

Axion-like particle and dark sector search at BESIII

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On behalf of the BESIII collaboration

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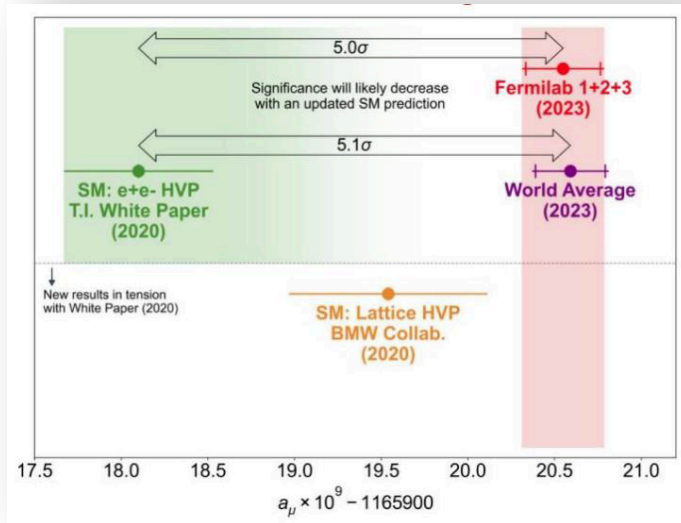
○ Standard Model (SM) is incredibly successful in describing the properties of visible matter

☑ *leptons, photon, Z, W[±], Higgs, quarks, mesons, hadrons*

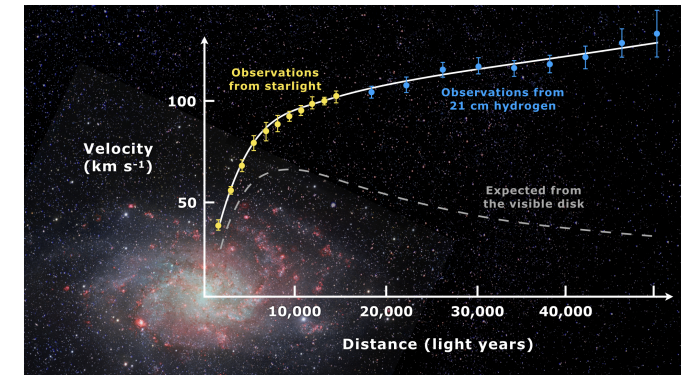
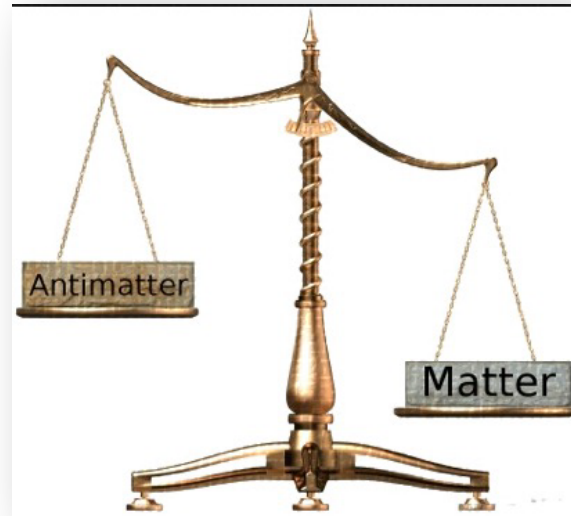
$g_\mu - 2$ anomaly

Matter-antimatter asymmetry

Galactic rotation curve

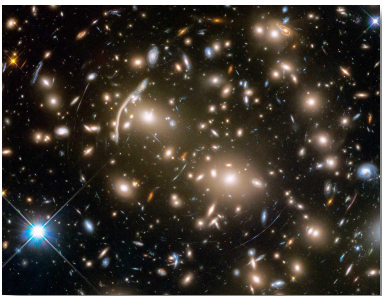
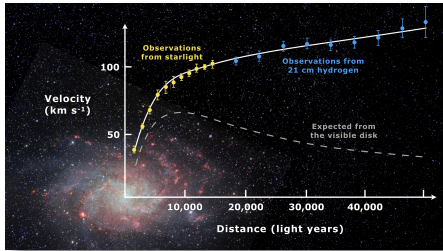


PRL 131, 161802(2023)



arXiv:astro-ph/0403324

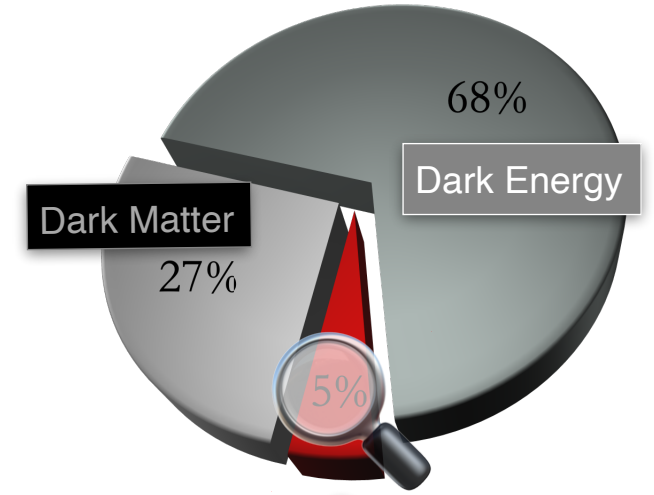
There is nothing new to be discovered in physics now.



Dark Matter



Physics BSM must exist!



Visible Matter of Elementary Particles

	Three generations of matter (fermions)			Interactions / Force carriers (bosons)	
	I	II	III		
mass	~2 MeV/c ²	~1.38 GeV/c ²	~173.1 GeV/c ²	0	~125.12 GeV/c ²
charge	2/3	2/3	2/3	0	0
spin	1/2	1/2	1/2	1	0
QUARKS	u up	c charm	t top	g gluon	H higgs
	d down	s strange	b bottom	γ photon	
LEPTONS	e electron	μ muon	τ tau	Z Z boson	
	ν _e electron neutrino	ν _μ muon neutrino	ν _τ tau neutrino	W W boson	
					SCALAR BOSONS
					GAUGE BOSONS / VECTOR BOSONS

Dark sector

○ A **collection of particles** that are not charged directly under the SM strong, weak, or electromagnetic forces.

● May interact with SM particle through **portal** interactions, not through gravitational effect not.

➔ Dark sector particles

- **Axion**: QCD axion and axion like particles
- **Muonphilic vector or scalar**
- **Dark photon γ'** : Massive or massless
- SUSY, dark Higgs, heavy neutrinos, dark fermion

The main
of this talk

mass → +2.3 MeV/c ²	+1.275 GeV/c ²	+173.07 GeV/c ²	0	+126 GeV/c ²
charge → 2/3	2/3	2/3	0	0
spin → 1/2	1/2	1/2	1	0
u up	c charm	t top	g gluon	H Higgs boson
+4.8 MeV/c ²	+95 MeV/c ²	+4.18 GeV/c ²	0	
-1/3	-1/3	-1/3	0	
1/2	1/2	1/2	1	
d down	s strange	b bottom	γ photon	
0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
-1	-1	-1	0	
1/2	1/2	1/2	1	
e electron	μ muon	τ tau	Z Z boson	
<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	80.4 GeV/c ²	
0	1/2	1/2	1	
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

Standard Model

Portal

Hidden Sector

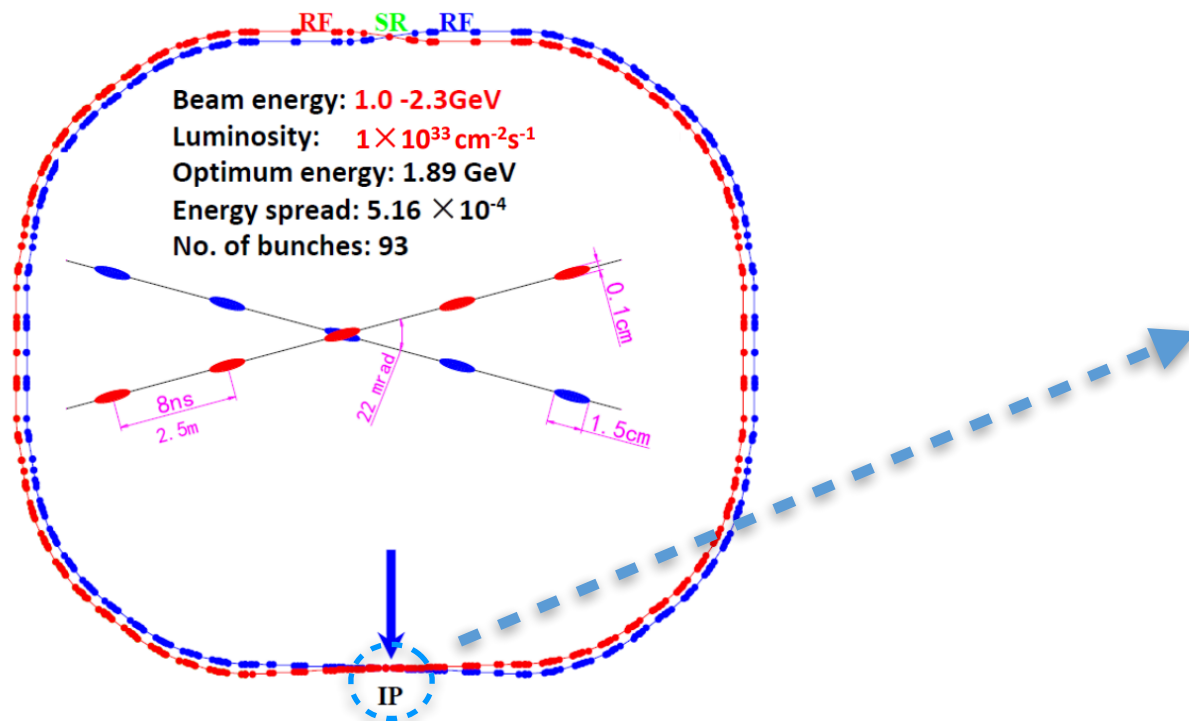
Merits of BSM search at BESIII:

