Quark production and thermalization of the longitudinally boost-invariant quark-gluon plasma

Sergio Barrera Cabodevila[1]

in collaboration with

Xiaojian Du, Carlos A. Salgado and Bin Wu

July 2024

IGFAE INSTITUTO GALEGO DE FÍSICA DE ALTAS ENERXÍAS 25 1999 2024

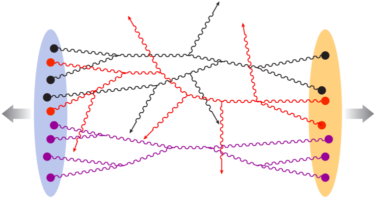
USC UNIVERSIDADE DE SANTIAGO DE COMPOSTELA · MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES · AGENCIA ESTATAL DE INVESTIGACIÓN · EXCELENCIA MARÍA DE MAEZTU 2024-2029 · XUNTA DE GALICIA · CENTROS CiGUS · Cofinanciado por la Unión Europea · Fondos Europeos

[1] sergio.barrera.cabodevila@usc.es

- After a heavy-ion collision, an out-of-equilibrium high-populated and quickly expanding system of gluons is produced at some time $\tau \sim 1/Q_s$, with Q_s the typical momentum.
- Up to this moment, the system can be described by CGC.

Phys. Rev. D 55 (1997). Jalilian-Marian et al.

Nucl. Phys. B 529 (1998). Kovchegov and Mueller



Ann. Rev. Nucl. Part. Sci. 60 (2010). Gelis et al.

- We aim to study the thermalization/hydrodynamization at $\tau \gtrsim Q_s^{-1}$.

How can we study the evolution?

- In the weak coupling limit, the bulk thermalization follows a bottom-up fashion (BMSS).

Phys. Lett. B 502 (2001). Baier et al.

- Previous works have performed a quantitative study of this system using the Effective Kinetic Theory (EKT).

JHEP 01 (2003). Arnold, Moore, and Yaffe

Phys. Rev. Lett. 115.18 (2015). Kurkela and Zhu

Phys. Rev. D 104.5 (2021). Du and Schlichting

- Our study uses the Boltzmann Equation in Diffusion Approximation (BEDA) as an alternative approach.

Physics Letters B 834 (2022). SBC, Salgado, and Wu

JHEP 06 (2024). SBC, Salgado, and Wu

- The QCD Boltzmann equation at leading order:

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f^a(\mathbf{x}; \mathbf{p}; t) = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f] \,, \quad f^a = \{f^g, f^q, f^{\bar{q}}\}$$

INSTITUTO GALEGO DE FÍSICA DE ALTAS ENERXÍAS (IGFAE) 25 1999 2024

- The QCD Boltzmann equation at leading order:

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f^a(\mathbf{x}; \mathbf{p}; t) = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f] \,, \quad f^a = \{f^g, f^q, f^{\bar{q}}\}$$

- We will restrict to the boost-invariant expansion, where the transverse plane derivative vanishes.

$$\left(\frac{\partial}{\partial \tau} - \frac{p_z}{\tau} \frac{\partial}{\partial p_z} \right) f^a(\mathbf{p}; t) = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f] \,, \quad f^a = \{f^g, f^q, f^{\bar{q}}\}$$

- The QCD Boltzmann equation at leading order:

$$(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}}) f^a(\mathbf{x};\mathbf{p};t) = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f]\,, \quad f^a = \{f^g, f^q, f^{\bar{q}}\}$$

- We will restrict to the boost-invariant expansion, where the transverse plane derivative vanishes.

$$\left(\frac{\partial}{\partial\tau} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f^a(\mathbf{p};t) = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f]\,, \quad f^a = \{f^g, f^q, f^{\bar{q}}\}$$

- The thermalization can be studied following the time evolution of the screening mass, the jet quenching parameter and the effective temperature and net quark chemical potential[1].

$$m_D^2 = m_D^2[f] \qquad \hat{q} = \hat{q}[f]$$

$$T_*(t) \equiv \frac{\hat{q}}{2\alpha_s N_c m_D^2 \ln\frac{\langle p_t^2\rangle}{m_D^2}} \qquad \mu_* = \mu_*[f]$$

[1] All quarks are assumed to have identical distribution. In general each flavour would have its own μ_* associated.

