Soft gluon resummation for associated top-quark pair production with a photon at the LHC

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In collaboration with

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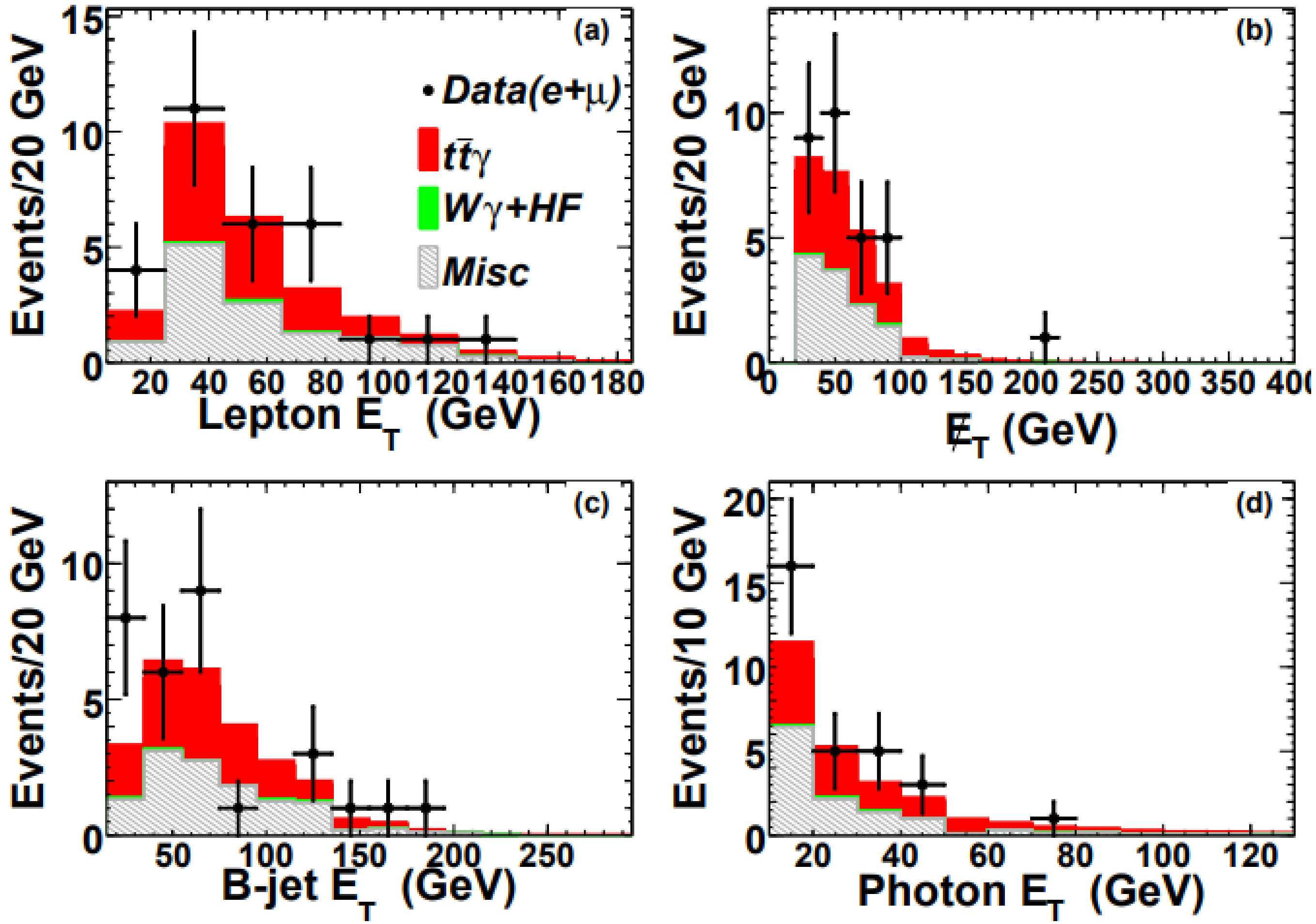
The tt̄γ process

- First evidence at Tevatron in 2011

CDF collaboration, Phys. Rev. D 84, 031104 (2011)



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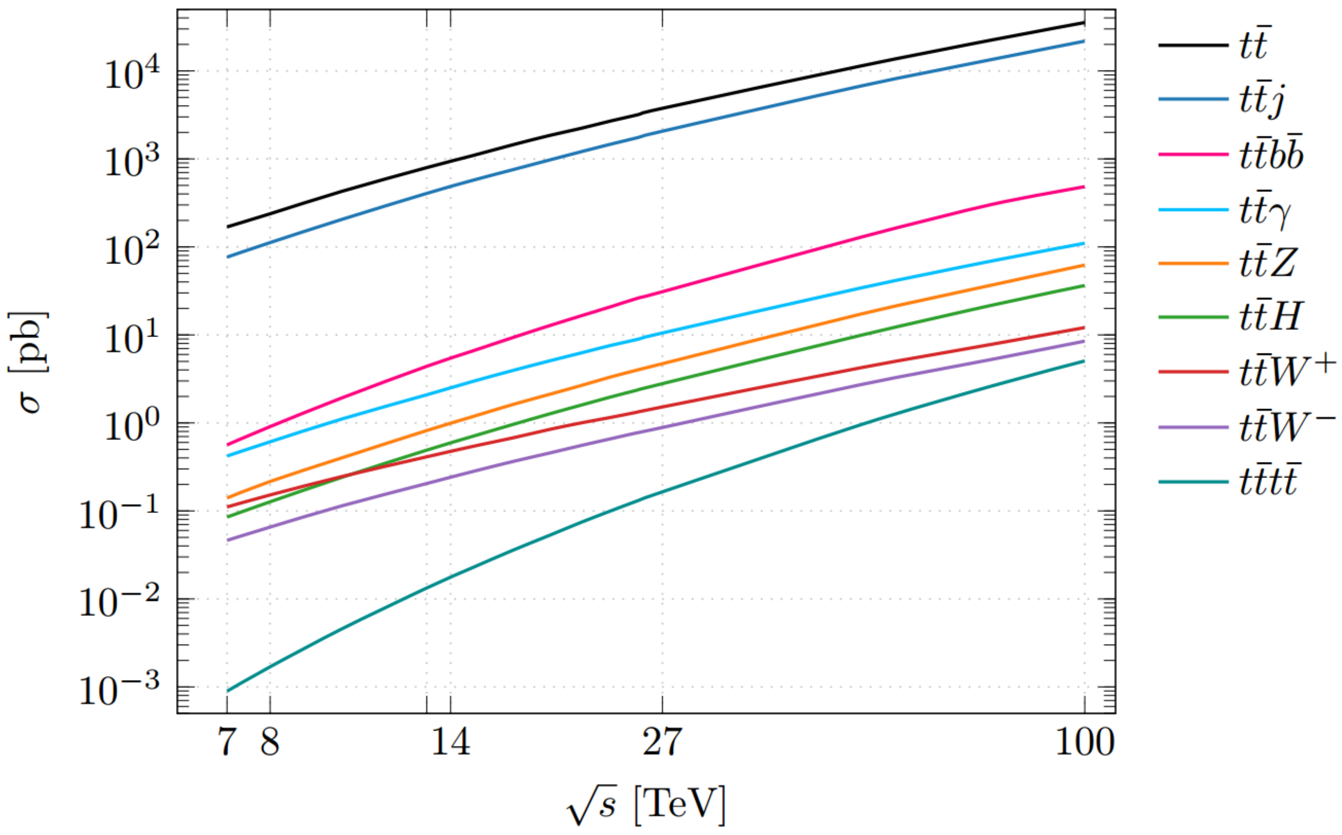
Snowmass Report 2021



