$

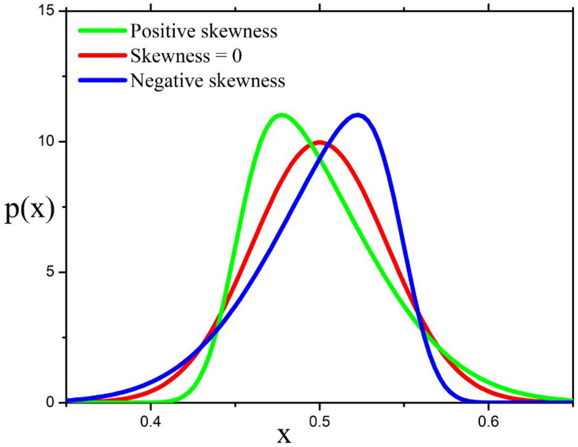
$\mathbf{e_x}$: along impact parameter

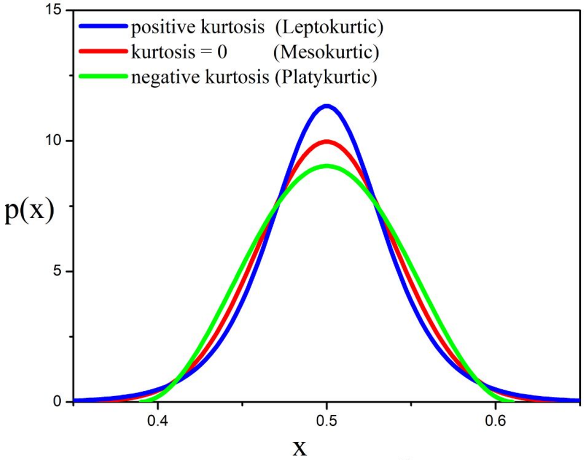
$\mathbf{e_y}$: perpendicular to impact parameter

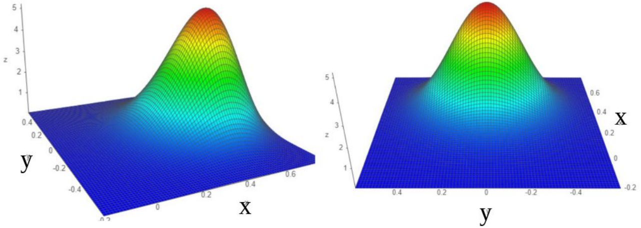


Motivation - Asymmetry of the v_2 Distribution

- Q-cumulants, $v_2\{2k\}$ (k = 1, 2, …) : good tool to study non-Gaussianities
- Non-Gaussianities : fine splitting between cumulants of different orders







- **Skewness** : degree of asymmetry of the distribution
- **Kurtosis** : degree of peakedness and flatness
- **Superskewness** : measure of asymmetry of tail

Motivation - Hydrodynamic Probes

- Hydrodynamic probes : **observed azimuthal angle correlations** ⟺ **initial-state geometry**

Phys. Rev. C 95 (2017) 014913

$$\boxed{\frac{v_2\{6\}-v_2\{8\}}{v_2\{4\}-v_2\{6\}} \approx \frac{1}{11} \approx 0.091}$$

→ ~~Centrality independent~~

(limited to leading order term)

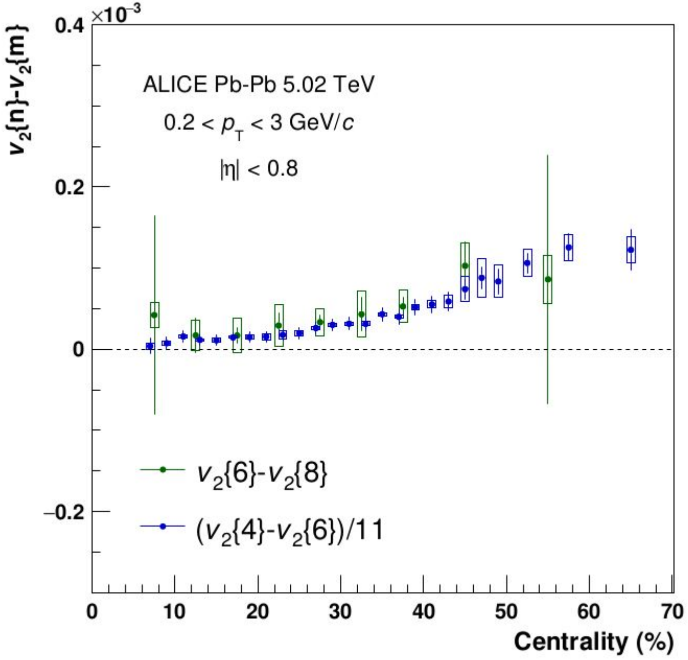
$$0.143 \pm 0.008(stat) \pm 0.014(syst)$$

: 20-25% centrality

$$0.185 \pm 0.005(stat) \pm 0.012(syst)$$

: 55-60% centrality

Phys. Lett. B 789 (2019) 643 (CMS)

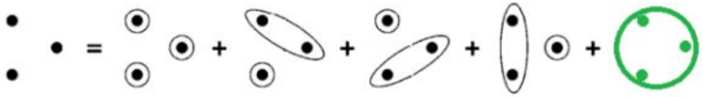


JHEP 07 (2018) 103

- Goal - to improve precision
- **Possible Solution** : **Introducing higher-order terms in the cumulant expansion of the v_2 distribution**
  - **large amount of statistics required** ✅

Measuring $v_n\{2k\}$ - Q-cumulant Method

- k-particle cumulant (k > 1) : **collective nature of flow**
  - suppresses non-flow



R. Kubo, J. Phys. Soc. Jpn. 17 (1962)

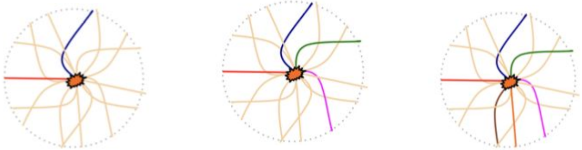
Q-Vector

$$Q_n = \sum_i e^{in\phi_i}$$

All-event average

$$<<2>> \equiv << e^{in(\phi_1-\phi_2)} >>$$

$$<<4>> \equiv << e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} >>$$



k = 1 k = 2 k = 3

[Courtesy : Y. Zhou, 2019]

