Theoretical results for top-pair and top+W production

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- **Higher-order soft-gluon corrections**
- $t\bar{t}$ **production**
- tW **production**
- $t\bar{t}W$ **production**

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Soft-gluon corrections

They are important for top-quark processes and they approximate known exact results at NLO and NNLO very well

partonic processes $a(p_a) + b(p_b) \to t(p_t) + X$

define $s=(p_a+p_b)^2$**,** $t=(p_a-p_t)^2$**,** $u=(p_b-p_t)^2$

For a $2 \to n$ **process with** $p_a+p_b \to p_t+p_2+\cdots+p_n$
we define the threshold variable $s_4=s+t+u-m_t^2-(p_2+\cdots+p_n)^2$
Also $s_4=(p_2+\cdots+p_n+p_g)^2-(p_2+\cdots+p_n)^2$ **where extra gluon with** p_g **emitted**

At partonic threshold $p_g \to 0$ **and thus** $s_4 \to 0$

Soft corrections $\left[\frac{\ln^k(s_4/m_t^2)}{s_4}\right]_+$ **with** $k \le 2n-1$ **for the order** α_s^n **corrections**

Resum these soft corrections for the double-differential cross section

Finite-order expansions $\to$ **no prescription needed or used**
(this avoids underestimating the size of the corrections)

Approximate NNLO (aNNLO) and/or approximate N^{3}LO (aN^{3}LO) predictions

for cross sections and differential distributions

Soft-gluon Resummation

$$d\sigma_{pp\to tX} = \sum_{a,b} \int dx_a\, dx_b\, \phi_{a/p}(x_a,\mu_F)\, \phi_{b/p}(x_b,\mu_F)\, d\hat{\sigma}_{ab\to tX}(s_4,\mu_F)$$

take Laplace transforms $d\hat{\sigma}_{ab\to tX}(N) = \int (ds_4/s)\, e^{-N s_4/s} d\hat{\sigma}_{ab\to tX}(s_4)$

and $\tilde{\phi}(N) = \int_0^1 e^{-N(1-x)}\, \phi(x)\, dx$ **with transform variable** N

Then

$$d\tilde{\sigma}_{ab\to tX}(N) = \tilde{\phi}_{a/a}(N_a,\mu_F)\, \tilde{\phi}_{b/b}(N_b,\mu_F)\, d\tilde{\hat{\sigma}}_{ab\to tX}(N,\mu_F)$$

Refactorization for the cross section

$$d\sigma_{ab\to tX}(N) = \tilde{\psi}_a(N_a,\mu_F)\, \tilde{\psi}_b(N_b,\mu_F)\, \tilde{J}(N,\mu_F)\, \mathrm{tr}\left\{H_{ab\to tX}\left(\alpha_s(\mu_R)\right)\, \tilde{S}_{ab\to tX}\left(\frac{\sqrt{s}}{N\mu_F}\right)\right\}$$

ψ_a, ψ_b $\to$ **collinear emission from incoming partons**

J $\to$ **collinear emission from final-state gluons or massless quarks (if any)**

$H_{ab\to tX}$ **is hard function** $\to$ **short distance**

$S_{ab\to tX}$ **is soft function** $\to$ **noncollinear soft gluons**

Thus

$$d\tilde{\hat{\sigma}}_{ab\to tX}(N) = \frac{\tilde{\psi}_{a/a}(N_a,\mu_F)\, \tilde{\psi}_{b/b}(N_b,\mu_F)}{\tilde{\phi}_{a/a}(N_a,\mu_F)\, \tilde{\phi}_{b/b}(N_b,\mu_F)}\, \tilde{J}(N,\mu_F)\ \mathrm{tr}\left\{H_{ab\to tX}\left(\alpha_s(\mu_R)\right)\, \tilde{S}_{ab\to tX}\left(\frac{\sqrt{s}}{N\mu_F}\right)\right\}$$

$S_{ab\to tX}$ **satisfies the renormalization group equation**

$$\left(\mu_R \frac{\partial}{\partial \mu_R} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{ab\to tX} = -\Gamma^{\dagger}_{S\,ab\to tX}\, S_{ab\to tX} - S_{ab\to tX}\, \Gamma_{S\,ab\to tX}$$

Soft anomalous dimension $\Gamma_{S\,ab\to tX}$ **controls the evolution of the soft function which gives the exponentiation of logarithms of** N

Renormalization group evolution $\rightarrow$ **resummation**

$$d\tilde{\hat{\sigma}}^{\rm resum}_{ab\to tX}(N) = \exp\left[\sum_{i=a,b} E_i(N_i)\right] \exp\left[\sum_{i=a,b} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\gamma_{i/i}(N_i)\right] \exp\left[E'(N)\right]$$
$$\times {\rm tr}\left\{H_{ab\to tX}\left(\alpha_s(\sqrt{s})\right) \bar{P} \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\,\Gamma^{\dagger}_{S\,ab\to tX}(\alpha_s(\mu))\right] \tilde{S}_{ab\to tX}\left(\alpha_s\left(\frac{\sqrt{s}}{N}\right)\right) P \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\,\Gamma_{S\,ab\to tX}(\alpha_s(\mu))\right]\right\}$$