- High statistic of data
- Clean background environment
- Angular coverage is almost 4π

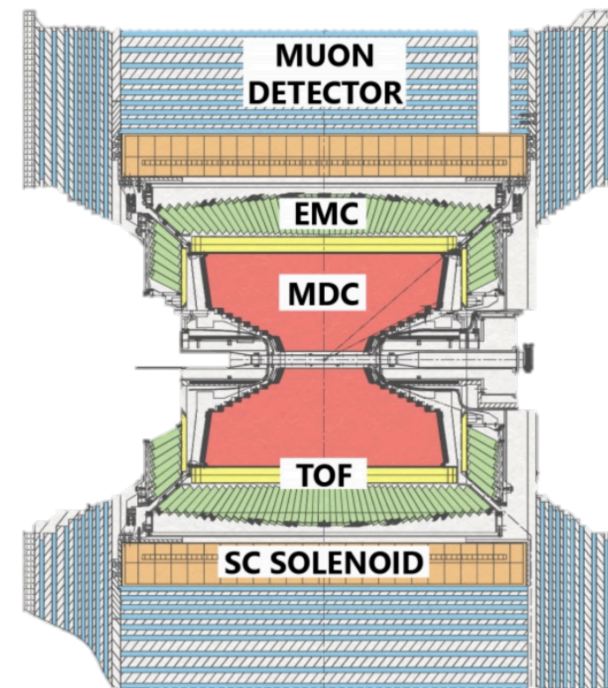
If their mass are in the **MeV-GeV** range
→ Accessible at **BEPCII and BESIII** experiment.

BESIII Experiment

Beijing Electron-Positron Collider II



Beijing Spectrometer III Detector



NIMA 614, 345 (2010)
 CPC 44, 040001 (2020)

BESIII has collected the largest data samples of **10 billion**
J/ψ, **2.7 billion ψ(3686)** on threshold in the world

Axion-like particles (ALPs) search

📍 Search for an axion-like particle in $J/\psi \rightarrow \gamma a, a \rightarrow \gamma\gamma$



Using 2.7B $\psi(3686)$ data

Using 10B J/ψ data

What is ALP

○ QCD axion (a)

- Predicted by the Peccei-Quinn (PQ) solution to the strong CP problem, could also be a dark matter candidate

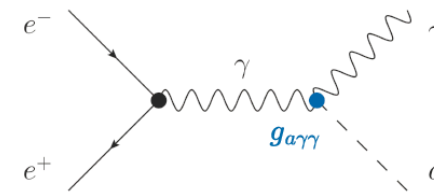
PRL. 40, 223 (1978)
PRL. 40, 279 (1978)

○ Axion-like particles (ALPs)

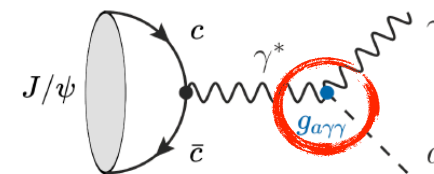
- Have the same quantum numbers as the QCD axion, but could have **arbitrary masses and couplings**.
- The ALP-photon coupling $g_{a\gamma\gamma}$ is mostly discussed \rightarrow ALP decays to two photon
- Experimental bounds on $g_{a\gamma\gamma}$ with m_a range of $\text{MeV}/c^2 - \text{GeV}/c^2$ mainly from e^+e^- colliders

PLB 753,482 (2016)

■ Non-resonant ALP production



■ Resonant ALP production



JHEP. 06, 091 (2019)

This talk:

ALP is searched via
 $J/\psi \rightarrow \gamma a, a \rightarrow \gamma\gamma$

Using 2.7B $\psi(2S)$ Data, in
 $0.165 < m_a < 2.84 \text{ GeV}/c^2$
[PLB 838\(2023\) 137698](#)

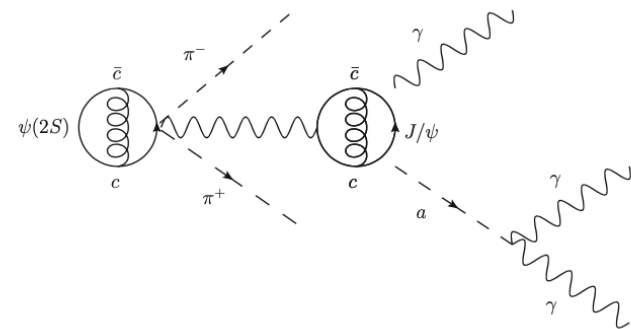
Using 10B J/ψ Data, in
 $0.18 < m_a < 2.85 \text{ GeV}/c^2$
[arXiv:2404.04640](#)

Accepted by PRD(L)

Search for an axion-like particle in radiative J/ψ decays

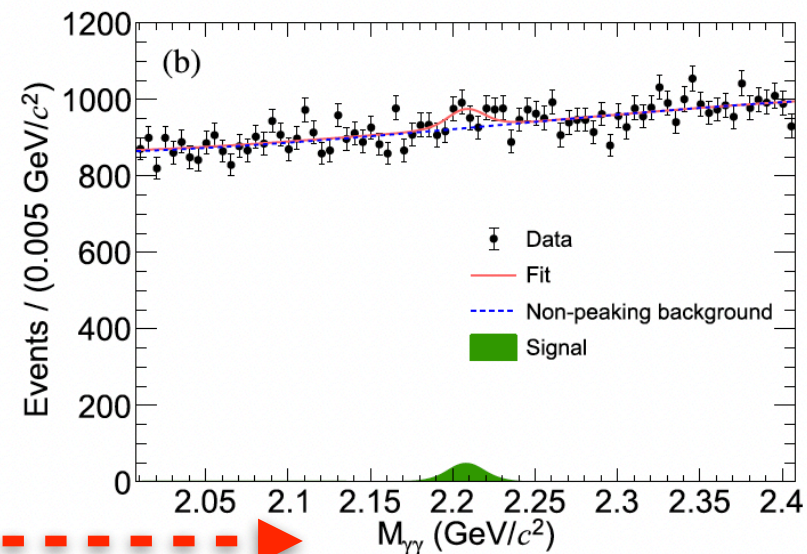
PLB 838 137698 (2023)

- Data samples: **2.7B** $\psi(3686)$ events
- Strategy: Search for $J/\psi \rightarrow \gamma a, a \rightarrow \gamma\gamma$ with J/ψ sample obtained from $\psi(3686) \rightarrow \pi^+\pi^- J/\psi$
 - $\psi(3686)$ decay: avoid the pollution from non-resonant production $e^+e^- \rightarrow \gamma\gamma\gamma$



$$\frac{\mathcal{B}(J/\psi \rightarrow \gamma a)}{\mathcal{B}(J/\psi \rightarrow e^+ e^-)} = \frac{m_{J/\psi}^2}{32\pi\alpha} g_{a\gamma\gamma}^2 \left(1 - \frac{m_a^2}{m_{J/\psi}^2}\right)^3$$

- Decay width of $a \rightarrow \gamma\gamma$: $\Gamma_a = \frac{g_{a\gamma\gamma}^2 m_a^3}{64\pi}$
- Taking $g_{a\gamma\gamma} \sim 10^{-4} \text{ GeV}^{-1}$, $m_a \sim \text{GeV}$, the lifetime of ALP is short in the detector
 - ➔ Assume $\text{BF}(a \rightarrow \gamma\gamma) \sim 100\%$
- Three $\gamma\gamma$ combinations per event, **extract signal from $M_{\gamma\gamma}$ distribution**
 - 674 mass hypotheses



