Soft-gluon effective coupling

perturbative results and the large n_F limit to all orders

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Stefano Catani

Outline

- Soft-gluon effective coupling introduction
- NLO
- NNLO and beyond
- Conformal relation
- Large n_F limit to all orders
- Conclusions

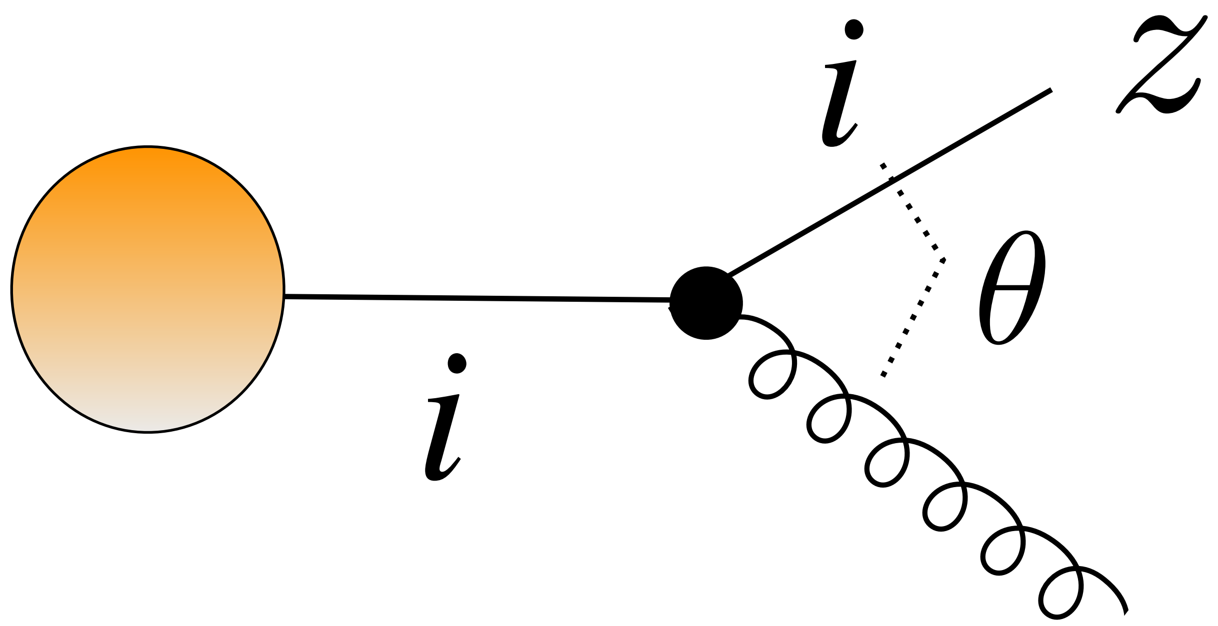
- Hard scattering observables sensitive to soft-gluon effects
  - originate from boundaries of phase space
  - real-radiation strongly suppressed : unbalance virtual-radiation
  - generate large logarithmic corrections $\alpha_s^n L^{2n}$

low transverse momentum $\log \frac{q_T}{Q}$

threshold $\log \left(1 - \frac{Q^2}{\hat{s}}\right)$

- Need to be resumed to some logarithmic accuracy to improve the convergence of the perturbative expansion

- One way to resum large logs (and more) → **Parton Shower Monte Carlo**



collinear branching driven by splitting function

$$dw_i^{DL} = \frac{\alpha_S}{2\pi} P_{ii}(z)\, dz \frac{d\theta^2}{\theta^2} \simeq C_i \frac{\alpha_S}{\pi} \frac{dz}{1-z} \frac{dq_T^2}{q_T^2}$$

soft and collinear emission (DL accuracy)

$$P_{ii}\left(\alpha_S; z\right) = \frac{1}{1-z} A_i\left(\alpha_S\right) + \ldots$$