The $t\bar{t}\gamma$ process

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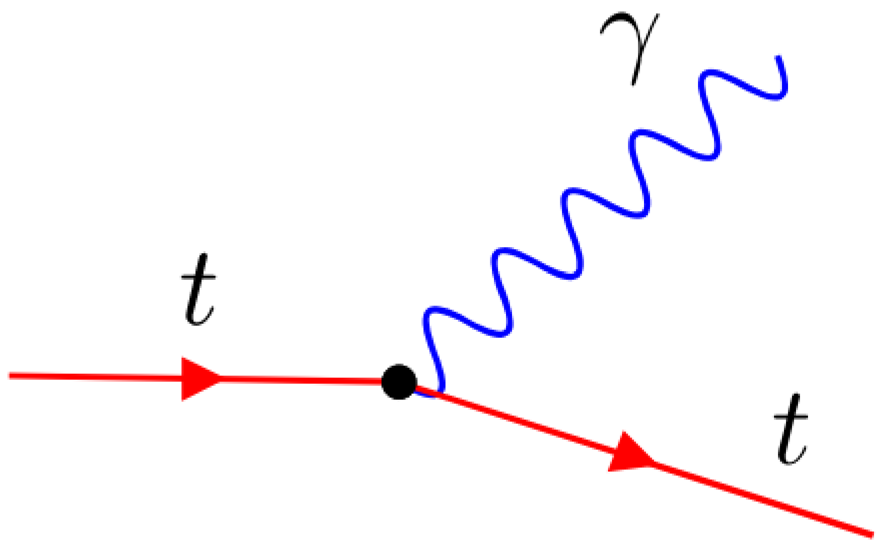
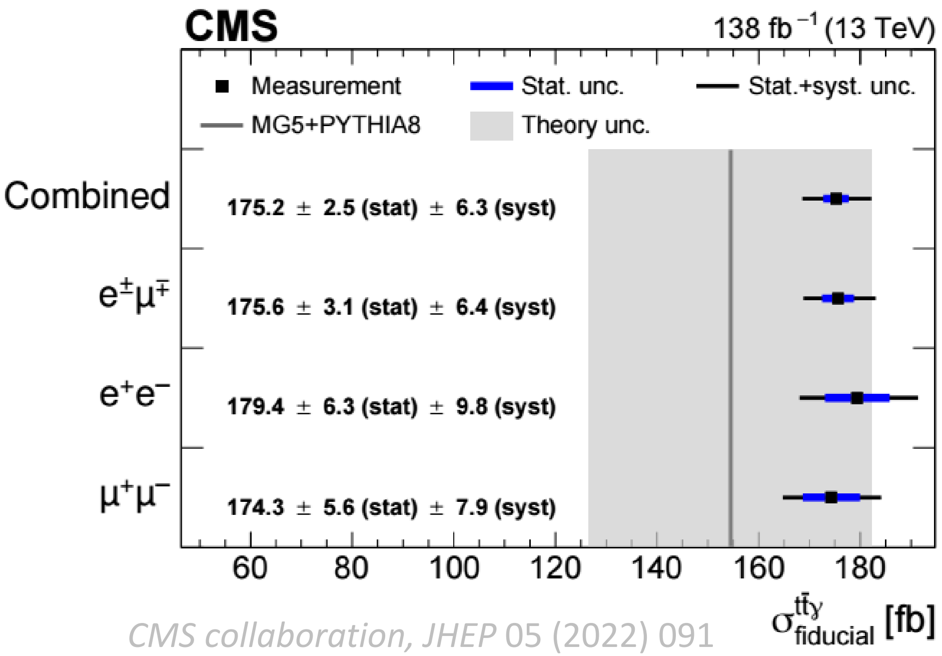
- Probes the top-photon coupling
  → test of Standard Model
  → sensitive to new physics

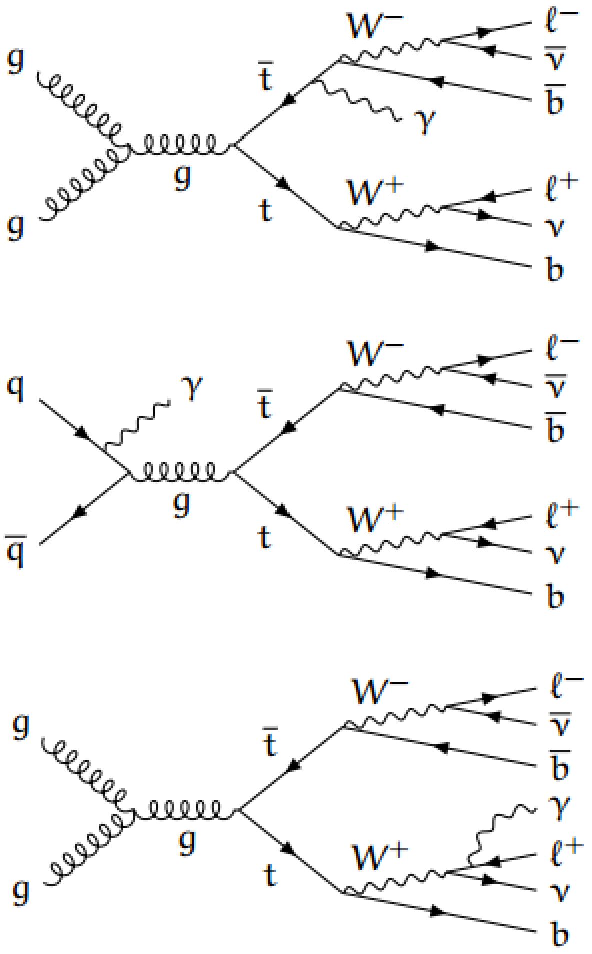


Feynman diagrams generated with FeynGame
[Harlander, Klein, Lipp, Comput. Phys. Commun. 256 (2020) 107465]