The maximum local significance is 2.6σ at $m_a = 2208 \text{ MeV}/c^2$

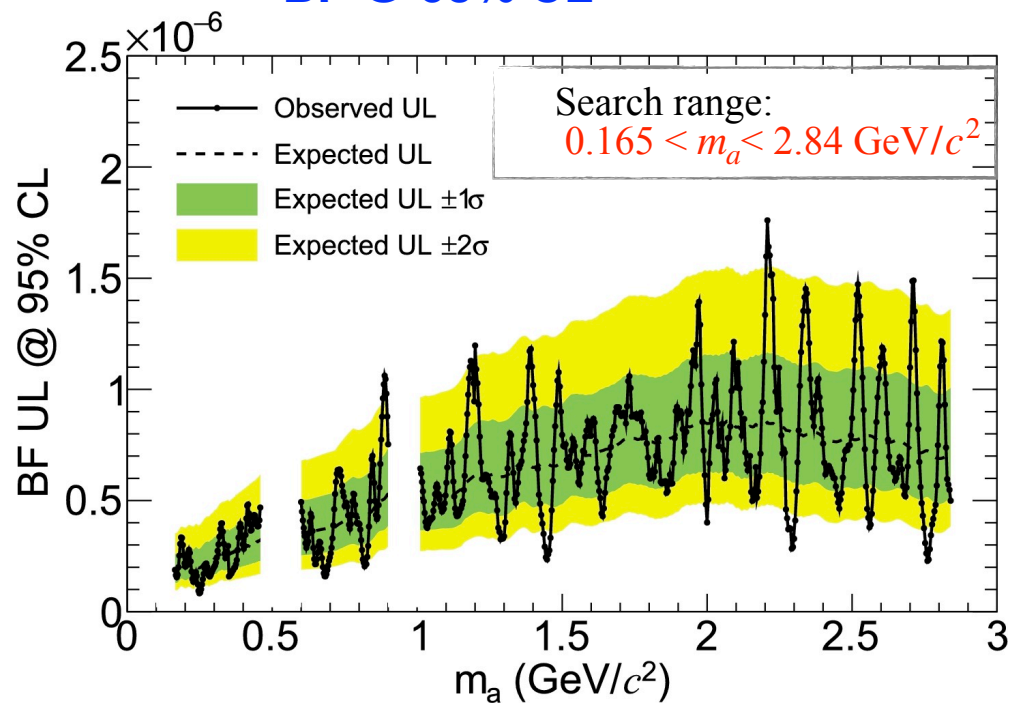
Search for an axion-like particle in radiative J/ψ decays

- No significant ALP signal is observed

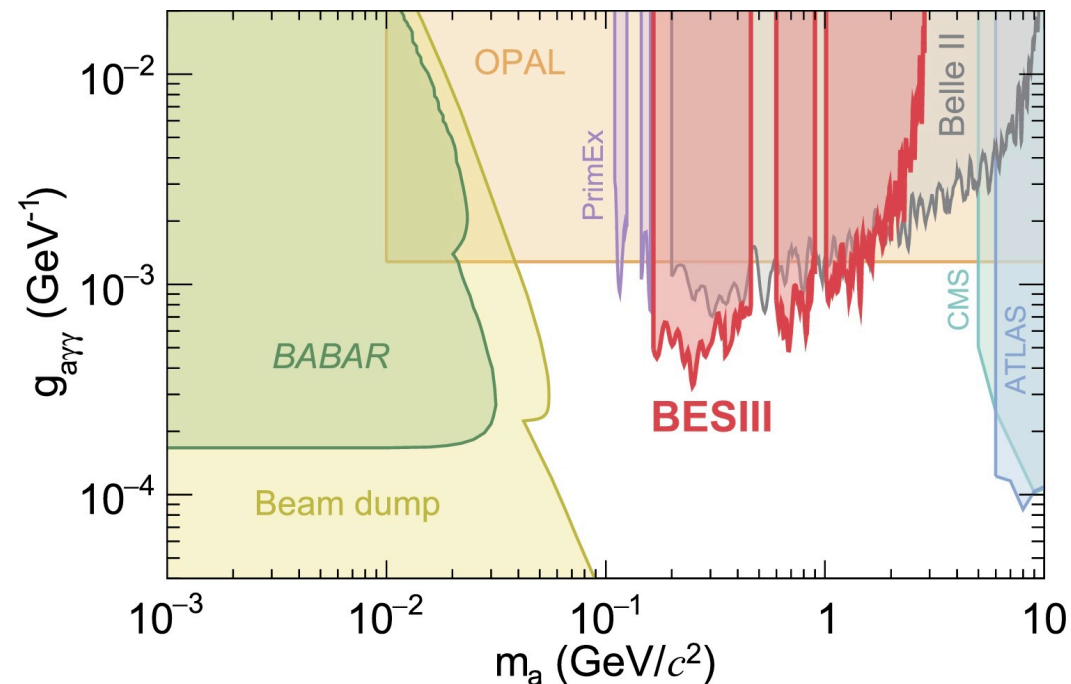
$$\frac{\mathcal{B}(J/\psi \rightarrow \gamma a)}{\mathcal{B}(J/\psi \rightarrow e^+e^-)} = \frac{m_{J/\psi}^2}{32\pi\alpha} g_{a\gamma\gamma}^2 \left(1 - \frac{m_a^2}{m_{J/\psi}^2}\right)^3$$



BF @ 95% CL



ALPs-photon Coupling



Search for an axion-like particle in radiative J/ψ decays

[arXiv:2404.04640](https://arxiv.org/abs/2404.04640)

Accepted by Phys. Rev. D (Letter)

○ Data samples: 10B J/ψ events

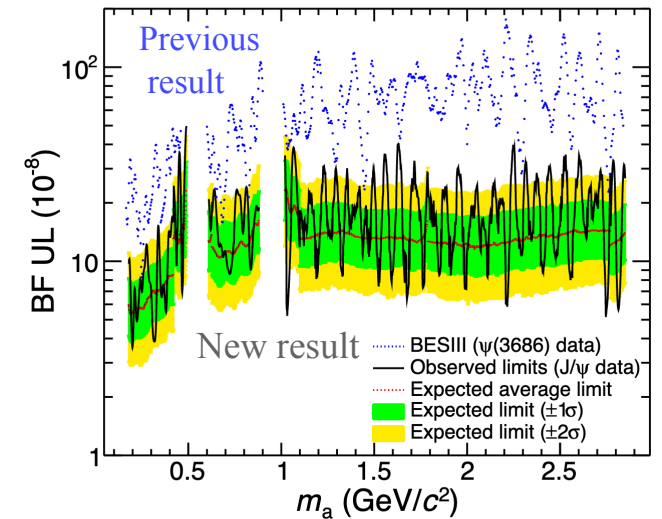
NEWEST RESULT!

○ Strategy: Search for $J/\psi \rightarrow \gamma a, a \rightarrow \gamma\gamma$ with J/ψ data on threshold

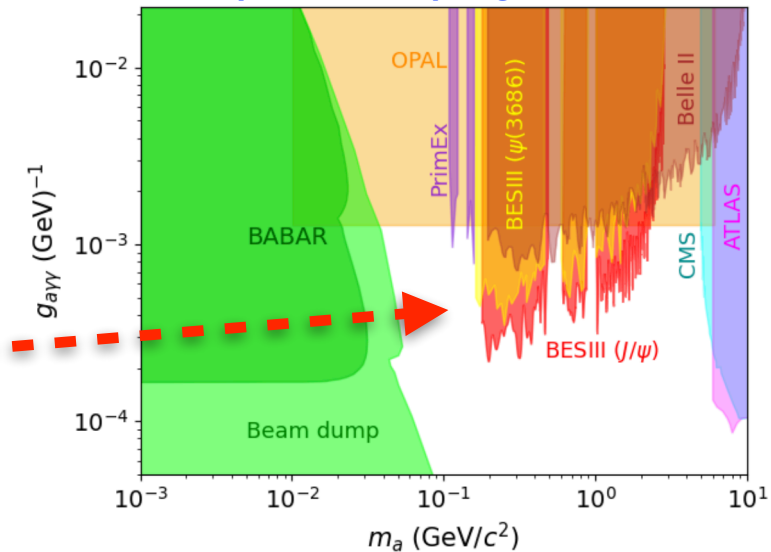
- Non-resonant process $e^+e^- \rightarrow \gamma a$ is indistinguishable from $J/\psi \rightarrow \gamma a$
- The ratio, $\sigma_{non-res}/\sigma_{res}$ is calculated to be 4.4%
- The contribution from $e^+e^- \rightarrow \gamma a$ is subtracted from the signal yields (interference between $\sigma_{non-res}, \sigma_{res}$ is negligible).