soft limit of splitting function

cusp anomalous dimension

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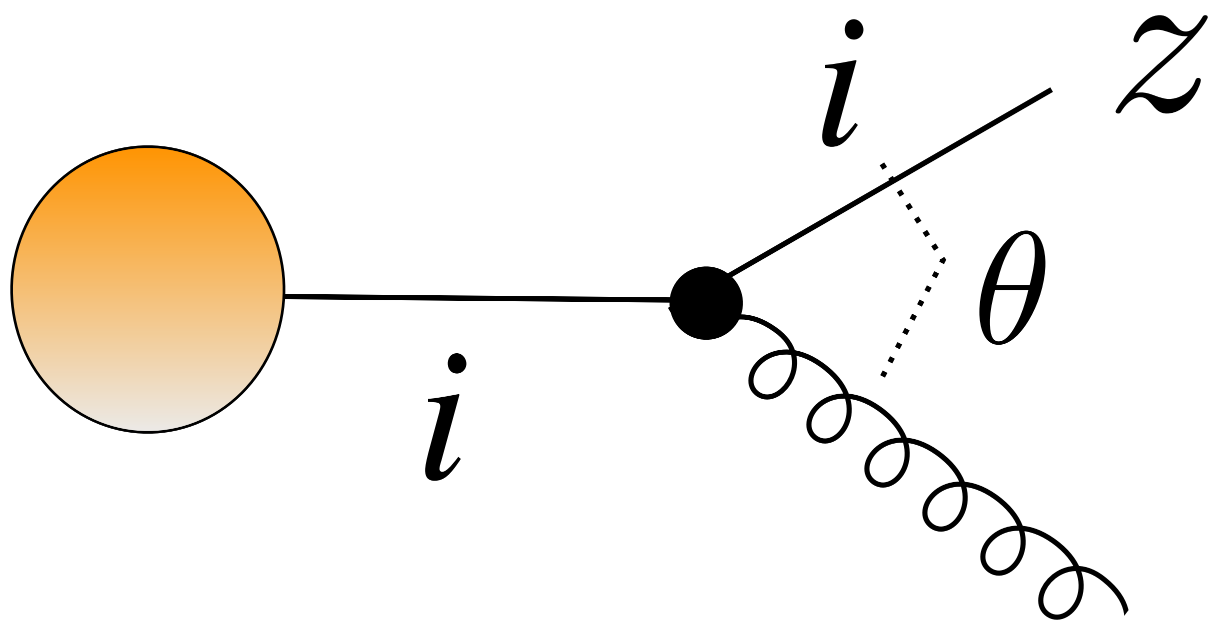

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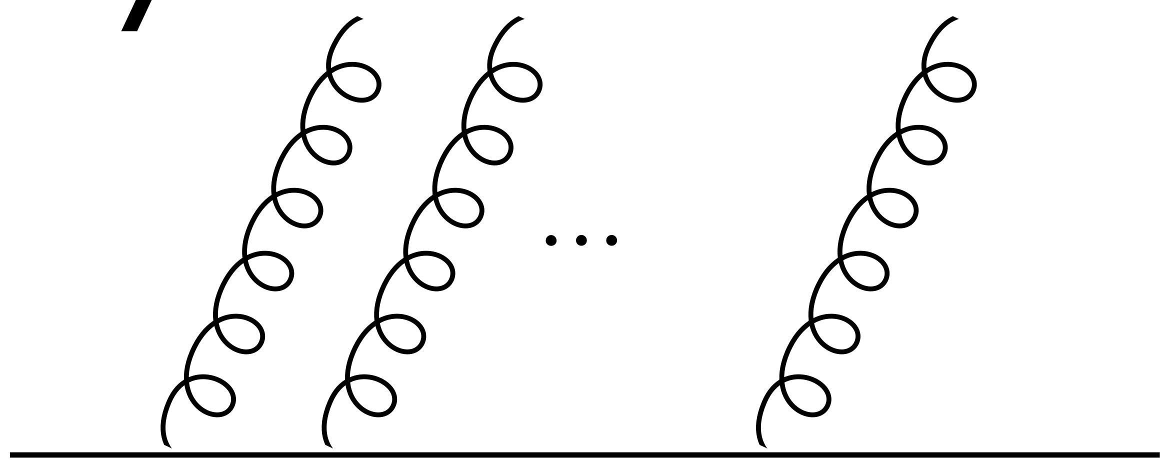
- The resummation of soft-collinear terms at LL achieved by

$$C_i \frac{\alpha_S}{\pi} \rightarrow C_i \frac{\alpha_S\left(q_T^2\right)}{\pi}$$

coupling evaluated at q_T (resum)



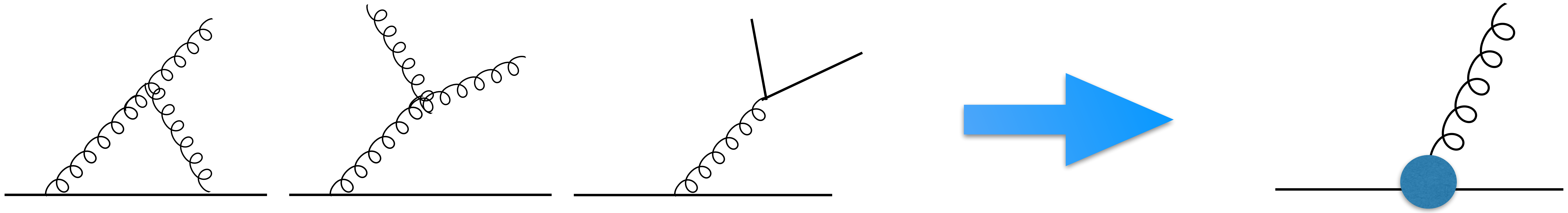
▸ The resummation of soft-collinear terms at NLL achieved by (MC@NLL)

$$C_i \frac{\alpha_S}{\pi} \to \mathcal{A}_i^{CMW}\left(\alpha_S\left(q_T^2\right)\right) = C_i \frac{\alpha_S^{CMW}\left(q_T^2\right)}{\pi} = C_i \frac{\alpha_S\left(q_T^2\right)}{\pi}\left(1 + \frac{\alpha_S\left(q_T^2\right)}{2\pi} K\right)$$

Catani, Marchesini, Webber (1991)

