Realistic extraction of polarisation and spin-correlation coefficients in di-boson processes at the LHC

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Polarization and Spin correlations – WHY?

LHC luminosities accumulated in Run 2 (≈ 150 fb^{-1}) and foreseen in next runs (300 fb^{-1} in Run 3, and 3000 fb^{-1} in High-Lumi) at 13/14 TeV CoM energy enable precise measurements of EW processes: multi-boson production

- are crucial probes of interplay between gauge and scalar sector in SM
- provide discrimination power between SM and new physics (NP)
- **allow to construct quantum observables for entanglement tests**

The theoretical study and the experimental extraction of such pseudo-observables is thus of prime importance for the present and upcoming physics programme of the LHC

We cannot directly measure the spin state of EW bosons but we can *extract spin-sensitive coefficients from angular distributions that reflect polarisation state of the decayed V boson*

Angular dependence for boson system

At tree-level, decay of a single resonant boson ($\vartheta^* \varphi^*$ are angles in V rest frame, w.r.t. V direction in some Lorentz frame)

$$\frac{d\sigma}{d\cos\theta\, d\phi\, d\mathcal{X}} = \frac{d\sigma}{d\mathcal{X}}\left[\frac{1}{4\pi} + \sum_{l=1}^{2}\sum_{m=-l}^{l} \alpha_{lm}(\mathcal{X})\, Y_{lm}(\theta,\phi)\right]$$

8 independent coefficients extracted through projections

with no cut they represent a complete basis for the l ≤ 2 angular structure

At tree-level, decay of 2 resonant boson (e.g. ZZ):

$$\frac{d\sigma}{d\cos\theta_1\, d\phi_1\, d\cos\theta_3\, d\phi_3\, d\mathcal{X}} = \frac{d\sigma}{d\mathcal{X}}\left[\frac{1}{(4\pi)^2} + \right.$$

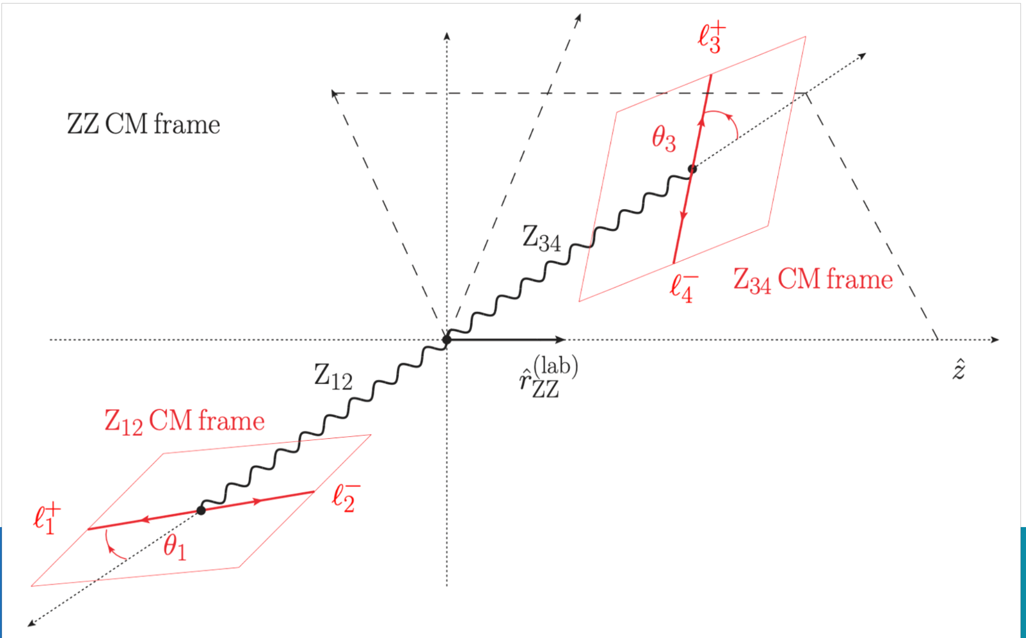
$$+\frac{1}{4\pi}\sum_{l=1}^{2}\sum_{m=-l}^{l} \alpha^{(1)}_{lm}(\mathcal{X})\, Y_{lm}(\theta_1,\phi_1)$$

$$+\frac{1}{4\pi}\sum_{l=1}^{2}\sum_{m=-l}^{l} \alpha^{(3)}_{lm}(\mathcal{X})\, Y_{lm}(\theta_3,\phi_3)$$

$$\left. + \sum_{l_1=1}^{2}\sum_{l_3=1}^{2}\sum_{m_1=-l_1}^{l_1}\sum_{m_3=-l_3}^{l_3} \gamma_{l_1 m_1 l_3 m_3}(\mathcal{X})\, Y_{l_1 m_1}(\theta_1,\phi_1)\, Y_{l_3 m_3}(\theta_3,\phi_3)\right]$$

