Top quark mass calibration for MC event generators

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Update of previous study: [Butenschoen, Dehnadi, Hoang, VM, Preisser, Stewart, PRL 117 (2016) 23, 232001]

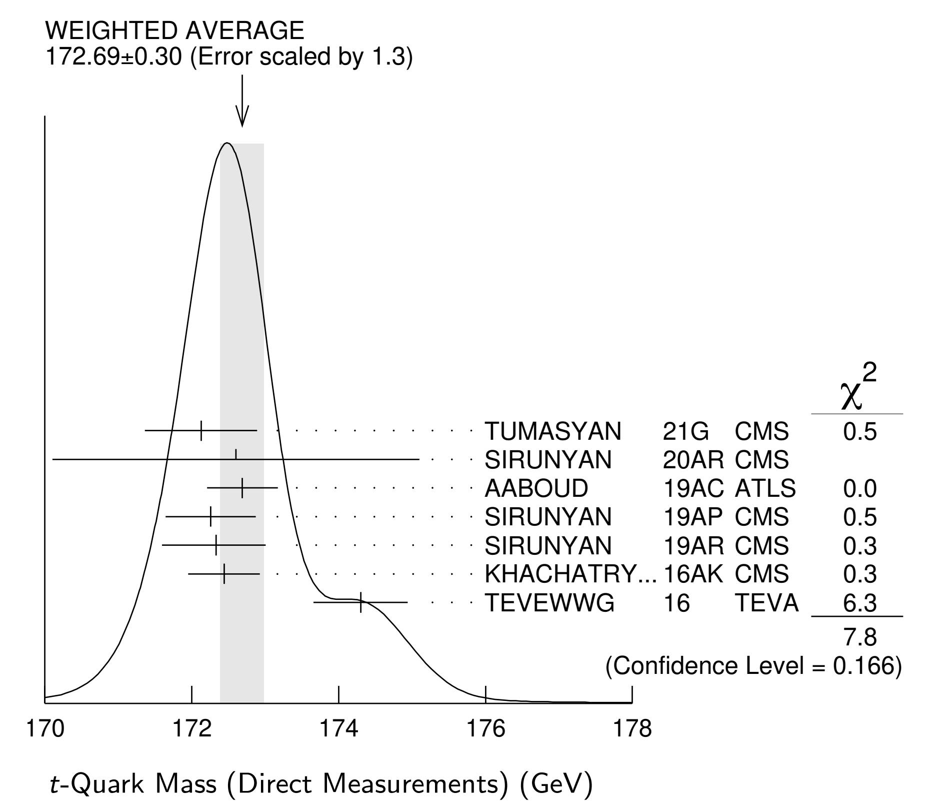
Motivation

The top quark is the heaviest particle found so far

Only quark capable of escaping infrared slavery

Direct measurements correspond to the MC top quark parameter

not the pole mass
related to a short-distance mass?



Current world average

$$m_t^{\mathrm{MC}} = (172.69 \pm 0.30)\ \mathrm{GeV}$$

Relation to a short-distance mass

For the NLL-precise coherent branching shower one can show

$$m_t^{\mathrm{MC}} - m_t^{\mathrm{pole}} \propto Q_0 \times \alpha_s(Q_0)$$

transverse
momentum
shower cut

[Hoang, Plätzer, Samitz
JHEP10 (2018) 200]

Q_0 acts as a IR factorization scale and a resolution parameter

Therefore, natural to associate $m_t^{\mathrm{MC}} \approx m_t^{\mathrm{MSR}}(R = Q_0)$ [Hoang]

MSR mass: [Hoang, Jain, Scimemi, Stewart;
Hoang, Jain, Lepenik, VM, Preisser, Scimemi, Stewart]

In this talk we make this relation more quantitative

The MSR mass

$$m_t^{\rm pole} - \overline{m}_t = \overline{m}_t \sum_{n=1} a_n^{\overline{\rm MS}}(n_\ell = 5, n_h = 1)\left[\frac{\alpha_s^{(6)}(\overline{m}_t)}{4\pi}\right]^n$$

The series has an $\mathcal{O}(\Lambda_{\rm QCD})$ ambiguity (renormalon)

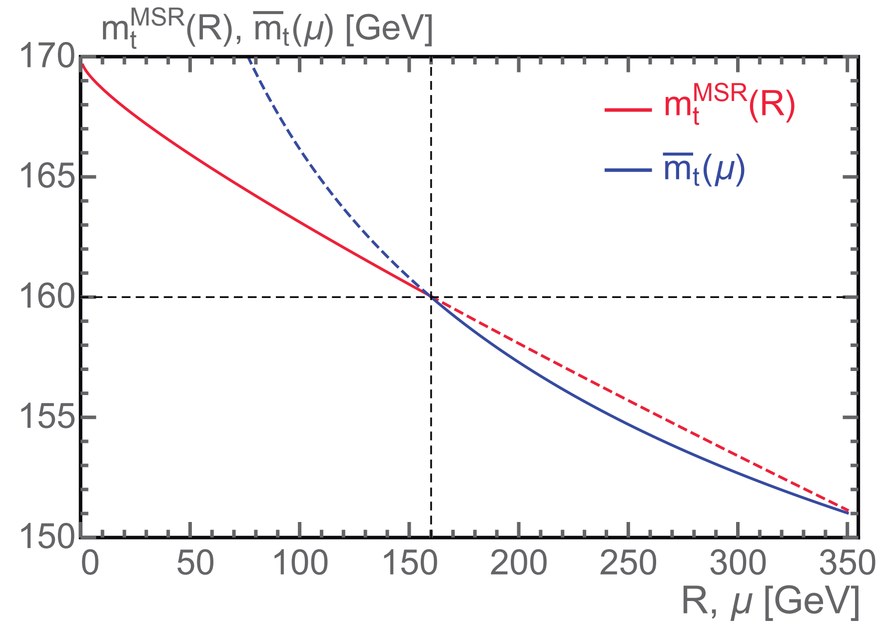
$\overline{\rm MS}$ is a high-energy mass, not adequate for threshold problems

$$m_t^{\rm pole} - m_t^{\rm MSR}(R) = R \sum_{n=1} a_n^{\overline{\rm MS}}(n_\ell = 5, 0)\left[\frac{\alpha_s^{(5)}(R)}{4\pi}\right]^n$$

same ambiguity, but now it is an n_ℓ-quantity

MSR adequate for problems in which m_t is no longer dynamic

R-evolution



Use REvolver library to match & RG-evolve masses and coupling

[Hoang, Lepenik, VM, 2102.01085]

Setup

Observables

$$\begin{cases} \tau_2 = \dfrac{1}{Q} \min\limits_{\vec{n}_t} \sum\limits_i (E_i - |\vec{n}_t \cdot \vec{p}_i|) & \text{2-jettiness} \\ \tau_s = \rho_a + \rho_b & \text{sum of hemisphere masses} \\ \tau_m = \tau_s + \dfrac{1}{2}\tau_s^2 & \text{modified jet mass} \end{cases}$$

The three differ by mass and kinematic power corrections only

Same factorisation theorem, different power corrections

Setup

$$\text{Observables}\ \left\{\begin{array}{l} \tau_2 = \dfrac{1}{Q}\min_{\vec{n}_t}\displaystyle\sum_i (E_i - |\vec{n}_t \cdot \vec{p}_i|) \\[2ex] \tau_s = \rho_a + \rho_b \\[2ex] \tau_m = \tau_s + \dfrac{1}{2}\tau_s^2 \end{array}\right.$$