Latest experimental results



CMS collaboration, JHEP 05 (2022) 091



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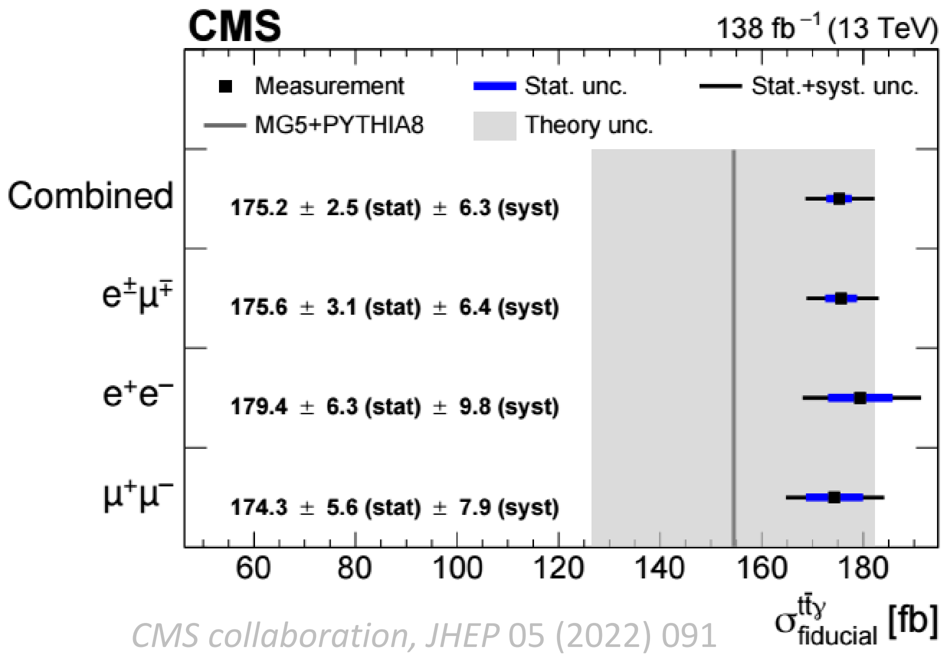

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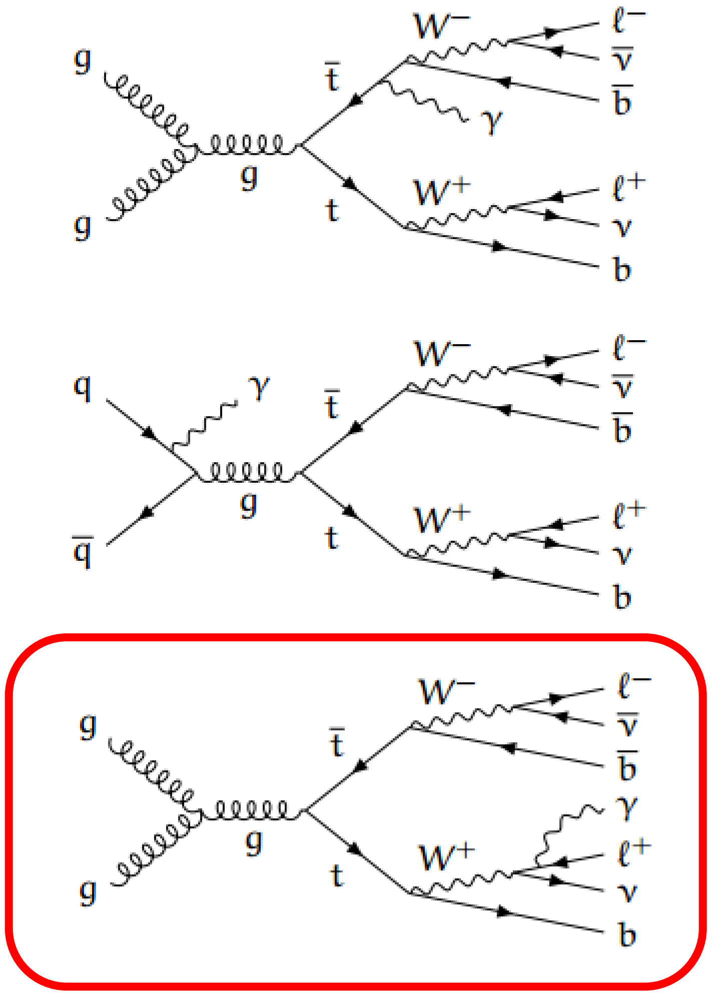

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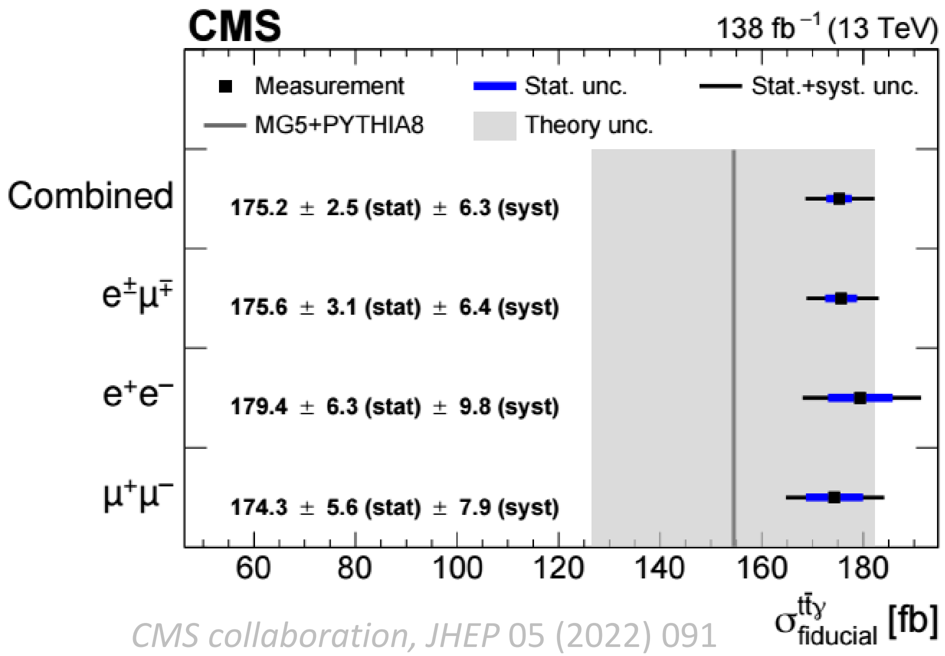


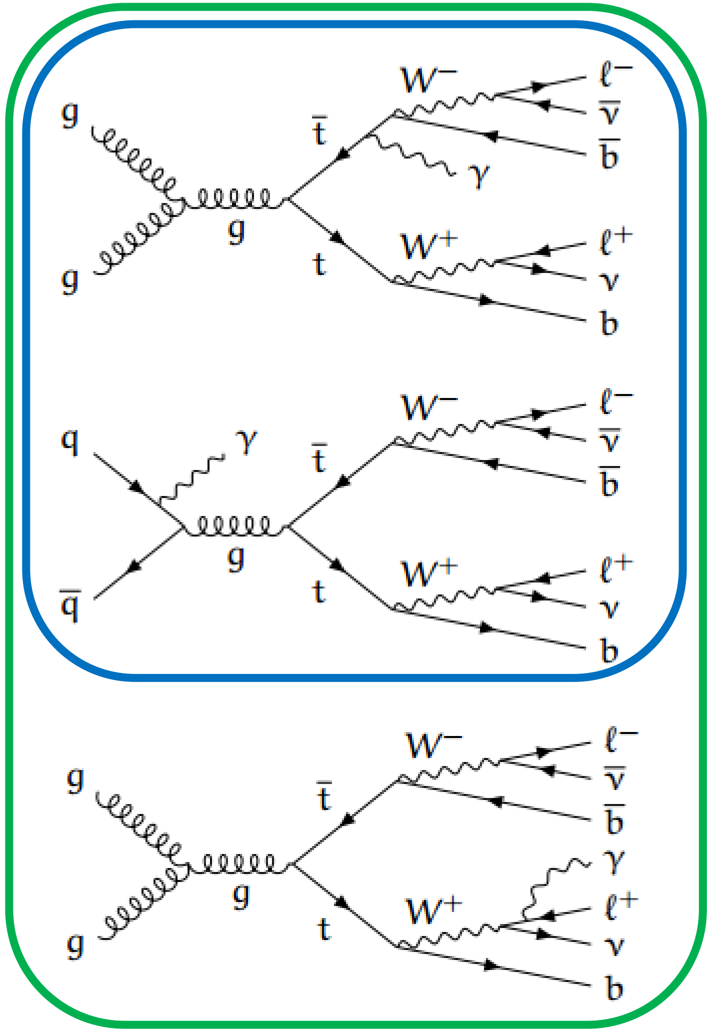
CMS collaboration, JHEP 05 (2022) 091

$$\sigma_{t\bar{t}\gamma\ \text{production}} = 322^{+16}_{-15}\ \text{fb} = 322 \pm 5\ (\text{stat}) \pm 15 (\text{syst})\ \text{fb}$$