✓ The 95% CL. upper limits of $\mathcal{B}(J/\psi \rightarrow \gamma a)$ is set based on 10B data

Most stringent constraints to date for $0.18 < m_a < 2.85$ GeV



ALPs-photon Coupling



(Other)

Dark sector search

✓ Search for fully invisible decays

- Massive dark photon
- Muonphilic scalar X_0 or vector X_1
- $\Lambda, \eta, \eta', \omega, \phi$

PRD 105, L071101 (2022)
PRD 98, 032001 (2018)
PRD 87, 012009 (2013)

 This talk

✓ Other searches with invisible signatures

- FCNC process:

$\Sigma^+ \rightarrow p + \text{invisible}, \Lambda_c^+ \rightarrow p + \text{invisible}, D^0 \rightarrow \pi^0 \nu \bar{\nu}$

- $J/\psi \rightarrow \gamma + \text{invisible}$

PLB 852, 138614 (2024)
PRD 105, L071102 (2022)
PRD 105, 106, 072008 (2022)

PRD 101, 112005 (2020)

✓ Visible dark photon searches

- $e^+e^- \rightarrow \gamma\gamma', \gamma' \rightarrow l^+l^-$
- $J/\psi \rightarrow \eta\gamma', \gamma' \rightarrow e^+e^-$

PLB 774, 252(2017)

PRD 99, 012006 (2019)

What is Dark photon γ'

○ An extra Abelian gauge group $U(1)_D$:

- A minimal extension to SM, causing the associated spin-one boson the dark photon



- **Massive γ'** , if the symmetry is spontaneously **broken**
- **Massless γ'** , if the symmetry is **unbroken**

- The dark photon has a **kinetic mixing** with SM photon through

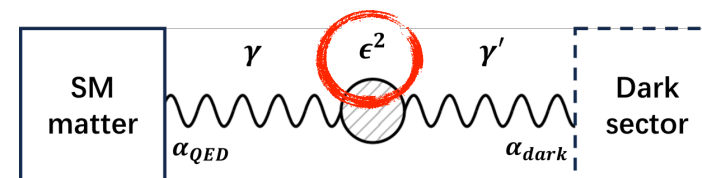
$$\rightarrow \frac{1}{2} \epsilon F'_{\mu\nu} F^{\mu\nu}$$

PLB, 196 (1986)

with a kinetic mixing parameter $\epsilon \sim 10^{-3}$ (empirical, very small)

- ϵ : controls the coupling strength
- $F'_{\mu\nu}$: field strengths of the dark photon

- dark



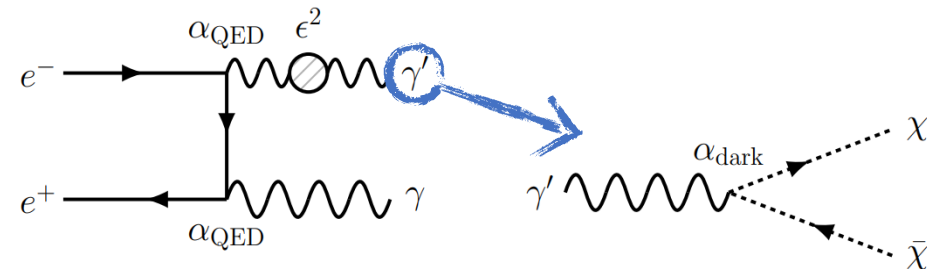
Accessible at high intensity facilities like BESIII

Phys. Rev. D 102, 015023 (2020).

Search for Massive dark photon with $e^+e^- \rightarrow \gamma\gamma'$

[PLB 839 \(2023\) 137785](#)

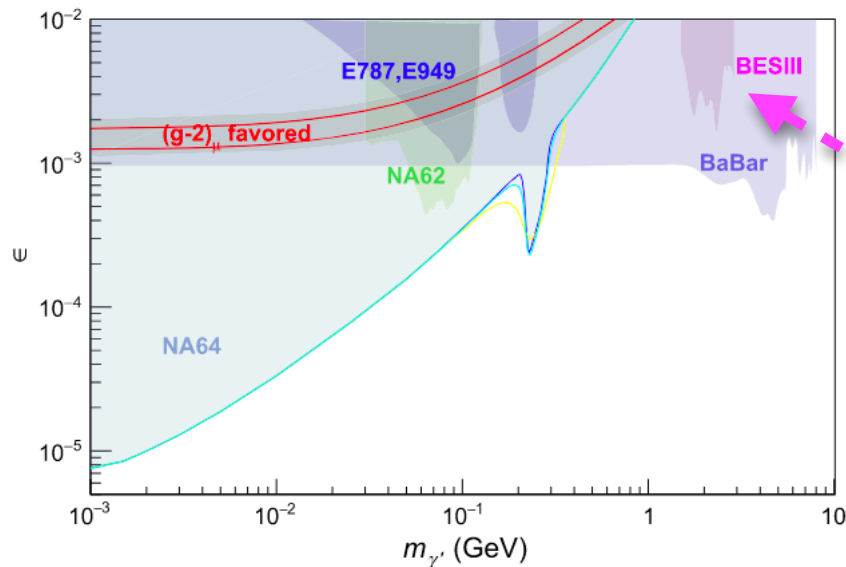
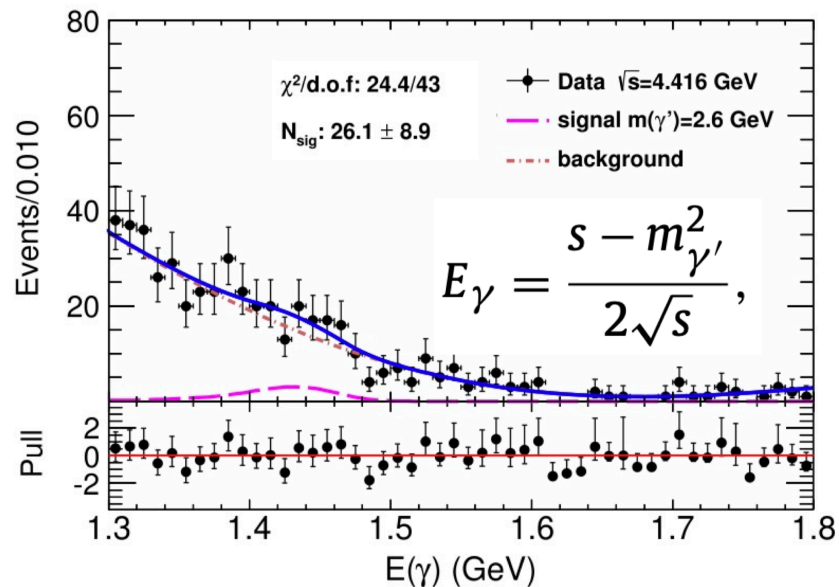
- Data samples: $14.9 \text{ fb}^{-1} e^+e^-$ annihilation data at $\sqrt{s} = 4.13 \sim 4.60 \text{ GeV}$
- Massive dark photon γ' is explored via $e^+e^- \rightarrow \gamma\gamma', \gamma' \rightarrow \text{invisible}$
- Search for single photon signals in $1.3 < E(\gamma) < 1.8 \text{ GeV}$ corresponding to $1.5 < m_{\gamma'} < 2.9 \text{ GeV}$ 29 different γ' mass hypotheses
- Extract the signal through SM photon energy spectrum



- If $m_\chi < m_{\gamma'}/2$, $\gamma' \rightarrow \chi\chi$
 - Invisible signature

Maximum global significance 2.2σ

Mixing parameter constraint



- ➔ 90% CL upper limits of ϵ are $(1.6 - 5.7) \times 10^{-3}$ in the GeV mass region,
- ➔ Consistent with what already excluded by BaBar
- ➔ More competitive results with 20 fb^{-1} data taken at 3.773 GeV can be provided in the future.

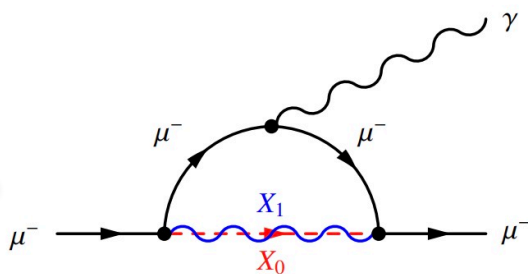
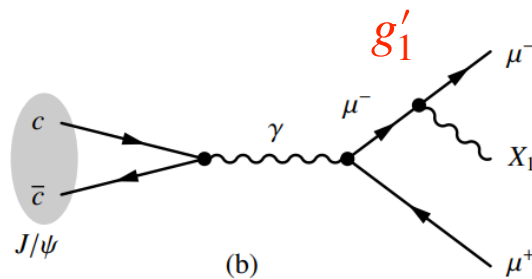
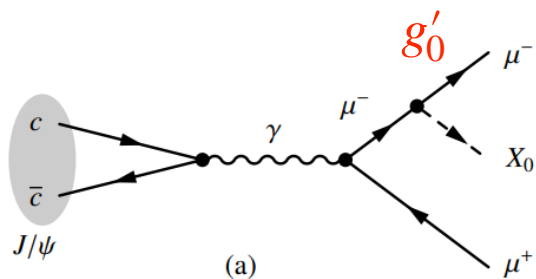
Muonphilic scalar X_0 or vector X_1

➔ Similar to the previous dark photon, an extra $U(1)$ group is added as a minimal extension to the SM

- $U(1)_{L_\mu-L_\tau}$ model : a new massive vector boson X_1 or scalar boson X_0 only couple to the second or third generations of leptons ($\mu, \nu_\mu, \tau, \nu_\tau$) with the coupling strength g'_1, g'_0
- The light muonphilic scalar or vector particles can contribute to the muon anomalous magnetic moment and explain the $(g-2)_\mu$ anomaly

JHEP10(2020)207
Mod. Phys. Lett. A 06, 527 (1991).
PRD 43, R22 (1991)

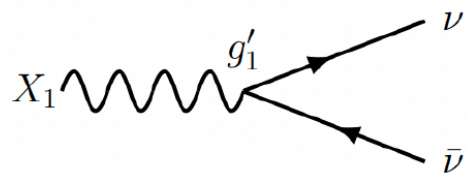
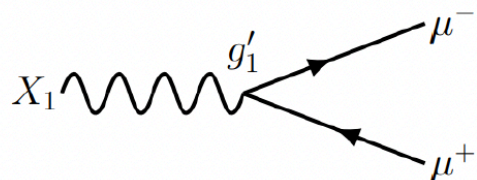
Eur. Phys. J. C 81, 861 (2021).