Easiest mass correction: lower endpoint

$$\tau_{2,\min} = 1 - \sqrt{1 - 4\hat{m}_t^2} = 2\hat{m}_t^2 + 2\hat{m}_t^4 + \mathcal{O}(\hat{m}_t^6)$$

$$\tau_{s,\min} = 2\hat{m}_t^2$$

$$\tau_{m,\min} = 2\hat{m}_t^2 + 2\hat{m}_t^4$$

$$\hat{m}_t \equiv \frac{m_t}{Q}$$

Setup

Affected by $\hat{m}_t^2$ corrections in measurement function

sizable due to soft and non-perturbative effects

Not affected by such corrections

Observables

$$\tau_2 = \frac{1}{Q} \min_{\vec{n}_t} \sum_i (E_i - |\vec{n}_t \cdot \vec{p}_i|)$$ 2-jettiness

$$\tau_s = \rho_a + \rho_b$$ sum of hemisphere masses

$$\tau_m = \tau_s + \frac{1}{2}\tau_s^2$$ modified jet mass

We implement these mass corrections

τ_m : important diagnosis tool for our theoretical treatment

Setup

Compute the same observable (at hadron level) in

parton shower MC

- Pythia 8.205
- herwig 7.2.1
- sherpa 2.2.11

Larger sensitivity to m_t in the peak region

Observables

$$\tau_2 = \frac{1}{Q} \min_{\vec{n}_t} \sum_i (E_i - |\vec{n}_t \cdot \vec{p}_i|)$$ 2-jettiness

$$\tau_s = \rho_a + \rho_b$$ sum of hemisphere masses

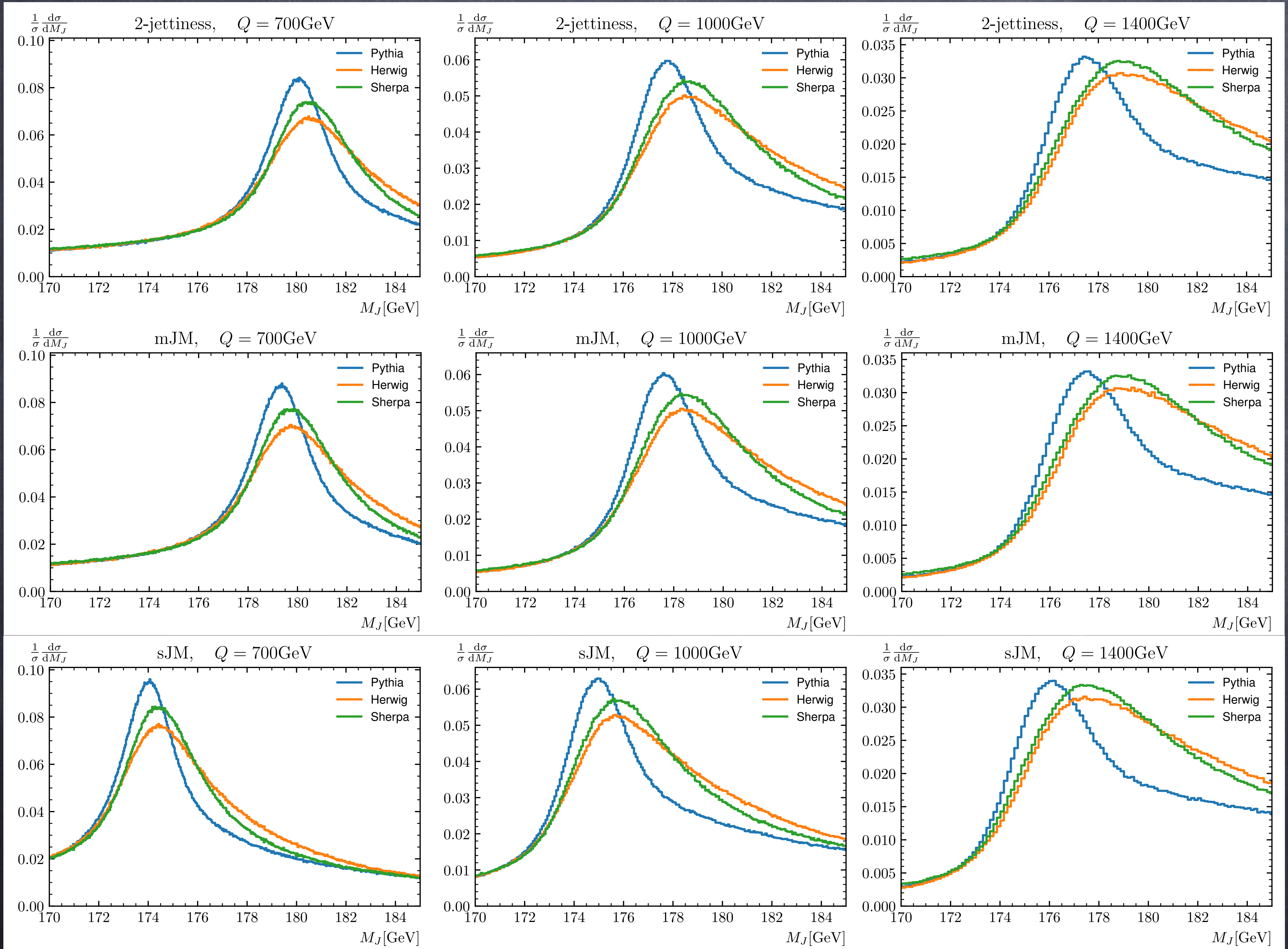
$$\tau_m = \tau_s + \frac{1}{2}\tau_s^2$$ modified jet mass