- In diffusion approximation, the $2 \leftrightarrow 2$ collision kernel can be expressed as a Fokker-Planck equation plus an additional source term.

Phys. Lett. B 475 (2000). Mueller

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

$$C^a_{2\leftrightarrow 2} = \frac{1}{4}\hat{q}_a(t)\nabla_{\mathbf{p}} \cdot \left[\nabla_{\mathbf{p}} f^a + \frac{\mathbf{v}}{T^*(t)} f^a(1+\epsilon_a f^a)\right] + \mathcal{S}_a$$

$$\mathcal{S}_q = \frac{2\pi\alpha_s^2 C_F^2 \ln\frac{\langle p_t^2\rangle}{m_D^2}}{p}\left[\mathcal{I}_c f^g(1-f^q) - \bar{\mathcal{I}}_c f^q(1+f^g)\right],$$

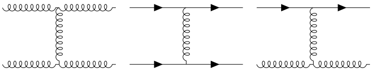
$$\mathcal{S}_{\bar{q}} = \mathcal{S}_q\big|_{F\leftrightarrow\bar{F}}, \qquad \mathcal{S}_g = -\frac{N_f}{2C_F}(\mathcal{S}_q + \mathcal{S}_{\bar{q}}),$$

- In diffusion approximation, the $2 \leftrightarrow 2$ collision kernel can be expressed as a Fokker-Planck equation plus an additional source term.

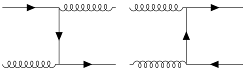
Phys. Lett. B 475 (2000). Mueller

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

Fokker-Planck term



Source term

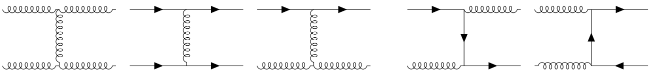


- In diffusion approximation, the $2 \leftrightarrow 2$ collision kernel can be expressed as a Fokker-Planck equation plus an additional source term.

Phys. Lett. B 475 (2000). Mueller

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

Fokker-Planck term | Source term



- The gluon distribution function is known to diverge at small p, $f \propto 1/p$, for over-occupied systems, which is interpreted as the onset of Bose-Einstein Condensation (BEC).

Nucl. Phys. A 920 (2013). Blaizot, Liao, and McLerran

- The presence of BEC can be study numerically by choosing the appropriate boundary conditions with[2] $\dot{n}_c \propto (\lim_{p\to 0} pf - T_*)$.

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

[2] $n_c \equiv$ number density of the BEC.

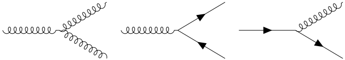
- The $1 \leftrightarrow 2$ kernel can be computed in the deep LPM regime

Nucl. Phys. B 483 (1997). Baier et al.

Phys. Rev. D 78 (2008). Arnold and Dogan

$$C^a_{1\leftrightarrow 2} = \int_0^1 \frac{dx}{x^3} \sum_{b,c} \left[\frac{\nu_c}{\nu_a} C^c_{ab}(\mathbf{p}/x; \mathbf{p}, \mathbf{p}(1-x)/x) - \frac{1}{2} C^a_{bc}(\mathbf{p}; x\mathbf{p}, (1-x)\mathbf{p}) \right]$$

- The $C^a_{bc}(\mathbf{p}; x\mathbf{p}, (1-x)\mathbf{p})$ describes the collinear splitting $a \leftrightarrow bc$.
- The three possible processes involved are the three QCD interaction vertices.



- Will the BEC still appear in initially over-populated system after including inelastic collisions?

- At small p, the $g \leftrightarrow gg$ and $g \leftrightarrow q\bar{q}$ are the dominant processes in the production of gluons and (anti)quarks, respectively.
- The distributions of gluons and quarks quickly fill a thermal distribution up to small soft momentum p_s

$$f^g(p) \approx \frac{T_*(v_z)}{p} \qquad \text{for } p \lesssim p_g$$

$$f^q(p) \approx \frac{1}{e^{-\frac{\mu_*(v_z)}{T_*(v_z)}} + 1} \qquad \text{for } p \lesssim p_q$$

with $T_* \equiv \int dv_z T_*(v_z)$. At early times, p_s is given by

$$p_g \equiv [m_g^4(v_z)\tau/2]^{\frac{2}{5}} \hat{q}_A^{\frac{1}{5}} \qquad p_q \equiv [m_q^4(v_z)\tau/2]^{\frac{2}{5}} \hat{q}_F^{\frac{1}{5}},$$

where m_g and m_q are the thermal masses of gluons and quarks, respectively.