$$\sigma_{t\bar{t}\gamma} = 793 \pm 38\ \text{fb} = 793 \pm 5\ (\text{stat})\ ^{+38}_{-37}\ (\text{syst})\ \text{fb}$$

*Single-lepton and dilepton channels

ATLAS collaboration, arXiv:2403.09452



Theoretical predictions

- NLO QCD

Duan, Ma, Zhang, Han, Guo, Wang ‘09, ‘11 | Maltoni, Pagani, Tsinikos ’16

- NLO QCD + top-quark decays

Melnikov, Schulze, Scharf ‘11

- NLO QCD + parton shower

Kardos, Trocsanyi ‘15

- NLO QCD + EW corrections

Duan, Zhang, Wang, Song, Li ‘17 | Pagani, Shao, Tsinikos, Zaro ‘21

- NLO QCD + off-shell effects

Bevilacqua, Hartanto, Kraus, Weber, Worek ‘18, ‘19, ‘20

- Approximate NNLO

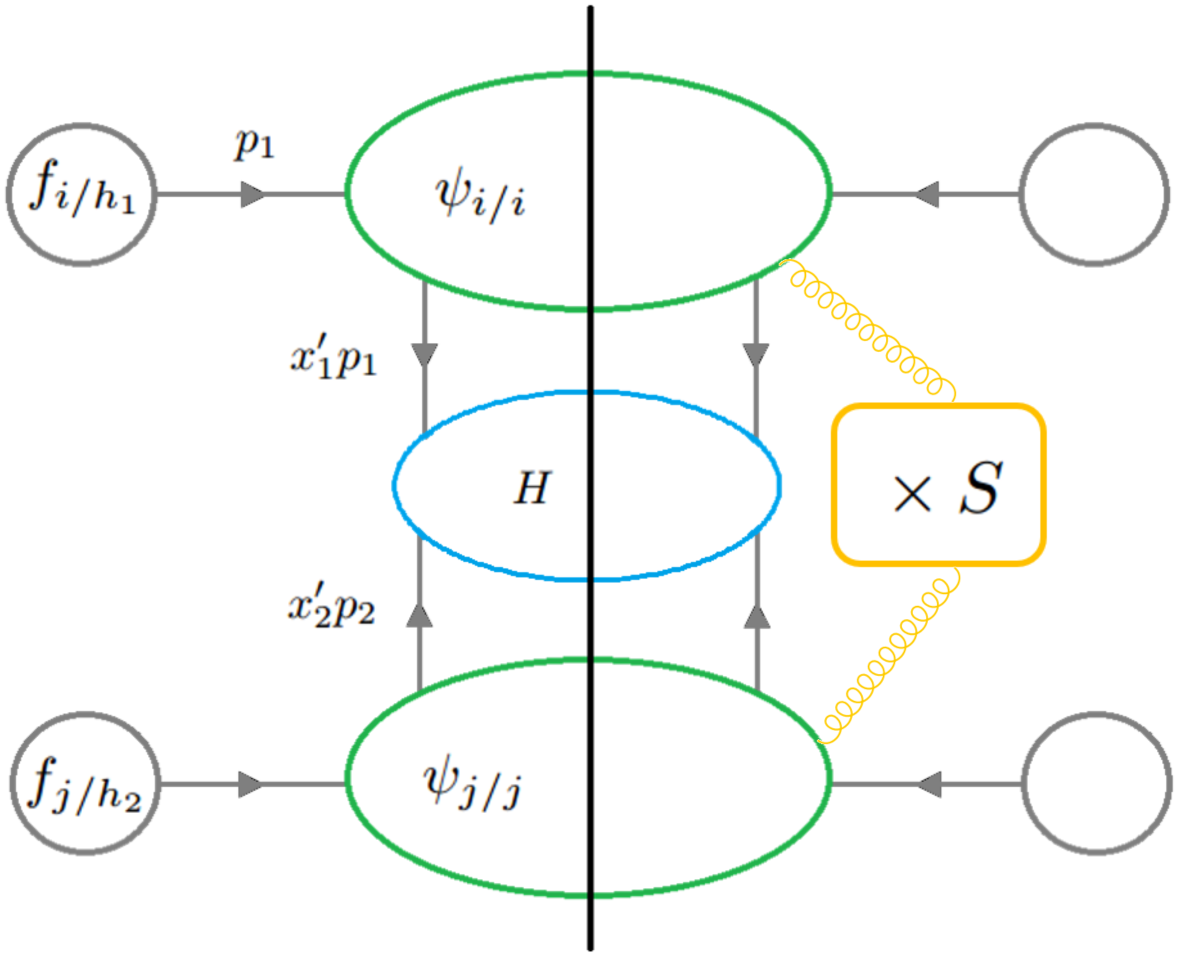
Kidonakis, Tonero ’23

- Complete NLO in top-quark pair production

Stremmer, Worek ‘24

Resummation

- Takes into account all orders in the soft-gluon emission limit
- Factorization of the amplitude close to kinematic threshold
- Factorization of the phase space in Mellin space



Invariant mass threshold resummation

$$\hat{\rho} = \frac{Q^2}{s} \to 1 \qquad \alpha_s^n \left[\frac{\log^m(1-\hat{\rho})}{1-\hat{\rho}}\right]_+, \quad m \leq 2n-1$$

Threshold variable

$$\frac{d\sigma_{pp\to t\bar{t}B}}{dQ^2}(\rho) = \sum_{i,j} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} \rho^{-N} \tilde{f}_{i/h_1}(N+1) \tilde{f}_{j/h_2}(N+1) \frac{d\tilde{\sigma}_{ij\to t\bar{t}B}}{dQ^2}(N), \quad \rho = \frac{Q^2}{S}$$

$$\frac{d\tilde{\sigma}_{ij\to t\bar{t}B}}{dQ^2}(N) = \int dPS_3 \mathrm{Tr}\left[\tilde{\mathbf{H}}_{ij\to t\bar{t}B}(N) \bar{\mathbf{U}}_{ij\to t\bar{t}B}(N+1) \bar{\mathbf{S}}_{ij\to t\bar{t}B}\left(\alpha_S\left(\frac{Q^2}{(N+1)^2}\right)\right) \mathbf{U}_{ij\to t\bar{t}B}(N+1)\right] \Delta_i(N+1)\Delta_j(N+1).$$

$$\alpha_s^n \left[\frac{\log^m(1-\hat{\rho})}{1-\hat{\rho}}\right]_+ \leftrightarrow \sum_k c_k \log^k N$$