Monte Carlo predictions

$$M_J = Q\sqrt{\frac{\tau}{2}} = m_t + \mathcal{O}(m_t^2, \Gamma_t, \alpha_s)$$

better meaning to the peak position

$$m_t^{\mathrm{MC}} = 173\,\mathrm{GeV}$$



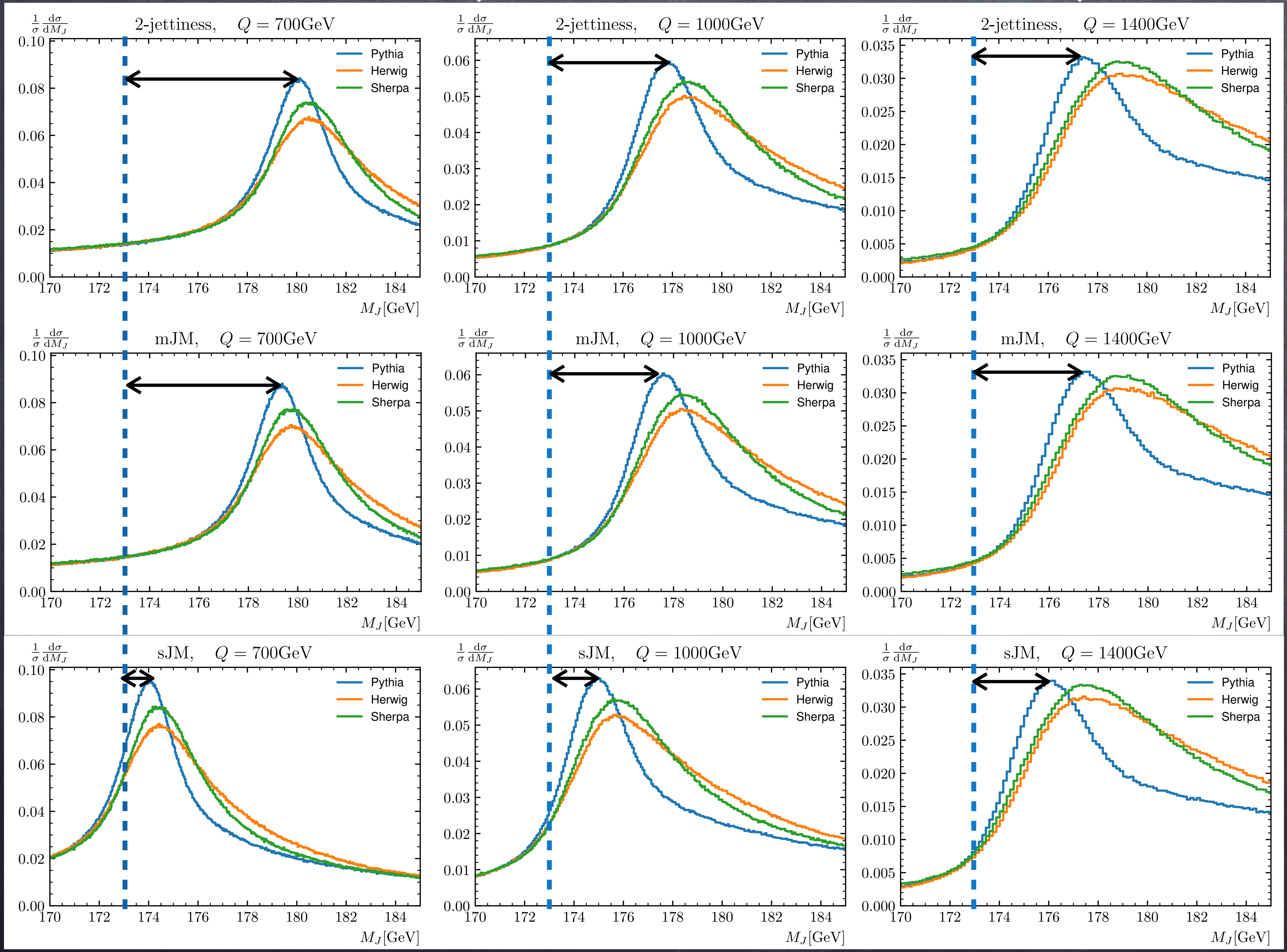
Q = c.o.m. energy

modified jet mass

sum of jet masses

Monte Carlo predictions

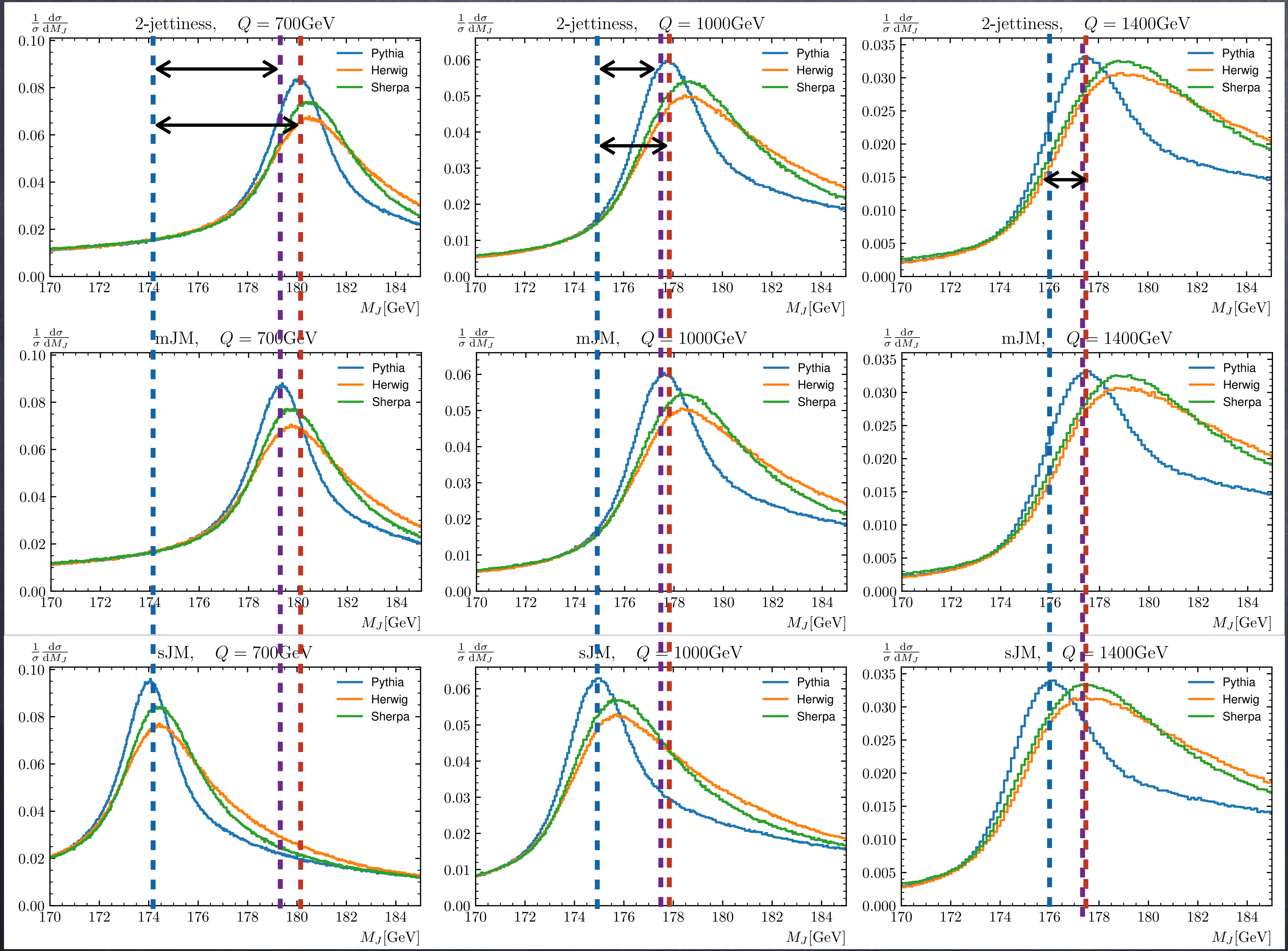
peak far from mass position due to soft and collinear radiation, and non-perturbative effects (Q-dependent)



Monte Carlo predictions

mass power corrections have
large impact in peak position

$$\tau_{2,\min} = 1 - \sqrt{1 - 4\hat{m}_t^2} = 2\hat{m}_t^2 + 2\hat{m}_t^4 + \mathcal{O}(\hat{m}_t^6)$$