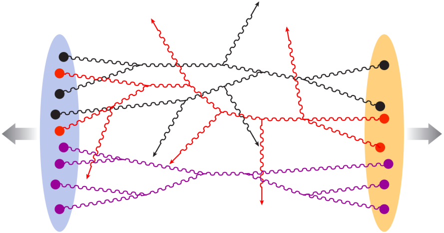
- This behavior implies that $\dot{n}_c = 0$, so no BEC is observed as in the spatially homogeneous case.

Nucl. Phys. A 961 (2017). Blaizot, Liao, and Mehtar-Tani

JHEP 06 (2024). SBC, Salgado, and Wu

From now on, we will work under the following assumptions:

- We work in the weak coupling limit, $\alpha_s \ll 1$.
- At initial time, $\tau \sim Q_s^{-1}$, the system is composed exclusively by gluons.
- The gluon distribution at initial time is:
  - isotropic in momentum space, with typical momentum Q_s.
  - highly populated, $f^g \sim \frac{1}{\alpha_s}$.

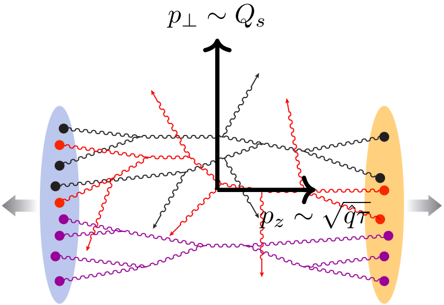


Ann. Rev. Nucl. Part. Sci. 60 (2010). Gelis et al.

Since we are assuming longitudinal boost invariance as for Bjorken hydrodynamics:

Phys. Rev. D 27 (1 Jan. 1983). Bjorken

- All partons moving along the transverse plane their momentum will not be modified by the expansion.
  - Typical momentum of this partons will be $p_\perp \sim Q_s$.
- Partons moving in the longitudinal direction will run away from the system due to the expansion.
  - Typical momentum of this partons will be given by the broadening of the transverse plane partons, $p_z \sim \sqrt{\hat{q}\tau}$.

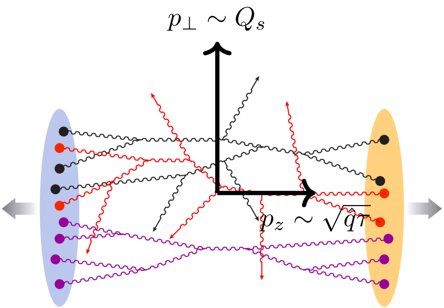


Ann. Rev. Nucl. Part. Sci. 60 (2010). Gelis et al.

Since we are assuming longitudinal boost invariance as for Bjorken hydrodynamics:

Phys. Rev. D 27 (1 Jan. 1983). Bjorken

- All partons moving along the transverse plane their momentum will not be modified by the expansion.
  - Typical momentum of this partons will be $p_\perp \sim Q_s$.
- Partons moving in the longitudinal direction will run away from the system due to the expansion.
  - Typical momentum of this partons will be given by the broadening of the transverse plane partons, $p_z \sim \sqrt{\hat{q}\tau}$.



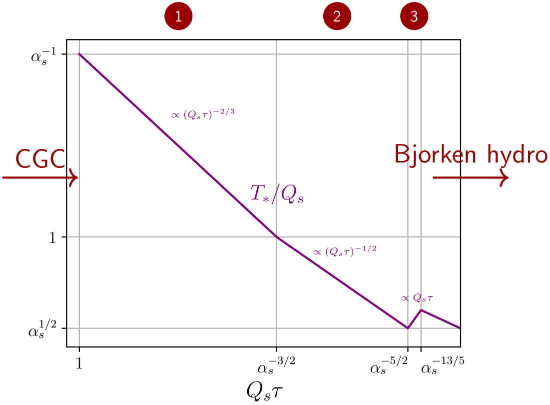
Ann. Rev. Nucl. Part. Sci. 60 (2010). Gelis et al.