Single-event average over all particles

$$<2> \equiv < e^{in(\phi_1-\phi_2)} > \quad \Longrightarrow <2> \equiv \frac{|Q_n|^2 - M}{M(M-1)}$$

$$<4> \equiv < e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} >$$

$$\Longrightarrow <4> \equiv \frac{|Q_n|^4 + |Q_{2n}|^2 - 2Re[Q_{2n}Q_n^*Q_n^*]}{M(M-1)(M-2)(M-3)}$$

$$-2\frac{2(M-2)|Q_n|^2 - M(M-3)}{M(M-1)(M-2)(M-3)}$$

$v_n\{10\}$ from Q-cumulants

- **10-particle azimuthal correlator** $\langle\langle 10\rangle\rangle = \langle\langle e^{in(\phi_1+\phi_2+\phi_3+\phi_4+\phi_5-\phi_6-\phi_7-\phi_8-\phi_9-\phi_{10}}\rangle\rangle$
- Recurrence relation (Phys. Rev. C 104 (2021) 034906) :

$$c_n\{2k\} = \langle\langle 2k\rangle\rangle - \sum_{m=1}^{k-1}\begin{pmatrix} k \\ m \end{pmatrix}\begin{pmatrix} k-1 \\ m \end{pmatrix}\langle\langle 2m\rangle\rangle\, c_n\{2k-2m\}$$

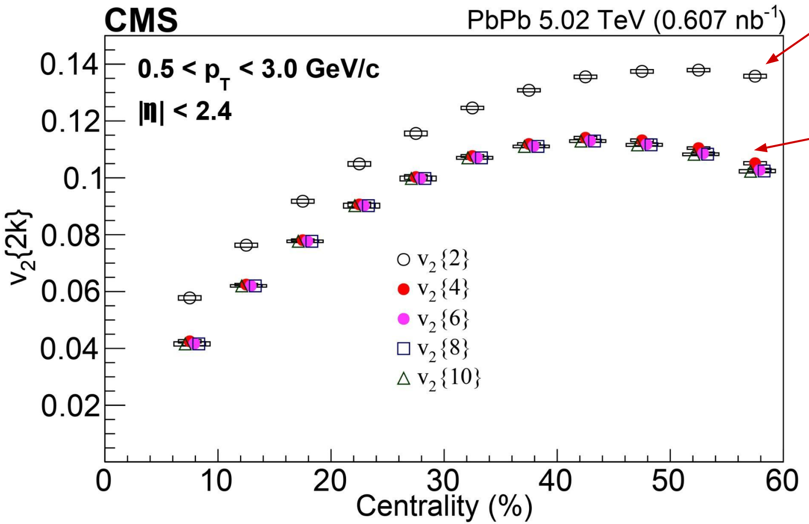
$$c_n\{10\} = \langle\langle 10\rangle\rangle - 25.\,\langle\langle 2\rangle\rangle\,\langle\langle 8\rangle\rangle - 100.\,\langle\langle 4\rangle\rangle\,\langle\langle 6\rangle\rangle + 400.\,\langle\langle 6\rangle\rangle\,\langle\langle 2\rangle\rangle^2 + 900.\,\langle\langle 2\rangle\rangle\,\langle\langle 4\rangle\rangle^2 \\ -3600.\,\langle\langle 4\rangle\rangle\,\langle\langle 2\rangle\rangle^3 + 2800.\,\langle\langle 2\rangle\rangle^5$$

$$\boxed{v_n\{10\} = \sqrt[10]{\frac{1}{456}c_n\{10\}}}$$

First-time measurement by CMS - huge amount of statistics!

$v_n\{2k\}$ (k = 1,...,5) vs centrality

JHEP02 (2024) 106



$v_2\{2\}$

$v_2\{2k\}$

Clear splitting between $v_2\{2\}$ and $v_2\{2k\}$ - larger towards more peripheral

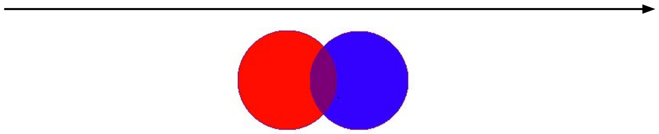
$$\boxed{v_2\{2\} > v_2\{4\} \gtrsim v_2\{6\} \gtrsim v_2\{8\} \gtrsim v_2\{10\}}$$

(v_2 variance)

Flow fluctuations : $v_2\{2\}^2 \approx v_2\{2k\}^2 + 2\sigma_v^2 (k > 1)$

- systematic uncertainties ~ 2 orders of magnitude greater than statistical ones - **high precision measurement**

Decreasing overlap region



First-time measurement of $v_2\{10\}$ by CMS

$v_n\{2k\}$ (k = 1,...,5) vs centrality

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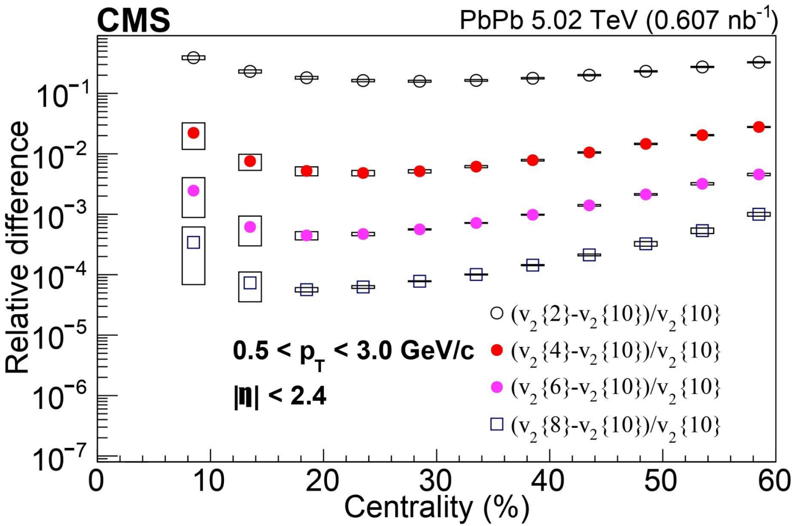
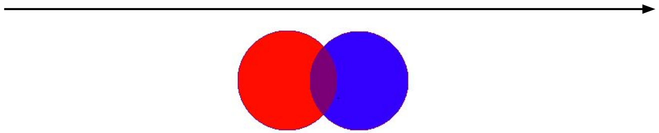


Decreasing overlap region



Fine splitting observed

$$v_2\{2\} > v_2\{4\} \gtrsim v_2\{6\} \gtrsim v_2\{8\} \gtrsim v_2\{10\}$$

Signature of non-Gaussian fluctuations

- The relative difference between adjacent $v_2\{2k\} - v_2\{10\}$ values decreases by about an order of magnitude for each increment in k

Introducing a new Hydrodynamic Probe

- $v_2\{2k\}$: Taylor expansion through **central moments** of the v_2 distribution (upto **5th moment**)

PRC 95 (2017) 014913,
PRC 99 (2019) 014907

(mean, variance, skewness, kurtosis, superkurtosis)

+

Assuming $\sigma_x^2 = \sigma_y^2$ ⟹ **hydrodynamic probes in terms of $v_2\{2k\}$**