Invariant mass threshold

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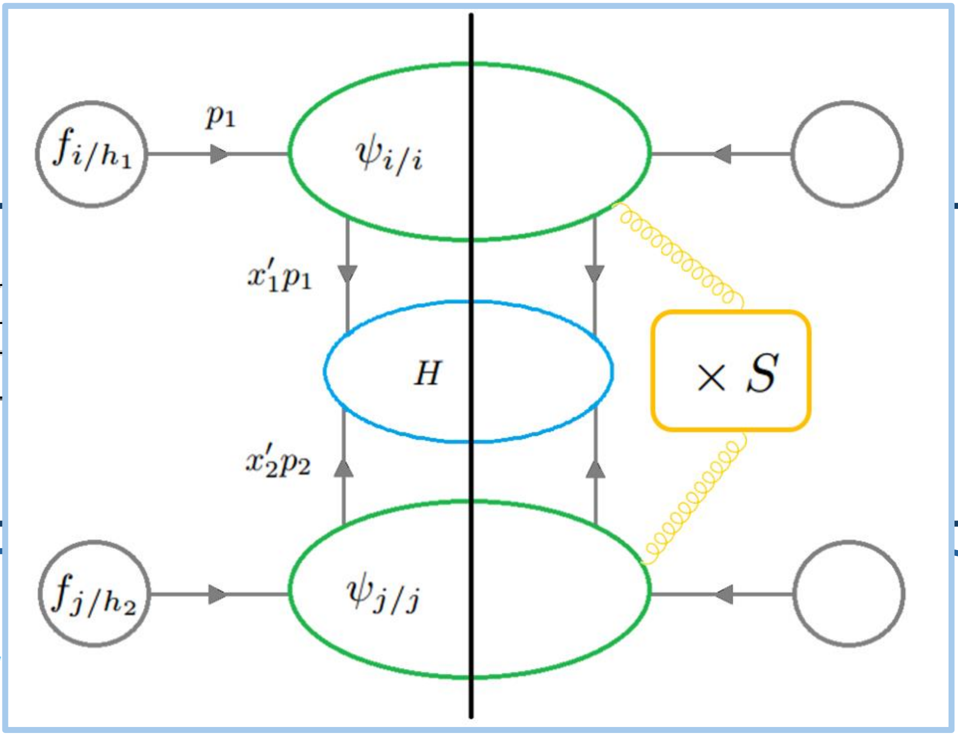

$$\frac{\mathrm{d}\sigma_{pp \to t\bar{t}B}}{\mathrm{d}Q^2}(\rho) = \sum_{i,j} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \frac{\mathrm{d}N}{2\pi \mathrm{i}} \rho^{-N} \tilde{f}_{i/h_1}(N +$$

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$$\left. \mathbf{U}_{ij \to t\bar{t}B}(N+1) \right] \Delta_i(N+1) \Delta_j(N+1).$$

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Ingredients for NNLL resummation

$$\tilde{\mathbf{H}}_{ij\to t\bar{t}B} = \tilde{\mathbf{H}}^{(0)}_{ij\to t\bar{t}B} + \frac{\alpha_S(\mu_R^2)}{\pi}\tilde{\mathbf{H}}^{(1)}_{ij\to t\bar{t}B} + \dots .$$

$$\frac{d\tilde{\sigma}_{ij\to t\bar{t}B}}{dQ^2}(N) = \int dPS_3 \mathrm{Tr}\left[\tilde{\mathbf{H}}_{ij\to t\bar{t}B}(N)\bar{\mathbf{U}}_{ij\to t\bar{t}B}(N+1)\bar{\mathbf{S}}_{ij\to t\bar{t}B}\left(\alpha_S\left(\frac{Q^2}{(N+1)^2}\right)\right)\mathbf{U}_{ij\to t\bar{t}B}(N+1)\right]\Delta_i(N+1)\Delta_j(N+1).$$

$$\bar{\mathbf{S}}_{ij\to t\bar{t}B}\left(\alpha_S\left(\frac{Q^2}{N^2}\right)\right) \approx \bar{\mathbf{S}}^{(0)}_{ij\to t\bar{t}B} + \frac{\alpha_S(\mu_R^2)}{\pi}\frac{\bar{\mathbf{S}}^{(1)}_{ij\to t\bar{t}B}}{1-2\lambda}$$

$$\lambda = \alpha_S(\mu_R^2) b_0 \log(N)$$

$$\bar{\mathbf{\Gamma}}_{ij\to t\bar{t}B} = \frac{\alpha_S(\mu_R^2)}{\pi}\bar{\mathbf{\Gamma}}^{(1)}_{ij\to t\bar{t}B} + \left(\frac{\alpha_S(\mu_R^2)}{\pi}\right)^2\bar{\mathbf{\Gamma}}^{(2)}_{ij\to t\bar{t}B} + \dots$$

$$\mathbf{U}_{ij\to t\bar{t}B}\left(\frac{Q}{N},\mu_R\right) = \mathrm{P}\exp\left(\int_{\mu_R}^{\frac{Q}{N}}\frac{d\lambda}{\lambda}\bar{\mathbf{\Gamma}}_{ij\to t\bar{t}B}(\alpha_S(\lambda^2))\right)$$

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Matching to NLO

$$\sigma^{\text{fixed order + res}}_{pp\to Y}(\rho) = \sigma^{\text{fixed order}}_{pp\to Y}(\rho) + \sum_{i,j}\int_{C_T}\frac{\mathrm{d}N}{2\pi\mathrm{i}}\rho^{-N}\tilde{f}_{i/h_1}(N+1)\tilde{f}_{j/h_1}(N+1) \left[\tilde{\sigma}^{\text{res}}_{ij\to Y}(N) - \tilde{\sigma}^{\text{res}}_{ij\to Y}(N)\Big|_{\text{fixed order}}\right].$$

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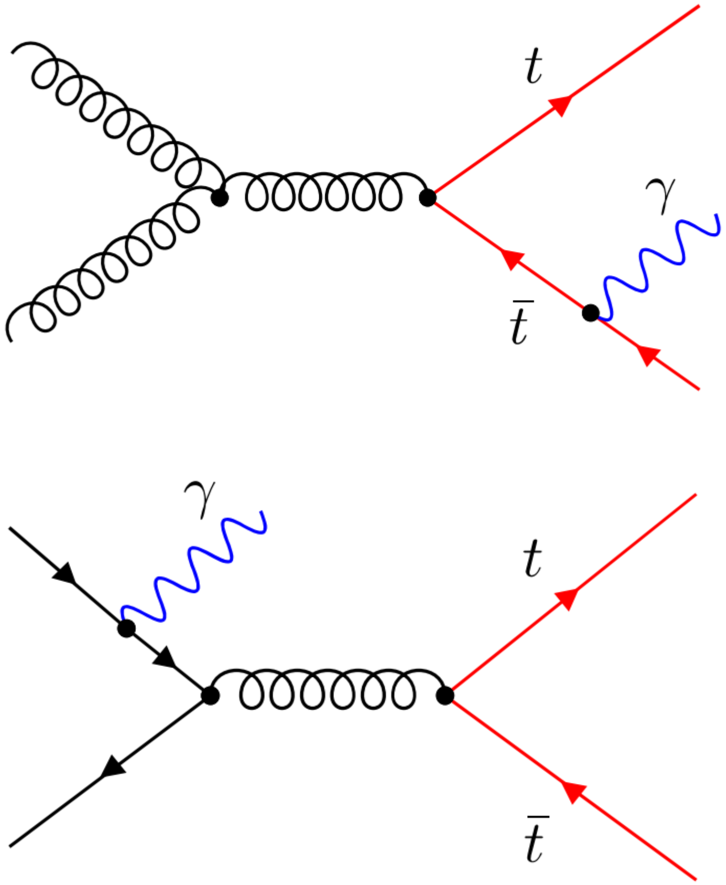
Approximate NNLO corrections (aNNLO)