- From now on, we will divide our momentum phase space into **soft partons**, with typical momentum $p \sim \sqrt{\hat{q}\tau}$, and **hard partons**, with typical momentum $p \sim Q_s$.

- The parametric behavior for a Yang-Mills/pure gluon system is well known acording to BMSS.

Phys. Lett. B 502 (2001). Baier et al.

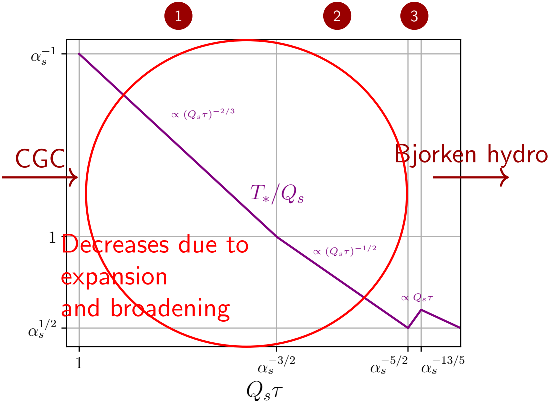
- We find that the macroscopic system behavior does not change parametrically when quarks are introduced. The three known stages of thermalization persist.



- The parametric behavior for a Yang-Mills/pure gluon system is well known acording to BMSS.

Phys. Lett. B 502 (2001). Baier et al.

- We find that the macroscopic system behavior does not change parametrically when quarks are introduced. The three known stages of thermalization persist.

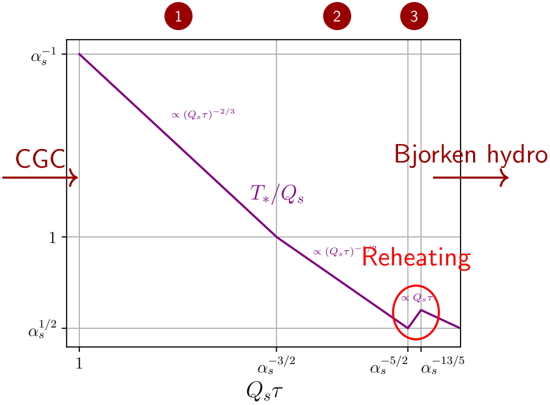


The bottom-up thermalization I

- The parametric behavior for a Yang-Mills/pure gluon system is well known acording to BMSS.

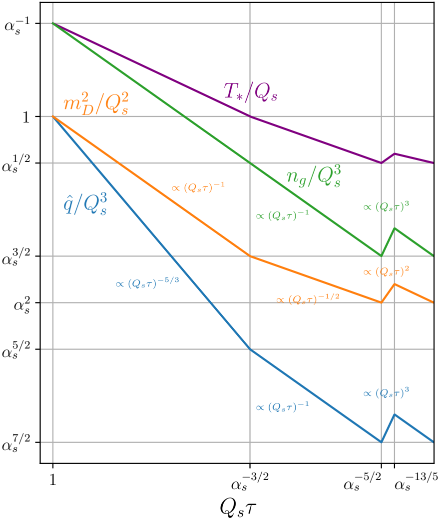
Phys. Lett. B 502 (2001). Baier et al.

- We find that the macroscopic system behavior does not change parametrically when quarks are introduced. The three known stages of thermalization persist.