Can be accessible via $J/\psi \rightarrow \mu^+ \mu^- X_{0,1}$
with $X_{0,1}$ invisible at BESIII

Three cases of muonphilic particles

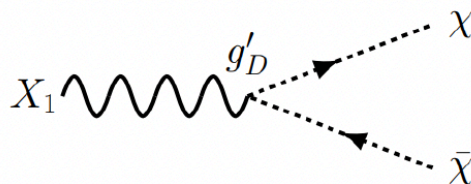
“vanilla” $L_\mu - L_\tau$ model



X_1 only couples with SM particles

- $\mathcal{B}(X_1 \rightarrow \nu\bar{\nu}) = 33 \sim 100\%$

“invisible” $L_\mu - L_\tau$ model

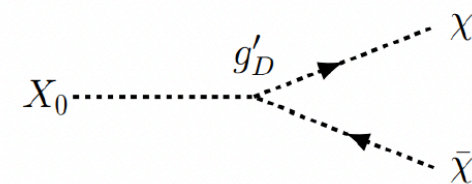


X_1 couples with DM particle χ with coupling

g'_D

- $m_\chi < m_{X_1}/2$
- $g'_D \gg g'_1$
- $\mathcal{B}(X_1 \rightarrow \chi\bar{\chi}) \sim 100\%$

“scalar” $U(1)$ model



X_0 long-lived

- Assume the X_0 is long lived or only decay to invisible final states

Search for X_0, X_1 via $J/\psi \rightarrow \mu^+\mu^-X_{0,1}$ with $X_{0,1}$ is invisible

Search for a muonphilic scalar X_0 or vector X_1 via $J/\psi \rightarrow \mu^+\mu^- + \text{invisible decays}$ [PRD 109, L031102 \(2024\)](#)

○ Data samples: **9B** J/ψ events

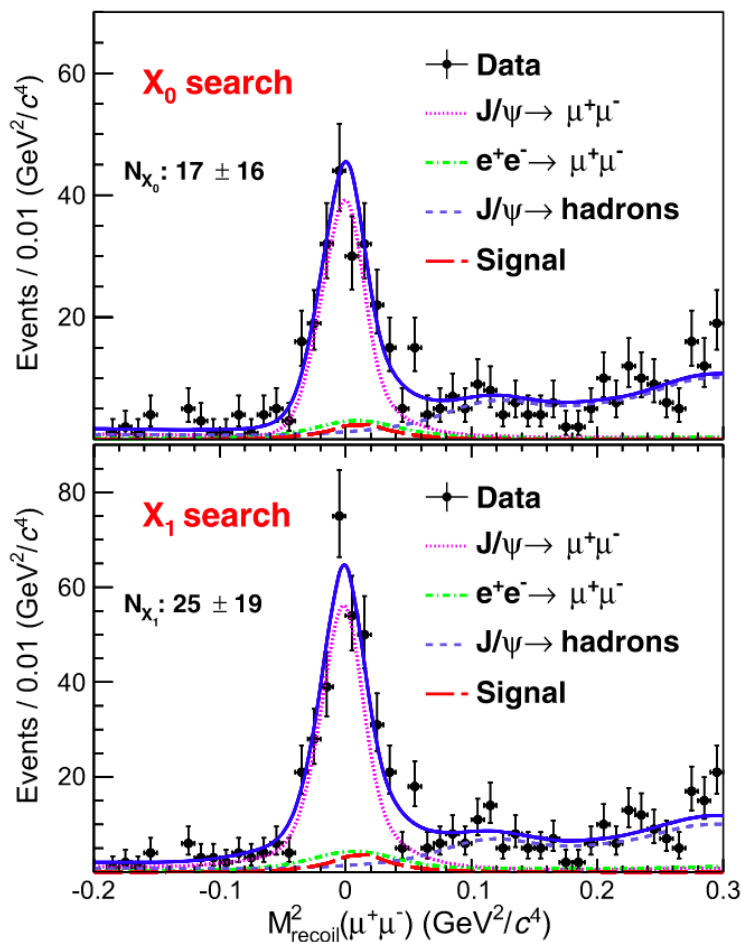
○ Signal extraction: A series of unbinned maximum likelihood fits are performed to $M_{\text{recoil}}^2(\mu^+\mu^-)$ or $M_{\text{recoil}}(\mu^+\mu^-)$

$$M_{\text{recoil}}^2(\mu^+\mu^-) = (p_J - p_{\mu^+} - p_{\mu^-})^2$$

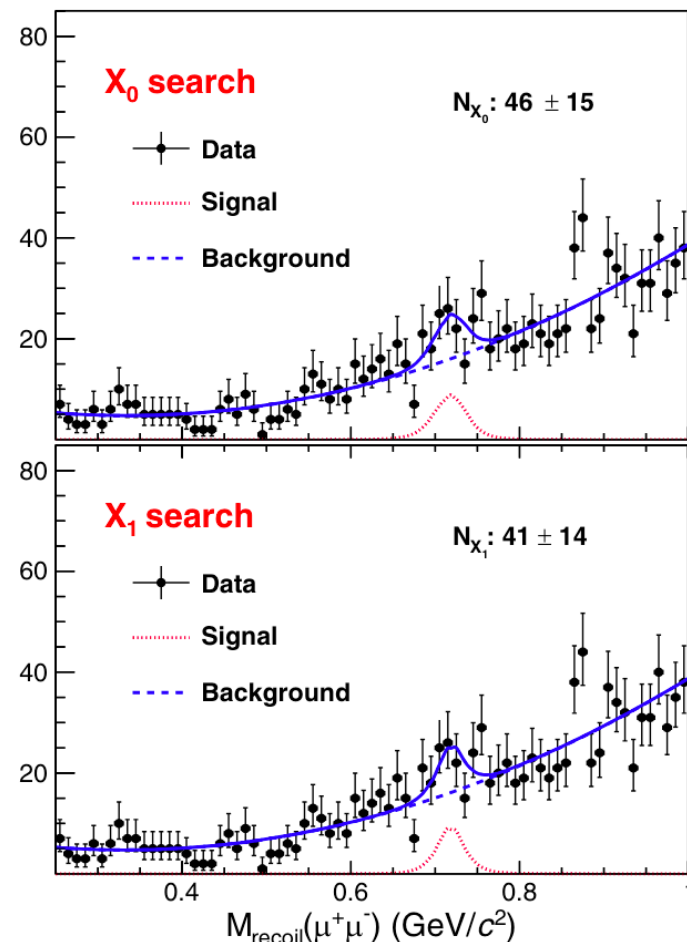
○ The maximum local significance is 2.5σ at $M(X_{0,1}) = 720 \text{ MeV}/c^2$

○ **No evidence** for $J/\psi \rightarrow \mu^+\mu^- + X_{0,1}$ signals

■ **Low mass region**, with $M(X_{0,1}) = 120 \text{ MeV}/c^2$



■ **High mass region**, with $M(X_{0,1}) = 720 \text{ MeV}/c^2$



Search for a muonphilic scalar X_0 or vector X_1 via $J/\psi \rightarrow \mu^+ \mu^- +$ invisible decays