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{5}{9} n_F$$

NLL



soft effective coupling at NLL

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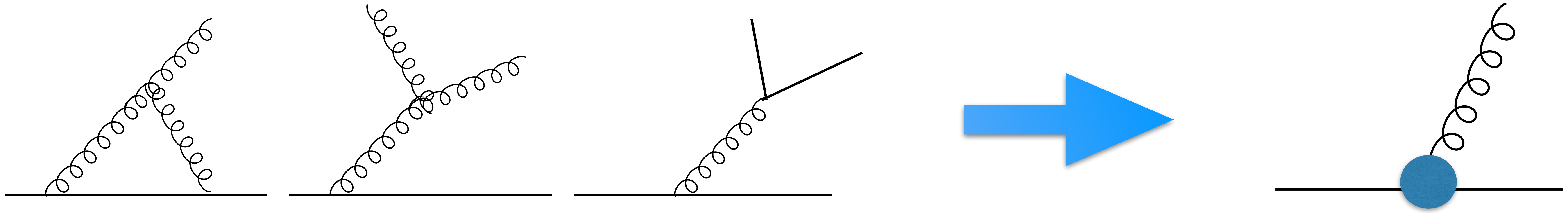


soft effective coupling at NLL

- Up to 2-loops, the soft-gluon effective coupling is still given by the cusp anomalous dimension

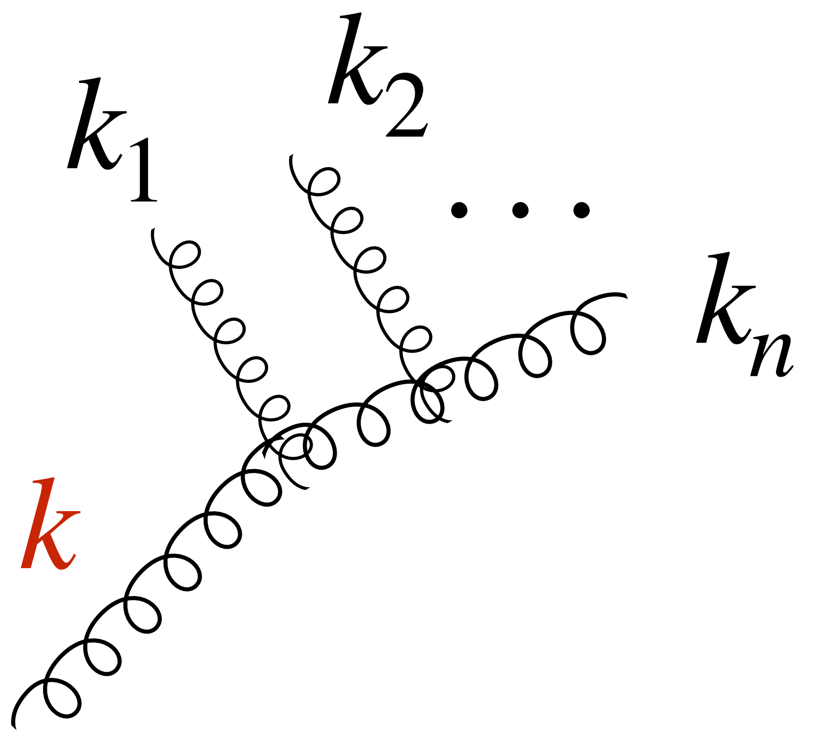
$$A_i^{(1)} = C_i, \quad A_i^{(2)} = \frac{1}{2} C_i \left[C_A\left(\frac{67}{18} - \frac{1}{6}\pi^2\right) - \frac{5}{9} N_f\right] \equiv \frac{1}{2} C_i K$$

  - Higher orders of cusp known $A_i^{(3)}$ $A_i^{(4)}$

- But, cusp=soft coupling beyond 2-loops?

- all-order definition in terms of a prob. density *(web)* Banfi, El-Menoufi, Monni (2018)

$$w\left(k;\epsilon\right)=\sum_{n=1}^{\infty}\int\left(\prod_{i=1}^{n}\left[dk_i\right]\right)\tilde{M}_s^2\left(k_1,\ldots,k_n\right)(2\pi)^d\delta^{(d)}\left(k-\sum_i k_i\right)$$



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depends only on k_T and $m_T^2=k_T^2+k^2$ and

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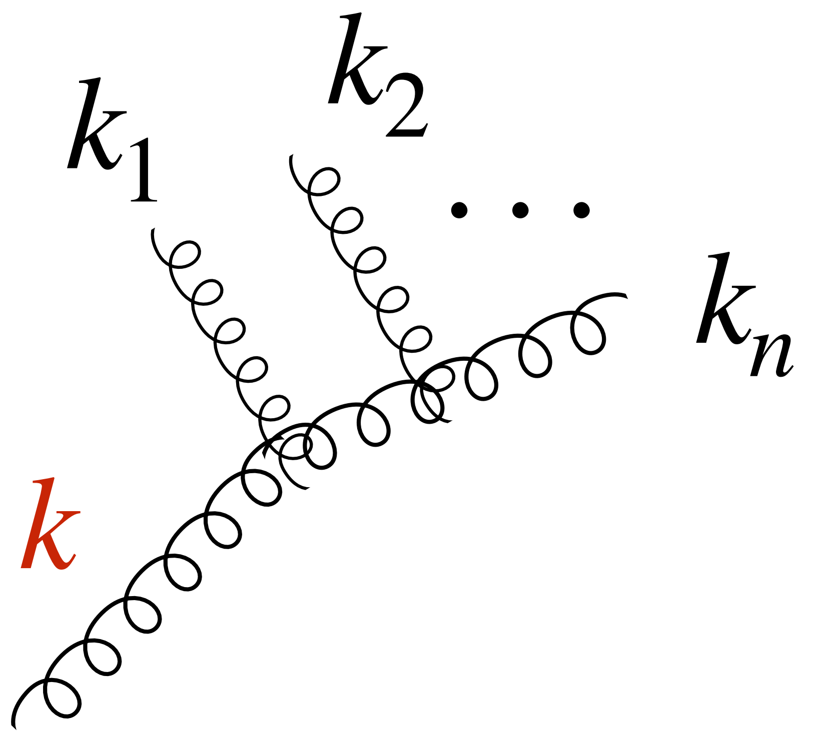