- Expansion in α_s of the NNLL cross section in Mellin space
- Keep only relative order α_s^2 terms

Setup



We use the same setup as ATLAS analysis:

- Default scale choice:

$$\mu_R = \mu_F = \mu_0$$

$$\mu_0 = \frac{H_T}{2} = \frac{1}{2}\sum_i \sqrt{m_i^2 + p_{T,i}^2}$$

- Default PDF choice:

NNPDF3.0

- Default cuts:

$$p_T^\gamma > 15 \text{ GeV}$$

Results

$\mu_0 = H_T/2 \quad \text{NNPDF3.0}$

	σ [fb]	$+\delta\sigma$	$-\delta\sigma$	$\mathcal{K}^i_{t\bar{t}\gamma} = \sigma^i_{t\bar{t}\gamma}/\sigma^{\text{NLO}}_{t\bar{t}\gamma}$
NLO	3.01(2)	+15.0%	-14.1%	-
NNLL	3.48(2)	+9.5%	-10.0%	1.16
aNNLO	3.35(2)	+10.8%	-11.0%	1.11

NLO predictions obtained with mg5_aMC@NLO

The higher-order soft-gluon corrections:

- Reduce the scale dependence by 5% (4% when keeping only NNLO)
- Their size is 16% of the NLO cross section (11% from NNLO)

Results

$\mu_0 = H_T/2$ NNPDF3.0

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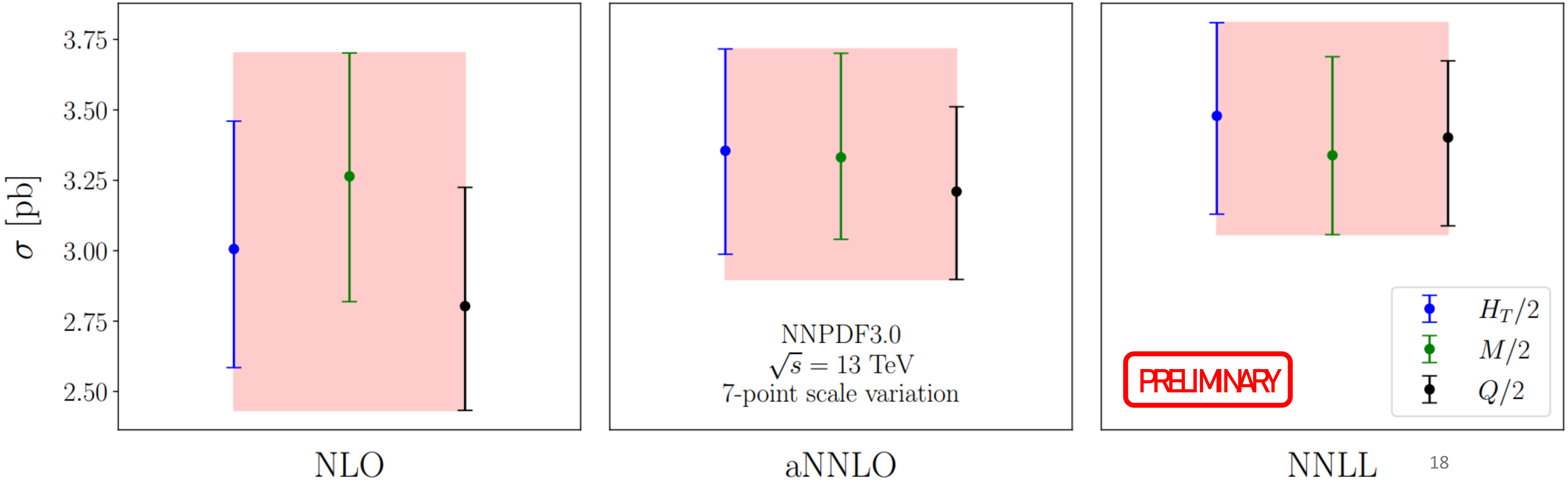
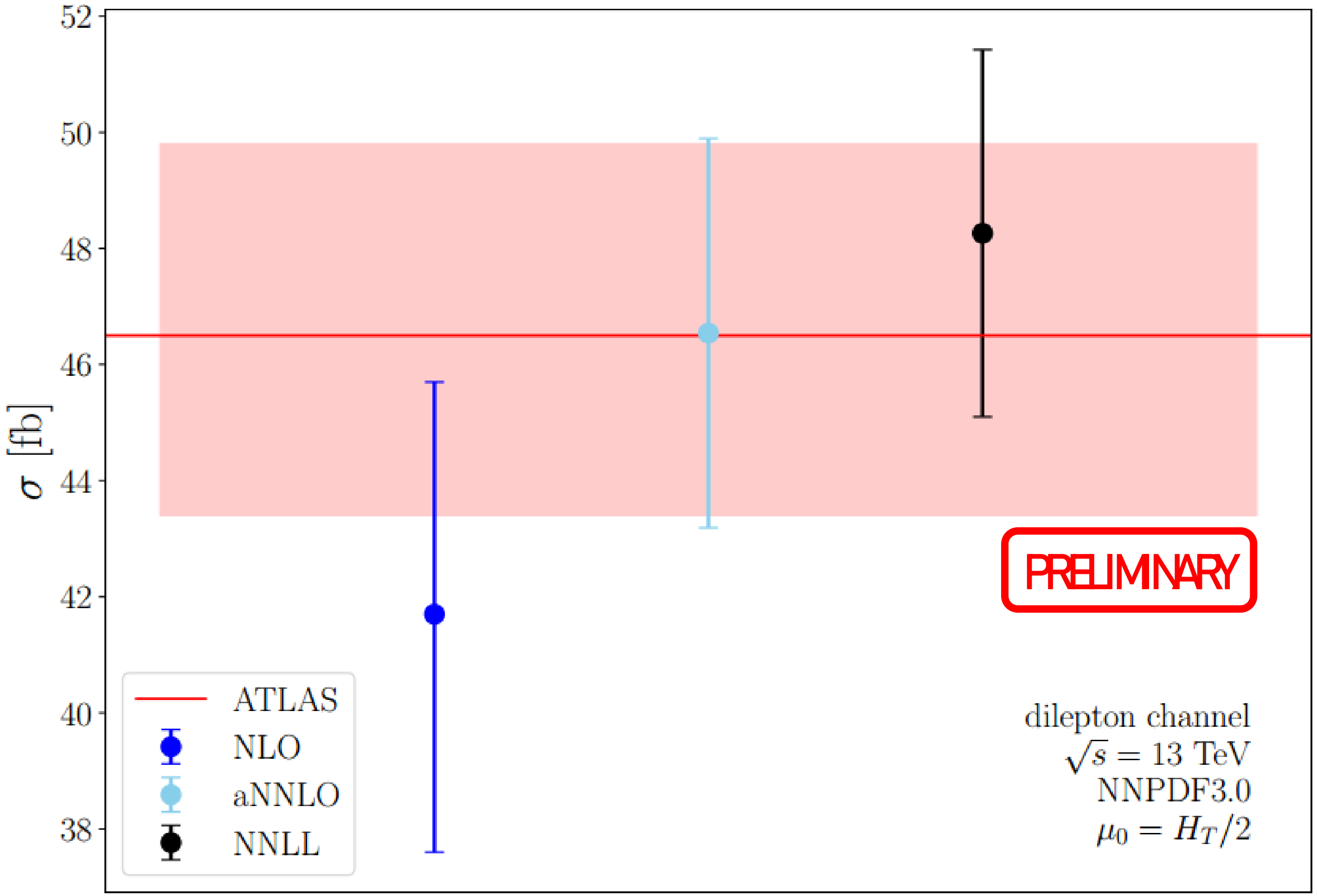