$$\tau_{m,\min} = 2\hat{m}_t^2 + 2\hat{m}_t^4$$

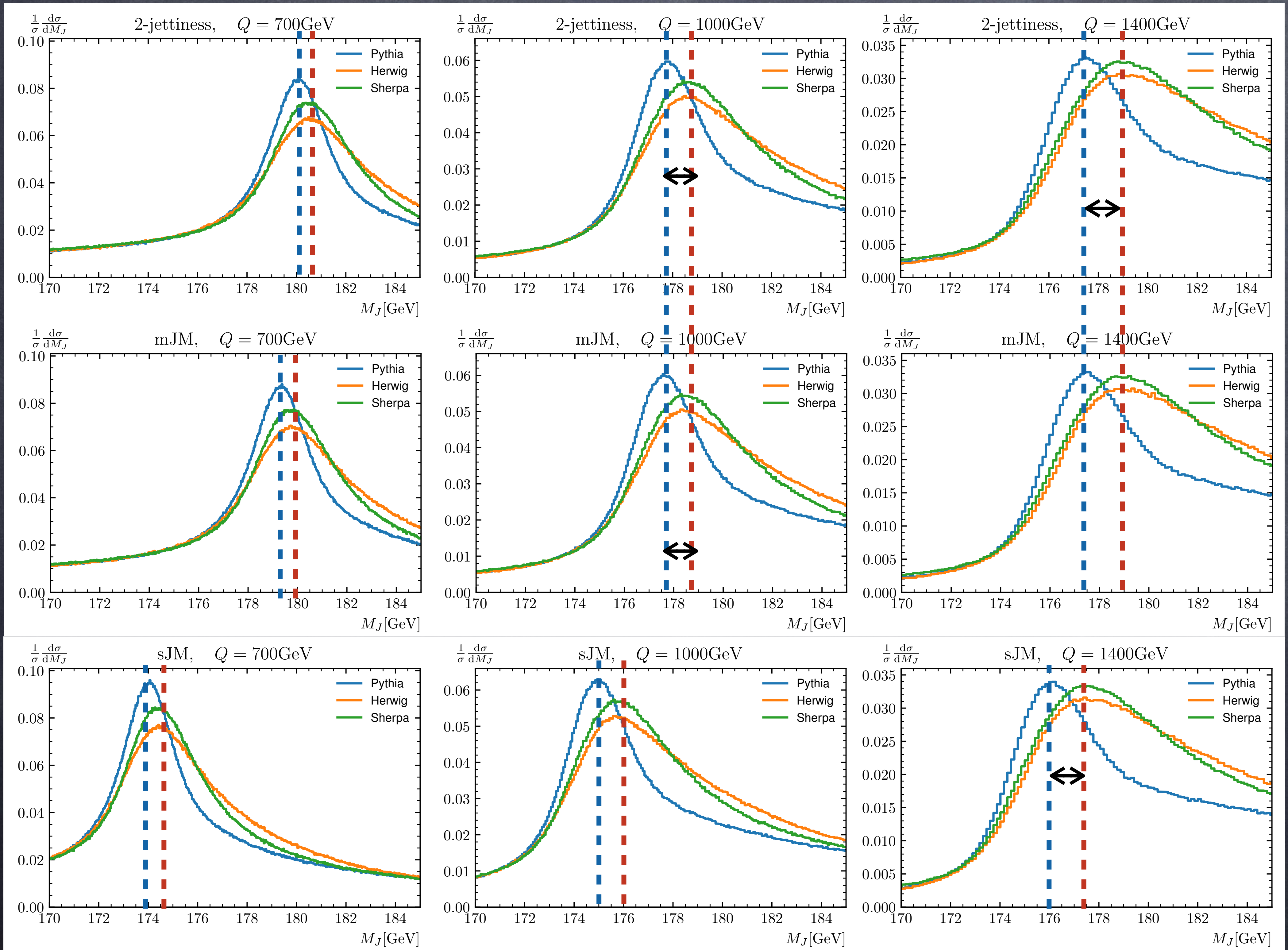
$$\tau_{s,\min} = 2\hat{m}_t^2$$

Monte Carlo predictions

different MCs have different peak positions

caused by different description of hadronization

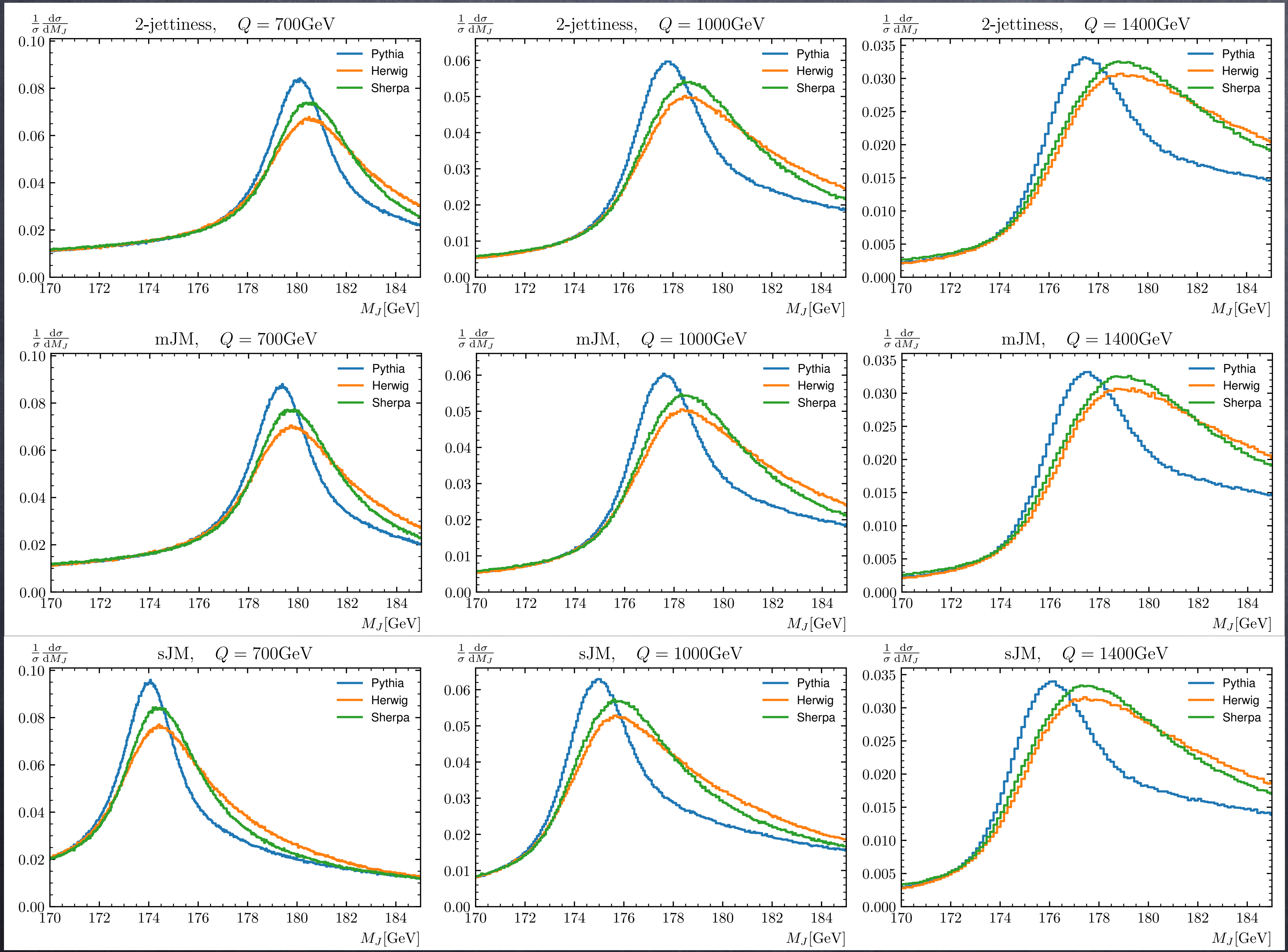
$$m_t^{\mathrm{MC}} = 173\,\mathrm{GeV}$$



Monte Carlo predictions

also the shapes of the distributions are different

spoiler: these differences do not affect the calibration results



Theoretical description

Compute the same observable (at hadron level) in

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  - Pythia 8.205
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  - sherpa 2.2.11
- first-principle QCD
  - bHQET
  - SCET
  - FO QCD

Theoretical description at N^{2}LL + NLO

includes

- mass corrections in
  - lower endpoint position
  - singular structures
  - measurement function
- effects of finite top width Γ_t through Breit-Wigner
- kinematical power corrections
- soft and mass renormalon subtractions