The soft anomalous dimensions Γ_S **and the hard and soft functions are in general matrices in the space of color exchanges in the hard scattering**

Top processes studied - total and differential cross sections

Top pair

$t\bar{t}$ **aN^{3}LO (total; top p_T, y, and double-differential; also A_{FB})**
$t\bar{t}$ **aN^{3}LO + EW (total; top p_T, y)**
$t\bar{t}$ **SMEFT aNNLO (total; top p_T)**

Top-pair+X

$t\bar{t}\gamma$ **aNNLO + EW (total; top p_T, y)**
$t\bar{t}W$ **aN^{3}LO + EW (total; top p_T, y)**

Single top

t**- and** s**-channel aNNLO (total; top p_T) and aN^{3}LO (total)**
tW **aN^{3}LO (total; p_T, y for top and W)**

Single-top+X

tqH **aNNLO (total; top p_T, y)**
$tq\gamma$ **aNNLO (total; top p_T, y)**
tqZ **aNNLO (total; top y)**

Single-top BSM

$t\gamma$, tZ, tZ' **aNNLO (total; top p_T, y)**
tg **aNNLO (total)**
tH^- **aNNLO (total; top p_T, y) and aN^{3}LO (total)**

$t\bar{t}$ production

Soft anomalous dimension matrix is 2×2 for $q\bar{q} \to t\bar{t}$ channel and 3×3 for $gg \to t\bar{t}$ channel

I calculated them at one loop in the mid-90's and at two loops fifteen years ago

more recent partial results at three loops

Four-loop massive cusp anomalous dimension from asymptotics [NK, PRD 107, 054006 (2023)] contributes to 4-loop result

NLO expansions agree with exact NLO results very well

NNLO expansions (aNNLO) predicted the exact NNLO results to high accuracy (percent or per mille) for total cross sections and top-quark p_T and rapidity distributions

aN^{3}LO is the state of the art

electroweak corrections also included

$t\bar{t}$ production at aN^{3}LO QCD + NLO EW at LHC energies

(in collaboration with Marco Guzzi and Alberto Tonero)

$t\bar{t}$ total cross sections at LHC energies with MSHT20 aN^{3}LO pdf						
σ in pb	5.02 TeV	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
LO QCD	$40.0^{+14.9+1.1}_{-10.1-1.2}$	103^{+35+3}_{-24-3}	146^{+48+3}_{-34-4}	$469^{+133+9}_{-97\ -10}$	$518^{+145+10}_{-106-11}$	$553^{+153+11}_{-113-11}$
NLO QCD	$58.1^{+6.8+1.8}_{-7.8-2.0}$	151^{+17+4}_{-20-5}	215^{+25+5}_{-27-6}	700^{+80+15}_{-80-15}	775^{+89+16}_{-88-16}	828^{+94+16}_{-94-18}
NLO QCD+EW	$58.1^{+6.6+1.8}_{-7.8-2.0}$	150^{+17+4}_{-19-4}	214^{+25+6}_{-26-6}	698^{+78+14}_{-80-16}	772^{+88+16}_{-87-16}	825^{+92+16}_{-93-18}
NNLO QCD	$65.3^{+2.8+2.0}_{-4.4-2.2}$	$169^{+7\ +5}_{-11-5}$	$240^{+9\ +6}_{-15-7}$	781^{+27+16}_{-43-17}	864^{+30+18}_{-47-19}	922^{+32+18}_{-49-20}
NNLO QCD+EW	$65.3^{+2.8+2.0}_{-4.4-2.2}$	$168^{+7\ +5}_{-11-5}$	$239^{+9\ +6}_{-15-7}$	779^{+27+16}_{-43-17}	861^{+30+18}_{-47-19}	919^{+32+18}_{-49-20}
aN^{3}LO QCD	$68.2^{+2.1+2.1}_{-3.2-2.3}$	175^{+5+5}_{-7-5}	249^{+7+6}_{-9-7}	804^{+22+16}_{-17-17}	889^{+24+18}_{-19-20}	948^{+26+19}_{-21-21}
aN^{3}LO QCD+EW	$68.2^{+2.1+2.1}_{-3.2-2.3}$	174^{+5+5}_{-7-5}	248^{+7+6}_{-9-7}	802^{+22+16}_{-17-17}	886^{+24+18}_{-19-20}	945^{+26+19}_{-21-21}

aN^{3}LO QCD + NLO EW cross section with scale and pdf uncertainties is

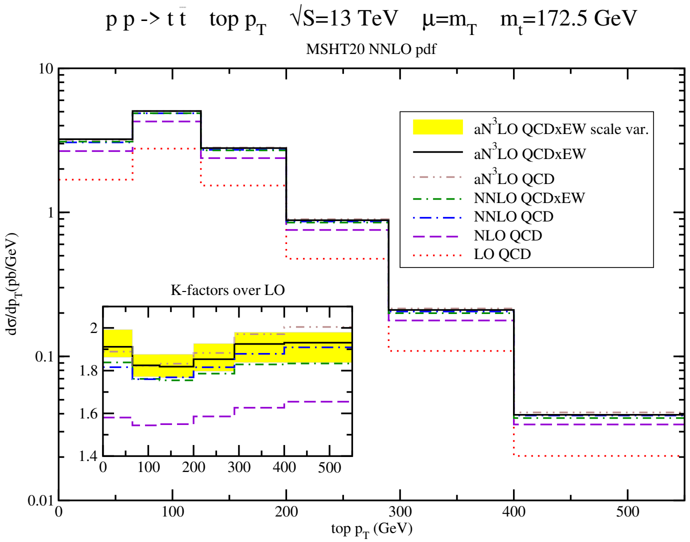
with MSHT20 NNLO pdf at 13 TeV: 836^{+23+17}_{-18-11} **pb ; at 13.6 TeV:** 925^{+25+18}_{-20-12} **pb**