Comparison to ATLAS



	σ_{dilepton} [fb]	$+\delta\sigma$	$-\delta\sigma$
NLO	41.7	+9.6%	-9.8%
NNLL	48.3	+6.5%	-6.5%
aNNLO	46.5	+7.2%	-7.2%
ATLAS	46.5	+7.1%	-6.7%

$$\sigma^{\text{aNNLO}}_{\text{dilepton}} = \mathcal{K}^{\text{aNNLO}}_{t\bar{t}\gamma}\,\sigma^{\text{NLO}}_{\text{dilepton}}$$

$$\sigma^{\text{NNLL}}_{\text{dilepton}} = \mathcal{K}^{\text{NNLL}}_{t\bar{t}\gamma}\,\sigma^{\text{NLO}}_{\text{dilepton}}$$

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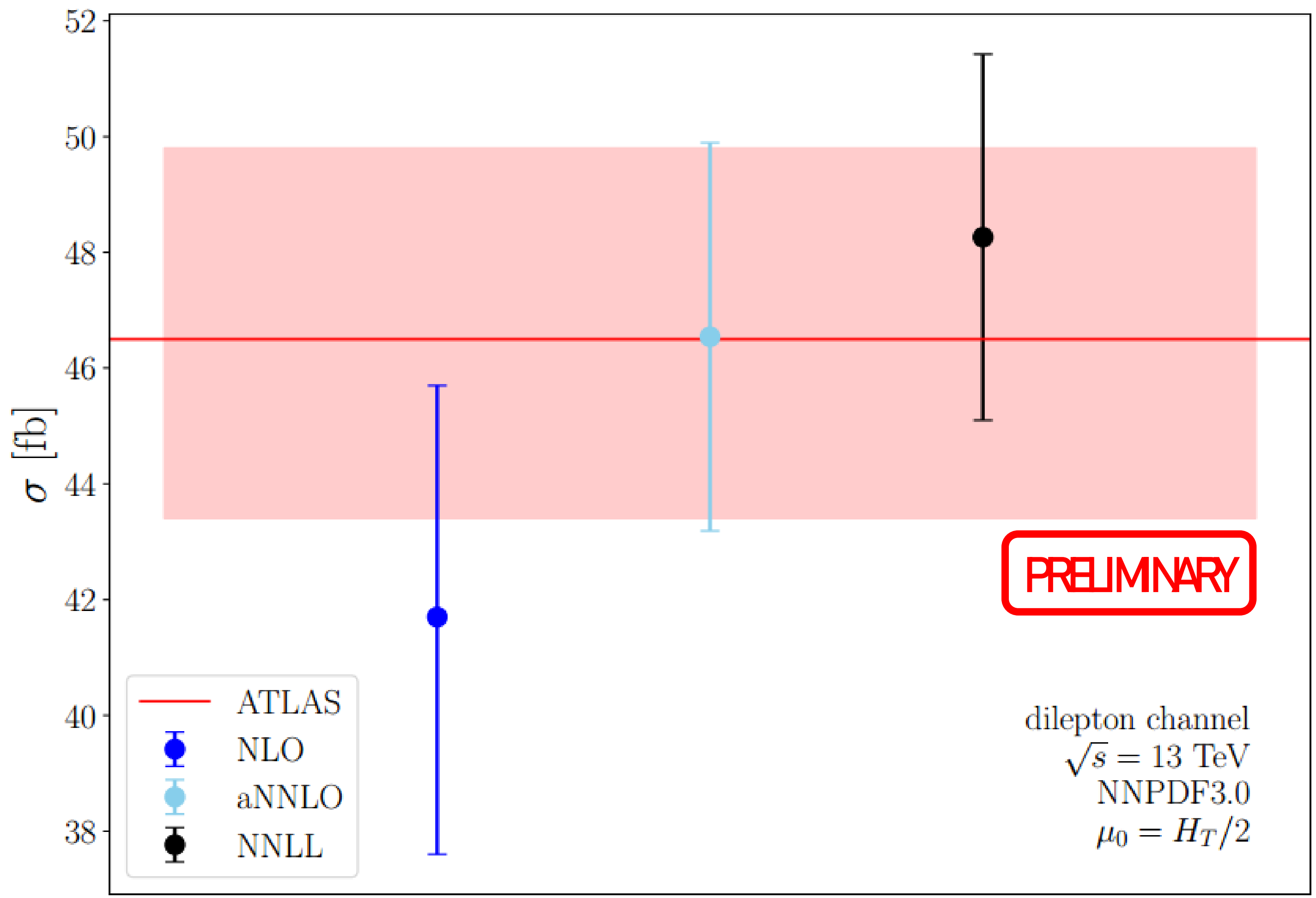