○ Constraints on the coupling $g'_{0,1}$

BarBar, CMS, Belle : $X_1 \rightarrow \mu^+ \mu^-$

Belle and BESIII: $X_1 \rightarrow \nu \bar{\nu}$

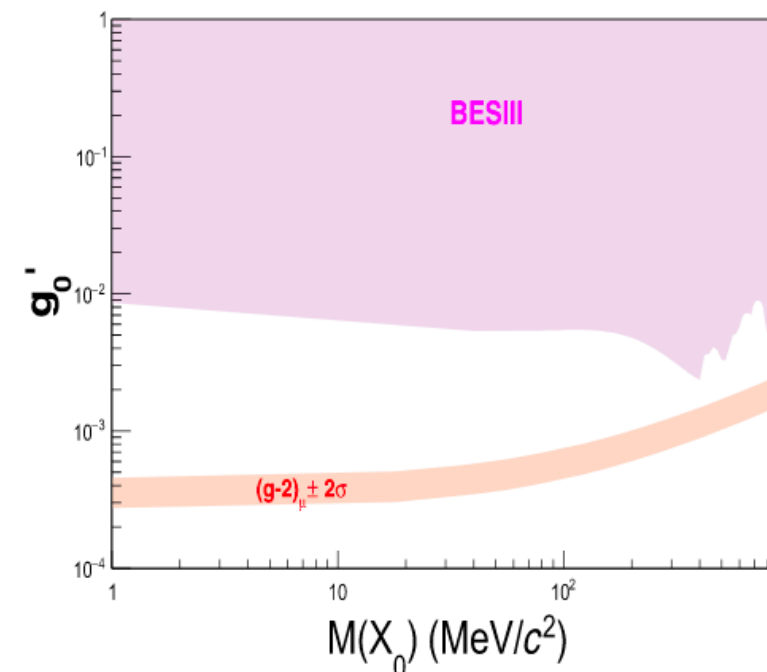
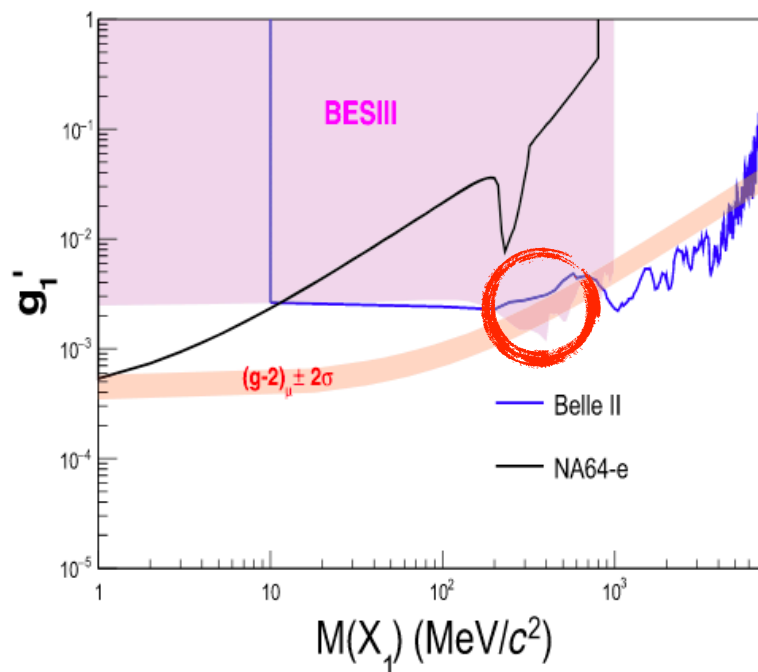
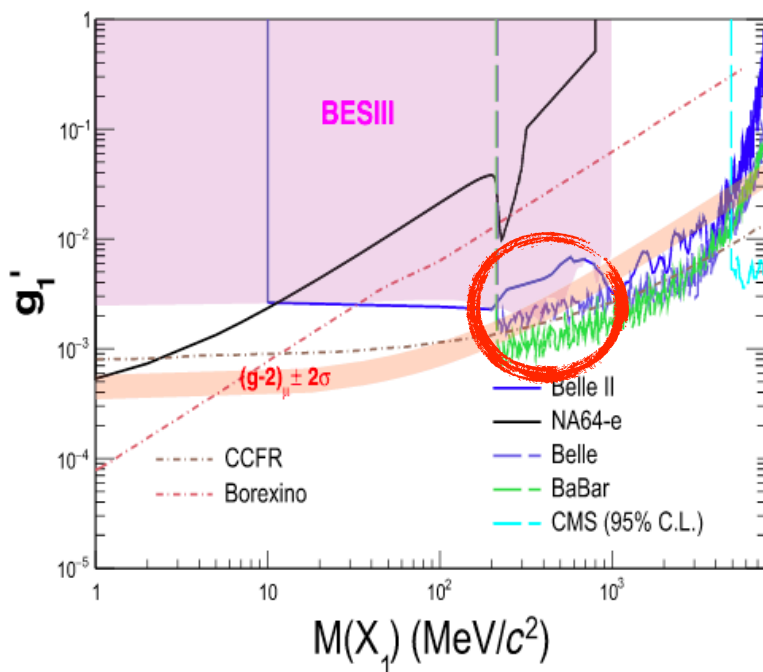
Better sensitivity in the range 200–860 MeV/c^2 is obtained

First constraint for the “scalar” invisible X_0 case

“vanilla” $L_\mu - L_\tau$ model

“invisible” $L_\mu - L_\tau$ model

“scalar” $U(1)$ model



Summary

➔ BESIII plays an active role in dark sector and axion-like particle search, with many first searches or best limits

Axion-like particles (ALPs) search

Search for an **axion-like** particle in radiative J/ψ decays

✓ Using 2.7B $\psi(3686)$ data PLB 838 137698 (2023)

✓ Using 10B J/ψ data Accepted by PRD(L)

➔ **Most stringent limits on $g_{a\gamma\gamma}$ for $0.165 < m_a < 2.85 \text{ GeV}/c^2$**

Dark sector search

✓ Search for a **muonphilic** scalar X_0 or vector X_1 via $J/\psi \rightarrow \mu^+\mu^- + \text{invisible}$ ➔

Stringent limits on the coupling $g'_{0,1}$ are set

✓ Search for Massive **dark photon** with $e^+e^- \rightarrow \gamma\gamma'$ PLB 839 (2023) 137785



Future

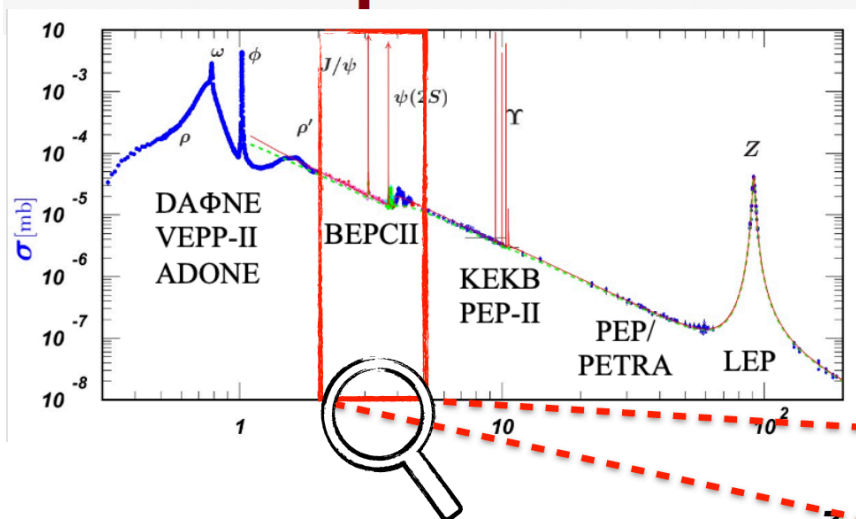
- BESIII has recently collected 20 fb^{-1} of $\psi(3770)$ data sample
 - ➔ More conclusive results are ongoing!

*Enlightening the dark,
coming is the future!*

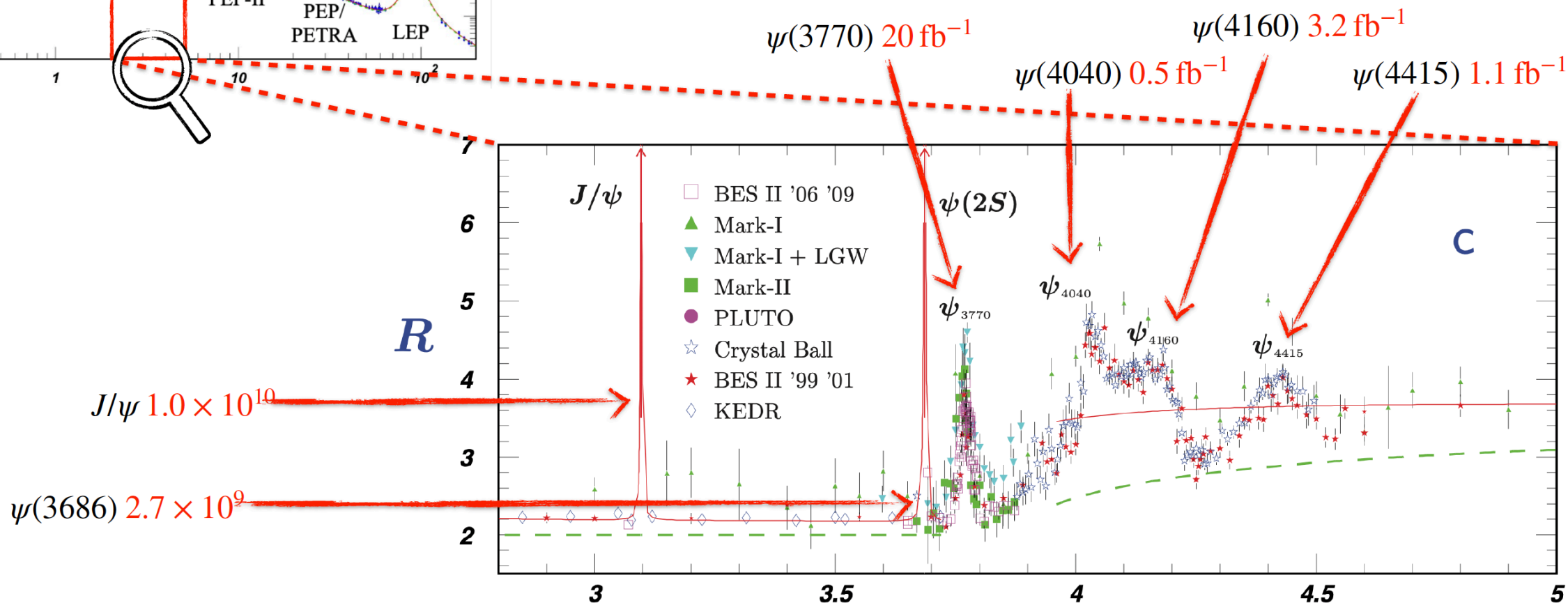
Thanks for your attention!