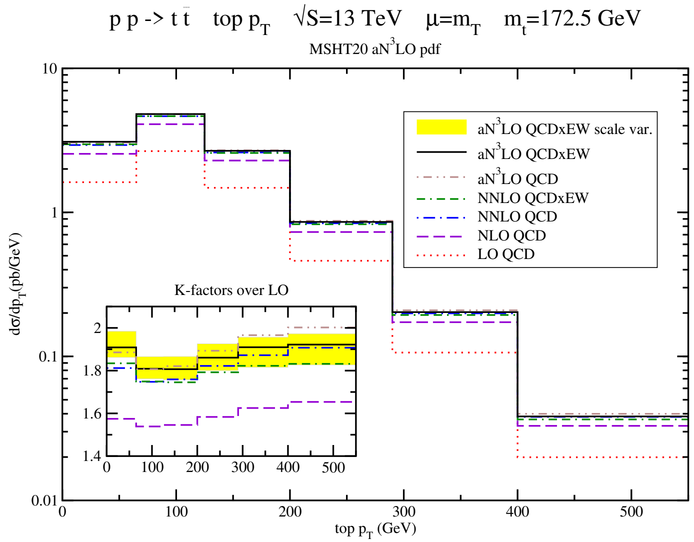
with CT18 NNLO pdf at 13 TeV: 842^{+23+18}_{-18-16} **pb ; at 13.6 TeV:** 932^{+25+20}_{-20-18} **pb**

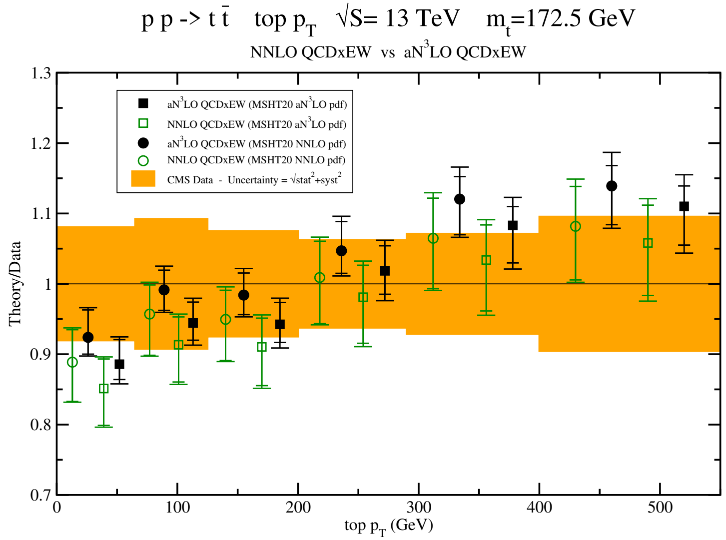
with NNPDF4.0 NNLO pdf at 13 TeV: 816^{+23+5}_{-18-4} **pb ; at 13.6 TeV:** 904^{+25+5}_{-20-5} **pb**

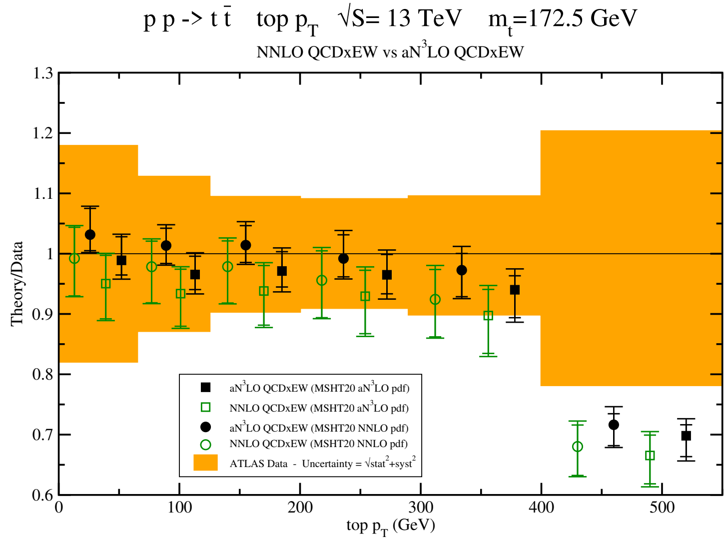
with PDF4LHC21 NNLO pdf at 13 TeV: 837^{+23+20}_{-18-16} **pb ; at 13.6 TeV:** 926^{+25+22}_{-20-17} **pb**