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depends only on k_T and $m_T^2 = k_T^2 + k^2$ and

- given the two variables, propose two definitions for soft-coupling

$$\widetilde{\mathscr{A}}_{T,i}\left(\alpha_S\left(\mu^2\right);\epsilon\right) = \frac{1}{2}\mu^2\int_0^{\infty} dm_T^2\, dk_T^2\,\delta\left(\mu^2 - k_T^2\right) w_i(k;\epsilon)$$

defined at fixed value of k_T

Banfi, El-Menoufi, Monni (2018)

suitable for q_T-related observables

$$\widetilde{\mathscr{A}}_{0,i}\left(\alpha_S\left(\mu^2\right);\epsilon\right) = \frac{1}{2}\mu^2\int_0^{\infty} dm_T^2\, dk_T^2\,\delta\left(\mu^2 - m_T^2\right) w_i(k;\epsilon)$$

defined at fixed value of m_T

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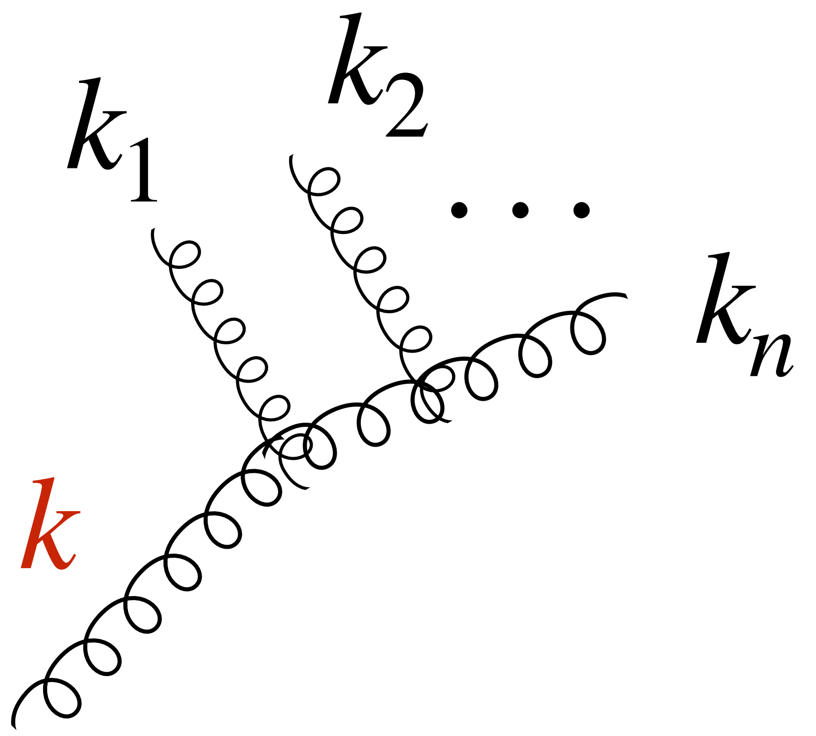

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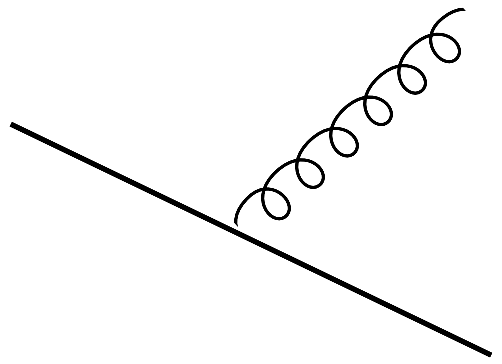
suitable for threshold-related observables

- measures intensity of soft emission at scale μ^2

- can take limit $\epsilon \to 0$ to obtain the physical couplings: keep *D*-dimensional
- at lowest order both couplings agree to all orders in ϵ