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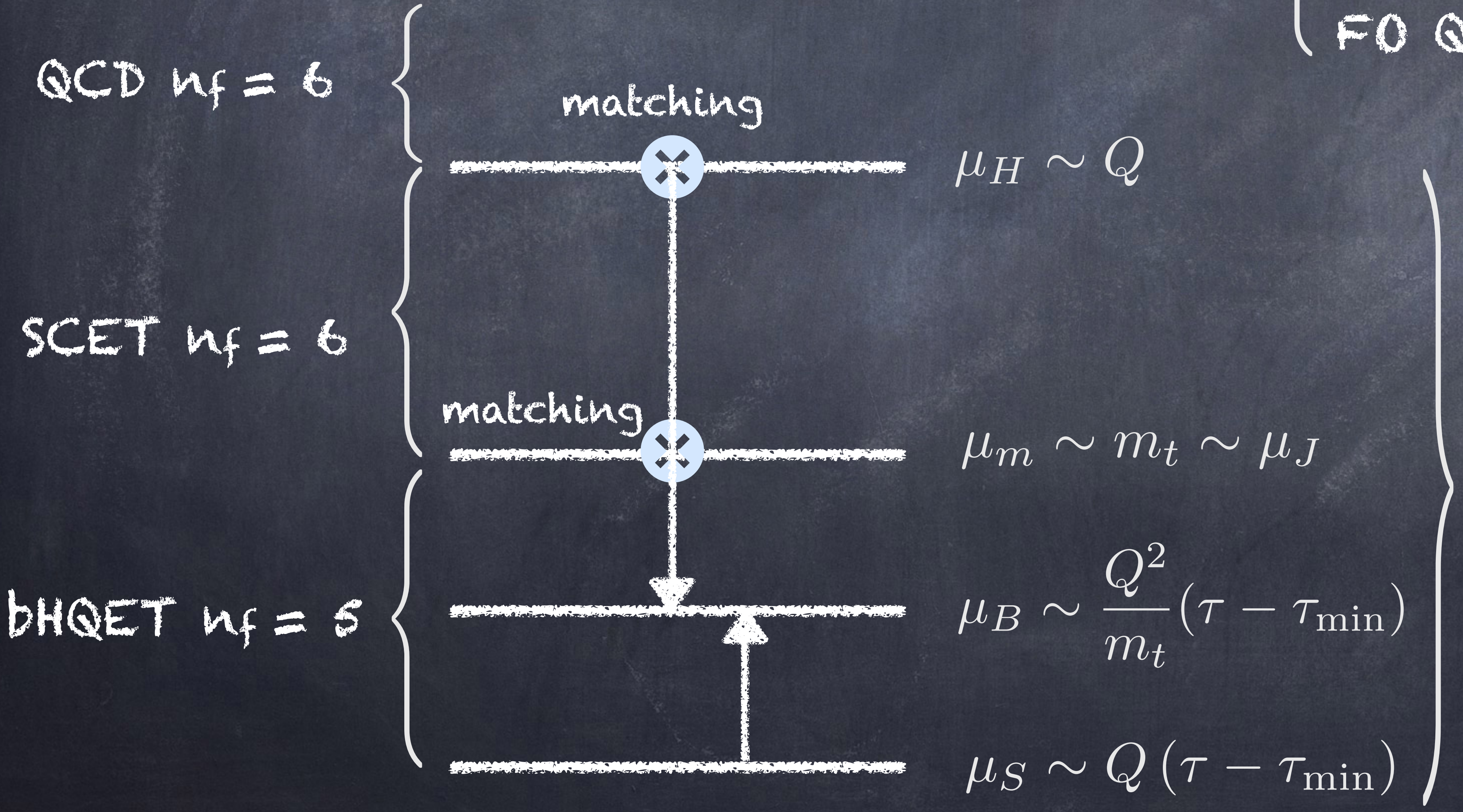
large mass sensitivity



large hierarchy

$\mu_H \gg \mu_m \gg \mu_B \sim \Gamma_t \gg \mu_S$

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large mass sensitivity

cross section [Fleming, Hoang, Mantry, Stewart]

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = Q^2 H_Q(Q,\mu_m)H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big)\int \mathrm{d}\ell\, B_\tau\Big(\frac{Q^2\tau - 2m^2 - Q\ell}{m},\Gamma_t,\mu\Big)S_\tau(\ell,\mu)$$

Worked out at $\mathrm{N^3LL}$ in [Bachu, Hoang, VM, Pathak, Stewart]

Jet function known at 3-loops [A. Clavero, R. Bruser, VM, M. Stahlhofen]

Rest of ingredients known at 2-loops

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RG evolution between matrix elements

soft renormalon subtraction

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RG evolution between matrix elements

soft renormalon subtraction

finite width effects

$$B_n(\hat{s},\Gamma_t,\mu) = \int_0^\infty \frac{\mathrm{d}\hat{s}'}{\pi}\frac{\Gamma_t}{(\hat{s}-\hat{s}')^2+\Gamma_t^2} B_n(\hat{s}',\mu)$$

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cross section [Fleming, Hoang, Mantry, Stewart]

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = Q^2 H_Q(Q,\mu_m) H_m\!\left(m,\frac{Q}{m},\mu_m,\mu\right)\int \mathrm{d}\ell \, \boxed{B_\tau\!\left(\frac{Q^2\tau-2m^2-Q\ell}{m},\Gamma_t,\mu\right)} S_\tau(\ell,\mu)$$

RG evolution between matrix elements

soft renormalon subtraction

finite width effects

mass renormalon subtraction: use MSR mass with $R \sim \Gamma_t$

Summary

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Observables

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- $\tau_s = \rho_a + \rho_b$ sum of hemisphere masses
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Gap subtraction scheme

- derivative of partonic soft function
- subtraction without derivatives

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used in previous calibration

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new in updated analysis

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  - bHQET
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  - FO QCD

Observables

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Gap subtraction scheme

- derivative of partonic soft function

new in updated analysis

- subtraction without derivatives

Non-perturbative effects

$$S(\ell, \mu_S) = \int \mathrm{d}k\, \hat{S}_\tau^{(5)}(\ell - k, \bar{\delta}, \mu_S) F(k - 2\hat{\Delta})$$

shape function

partonic soft function

renormalon subtraction

[Ligeti, Tackmann, Stewart, Phys. Rev. D 78, 114014]

$$F(k; \lambda, \{c_i\}, N) = \frac{1}{\lambda}\left[\sum_{n=0}^{N} c_n f_n\left(\frac{k}{\lambda}\right)\right]^2$$

model independent description: expansion using a basis function on Legendre Polynomial

normalisation: $\sum_i c_i^2 = 1$

Truncation at N = 3 sufficient for the calibration

most relevant non-perturbative parameter

$$\Omega_1(\lambda, \hat{\Delta}, N) = \frac{1}{2}\int_0^\infty \mathrm{d}k\, k\, F(k - 2\hat{\Delta}; \lambda, \{c_i\}, N)$$

Calibration procedure

Observables: binned cross section normalised to the fit window

$$f_{Q,i}(m_t;\{a\},\Delta_0,\lambda)=\frac{\int_{\tau_i}^{\tau_{i+1}}\mathrm{d}\tau\,\frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau}}{\int_{\tau_{\min}}^{\tau_{\max}}\mathrm{d}\tau\,\frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau}}$$

$$\chi^2(m_t;\{a\},\Delta_0,\lambda)=\sum_Q\sum_{\tau_{\min}\le\tau_i<\tau_{\max}}\frac{\left[f_{Q,i}^{\text{theo}}(m_t;\{a\},\Delta_0,\lambda)-f_{Q,i}^{\text{MC}}\right]^2}{\sigma_{Q,i}^2}$$