80 independent coefficients…

bosons are correlated if $\alpha^{(1)}_{\ell_1,m_1}\alpha^{(3)}_{\ell_3,m_3} \neq \gamma_{\ell_1 m_1 \ell_3 m_3}$



Angular coefficients and quantum entanglement

The differential cross section for the process $ZZ \to l^{+}_{1}\, l^{-}_{1}\, l^{+}_{2}\, l^{-}_{2}$ can be written with an explicit connection to the components of the spin density-operator of the two vector bosons [1]

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left\{\rho\left(\Gamma_1 \otimes \Gamma_2\right)^T\right\}$$

decay density matrix of a Z boson into charged leptons

The general spin state of the ZZ system is described by a density operator, ρ, acting on the (dim 9) Hilbert space defined by the three spin states of each Z

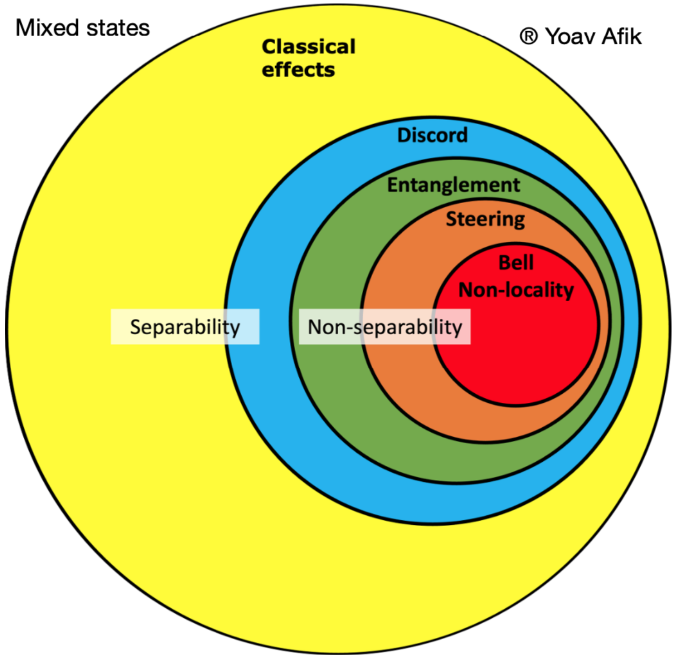
$$\rho = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{6}\left(\sqrt{2}A^{1}_{2,0}+2\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}\left(1-\sqrt{2}A^{1}_{2,0}\right) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{6}\left(\sqrt{2}A^{1}_{2,0}+2\right) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

[1] *J. A. Aguilar-Saavedra et al. - Phys. Rev. D 107, 016012*

Angular coefficients and quantum entanglement

Is the system entangled (non-classical)? What are the criterion?

The ZZ boson system is equivalent to a qutrit system the Peres-Horodecki condition for entanglement is not just sufficient (like for qubit, spin ½ system) , but also necessary



$$C_{2,1,2,-1} \neq 0 \qquad \text{or} \qquad C_{2,2,2,-2} \neq 0$$

Concurrence

$$\mathcal{C}(\rho) = \inf \left[\sum_i p_i c(|\psi_i\rangle) \right]$$

Entangled if > 0

$$(\mathcal{C}(\rho))^2 \geq 2 \max \left(0, \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[\rho_A^2\right], \mathrm{Tr}\left[\rho^2\right] - \mathrm{Tr}\left[\rho_B^2\right]\right) \equiv \mathcal{C}_{\mathrm{LB}}^2$$

$$(\mathcal{C}(\rho))^2 \leq 2 \min \left(1 - \mathrm{Tr}[\rho_A^2], 1 - \mathrm{Tr}[\rho_B^2]\right) \equiv \mathcal{C}_{\mathrm{UB}}^2$$

We can define even the conditions for the violation of Bell inequalities → we needs a strong quantum correlation, a the specific choice of the observables.