1.
$$\frac{v_2\{6\}-v_2\{8\}}{v_2\{4\}-v_2\{6\}} \approx \frac{1}{11} - \frac{1}{11}\frac{v_2\{4\}^2-12v_2\{6\}^2+11v_2\{8\}^2}{v_2\{4\}^2-v_2\{6\}^2+\cancel{\frac{(\sigma_y^2-\sigma_x^2)s_{30}}{3\overline{v_2}^3}}}$$

neglecting this term

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2.
$$\frac{v_2\{8\}-v_2\{10\}}{v_2\{6\}-v_2\{8\}} \approx \frac{3}{19} - \frac{1}{19}\frac{3v_2\{6\}^2-22v_2\{8\}^2+19v_2\{10\}^2}{v_2\{6\}^2-v_2\{8\}^2+\cancel{\frac{(\sigma_y^2-\sigma_x^2)s_{30}}{33\overline{v_2}^3}}}$$

introduction of higher order term

First-time measurement by CMS

neglecting this term

Taylor Expansion of Hydrodynamic Probes

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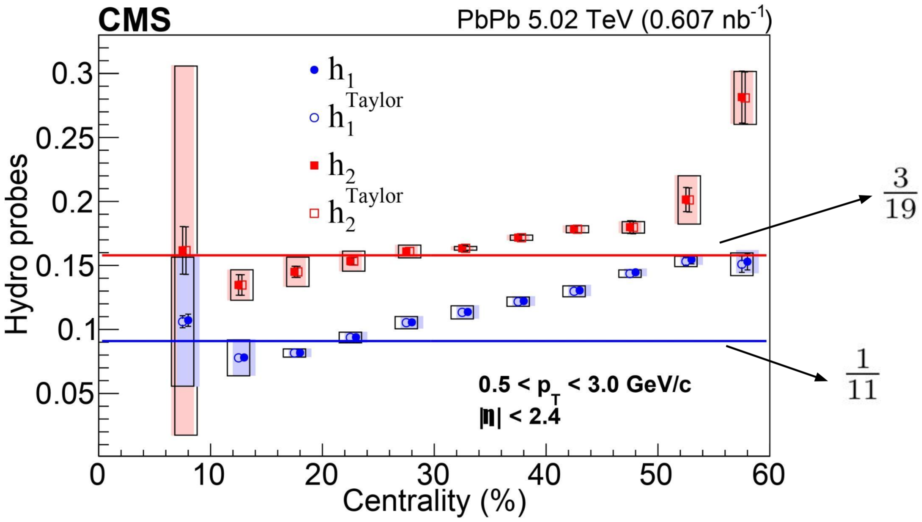

Clear dependence on centrality : Higher-order moments necessary to describe data

$$h_1 = \frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \qquad h_2 = \frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}}$$

Expansion using higher-order moments :

$$h_1^{Taylor} = \frac{1}{11} - \frac{1}{11}\frac{v_2\{4\}^2 - 12v_2\{6\}^2 + 11v_2\{8\}^2}{v_2\{4\}^2 - v_2\{6\}^2}$$

$$h_2^{Taylor} = \frac{3}{19} - \frac{1}{19}\frac{3v_2\{6\}^2 - 22v_2\{8\}^2 + 19v_2\{10\}^2}{v_2\{6\}^2 - v_2\{8\}^2}$$

First-time measurement by CMS

Taylor Expansion of Hydrodynamic Probes

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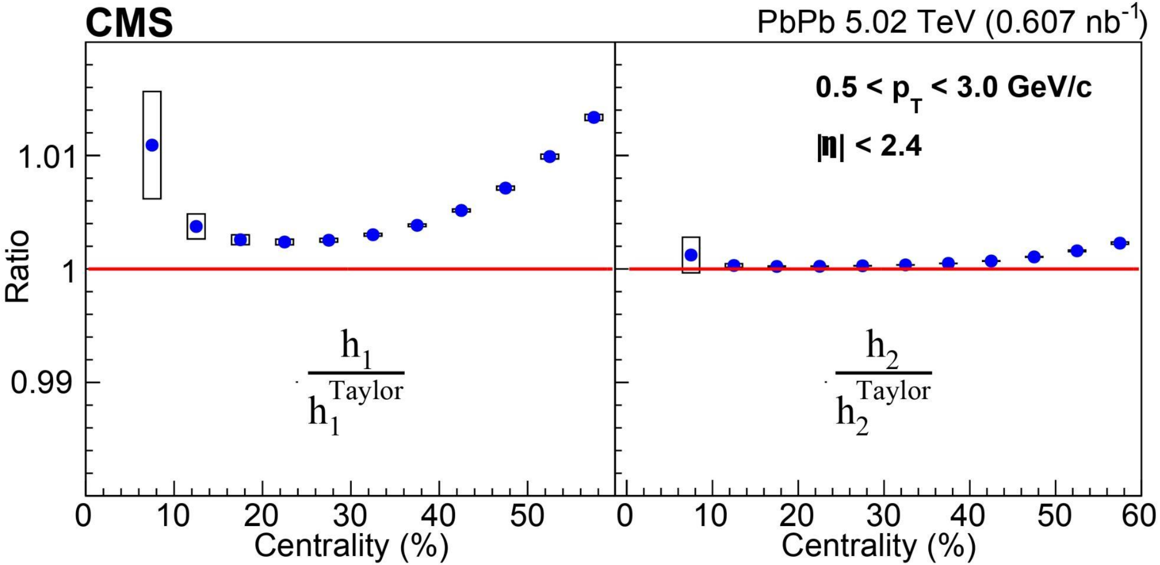


$$\frac{h_1}{h_1^{Taylor}} \approx 1.000 \pm 0.013$$
$$\frac{h_2}{h_2^{Taylor}} \approx 1.000 \pm 0.003$$

small contribution from the term $(\sigma_x^2 - \sigma_y^2)$, but still negligible

- Systematic uncertainties >> statistical uncertainties
- $\frac{h_2}{h_2^{Taylor}}$ is closer to unity

Standardized and Corrected Moments

- Non-Gaussian fluctuations explain fine-splitting : $v_2\{2\} > v_2\{4\} \gtrsim v_2\{6\} \gtrsim v_2\{8\} \gtrsim v_2\{10\}$
- "Standardized" skewness : $\gamma_1^{\text{exp}} = -2^{3/2}\frac{v_2^3\{4\}-v_2^3\{6\}}{\left[v_2^2\{2\}-v_2^2\{4\}^{3/2}\right]} \approx -2^{3/2}\frac{-s_{30}-O_N}{[2\sigma_x^2+O_D]^{3/2}} \approx \frac{s_{30}}{\sigma_x^3} = \gamma_1$
- Contributions from other moments :

Phys. Rev. C 99 (2019) 014907

non-negligible

$$O_N = \frac{3(\kappa_{40}+\kappa_{22})}{2\bar{v_2}} - \frac{3(p_{50}+2p_{32}+p_{14})}{4\bar{v_2}^2} + \frac{3(\sigma_y^2-\sigma_x^2)(s_{30}-2s_{12})}{2\bar{v_2}^2} + \dots$$

$$O_D = \frac{2}{\bar{v_2}}(s_{30}+s_{12}) + \frac{\kappa_{40}+2\kappa_{22}+\kappa_{04}}{2\bar{v_2}^2} + \frac{(\sigma_y^2-\sigma_x^2)^2}{\bar{v_2}^2} - 2\frac{(\sigma_y^2-\sigma_x^2)(s_{30}-s_{12})}{\bar{v_2}^3} + \dots$$