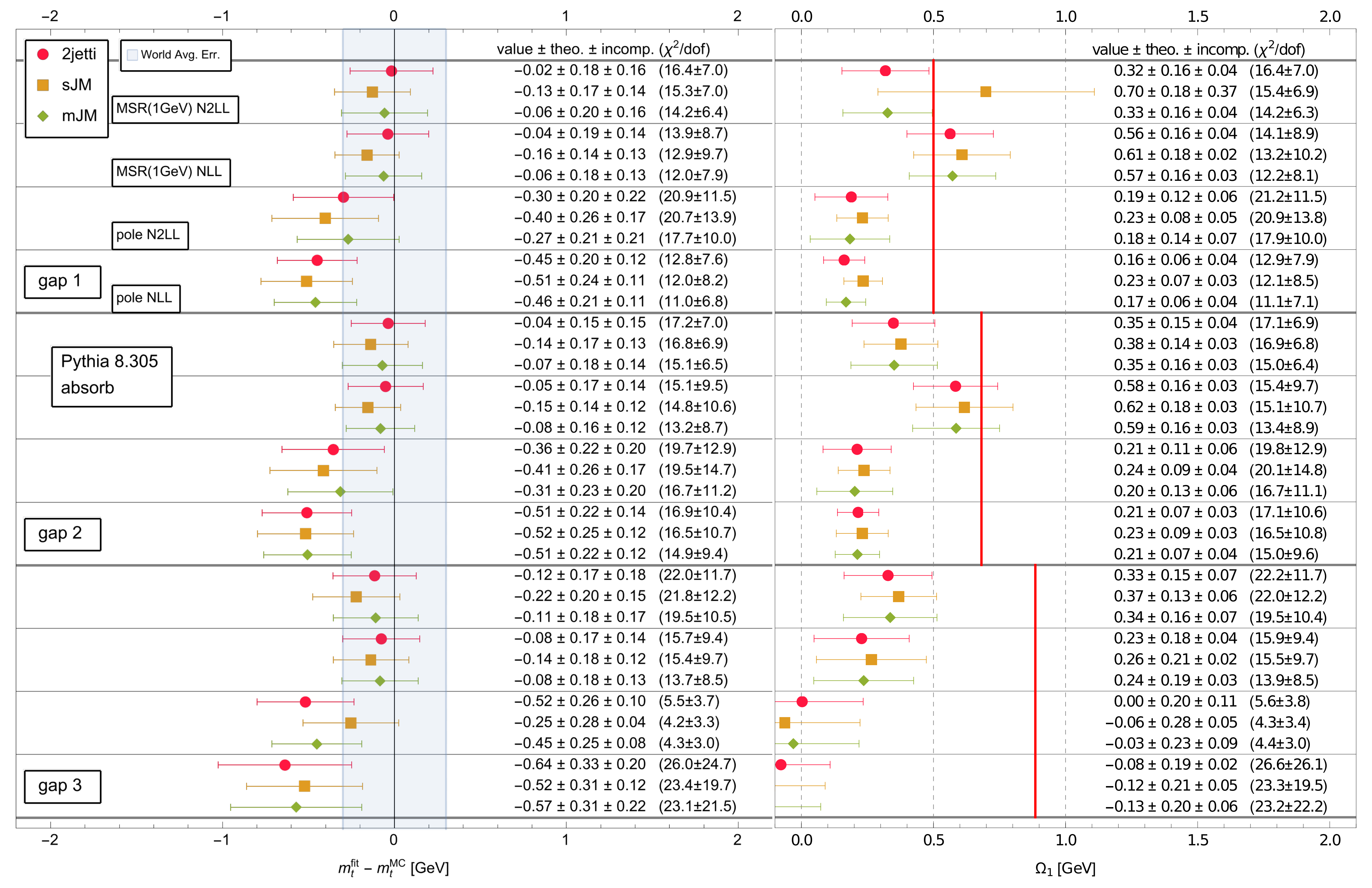
only MC statistical uncertainties

Use fortran-2008 code CALIPER paired with python analysis tools

Will quote only results for mass and Ω_1 only

Results

Results for Pythia with a fixed value of MC mass



value ± theo. ± incomp. (χ^2/dof)	value ± theo. ± incomp. (χ^2/dof)
−0.02 ± 0.18 ± 0.16 (16.4±7.0)	0.32 ± 0.16 ± 0.04 (16.4±7.0)
−0.13 ± 0.17 ± 0.14 (15.3±7.0)	0.70 ± 0.18 ± 0.37 (15.4±6.9)
−0.06 ± 0.20 ± 0.16 (14.2±6.4)	0.33 ± 0.16 ± 0.04 (14.2±6.3)
−0.04 ± 0.19 ± 0.14 (13.9±8.7)	0.56 ± 0.16 ± 0.04 (14.1±8.9)
−0.16 ± 0.14 ± 0.13 (12.9±9.7)	0.61 ± 0.18 ± 0.02 (13.2±10.2)
−0.06 ± 0.18 ± 0.13 (12.0±7.9)	0.57 ± 0.16 ± 0.03 (12.2±8.1)
−0.30 ± 0.20 ± 0.22 (20.9±11.5)	0.19 ± 0.12 ± 0.06 (21.2±11.5)
−0.40 ± 0.26 ± 0.17 (20.7±13.9)	0.23 ± 0.08 ± 0.05 (20.9±13.8)
−0.27 ± 0.21 ± 0.21 (17.7±10.0)	0.18 ± 0.14 ± 0.07 (17.9±10.0)
−0.45 ± 0.20 ± 0.12 (12.8±7.6)	0.16 ± 0.06 ± 0.04 (12.9±7.9)
−0.51 ± 0.24 ± 0.11 (12.0±8.2)	0.23 ± 0.07 ± 0.03 (12.1±8.5)
−0.46 ± 0.21 ± 0.11 (11.0±6.8)	0.17 ± 0.06 ± 0.04 (11.1±7.1)
−0.04 ± 0.15 ± 0.15 (17.2±7.0)	0.35 ± 0.15 ± 0.04 (17.1±6.9)
−0.14 ± 0.17 ± 0.13 (16.8±6.9)	0.38 ± 0.14 ± 0.03 (16.9±6.8)
−0.07 ± 0.18 ± 0.14 (15.1±6.5)	0.35 ± 0.16 ± 0.03 (15.0±6.4)
−0.05 ± 0.17 ± 0.14 (15.1±9.5)	0.58 ± 0.16 ± 0.03 (15.4±9.7)
−0.15 ± 0.14 ± 0.12 (14.8±10.6)	0.62 ± 0.18 ± 0.03 (15.1±10.7)
−0.08 ± 0.16 ± 0.12 (13.2±8.7)	0.59 ± 0.16 ± 0.03 (13.4±8.9)
−0.36 ± 0.22 ± 0.20 (19.7±12.9)	0.21 ± 0.11 ± 0.06 (19.8±12.9)
−0.41 ± 0.26 ± 0.17 (19.5±14.7)	0.24 ± 0.09 ± 0.04 (20.1±14.8)
−0.31 ± 0.23 ± 0.20 (16.7±11.2)	0.20 ± 0.13 ± 0.06 (16.7±11.1)
−0.51 ± 0.22 ± 0.14 (16.9±10.4)	0.21 ± 0.07 ± 0.03 (17.1±10.6)
−0.52 ± 0.25 ± 0.12 (16.5±10.7)	0.23 ± 0.09 ± 0.03 (16.5±10.8)
−0.51 ± 0.22 ± 0.12 (14.9±9.4)	0.21 ± 0.07 ± 0.04 (15.0±9.6)
−0.12 ± 0.17 ± 0.18 (22.0±11.7)	0.33 ± 0.15 ± 0.07 (22.2±11.7)
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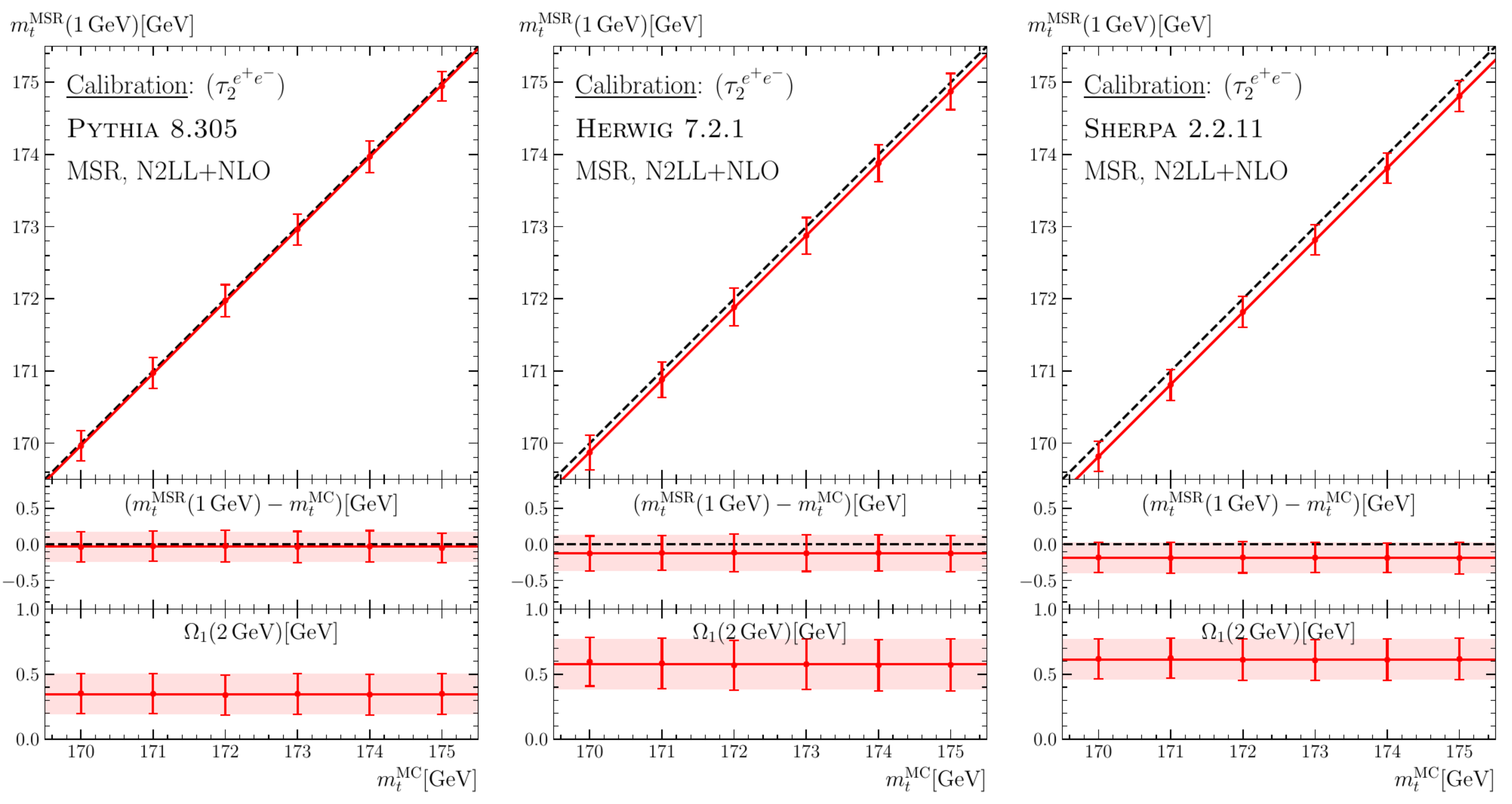
−0.08 ± 0.18 ± 0.13 (13.7±8.5)	0.24 ± 0.19 ± 0.03 (13.9±8.5)
−0.52 ± 0.26 ± 0.10 (5.5±3.7)	0.00 ± 0.20 ± 0.11 (5.6±3.8)
−0.25 ± 0.28 ± 0.04 (4.2±3.3)	−0.06 ± 0.28 ± 0.05 (4.3±3.4)
−0.45 ± 0.25 ± 0.08 (4.3±3.0)	−0.03 ± 0.23 ± 0.09 (4.4±3.0)
−0.64 ± 0.33 ± 0.20 (26.0±24.7)	−0.08 ± 0.19 ± 0.02 (26.6±26.1)
−0.52 ± 0.31 ± 0.12 (23.4±19.7)	−0.12 ± 0.21 ± 0.05 (23.3±19.5)
−0.57 ± 0.31 ± 0.22 (23.1±21.5)	−0.13 ± 0.20 ± 0.06 (23.2±22.2)