- can take limit $\epsilon \to 0$ to obtain the physical couplings: keep *D*-dimensional
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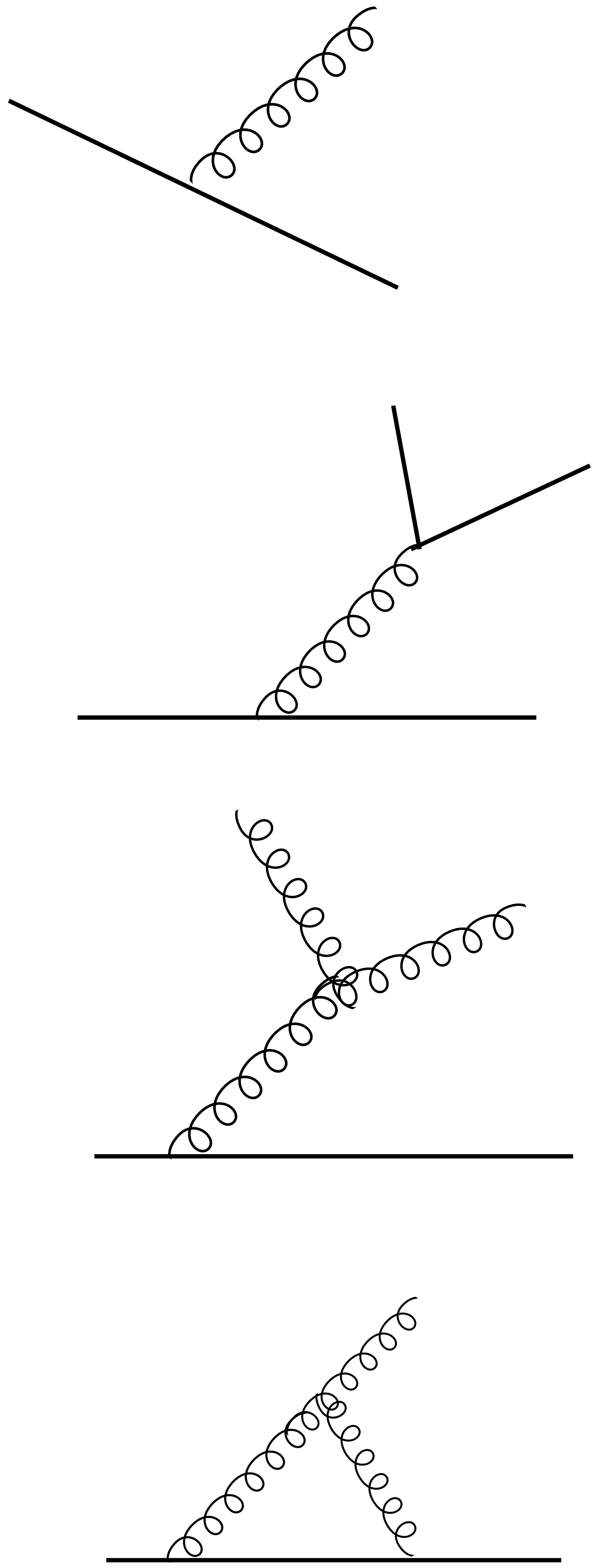
- we computed both couplings at α_s^2 (all orders in ϵ)



$$\widetilde{\mathscr{A}}_{T,i}^{(2)}(\epsilon) = A_i^{(2)} + \epsilon C_i \left[C_A \left(\frac{101}{27} - \frac{11\pi^2}{144} - \frac{7\zeta_3}{2} \right) + n_F \left(\frac{\pi^2}{72} - \frac{14}{27} \right) \right] + \epsilon^2 C_i \left[C_A \left(\frac{607}{81} - \frac{67\pi^2}{216} - \frac{77\zeta_3}{36} - \frac{7\pi^4}{120} \right) + n_F \left(\frac{5\pi^2}{108} - \frac{82}{81} + \frac{7\zeta_3}{18} \right) \right] + \mathcal{O}\left(\epsilon^3\right)$$

$$\widetilde{\mathscr{A}}_{0,i}^{(2)}(\epsilon) = A_i^{(2)} + \epsilon C_i \left[C_A \left(\frac{101}{27} - \frac{55\pi^2}{144} - \frac{7\zeta_3}{2} \right) + n_F \left(\frac{5\pi^2}{72} - \frac{14}{27} \right) \right] + \epsilon^2 C_i \left[C_A \left(\frac{607}{81} - \frac{67\pi^2}{72} - \frac{143\zeta_3}{36} - \frac{\pi^4}{36} \right) + n_F \left(\frac{5\pi^2}{36} - \frac{82}{81} + \frac{13\zeta_3}{18} \right) \right] + \mathcal{O}\left(\epsilon^3\right)$$

- agree with cusp anomalous dimension in 4 dimensions
  - ϵ terms different = effect at the next order

Why interested in D-dimensional expression?

$$\widetilde{\mathscr{A}}_{T,i}\left(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\right)=\widetilde{\mathscr{A}}_{0,i}\left(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\right)=A_i\left(\alpha_S\right)$$

- ‘D-dimensional’ $\beta = 0$

conformal point $\epsilon = \beta(\alpha_s)$

in QCD $$\frac{d\ln\alpha_s(\mu^2)}{d\ln\mu^2}=-\epsilon+\beta(\alpha_s(\mu^2))$$ $$\beta=-(\beta_0\alpha_s+\beta_1\alpha_s^2+\ldots)$$