- "Corrected" skewness : free from contributions of other moments (eg. kurtosis, superskewness, …)

$$\gamma_{1,corr}^{exp} = -2^{3/2}\frac{-s_{30}+3\frac{(\sigma_y^2-\sigma_x^2)(s_{30}-2s_{12})}{2\bar{v_2}^2}+o(>5)}{\left(2\sigma_x^2+\frac{(\sigma_y^2-\sigma_x^2)^2}{\bar{v_2}^2}+o(>5)\right)^{3/2}} = -2^{3/2}\frac{187v_2^3\{8\}-16v_2^3\{6\}-171v_2^3\{10\}}{[v_2^2\{2\}-40v_2^2\{6\}+495v_2^2\{8\}-456v_2^2\{10\}]^{3/2}}$$ ✅

Free of other moments (up to 5th order)

Standardized and Corrected Moments

- "Standardized" kurtosis : $\gamma_2^{exp} = -\frac{3}{2}\frac{v_2^4\{4\}-12v_2^4\{6\}+11v_2^4\{8\}}{[v_2^2\{2\}-v_2^2\{4\}]^2}$
- "Corrected" kurtosis :

$$\gamma_{2,corr}^{exp} = -\frac{3}{2}\frac{v_2^4\{4\}+24v_2^4\{6\}-253v_2^4\{8\}+228v_2^4\{10\}}{[v_2^2\{2\}-40v_2^2\{6\}+495v_2^2\{8\}-456v_2^2\{10\}]^2}$$

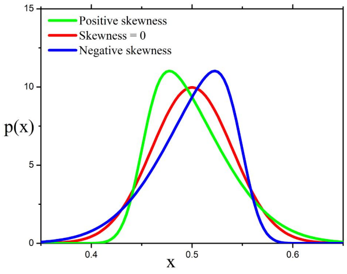
- "Standardized" superskewness : $\gamma_3^{exp} = 6\sqrt{2}\frac{3v_2^5\{6\}-22v_2^5\{8\}+19v_2^5\{10\}}{[v_2^2\{2\}-v_2^2\{4\}]^{5/2}}$
- "Corrected" superskewness :

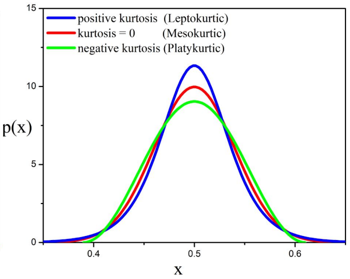
$$\gamma_{3,corr}^{exp} = 6\sqrt{2}\frac{3v_2^5\{6\}-22v_2^5\{8\}+19v_2^5\{10\}}{[v_2^2\{2\}-40v_2^2\{6\}+495v_2^2\{8\}-456v_2^2\{10\}]^{5/2}}$$

- **Free of other moments (up to 5th order)**
- **Additional constraints on initial-state geometry -** "Cleaning" conditions require elliptic power distribution, with : $\varepsilon_0 < 0.15$ and $v_n \propto \varepsilon_n$

[Phys. Rev. C 90, 024903 (2014)]

Standardized and Corrected Moments





First-time measurement by CMS

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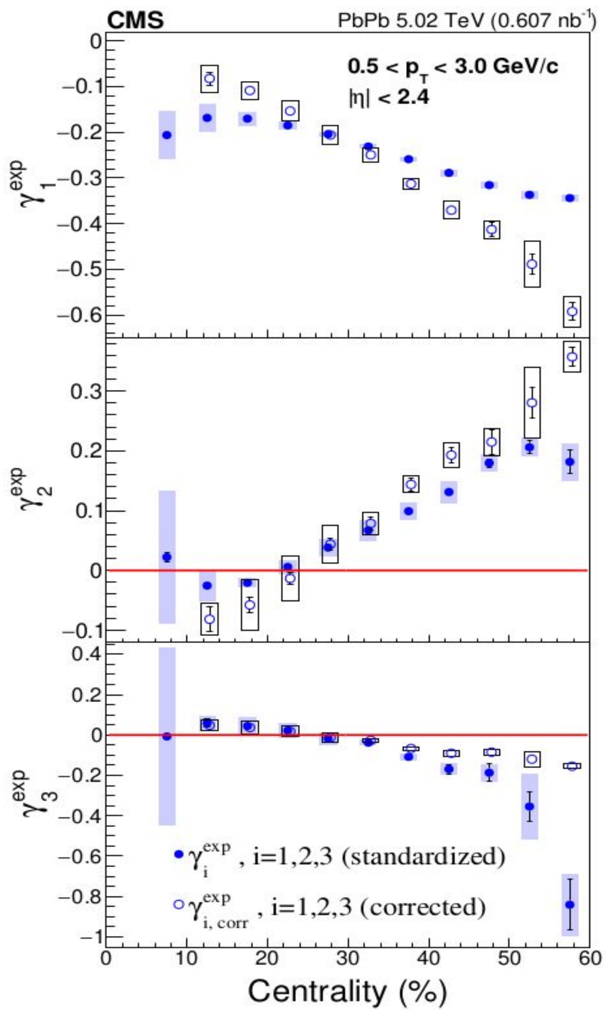
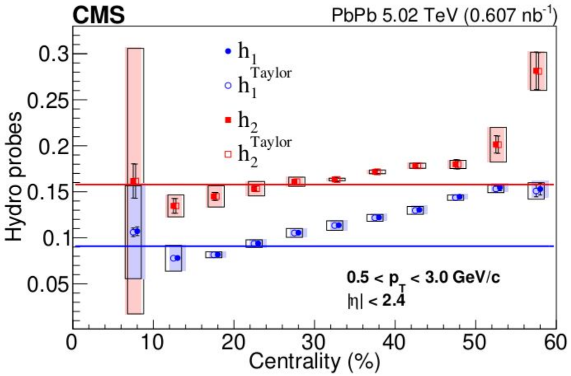
- Negative values of skewness for all centralities - v_2 distribution has longer tail to the left
  - corrected skewness is steeper
- Kurtosis - negative for most central events, positive towards peripheral
  - qualitatively agrees with theory predictions [Phys. Rev. C 99, 014907 (2019)]
- Superskewness - measured for the first time
  - positive towards more central region, then becomes negative