Results

Scan over several values of the MC top mass parameter

MC mass equals MSR mass within errors

$m_t^{\text{PYTHIA}} = m_t^{\text{MSR}}(1\text{GeV}) + 0.03(21)\text{GeV}$ $m_t^{\text{HERWIG}} = m_t^{\text{MSR}}(1\text{GeV}) + 0.12(25)\text{GeV}$ $m_t^{\text{SHERPA}} = m_t^{\text{MSR}}(1\text{GeV}) + 0.19(21)\text{GeV}$

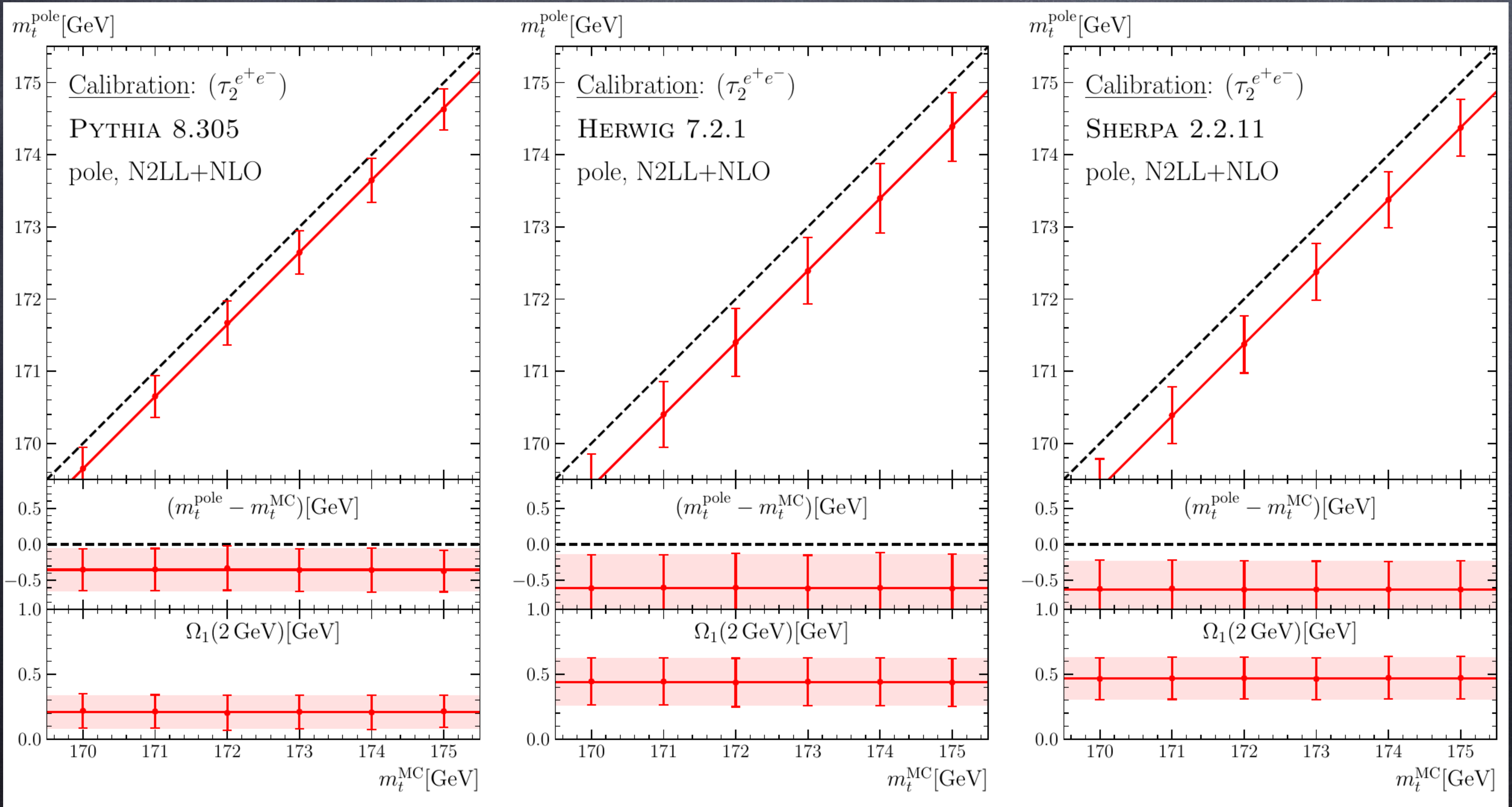


Results

Scan over several values of the MC top mass parameter

MC mass is NOT the pole mass (as assumed many times)

$m_t^{\text{PYTHIA}} = m_t^{\text{pole}} + 0.35(30)\text{GeV}$ $m_t^{\text{HERWIG}} = m_t^{\text{pole}} + 0.61(47)\text{GeV}$ $m_t^{\text{SHERPA}} = m_t^{\text{pole}} + 0.62(39)\text{GeV}$