Top p_T distributions in $t\bar{t}$ production at 13 TeV

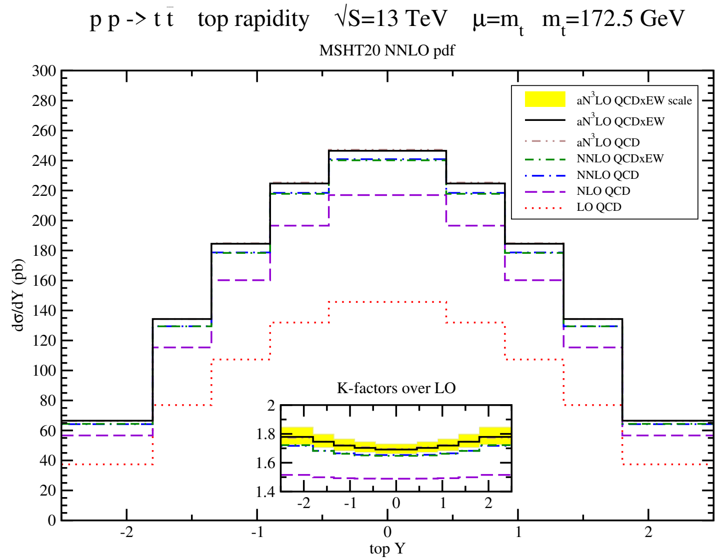


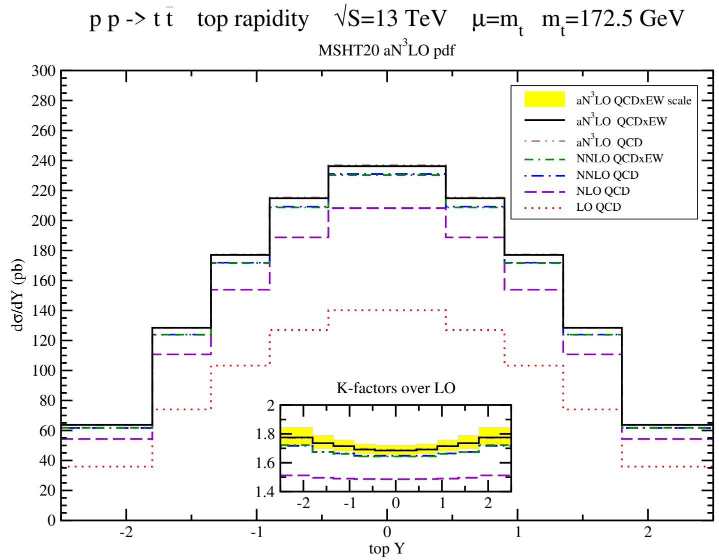


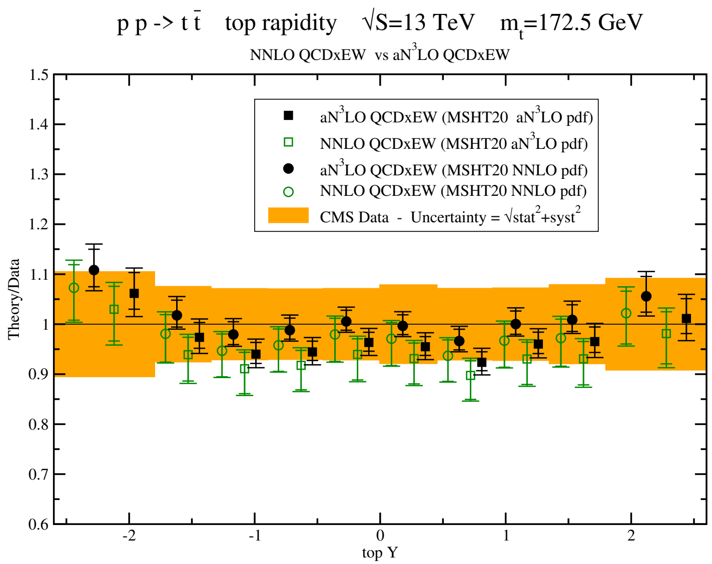


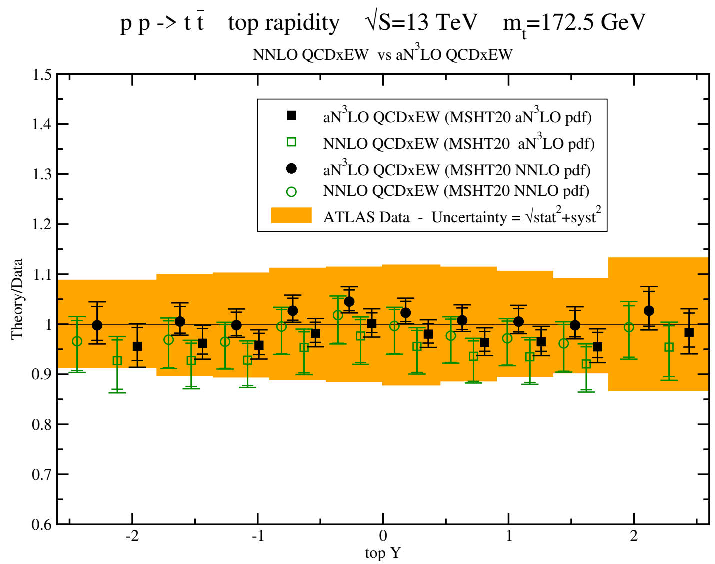


Top rapidity distributions in $t\bar{t}$ production at 13 TeV



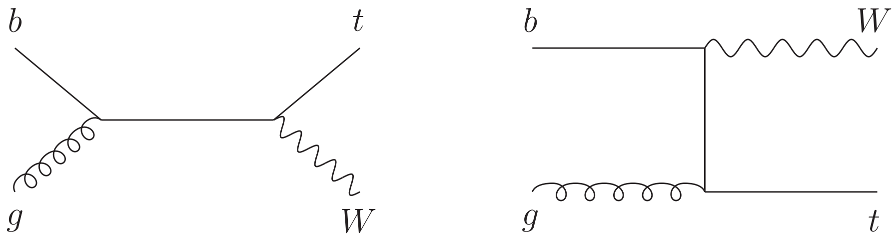






tW production

leading-order diagrams



At one loop $$\Gamma_S^{(1)\,bg\to tW}=C_F\left[\ln\left(\frac{m_t^2-t}{m_t\sqrt{s}}\right)-\frac{1}{2}\right]+\frac{C_A}{2}\ln\left(\frac{u-m_t^2}{t-m_t^2}\right)$$

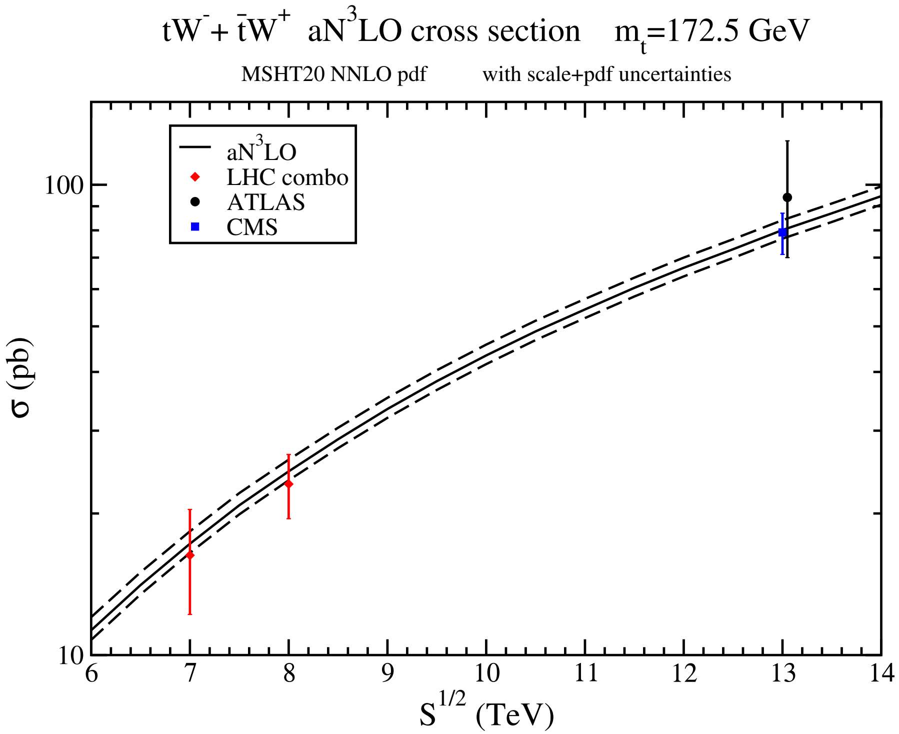
At two loops $$\Gamma_S^{(2)\,bg\to tW}=K_2\,\Gamma_S^{(1)\,bg\to tW}+\frac{1}{4}C_F C_A(1-\zeta_3)$$

At three loops

$$\Gamma_S^{(3)\,bg\to tW}=K_3\,\Gamma_S^{(1)\,bg\to tW}+\frac{1}{2}K_2 C_F C_A(1-\zeta_3)+C_F C_A^2\left[-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right]$$

tW production at LHC energies

(in collaboration with Nodoka Yamanaka)



The aN^{3}LO cross section for $tW^- + \bar{t}W^+$ with scale and pdf uncertainties is

with MSHT20 NNLO pdf at 13 TeV: $79.5^{+1.9+2.0}_{-1.8-1.4}$ **pb ; at 13.6 TeV:** $87.6^{+2.0+2.1}_{-1.9-1.5}$ **pb**

with MSHT20 aN^{3}LO pdf at 13 TeV: $77.3^{+1.9+2.0}_{-1.8-2.1}$ **pb ; at 13.6 TeV:** $85.6^{+2.0+2.2}_{-1.9-2.3}$ **pb**

with PDF4LHC21 pdf at 13 TeV: $79.3^{+1.9+2.2}_{-1.8-2.2}$ **pb ; at 13.6 TeV:** $87.9^{+2.0+2.4}_{-1.9-2.4}$ **pb**