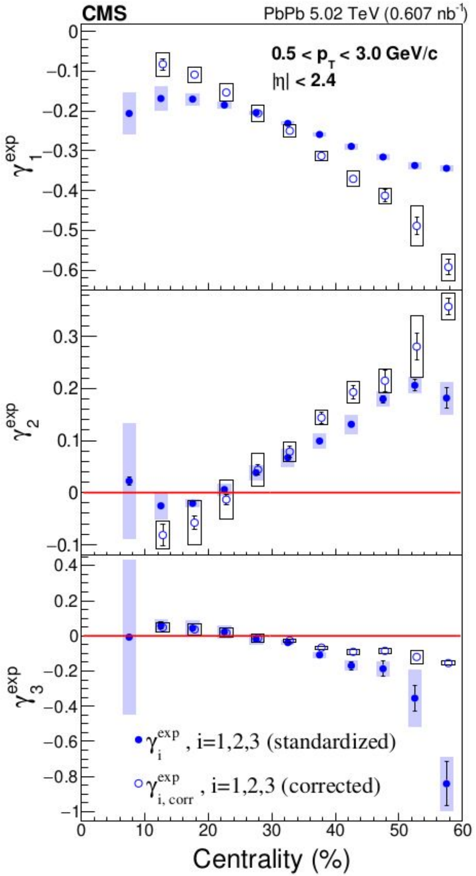
Summary



- Two hydrodynamics probes and first-time measurement of $v_2\{10\}$ performed with CMS PbPb data at 5.02 TeV energy

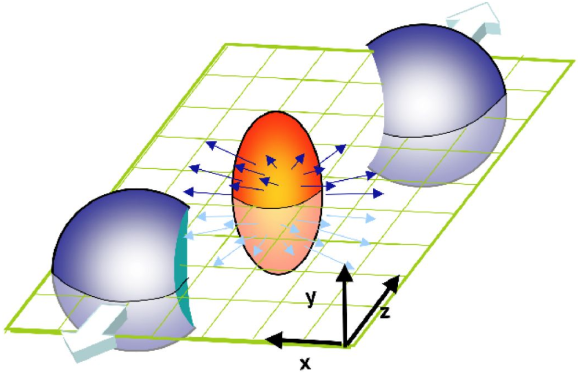
- High precision measurement of skewness, kurtosis and superskewness of the v_2 distribution

- Can provide novel constraints on the initial state geometry used in hydrodynamic calculations of the medium expansion in high energy nuclear collisions



Backup

$v_n\{2k\}$ from Q-cumulants



$$\langle 2m\rangle = \frac{1}{P_{M,2m}} \sum_{i_1\neq..i_{2m}=1}^{M} e^{in(\phi_{i_1}+\ldots+\phi_{i_m}-\phi_{i_{m+1}}-\ldots-\phi_{i_{2m}})}$$

$$P_{M,2m} = \frac{M!}{(M-2m)!}$$

Averaging over M particles in a single event

$\langle\langle\ldots\rangle\rangle$ Averaging over all events

Multi-particle correlations $\Longrightarrow$ **cumulants**

$$\langle\langle 2\rangle\rangle = \langle\langle e^{in(\phi_1-\phi_2)}\rangle\rangle \qquad \langle\langle 4\rangle\rangle = \langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle$$

$$\langle\langle 6\rangle\rangle = \langle\langle e^{in(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)}\rangle\rangle$$

$$\langle\langle 8\rangle\rangle = \langle\langle e^{in(\phi_1+\phi_2+\phi_3+\phi_4-\phi_5-\phi_6-\phi_7-\phi_8)}\rangle\rangle$$

$$c_n\{2\} = \langle\langle 2\rangle\rangle \qquad c_n\{4\} = \langle\langle 4\rangle\rangle - 2\cdot\langle\langle 2\rangle\rangle^2$$

$$c_n\{6\} = \langle\langle 6\rangle\rangle - 9\cdot\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle + 12\cdot\langle\langle 2\rangle\rangle^3$$

$$c_n\{8\} = \langle\langle 8\rangle\rangle - 16\cdot\langle\langle 6\rangle\rangle\langle\langle 2\rangle\rangle - 18\cdot\langle\langle 4\rangle\rangle^2 + \\ +144\cdot\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle^2 - 144\cdot\langle\langle 2\rangle\rangle^4$$

$$v_n\{2\} = \sqrt{c_n\{2\}} \qquad v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$

$$v_n\{6\} = \sqrt[6]{\frac{1}{4}c_n\{6\}} \qquad v_n\{8\} = \sqrt[8]{-\frac{1}{33}c_n\{8\}}$$

Measuring $v_n\{2k\}$ - Q-cumulant Method

General formulae :

$$c_n\{2k\} = \langle\langle 2k \rangle\rangle - \sum_{m=1}^{k-1} \binom{k}{m} \binom{k-1}{m} \langle\langle 2m \rangle\rangle c_n\{2k-2m\}$$

$$v_n = \sqrt[2k]{a_{2k}^{-1} c_n\{2k\}}.$$

$$a_{2k} = 1 - \sum_{m=1}^{k-1} \binom{k}{m} \binom{k-1}{m} a_{2k-2m}, \quad \text{with } a_2 = 1.$$

Phys. Rev. C 104 (2021) 034906
Phys. Rev. C 95 (2017) 014913
Phys.Rev.C 64 (2001) 054901
JHEP02 (2024) 106

Can be extended till any order

Example :

$$c_n\{2\} = \langle\langle 2 \rangle\rangle$$
$$c_n\{4\} = \langle\langle 4 \rangle\rangle - 2\,\langle\langle 2 \rangle\rangle^2$$

$$v_n\{2\} = \sqrt[2]{c_n\{2\}}$$
$$v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$

$$c_n\{10\} = \langle\langle 10 \rangle\rangle - 25\cdot\langle\langle 2 \rangle\rangle\langle\langle 8 \rangle\rangle - 100\cdot\langle\langle 4 \rangle\rangle\langle\langle 6 \rangle\rangle + 400\cdot\langle\langle 6 \rangle\rangle\langle\langle 2 \rangle\rangle^2 + 900\cdot\langle\langle 2 \rangle\rangle\langle\langle 4 \rangle\rangle^2$$
$$-3600\cdot\langle\langle 4 \rangle\rangle\langle\langle 2 \rangle\rangle^3 + 2880\cdot\langle\langle 2 \rangle\rangle^5$$

$$\boxed{v_n\{10\} = \sqrt[10]{\frac{1}{456} c_n\{10\}}}$$

First-time measurement by CMS - huge amount of statistics!