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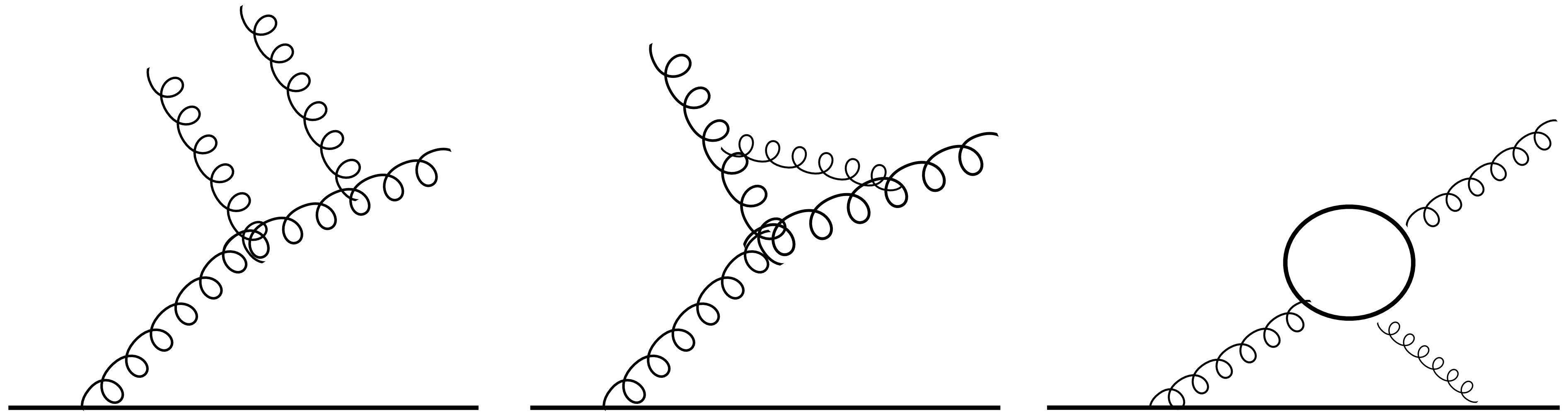
- An explicit third order computation would demand evaluation of

  - triple soft current (Born)
  - double soft current (one loop)
  - single soft current (two loops)



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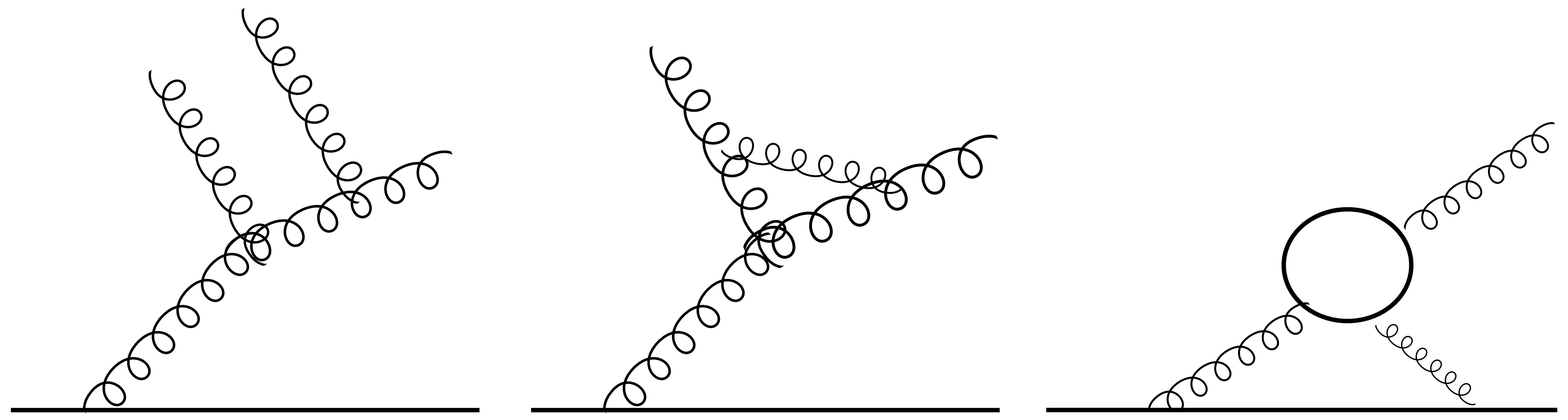


- Expanding conformal equation to third order one directly obtains

$$\mathscr{A}_i^{(3)}=A_i^{(3)}-\left(\beta_0\pi\right)^2\widetilde{\mathscr{A}}_i^{(1;2)}+\left(\beta_0\pi\right)\widetilde{\mathscr{A}}_i^{(2;1)}$$

- q_T

$$\mathscr{A}_{T,i}^{(3)} = A_i^{(3)} + C_i\left(\beta_0\pi\right)^2\frac{\pi^2}{12} + C_i\left(\beta_0\pi\right)\left[C_A\left(\frac{101}{27} - \frac{11\pi^2}{144} - \frac{7\zeta_3}{2}\right) + n_F\left(\frac{\pi^2}{72} - \frac{14}{27}\right)\right]$$

  - Agrees with previous result and 3rd order coefficient for q_T resummation

Banfi, El-Menoufi, Monni (2018)   Becher, Neubert (2011)