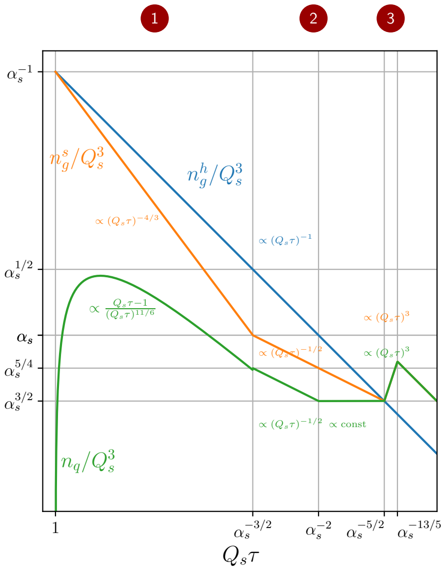
The bottom-up thermalization II

1 2 3



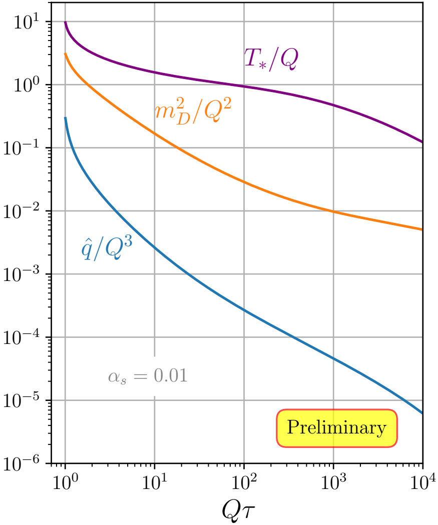
1. Initial stage is dominated by the rapid expansion of the hard gluons
2. Second stage starts when the soft gluon sector starts contributing dominantly to the screening.
   - Number density of hard gluons is still higher than the soft gluons.
3. Third stage corresponds to the thermalization of the soft sector.
   - Most of the gluons in the system are soft.
   - Remaining hard partons will radiate all their energy, heating up the soft sector.

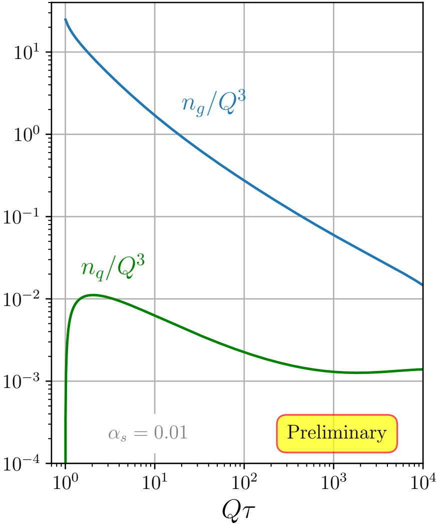
Parton number densities



1. First stage:
   - Soft gluon number decreases faster since $n_{g,s} \propto p_z^3$.
   - Quarks are produced by $g \to q\bar{q}$ and quickly reach their higher value.
2. Second stage:
   - The broadening pushes $p_z = \mathrm{const}$, slowing down the decrease of the number density.
   - At $Q\tau \sim \alpha_s^{-2}$ quark production is dominated by the soft gluons and $gg \to q\bar{q} \Rightarrow n_q \propto \mathrm{const}$.
3. In the third stage quark and soft gluon number density are parametrically equal, fitting a thermal distribution $\sim$ QGP.
   - The reheating affects g and q, increasing their number density as they grow parametrically as T^3.

Numerical simulation





Summary and outlook

- The Boltzmann Equation in Diffusion Approximation is a tool to study the evolution of the initial stages of a heavy ion collision.
- We have extended the parametric estimates from the BMSS by including quarks.
- The addition of quarks in the system do not affect to the parametric evolution of the system.
- Numerical simulation of the BEDA is being performed.
- Finer simulation should be run in order to identify the bottom-up stage.

Thanks!

Back-up

Explicit forms

- Jet quenching parameter

$$\hat{q}_a = 8\pi\alpha_s^2 C_a \ln\frac{\langle p_t^2\rangle}{m_D^2}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left[N_c f\,(1+f) + \frac{N_f}{2}F(1-F) + \frac{N_f}{2}\bar{F}(1-\bar{F})\right]$$

- Screening mass

$$m_D^2 = 8\pi\alpha_s\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{|\mathbf{p}|}\left(N_c f + \frac{N_f}{2}F + \frac{N_f}{2}\bar{F}\right)$$

- Integrals $\mathcal{I}_c$

$$\mathcal{I}_c = \int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{p}[f + F + f(F-\bar{F})], \qquad \bar{\mathcal{I}}_c = \mathcal{I}_c|_{F\leftrightarrow\bar{F}}$$

Numerical simulation with larger α_s