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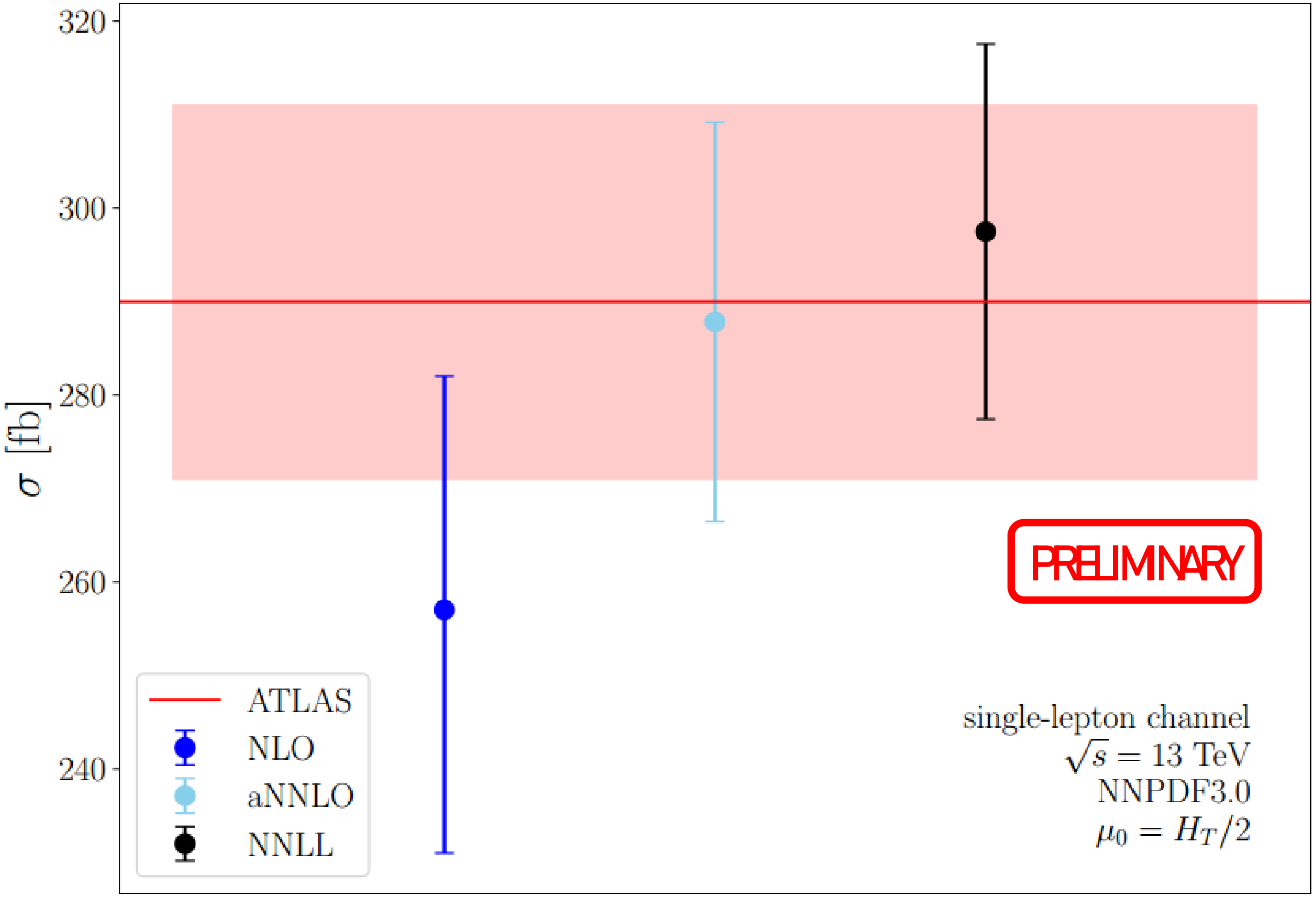

	$\sigma_{\text{single-lepton}}$ [fb]	$+\delta\sigma$	$-\delta\sigma$
NLO	257	+9.7%	-10.1%
NNLL	297	+6.7%	-6.7%
aNNLO	288	+7.4%	-7.4%
ATLAS	290	+7.2%	-6.6%

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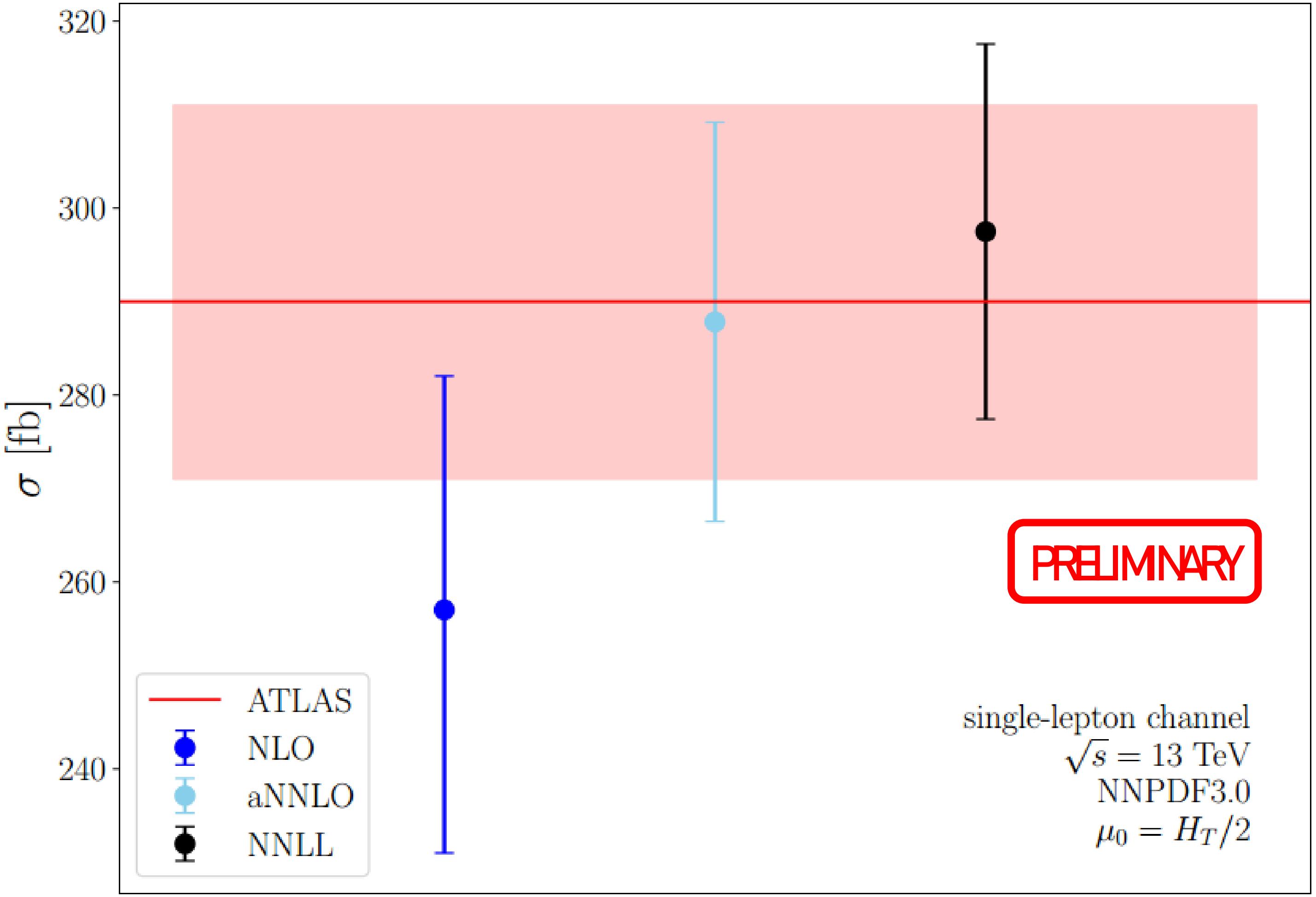

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Summary

- The inclusion of NNLL soft gluon corrections substantially reduces the dependence on the scales (the theoretical uncertainty from scale variation is estimated to ≈ 10%);

- Preliminary results that include NNLL soft gluon corrections bridge the gap between the theoretical predictions and the experimental results.

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- Completion of the comparison to ATLAS;
- Study of differential distributions.

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Thank you!