For further details see: J. A. Aguilar-Saavedra et al. - Phys. Rev. D 107, 016012 and Fabbrichesi at al. - Eur. Phys. J. C 83, 823 (2023)

Concurrence and quantum entanglement

$$\frac{d\sigma}{d\cos\theta_1\, d\phi_1\, d\cos\theta_3\, d\phi_3\, d\mathcal{X}} = \frac{d\sigma}{d\mathcal{X}}\Bigg[\frac{1}{(4\pi)^2} +$$

$$+\frac{1}{4\pi}\sum_{l=1}^{2}\sum_{m=-l}^{l} \alpha_{lm}^{(1)}(\mathcal{X})\, Y_{lm}(\theta_1,\phi_1) \quad \rightarrow f_a$$

$$+\frac{1}{4\pi}\sum_{l=1}^{2}\sum_{m=-l}^{l} \alpha_{lm}^{(3)}(\mathcal{X})\, Y_{lm}(\theta_3,\phi_3) \quad \rightarrow g_a$$

$$+\sum_{l_1=1}^{2}\sum_{l_3=1}^{2}\sum_{m_1=-l_1}^{l_1}\sum_{m_3=-l_3}^{l_3} \gamma_{l_1 m_1 l_3 m_3}(\mathcal{X})\, Y_{l_1 m_1}(\theta_1,\phi_1)\, Y_{l_3 m_3}(\theta_3,\phi_3)\Bigg] \quad \rightarrow h_{ab}$$

$$\rightarrow$$

$$\mathscr{C}_2 = 2\max\Bigg[-\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2,\; -\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2,\; 0\Bigg],$$

It is possible to compute the observable quantifying the entanglement in the gauge boson system once the coefficients f_a, g_a and h_{ab} are known

For further details see: J. A. Aguilar-Saavedra et al. - Phys. Rev. D 107, 016012 and Fabbrichesi at al. - Eur. Phys. J. C 83, 823 (2023)

Where are the issues: coefficient extraction

Ideal setting: coefficients extracted directly from data through projections up to $l_{1,3} = 2$ of the angular distributions $d\sigma/d\Omega_1 d\Omega_3$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_1\,\mathrm{d}\phi_1\,\mathrm{d}\cos\theta_3\,\mathrm{d}\phi_3\,\mathrm{d}\mathcal{X}}$$

Real setting (main consideration):

- assumes two spin-1 resonances being produced
- assumes two-body decays
- is not invariant under Lorentz boosts
- is not described by $l_{1,3} \leq 2$ if selection cuts are applied

→

Contribution of this talk:

Effect of high order correction

Effect of kinematic cuts

RESULTS – HO correction

ZZ			
	LO	NLO QCD	NLO EW
$\alpha^{(1)}_{1,0}$	$-0.00001(9)$	$-0.00097(10)$	$-0.00004(9)$
$\alpha^{(1)}_{2,0}$	$0.03009(11)$	$0.02794(13)$	$0.02960(11)$
$\alpha^{(3)}_{1,0}$	$0.00012(13)$	$-0.00086(15)$	$0.00018(14)$
$\alpha^{(3)}_{2,0}$	$0.03006(7)$	$0.02796(6)$	$0.02964(7)$
$\gamma_{1,0,1,0}$	$-0.00173(3)$	$-0.00148(3)$	$-0.00043(3)$
$\gamma_{2,0,2,0}$	$0.00188(2)$	$0.00168(2)$	$0.00187(2)$
$\alpha^{(1)}_{2,-2}$	$-0.00967(7)$	$-0.00993(9)$	$-0.00991(7)$
$\alpha^{(3)}_{2,-2}$	$-0.00973(4)$	$-0.01003(4)$	$-0.00996(4)$

Angular coefficients for **ZZ at LO, NLO QCD and NLO EW** accuracy for the full off-shell process.

All numbers have been computed integrating fully differential weights from POWHEG-BOX- RES multiplied by suitable combinations of spherical harmonics ($l \leq 2$) in the decay angles.

Numerical uncertainties are shown in parantheses. A fixed renormalisation and factorisation scale $\mu_F = \mu_R = M_Z$ is assumed, inclusive setup: $81\ \text{GeV} < M_{\ell^+\ell^-} < 101\ \text{GeV}$

- Higher-rank spherical harmonics appear with three-body decay: QED/QCD radiation of decay products
- QCD radiation sizeably changes longitudinal components, although angular expansion defined in ZZ CM frame, small EW effects
- QCD channel changes helicity structure, fraction of polarization components