- Threshold (new)

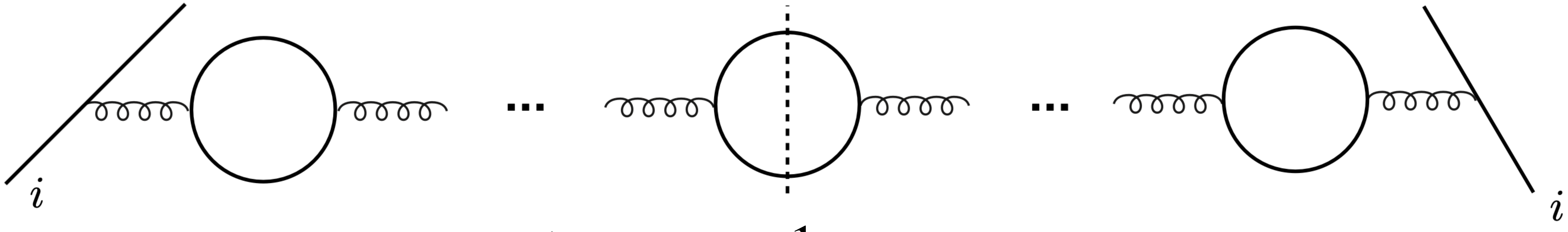
$$\mathscr{A}_{0,i}^{(3)} = A_i^{(3)} + C_i\left(\beta_0\pi\right)^2\frac{\pi^2}{12} + C_i\left(\beta_0\pi\right)\left[C_A\left(\frac{101}{27} - \frac{55\pi^2}{144} - \frac{7\zeta_3}{2}\right) + n_F\left(\frac{5\pi^2}{72} - \frac{14}{27}\right)\right]$$

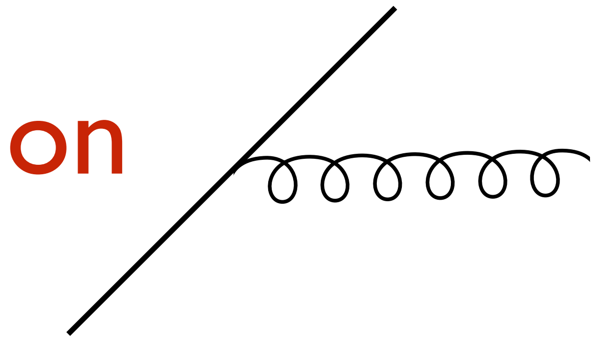
- Also computed for threshold at 4th order

Conformal relation

- Explicit check for n_F leading terms to all orders
- n_F terms can appear from:

(renormalized) bubble insertions



+ LO Renormalization  $\times \frac{1}{1+\frac{\beta_0}{\epsilon}\alpha_s}$

web: Both can be rearranged as geometric series (keep only n_F in beta)

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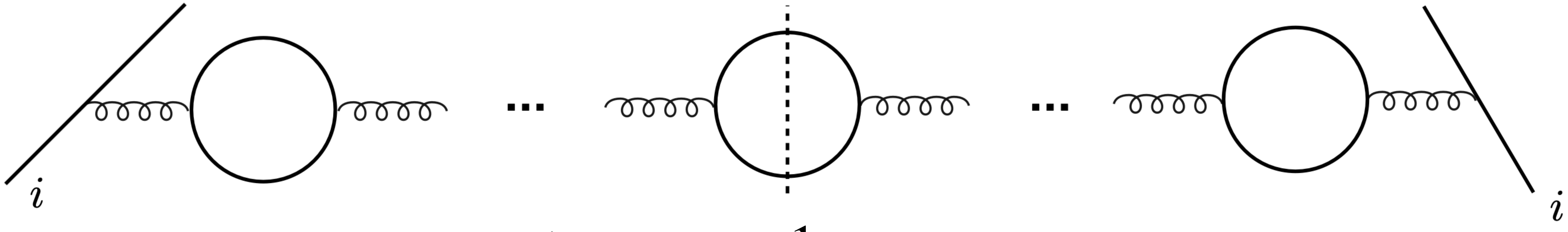
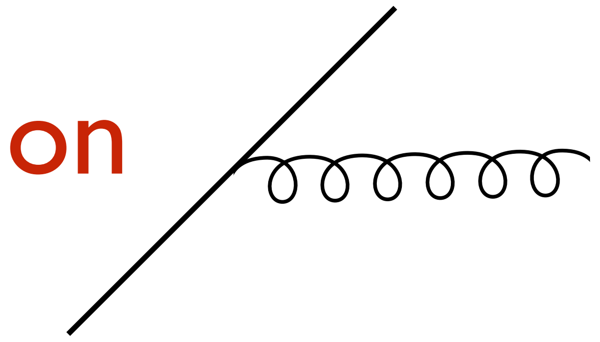


+ LO Renormalization  $\times \dfrac{1}{1+\frac{\beta_0}{\epsilon}\alpha_s}$

web: Both can be rearranged as geometric series (keep only n_F in beta)