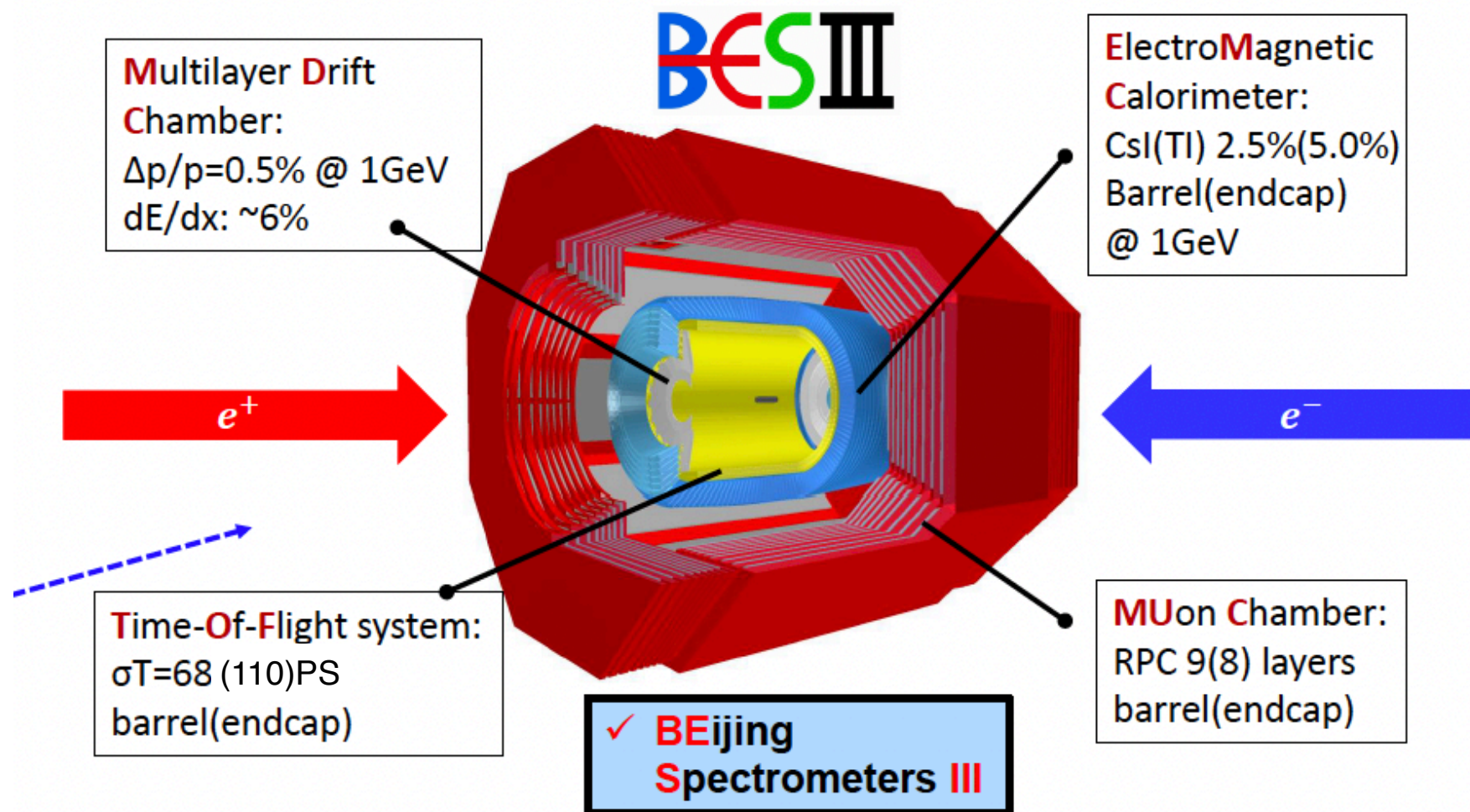
Back UP

Data samples at BESIII



- BESIII has collected the largest data samples of J/ψ and $\psi(3686)$ on the threshold in the world!
- $> 20 \text{ fb}^{-1}$ data above 4.0 GeV in total





Search for massless dark photon in $\Lambda_c^+ \rightarrow p\gamma'$

[PRD 105, 106, 072008 \(2022\)](#)

○ The FCNC process is highly **suppressed** by the GIM mechanism in the charm sector

➔ A massless dark photon γ' could induce FCNC process through higher dimensional operators

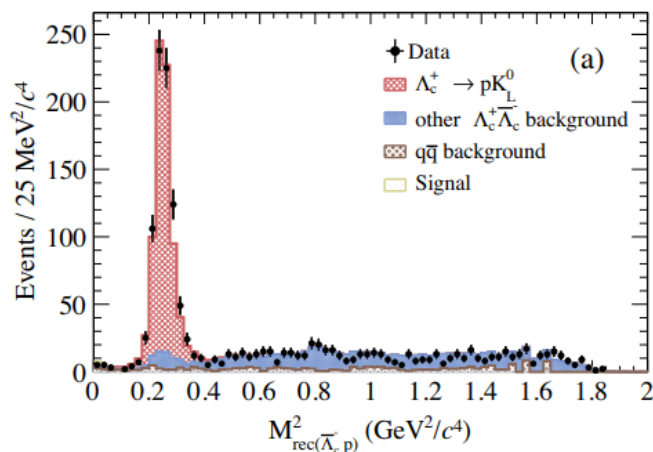
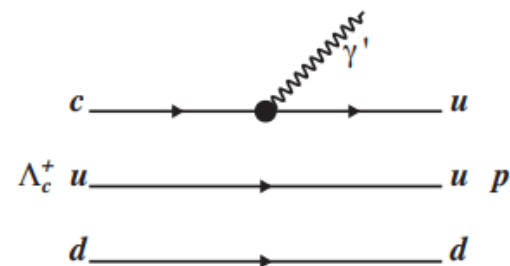
○ **Data samples:** $4.5 \text{ fb}^{-1} e^+e^-$ annihilation data at $\sqrt{s} = 4.6 \sim 4.7 \text{ GeV}$

○ **Strategy:** Double-Tag technique

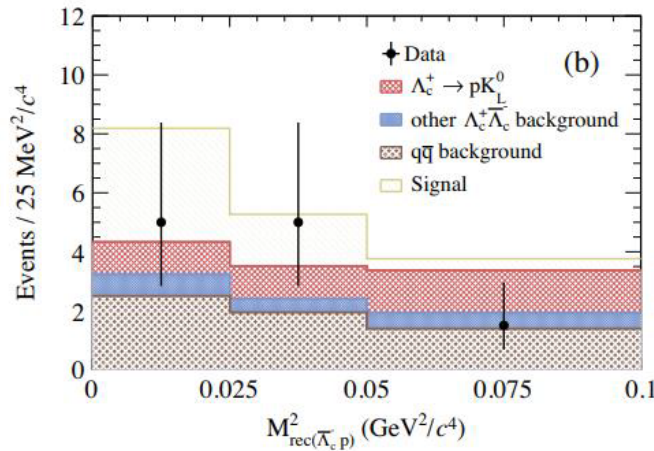
○ **Results:**

✓ No signal is observed

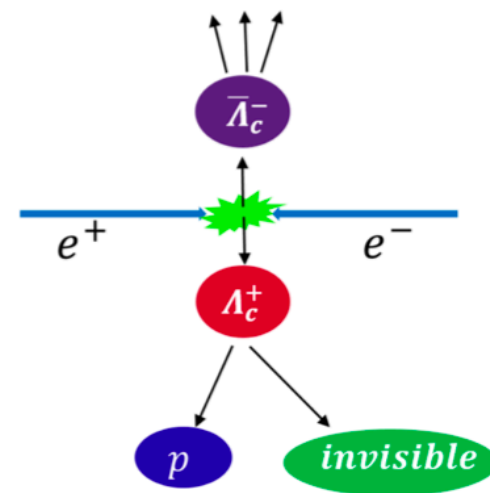
✓ 90% C.L. upper limit on BF is set $\mathcal{B}(\Lambda_c^+ \rightarrow p\gamma') < 8 \times 10^{-5}$