$t\bar{t}W$ production

(in collaboration with Chris Foster)

observation of $t\bar{t}W$ events at 7, 8, 13 TeV collisions at the LHC

measurements are significantly higher than theoretical predictions

QCD corrections at NLO are large, $\sim 47\%$ at 13.6 TeV

electroweak corrections are smaller but significant

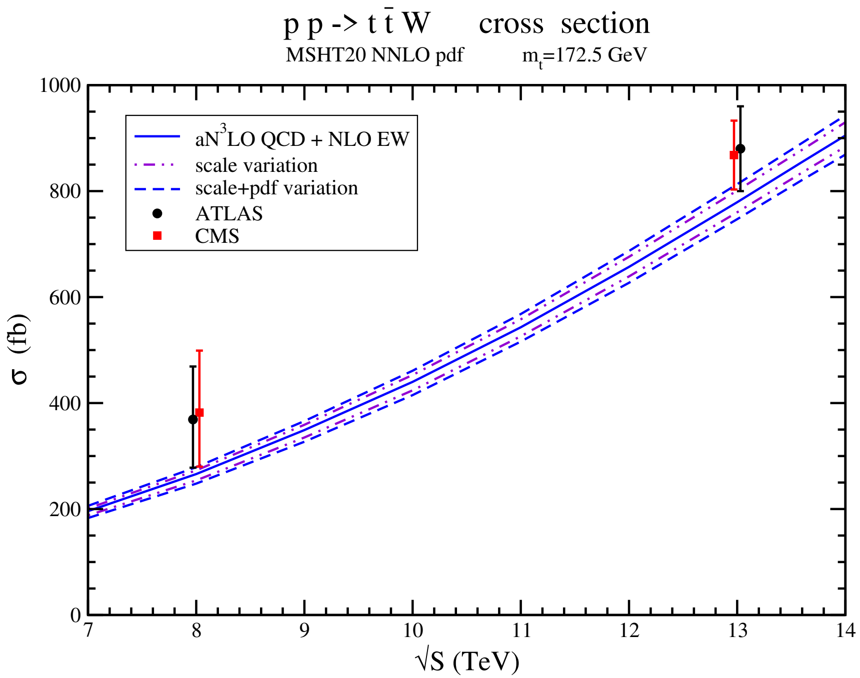
further improvement in theoretical accuracy by the inclusion of higher-order soft-gluon corrections

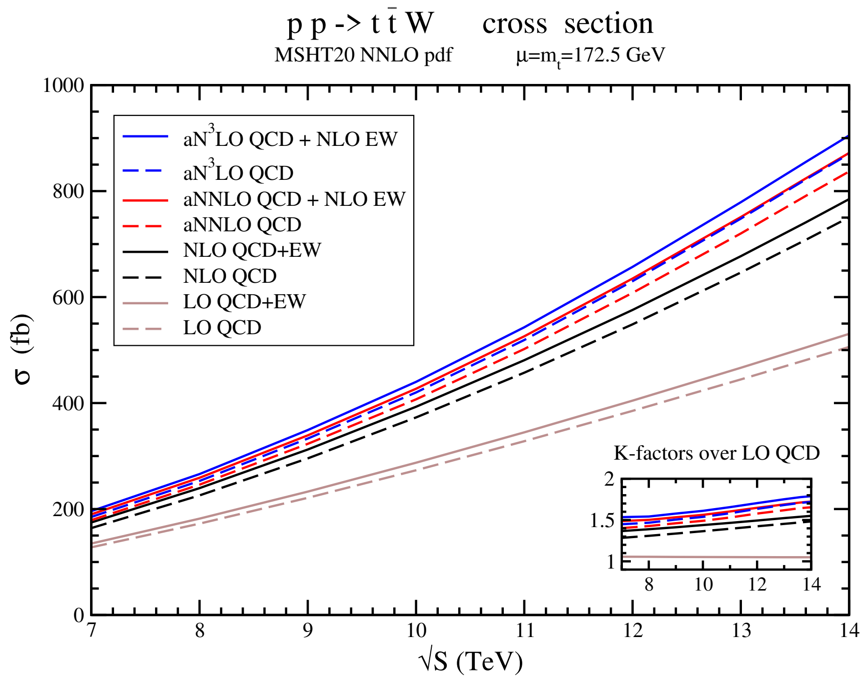
NLO expansions closely approximate exact NLO results for total cross sections and top-quark p_T and rapidity distributions

NNLO expansions (aNNLO) are consistent with (almost exact) NNLO results for total cross sections

approximate N^{3}LO (aN^{3}LO) QCD + NLO electroweak is state of the art

Cross sections for $t\bar{t}W$ production





large K-factors

improved agreement with data at aN^{3}LO

$t\bar{t}W$ cross sections

$t\bar{t}W$ cross sections (fb) in pp collisions at the LHC					
σ in fb	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
LO QCD	128^{+39}_{-28}	172^{+51}_{-36}	445^{+114}_{-84}	481^{+121}_{-90}	506^{+126}_{-94}
LO QCD+EW	135^{+41}_{-29}	182^{+53}_{-38}	467^{+119}_{-88}	505^{+127}_{-94}	531^{+132}_{-98}
NLO QCD	164^{+13}_{-17}	226^{+20}_{-23}	646^{+83}_{-74}	708^{+94}_{-82}	750^{+101}_{-88}
NLO QCD+EW	175^{+12}_{-17}	239^{+19}_{-23}	677^{+80}_{-74}	741^{+90}_{-82}	785^{+97}_{-88}
aNNLO QCD	179^{+6}_{-10}	246^{+9}_{-15}	720^{+29}_{-43}	791^{+32}_{-47}	837^{+34}_{-50}
aNNLO QCD + NLO EW	190^{+6}_{-10}	259^{+9}_{-15}	751^{+27}_{-43}	824^{+29}_{-47}	872^{+31}_{-50}
aN^{3}LO QCD	185^{+5}_{-8}	253^{+7}_{-12}	748^{+24}_{-19}	822^{+26}_{-20}	870^{+28}_{-21}
aN^{3}LO QCD + NLO EW	196^{+5}_{-8}	266^{+7}_{-12}	779^{+22}_{-19}	855^{+23}_{-20}	905^{+25}_{-21}