- find all-order expression and sum the series ✓ Beneke, Braun (1995)

$$\widetilde{\mathscr{A}}_{T,i}\left(\alpha_S;\epsilon=\beta_0^{(n_F)}\alpha_S\right)=\widetilde{\mathscr{A}}_{0,i}\left(\alpha_S;\epsilon=\beta_0^{(n_F)}\alpha_S\right)=A_i=C_i\frac{\alpha_S}{\pi}\frac{\Gamma(4+2\beta_0\alpha_S)}{6\Gamma(1-\beta_0\alpha_S)\,\Gamma^2(2+\beta_0\alpha_S)\,\Gamma(1+\beta_0\alpha_S)}$$

Conclusions

- Definition(s) of Soft-gluon effective coupling at higher orders
- Explicit results for NNLO
- N^3LO soft-coupling for threshold related observables
- Conformal relation (all orders)

$$\widetilde{\mathscr{A}}_{T,i}\Big(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\Big)=\widetilde{\mathscr{A}}_{0,i}\Big(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\Big)=A_i\left(\alpha_S\right)$$

- Checked to all orders in large n_F limit
- Towards improving the precision of PS : still need to account for soft wide-angle and collinear radiation but soft-collinear effective coupling is known


thank you
danke
謝謝
ngiyabonga
teşekkür ederim
gracias
merci
obrigado
dziękuję
bedankt
спасибо
go raibh maith agat
mochchakkeram
sukriya
kop khun krap
terima kasih
grazie
arigatô
dakujem
takk
dank je
tapadh leat
мерси
감사합니다
ありがとう
ευχαριστώ
рахмат
Баярлалаа
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welalin
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kia ora
vinaka
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blagodaram
misaotra
matondo
paldies
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mahalo
хвала
asante
manana
obrigada
tenki
murakoze
djiere dieuf
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дякую
mamnun
chokrane
trugarez
faafetai lava
kiitos
dankie
nanni
nandri
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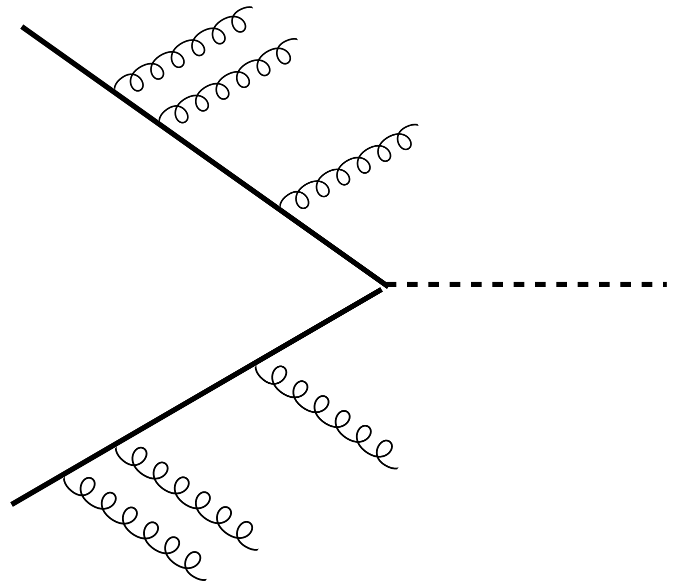
Backup

Full Soft-collinear radiation from hard partons

- Simpler for $c\bar{c} \to F$

DY $q\bar{q}$
Higgs gg



- Processes involving several hard partons more complicated (MC)
  - need to account for soft wide-angle emission and collinear radiation
- But soft coupling $\mathscr{A}_i$ not affected
  - intensity of soft-collinear radiation from parton i
  - exactly known to 4th order (for threshold related)
- All information for (threshold) resummation is contained in the d-dimensional version of the soft-coupling (work in progress)

- in general $\widetilde{\mathcal{A}}_{\mathcal{F},i}(\alpha_S(\mu^2);\epsilon) = \frac{1}{2}\,\mu^2 \int_0^\infty dm_T^2\, dk_T^2\, \delta\left(\mu^2 - \frac{k_T^2}{\mathcal{F}(k_T^2/m_T^2)}\right)\, w_i(k;\epsilon)$

measures intensity of soft emission at scale $\mu^2 = k_T^2/\mathcal{F}(k_T^2/m_T^2)$

- The difference between two definitions is given order by order $t = k_T^2/m_T^2$

$$\tilde{\mathscr{A}}_{\mathscr{F}_1,i}\left(\alpha_S\left(\mu^2\right);\epsilon\right) - \widetilde{\mathscr{A}}_{\mathscr{F}_2,i}\left(\alpha_S\left(\mu^2\right);\epsilon\right) = \sum_{n=2}^{+\infty} \frac{1}{\pi^n} \int_0^1 dt\left[\alpha_S^n\left(\mu^2 \mathscr{F}_1(t)\right) - \alpha_S^n\left(\mu^2 \mathscr{F}_2(t)\right)\right] \widehat{w}_{T,i}^{(n)}(t;\epsilon)$$

- For (D-dimensional) $\beta = 0$ all soft-couplings agree (with cusp)

$$\widetilde{\mathscr{A}}_{\mathscr{F},i}\left(\alpha_S;\epsilon = \beta\left(\alpha_S\right)\right) = A_i\left(\alpha_S\right)$$