<10> in terms of Q-Cumulants

- size of formula increases with the order k

$$c_n\{10\} = \langle\langle 10\rangle\rangle - 25\cdot\langle\langle 2\rangle\rangle\langle\langle 8\rangle\rangle - 100\cdot\langle\langle 4\rangle\rangle\langle\langle 6\rangle\rangle + 400\cdot\langle\langle 6\rangle\rangle\langle\langle 2\rangle\rangle^2 + 900\cdot\langle\langle 2\rangle\rangle\langle\langle 4\rangle\rangle^2$$
$$-3600\cdot\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle^3 + 2880\cdot\langle\langle 2\rangle\rangle^5$$

$$v_n\{10\} = \sqrt[10]{\frac{1}{456}c_n\{10\}}$$

$$< 10 >= \frac{|Q_n|^{10} - 20Re[Q_{2n}|Q_n|^6 Q_n^* Q_n^*] + 100|Q_{2n}|^2|Q_n|^6 + 30Re[Q_{2n}Q_{2n}|Q_n|^2 Q_n^* Q_n^* Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} +$$

$$+\frac{225|Q_{2n}|^4|Q_n|^2 - 300Re[Q_{2n}|Q_{2n}|^2|Q_n|^2 Q_n^* Q_n^*] + 40Re[Q_{3n}|Q_n|^4 Q_n^* Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} +$$

$$+\frac{600Re[Q_{3n}Q_n|Q_n|^2 Q_{2n}^* Q_{2n}^*] - 400Re[Q_{3n}|Q_n|^4 Q_n^* Q_{2n}^*] + 400|Q_{3n}|^2|Q_n|^4}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} +$$

$$+\frac{400Re[Q_{3n}|Q_{2n}|^2 Q_n^* Q_n^* Q_n^*] - 40Re[Q_{3n}Q_{2n}Q_n^* Q_n^* Q_n^* Q_n^* Q_n^*] - 600Re[Q_{3n}|Q_{2n}|^2 Q_n^* Q_{2n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} +$$

$$+\frac{400|Q_{3n}|^2|Q_{2n}|^2 - 800Re[|Q_{3n}|^2 Q_{2n}Q_n^* Q_n^*] - 60Re[Q_{4n}|Q_n|^2 Q_n^* Q_n^* Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} +$$

$$+\frac{600Re[Q_{4n}|Q_n|^2 Q_n^* Q_n^* Q_{2n}^*] - 900Re[Q_{4n}|Q_n|^2 Q_{2n}^* Q_{2n}^*] - 1200Re[Q_{4n}|Q_n|^2 Q_n^* Q_{3n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} +$$

$$+\frac{1200Re[Q_{4n}Q_n Q_{2n}^* Q_{3n}^*] + 900|Q_{4n}|^2|Q_n|^2 + 48Re[Q_{5n}Q_n^* Q_n^* Q_n^* Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} +$$

$$+\frac{720Re[Q_{5n}Q_n^* Q_{2n}^* Q_{2n}^*] - 480Re[Q_{5n}Q_n^* Q_n^* Q_n^* Q_{2n}^*] + 960Re[Q_{5n}Q_n^* Q_n^* Q_{3n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} +$$

$$+\frac{576|Q_{5n}|^2 - 960Re[Q_{5n}Q_{2n}^* Q_{3n}^*] - 1440Re[Q_{5n}Q_n^* Q_{4n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} +$$

$$+\frac{300Re[Q_{2n}|Q_n|^4 Q_n^* Q_n^*] - 25|Q_n|^8 - 900|Q_{2n}|^2|Q_n|^4 - 150Re[Q_{2n}Q_{2n}Q_n^* Q_n^* Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} +$$

$$+\frac{900Re[Q_{2n}|Q_{2n}|^2 Q_n^* Q_n^*] - 225|Q_{2n}|^4 - 400Re[Q_{3n}|Q_n|^2 Q_n^* Q_n^* Q_n^*] + 2400Re[Q_{3n}|Q_n|^2 Q_n^* Q_{2n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} +$$

$$+\frac{300Re[Q_{4n}Q_n^* Q_n^* Q_n^* Q_n^*] - 1200Re[Q_{3n}Q_n Q_{2n}^* Q_{2n}^*] - 1600|Q_{3n}|^2|Q_n|^2 - 1800Re[Q_{4n}Q_n^* Q_n^* Q_{2n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} +$$

$$+\frac{900Re[Q_{4n}Q_{2n}^* Q_{2n}^*] + 2400Re[Q_{4n}Q_n^* Q_{3n}^*] - 900|Q_{4n}|^2}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} +$$

$$+\frac{200|Q_n|^6 - 1200Re[Q_{2n}|Q_n|^2 Q_n^* Q_n^*] + 1800|Q_{2n}|^2|Q_n|^2}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-7)(M-8)} +$$

$$+\frac{800Re[Q_{3n}Q_n^* Q_n^* Q_n^*] - 2400Re[Q_{3n}Q_n^* Q_{2n}^*] + 800|Q_{3n}|^2}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-7)(M-8)} +$$

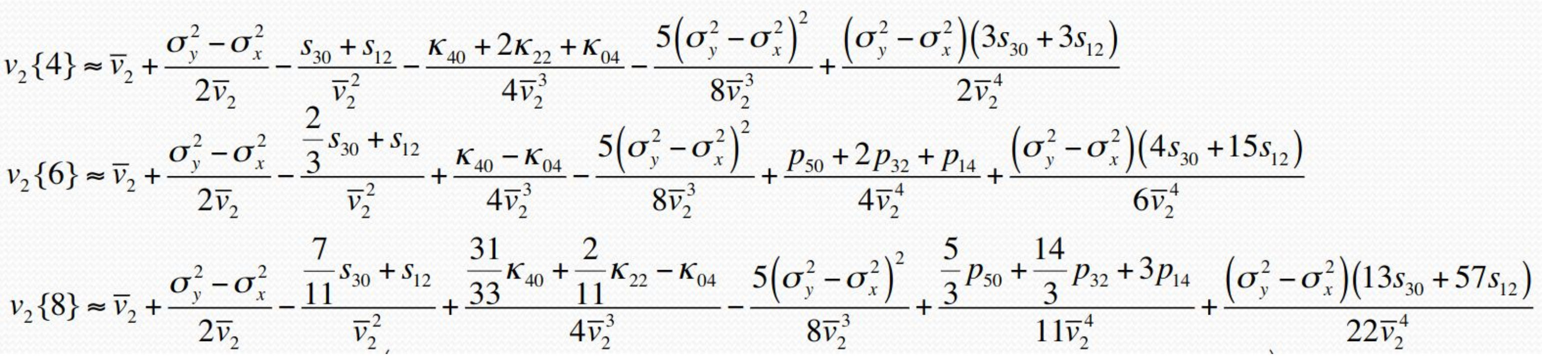
$$+\frac{1200Re[Q_{2n}Q_n^* Q_n^*] - 600|Q_n|^4 - 600|Q_{2n}|^2}{M(M-1)(M-2)(M-3)(M-5)(M-6)(M-7)} +$$

$$+\frac{600|Q_n|^2}{M(M-1)(M-3)(M-4)(M-5)(M-6)} - \frac{120}{(M-1)(M-2)(M-3)(M-4)(M-5)}$$

Taylor Expansion of Hydrodynamic Probes

- **Moments of v_2 distribution :**
  - Variance - 2nd moment $\sigma_x^2 = \langle (v_x - < v_x >)^2 \rangle$, $\sigma_y^2 = \langle v_y^2 \rangle$ - (i)
  - Skewness - 3rd moment $s_{30} = \langle (v_x - < v_x >)^3 \rangle$, $s_{12} = \langle (v_x - < v_x >) v_y^2 \rangle$ - (ii)
  - Kurtosis - 4th moment $\kappa_{40} = \left\langle (v_x - \langle v_x \rangle)^4 \right\rangle$, $\kappa_{22} = \left\langle (v_x - \langle v_x \rangle)^2 v_y^2 \right\rangle - \sigma_x^2 \sigma_y^2$ - (iii)