At 13.6 TeV

- **NLO QCD corrections → 47%**
- **aNNLO QCD corrections → 17%**
- **aN^{3}LO QCD corrections → 6%**
- **electroweak NLO corrections → 7%**

Total aN^{3}LO QCD+NLO EW cross section is 78% bigger than LO QCD

Comparison with 8 and 13 TeV CMS and ATLAS data

NLO and even aNNLO results are not sufficient

we need aN^{3}LO corrections to describe the data

At 8 TeV, measurements from

CMS: 382^{+117}_{-102} **fb**

and from

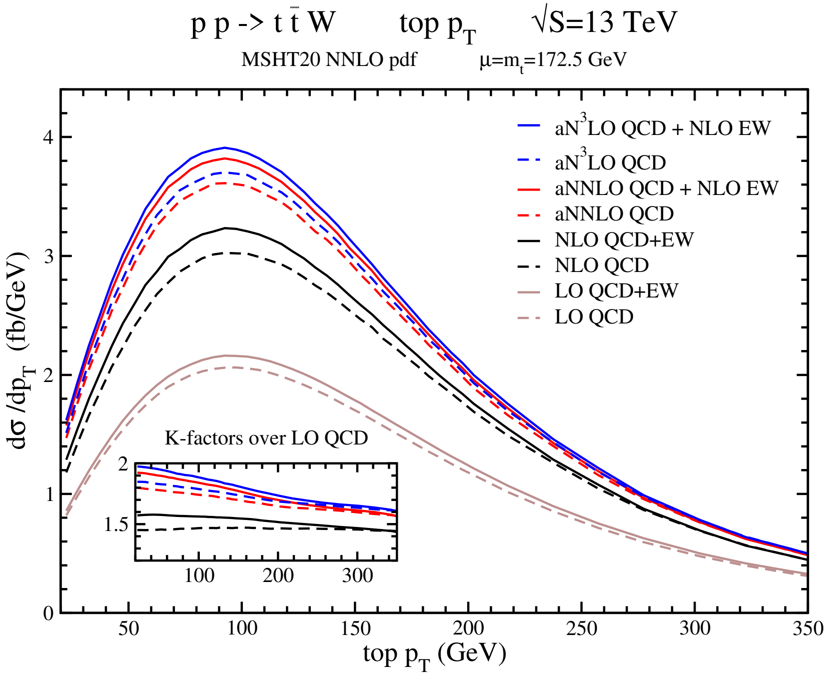
ATLAS: 369^{+100}_{-91} **fb**

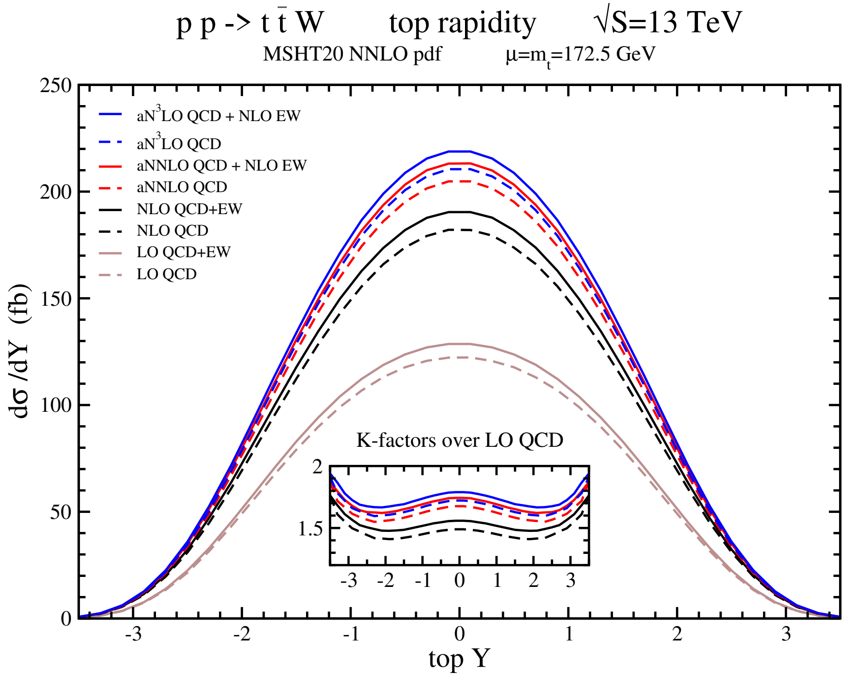
Theoretical prediction is
aN^{3}LO QCD + NLO EW: $266^{+7}_{-12}{}^{+6}_{-6}$ **fb**