Spin correlations for $\gamma_{2,0,2,0}$ in ZZ

Projection on $l = 2$ sph. Harm related to longitudinal production:

$$\int d\Omega_1 \int d\Omega_3 \, Y_{2,0}(\theta_1, \phi_1) Y_{2,0}(\theta_3, \phi_3) \frac{1}{\sigma} \frac{d\sigma}{d\Omega_1 d\Omega_3} = \gamma_{2,0,2,0}$$

$$f_L^{(i)} = \frac{1}{3}\left(1 - 4\sqrt{5\pi}\,\alpha_{2,0}^{(i)}\right), \quad f_{LL} = \frac{1}{3}\left(1 - 4\sqrt{5\pi}\,\alpha_{2,0}^{(1)} - 4\sqrt{5\pi}\,\alpha_{2,0}^{(3)} + 80\pi\,\gamma_{2,0,2,0}\right)$$

Level of correlation between the two bosons, R_c [ATLAS 2211.09435]

$$R_c = \frac{f_{LL}}{f_L^{(1)} f_L^{(3)}} = \frac{1 - 4\sqrt{5\pi}\,\alpha_{2,0}^{(1)} - 4\sqrt{5\pi}\,\alpha_{2,0}^{(3)} + 80\pi\,\gamma_{2,0,2,0}}{1 - 4\sqrt{5\pi}\,\alpha_{2,0}^{(1)} - 4\sqrt{5\pi}\,\alpha_{2,0}^{(3)} + 80\pi\,\alpha_{2,0}^{(1)}\,\alpha_{2,0}^{(3)}}$$

	ZZ			
	LO	NLO QCD	NLO EW	DPA
$\gamma_{2,0,2,0}$	0.00188(2)	0.00168(2)	0.00187(2)	0.00168(2)

RESULTS – kinematic cuts

	ZZ	
	inclusive	ATLAS fiducial
$\alpha_{1,0}^{(1)}$	$-0.00093(5)$	$-0.00025(7)$
$\alpha_{2,0}^{(1)}$	$0.02807(5)$	$0.02075(7)$
$\alpha_{1,0}^{(3)}$	$-0.00102(5)$	$-0.00141(7)$
$\alpha_{2,0}^{(3)}$	$0.02794(5)$	$0.02371(7)$
$\gamma_{1,0,1,0}$	$-0.00146(2)$	$-0.00145(2)$
$\gamma_{2,0,2,0}$	$0.00167(1)$	$0.00195(2)$
$\alpha_{2,-2}^{(1)}$	$-0.01000(2)$	$-0.02614(3)$
$\alpha_{2,-2}^{(2)}$	$-0.01001(2)$	$-0.02223(4)$

Angular coefficients for **ZZ NLO QCD** accuracy for the full off-shell process.

All numbers have been computed integrating fully differential weights from POWHEG-BOX- RES multiplied by suitable combinations of spherical harmonics ($l \leq 2$) in the decay angles.

Numerical uncertainties are shown in parantheses. A fixed renormalisation and factorisation scale $\mu_F = \mu_R = M_Z$ is assumed.

- Angular expansion ($l \leq 2$ spherical harmonics) valid fully differentially and after integration, if no cuts are applied on decay products
- Fiducial cut triggers angular modulation $\mathbf{f_{cut}}$ which is not a combination of $l \leq 2$ spherical harmonics:

$$\int_{-1}^{1} d\cos\theta^* \int_{0}^{2\pi} d\phi^* \; Y_{\ell m}(\theta^*, \phi^*) \frac{d\sigma}{d\cos\theta^* \, d\phi^*} f_{\rm cut}(\theta^*, \phi^*) \neq \sigma_{\rm cut}\alpha_{\ell m}$$

Conclusion

- Polarizations are framed in the spin density matrix and correlations can also be used to measure <u>quantum entanglement</u>
- This is a way of testing of quantum mechanics beyond electrodynamics with *weak and strong interactions* in presence of states with more than 2 *dof* (polarization of massive spin-1 particle)
- Angular decay structure gives access to spin-density matrix (*aka quantum state tomography*)
- For VV systems, expansion for $l \leq 2$ in spherical harmonics has limited validity:
    - fiducial cuts disrupt $l \leq 2$ expansion, extrapolation needed
    - NLO correction sizeably change the angular coefficients
    - All entanglement observables so far constructed at LO: higher orders unavoidable!

Outlook

- Similar studies on decay of the decay of a SM Higgs-boson into massless charged leptons
- Angular acceptance induced by fiducial cuts and neutrino reconstruction in the SM and in the presence of SMEFT effects at NLO QCD accuracy for the full off-shell W+Z process

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