Phys. Rev. C 95, 014913 (2017), Phys. Rev. C 64, 054901 (2001) :



$$v_2\{4\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{s_{30} + s_{12}}{\bar{v}_2^2} - \frac{\kappa_{40} + 2\kappa_{22} + \kappa_{04}}{4\bar{v}_2^3} - \frac{5\left(\sigma_y^2 - \sigma_x^2\right)^2}{8\bar{v}_2^3} + \frac{\left(\sigma_y^2 - \sigma_x^2\right)\left(3s_{30} + 3s_{12}\right)}{2\bar{v}_2^4}$$

- (iv)

$$v_2\{6\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{\frac{2}{3}s_{30} + s_{12}}{\bar{v}_2^2} + \frac{\kappa_{40} - \kappa_{04}}{4\bar{v}_2^3} - \frac{5\left(\sigma_y^2 - \sigma_x^2\right)^2}{8\bar{v}_2^3} + \frac{p_{50} + 2p_{32} + p_{14}}{4\bar{v}_2^4} + \frac{\left(\sigma_y^2 - \sigma_x^2\right)\left(4s_{30} + 15s_{12}\right)}{6\bar{v}_2^4}$$

- (v)

$$v_2\{8\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{\frac{7}{11}s_{30} + s_{12}}{\bar{v}_2^2} + \frac{\frac{31}{33}\kappa_{40} + \frac{2}{11}\kappa_{22} - \kappa_{04}}{4\bar{v}_2^3} - \frac{5\left(\sigma_y^2 - \sigma_x^2\right)^2}{8\bar{v}_2^3} + \frac{\frac{5}{3}p_{50} + \frac{14}{3}p_{32} + 3p_{14}}{11\bar{v}_2^4} + \frac{\left(\sigma_y^2 - \sigma_x^2\right)\left(13s_{30} + 57s_{12}\right)}{22\bar{v}_2^4}$$

- (vi)

Taylor Expansion of Hydrodynamic Probes

$$\frac{v_2\{6\}-v_2\{8\}}{v_2\{4\}-v_2\{6\}} \approx \frac{1}{11}\left(1-\frac{4\kappa_{40}+\frac{8(p_{50}+p_{32})}{\bar{v}_2}}{2\bar{v}_2 s_{30}+3(\kappa_{40}+\kappa_{22})+\frac{3(p_{50}+2p_{32}+p_{14})-2(\sigma_y^2-\sigma_x^2)(5s_{30}-6s_{12})}{2\bar{v}_2}}\right)$$

Using eqs. (iv), (v) and (vi) :

$$\underbrace{\frac{v_2\{6\}-v_2\{8\}}{v_2\{4\}-v_2\{6\}}}_{h_1} \approx \frac{1}{11} - \underbrace{\frac{1}{11}\frac{v_2\{4\}^2-12v_2\{6\}^2+11v_2\{8\}^2}{v_2\{4\}^2-v_2\{6\}^2+\cancel{\frac{(\sigma_y^2-\sigma_x^2)s_{30}}{3v_2^3}}}}_{h_1^{\text{Taylor}}}$$

negligible

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Taylor Expansion of Hydrodynamic Probes

- 5th moment - $p_{50}=\left\langle \left(v_x-\langle v_x\rangle\right)^5\right\rangle-10\sigma_x^2 s_{30}$

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Phys. Rev. C 64, 054901 (2001)

$$p_{32}=\left\langle \left(v_x-\langle v_x\rangle\right)^3 v_y^2\right\rangle-\sigma_y^2 s_{30}-3\sigma_x^2 s_{12} \qquad p_{14}=\left\langle \left(v_x-\langle v_x\rangle\right)v_y^4\right\rangle-6\sigma_y^2 s_{12}$$

$$v_2\{10\}\approx\bar{v}_2+\frac{\sigma_y^2-\sigma_x^2}{2\bar{v}_2}-\frac{\frac{12}{19}s_{30}+s_{12}}{\bar{v}_2^2}+\frac{\frac{53}{57}\kappa_{40}+\frac{4}{19}\kappa_{22}-\kappa_{04}}{4\bar{v}_2^3}-\frac{5\left(\sigma_y^2-\sigma_x^2\right)^2}{8\bar{v}_2^3}+\frac{\frac{163}{60}p_{50}+\frac{47}{6}p_{32}+\frac{21}{4}p_{14}}{19\bar{v}_2^4}+\frac{\left(\sigma_y^2-\sigma_x^2\right)\left(11s_{30}+\frac{99}{2}s_{12}\right)}{19\bar{v}_2^4}$$

$$\frac{v_2\{8\}-v_2\{10\}}{v_2\{6\}-v_2\{8\}}\approx\frac{3}{19}-\frac{88p_{50}}{95\left[4\bar{v}_2^2 s_{30}-2\bar{v}_2(\kappa_{40}-3\kappa_{22})-13(p_{50}+10p_{32}-3p_{14})-2(\sigma^2-\sigma_x^2)(5s_{30}-6s_{32})\right]}$$

New hydrodynamic probe :

$$\frac{v_2\{8\}-v_2\{10\}}{v_2\{6\}-v_2\{8\}}\approx\frac{3}{19}-\frac{1}{19}\,\frac{3v_2\{6\}^2-22v_2\{8\}^2+19v_2\{10\}^2}{v_2\{6\}^2-v_2\{8\}^2+\cancel{\frac{(\sigma_y^2-\sigma_x^2)s30}{33\overline{v_2}^3}}}$$

h_2

h_2^{Taylor}

Standardized and Corrected Moments

- Condition for "cleaning" : $s_{12} \approx \frac{s_{30}}{3} \quad \kappa_{22} \approx \frac{\kappa_{40}}{3} \quad p_{32} \approx p_{14} \approx \frac{p_{50}}{5}$