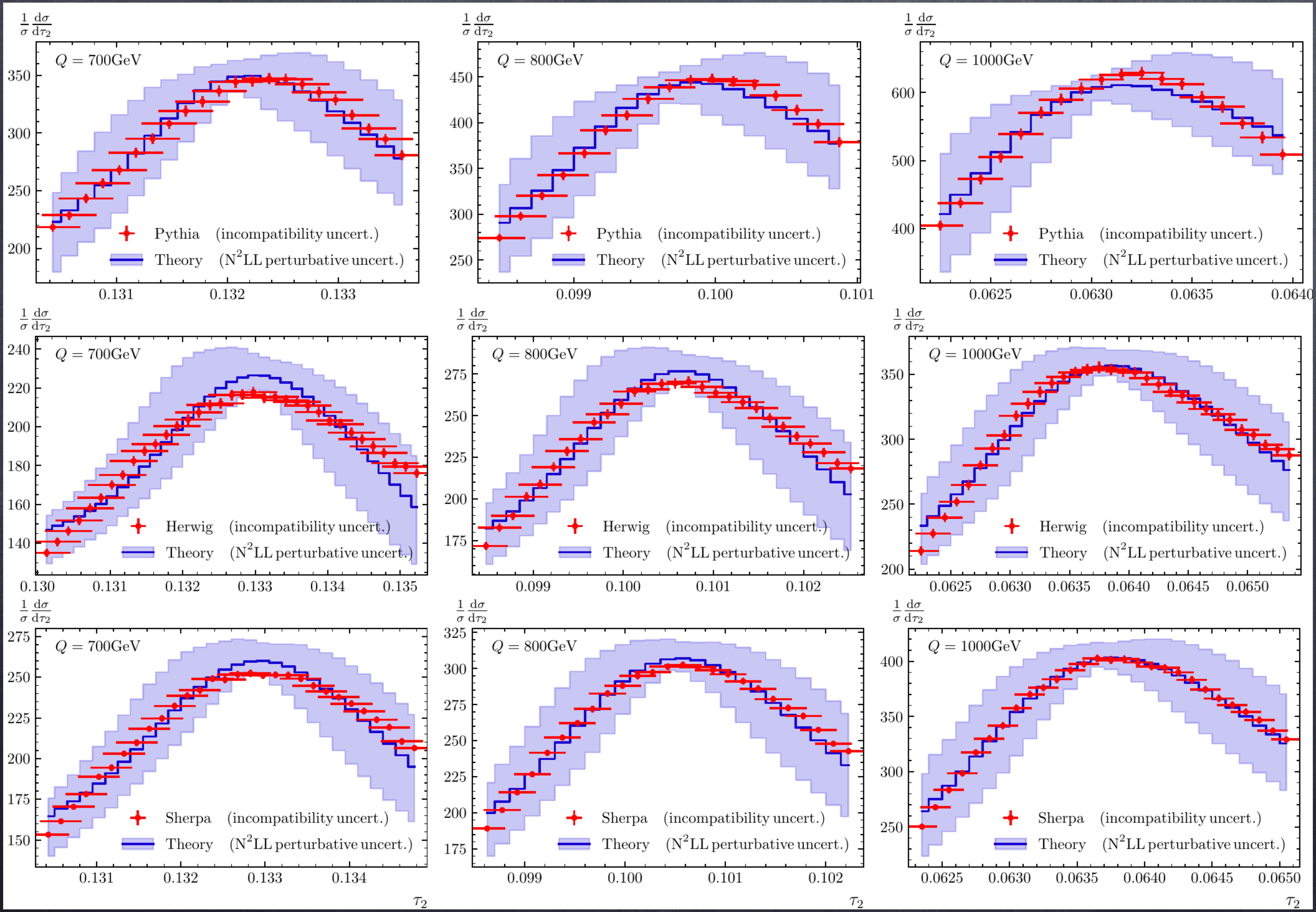
Conclusions

- Calibration setup is robust: insensitive to observable, gap and even MC
- All sources of mass power corrections must be under control
- MC top quark mass is definitely not the pole mass

Outlook

- Upgrade calibration to N^3LL
- Include more observables (C-parameter)
- Include sub-leading unstable top effects
- Formalism can be applied to massless event shape peak fits

Comparison theory vs MC



Further details

Computation of pert. uncertainties: random scan

fix $\alpha_s^{(5)}(m_Z) = 0.118$ and $\Gamma_t = 1.4\,\mathrm{GeV}$

Incompatibility uncertainty: comparison of different datasets

fix λ and fit for mass, angles and Δ_0. Scan over m_t^{MC}

Hadronization models very different in the three MC

… therefore the value of λ must be chosen differently

Summary: find λ-, observable- and gap scheme-independent results

find Ω_1 is m_t-independent

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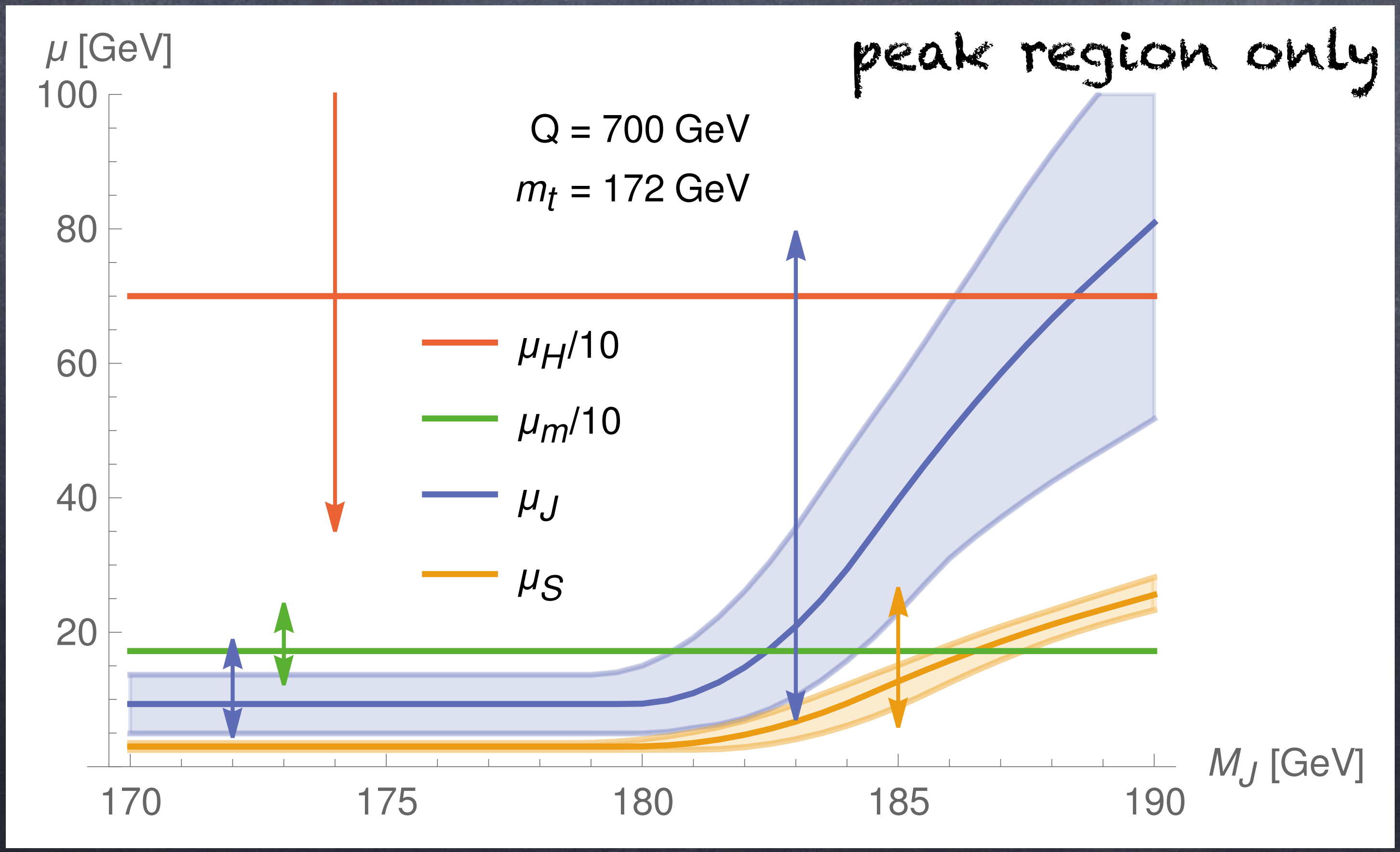
profile functions and their variation


peak region only