At 13 TeV, CMS finds 868 ± 65 **fb with** $t\bar{t}W^+$ 553 ± 42 **fb and** $t\bar{t}W^-$ 343 ± 36 **fb**
while ATLAS finds 880 ± 80 **fb with** $t\bar{t}W^+$ 583 ± 58 **fb and** $t\bar{t}W^-$ 296 ± 40 **fb**

Theoretical prediction is
aN^{3}LO QCD + NLO EW: $779^{+22}_{-19}{}^{+12}_{-13}$ **fb with** $t\bar{t}W^+$ $517^{+14}_{-12}{}^{+8}_{-9}$ **fb and** $t\bar{t}W^-$ $262^{+8}_{-7}{}^{+4}_{-4}$ **fb**

Top-quark p_T and rapidity distributions in $t\bar{t}W$ production at 13 TeV

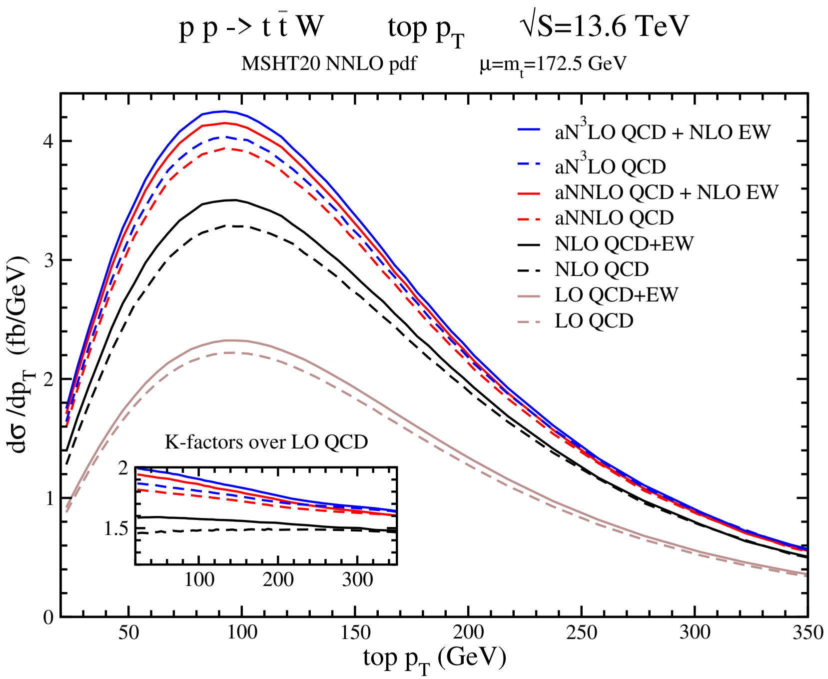


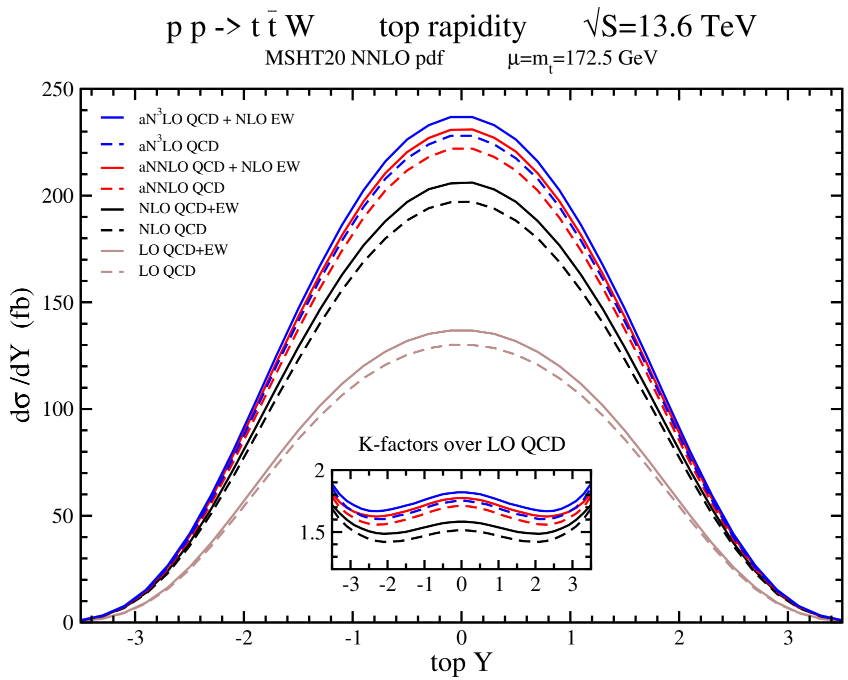


K-factors decrease at larger top p_T

K-factors increase at larger rapidities

Top-quark p_T and rapidity distributions in $t\bar{t}W$ at 13.6 TeV





K-factors decrease at larger top p_T

K-factors increase at larger rapidities

Summary

- **higher-order corrections for top-quark production processes**
- $t\bar{t}$ **production**
- tW **production**
- $t\bar{t}W$ **production**
- **soft-gluon resummation and aNNLO, aN^{3}LO expansions**
- **results for total cross sections and differential distributions**
- **higher-order corrections further enhance and improve the theoretical predictions**
- **good agreement with LHC data**