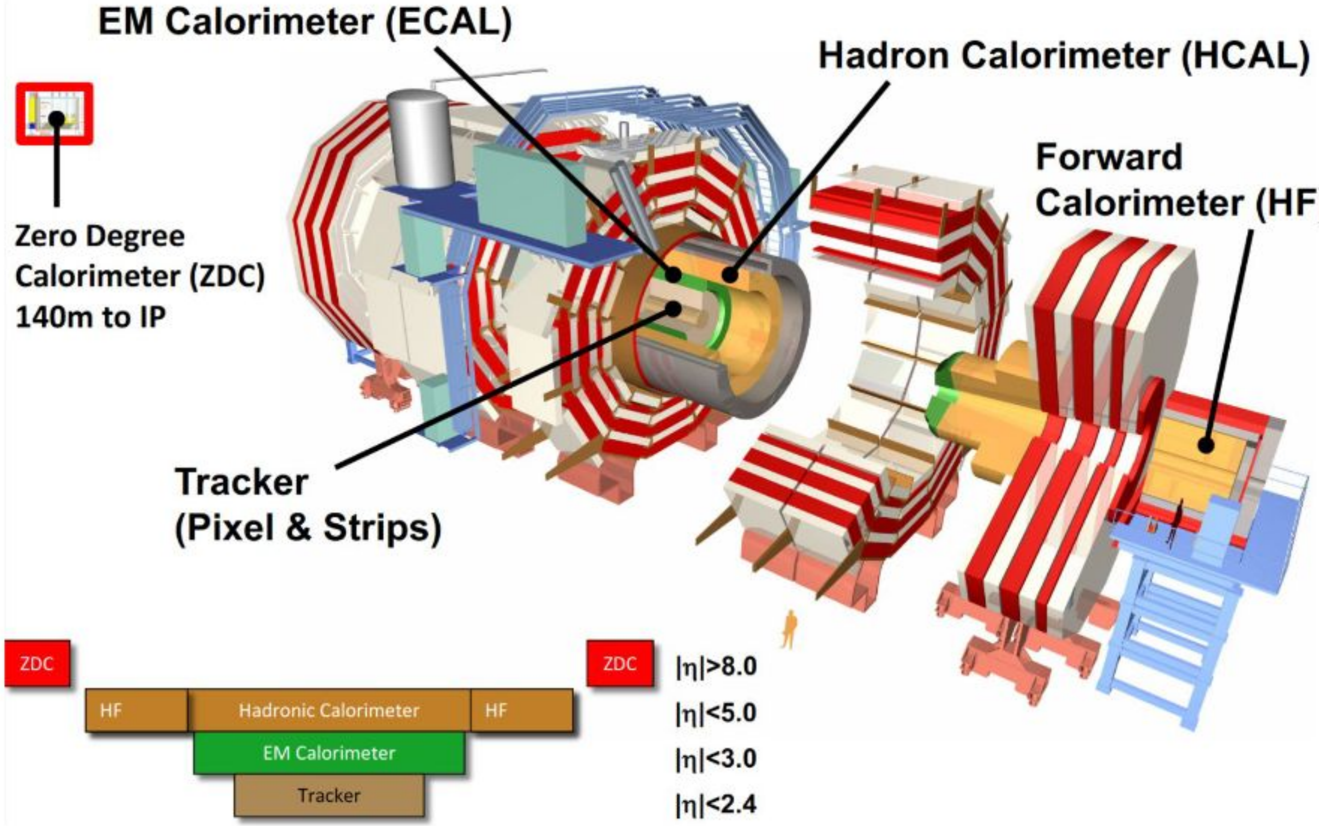
$$\varepsilon_0 \equiv \langle y_j^2 - x_j^2 \rangle / \langle y_j^2 + x_j^2 \rangle$$

Elliptic power distribution, ellipticity parameter : $\varepsilon_0 < 0.15$

Phys. Rev. C 90, 024903 (2014)

$$p(\varepsilon_x, \varepsilon_y) = \frac{\alpha}{\pi}(1 - \varepsilon_0^2)^{\alpha+\frac{1}{2}} \frac{(1-\varepsilon_x^2-\varepsilon_y^2)^{\alpha-1}}{(1-\varepsilon_0\varepsilon_x)^{2\alpha+1}}$$

The CMS Detector



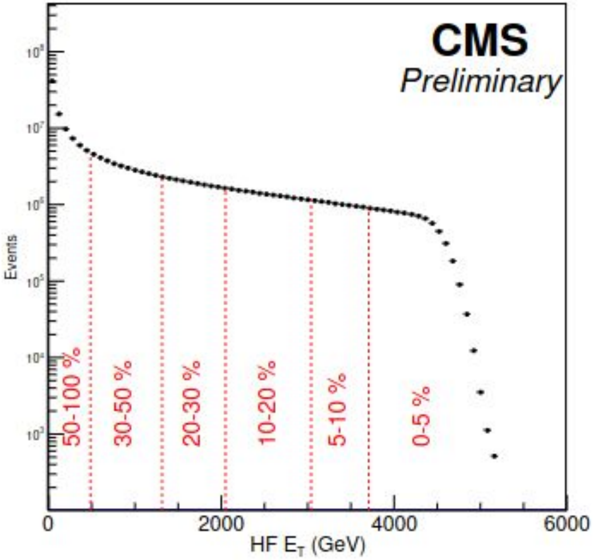
Track and Event Selections

- **Data**
  - 2018 PbPb Minimum Bias events

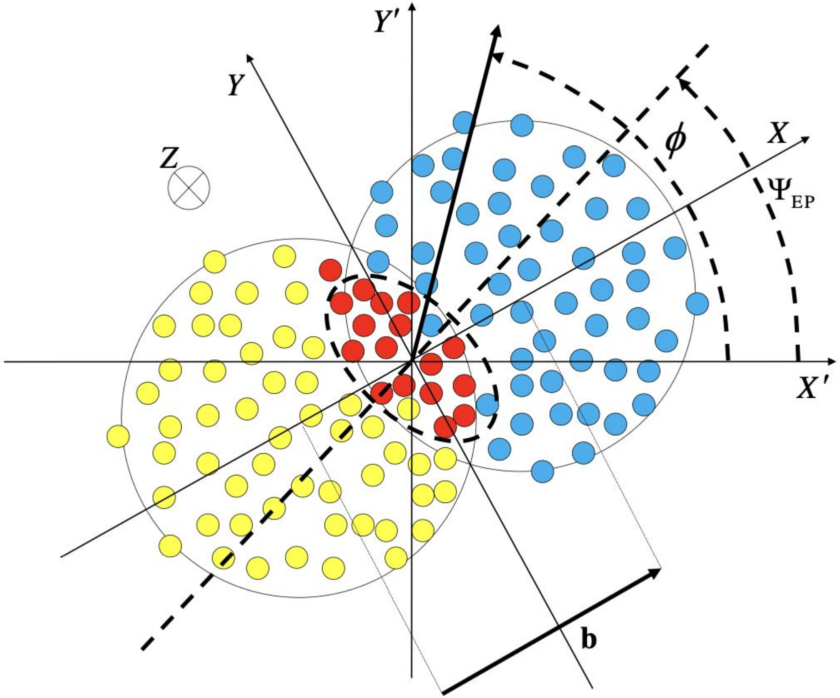
- **Track selections**
  - packedPFCandidates
  - $0.5 < p_T < 3.0$ GeV/c
  - $|\eta| < 2.4$
  - highPurity
  - DCA < 3.0
  - Nhits >= 11
  - $dp_T/p_T < 0.1$
  - $chi^2/ndof/Nlayers < 0.18$

- **Event selections**
  - primaryVertexFilter
  - clusterCompatibilityFilter
  - hfCoincFilter2Th4

Centrality Calibration from Transverse Energy Distribution



Non-central AA Collision in the Transverse Plane



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