Cutdown the peaking backgrounds

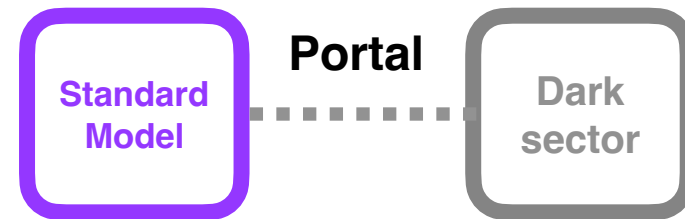


10 hadronic decay modes



○ Motivation on Dark sector

- The dark sector could be light and communicate with the visible sector through a feeble **portal** interaction
- The dark sector models can be classified based on the mediator particle or “portal”



$$\mathcal{L} \left(\begin{array}{ll} -\frac{\varepsilon}{2 \cos \theta_W} B_{\mu\nu} F'^{\mu\nu} & \text{Vector portal} \\ (\mu\phi + \lambda\phi^2) H^\dagger H, & \text{Higgs portal} \\ y_n L H N, & \text{Neutrino portal} \\ \frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}, & \text{Axion portal} \end{array} \right.$$

Dark photon γ' , kinematic mixing with γ, Z

Axion, coupling to DM

- Beyond these, certain anomaly gauged $U(1)_{L_\mu - L_\tau}$ model

Search for an **axion-like** particle in radiative J/ψ decays

PLB 838 137698 (2023)

○ **Data samples:** 2.7B $\psi(3686)$ events

○ **Strategy:**

Search for $J/\psi \rightarrow \gamma a, a \rightarrow \gamma\gamma$ with $\psi(3686) \rightarrow \pi^+\pi^-J/\psi$ decays

● Search range: $0.165 < m_a < 2.84 \text{ GeV}/c^2$

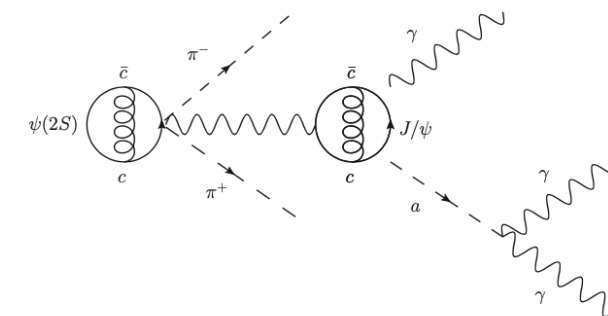
● a : negligible decay width and lifetime \triangleright decay width $\Gamma_a = g_{a\gamma\gamma}^2 m_a^3 / 64\pi$

● $\psi(3686)$ decay

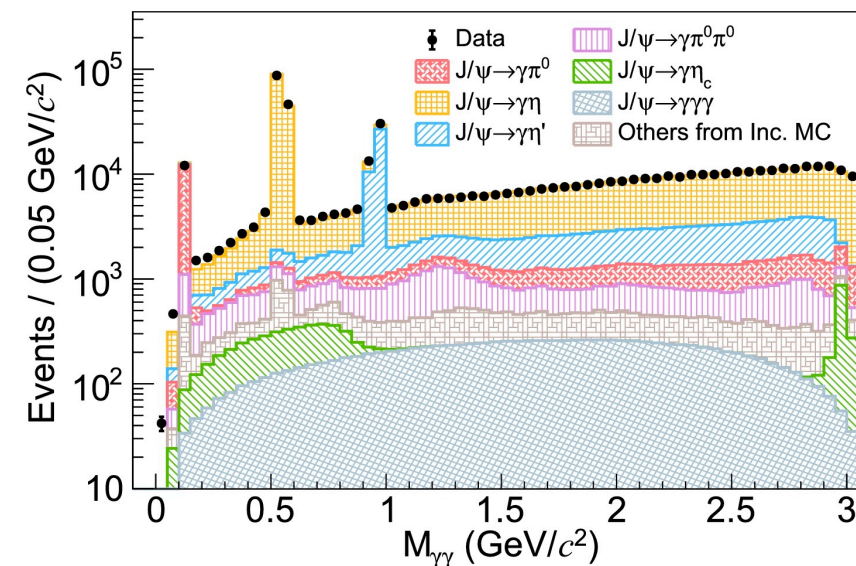
★ preclude the pollution from non-resonant production , avoid large QED background: $e^+e^- \rightarrow \gamma\gamma(\gamma)$

● Three $\gamma\gamma$ combinations per event, perform unbinned maximum-likelihood fits on $M_{\gamma\gamma}$

● Exclude mass intervals around π^0, η, η' peaks when extracting the signal



$$\mathcal{B}(J/\psi \rightarrow \gamma a) = \frac{m_{J/\psi}^2}{32\pi\alpha_{\text{em}}} g_{a\gamma\gamma}^2 \left(1 - \frac{m_a^2}{m_{J/\psi}^2}\right)^3 \mathcal{B}(J/\psi \rightarrow e^+e^-),$$



Search for an axion-like particle in radiative J/ψ decays

[arXiv:2404.04640](https://arxiv.org/abs/2404.04640)

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Table 1

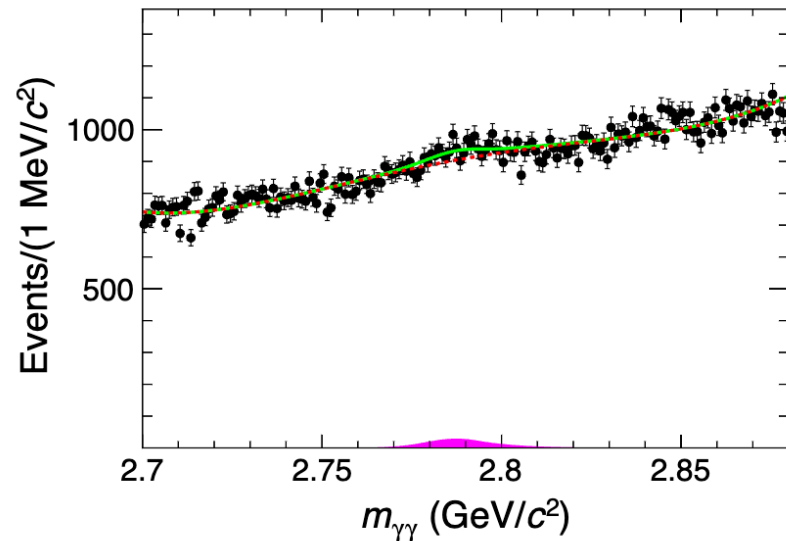
The $M_{\gamma\gamma}$ fit intervals for various m_a points.

m_a points (GeV/c^2)	$M_{\gamma\gamma}$ fit intervals (GeV/c^2)
0.165 - 0.35	0.06 - 0.45
0.35 - 0.75	0.25 - 0.85
0.75 - 1.20	0.65 - 1.30
1.20 - 2.84	$(m_a - 0.2) - (m_a + 0.2)$

674 hypothesis

TABLE I. The fit intervals of $m_{\gamma\gamma}$ for various m_a points.

m_a range (GeV/c^2)	$m_{\gamma\gamma}$ fit interval (GeV/c^2)	Polynomial function order
0.180 - 0.420	0.16, 0.46	4 th
0.421 - 0.490	0.39, 0.51	5 th
0.610 - 0.880	0.59, 0.90	5 th
1.020 - 1.099	1.00, 1.20	5 th
1.100 - 2.770	$m_a - 0.10, m_a + 0.10$	3 rd
2.772 - 2.850	2.70, 2.88	4 th



The largest value of upward local significance is determined to be 3.5σ at $m_a = 2.786 \text{ GeV}/c^2$

Search for Massive dark photon with $e^+e^- \rightarrow \gamma\gamma'$

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$$\sigma(e^+e^- \rightarrow \gamma\gamma') = \frac{2\pi\alpha^2}{s} \epsilon^2 \left(1 - \frac{m_{\gamma'}^2}{s}\right) \times \left(1 + \frac{2\frac{m_{\gamma'}^2}{s}}{\left(1 - \frac{m_{\gamma'}^2}{s}\right)^2}\right) \log \frac{(1 + \cos\theta_c)^2}{(1 + \cos\theta_c)^2 - 2\cos\theta_c}$$

$\cos\theta_c = 0.6$ is the $\cos\theta$ cut for the signal photon polar angle

Search for **single photon signals** in $1.3 < E(\gamma) < 1.8$ GeV

corresponding to $1.5 < m_{\gamma'} < 2.9$ GeV

- **Low $E(\gamma)$ region** \rightarrow low trigger efficiency & high background level
- **High $E(\gamma)$ region** \rightarrow saturation of the EMC electronics

Search for a muonphilic scalar X_0 or vector X_1 via $J/\psi \rightarrow \mu^+ \mu^- +$ invisible decays

Decay width

$$|\mathcal{M}_{\mu\mu X_0}|^2 = \left(\frac{2}{3}e^2 g_0 \frac{f_J}{m_J}\right)^2 \frac{-8}{3 m_J^2 (m_J - 2 E_-)^2 (-2 E_- - 2 E_X + m_J)^2} \left(-4 m_\mu^2 (4 E_-^2 (m_X^2 - 2 E_X m_J) \right. \\ \left. + E_- (-8 E_X^2 m_J + 4 E_X (m_X^2 + 2 m_J^2) - 4 m_X^2 m_J) - E_X^2 (m_X^2 - 6 m_J^2) - 2 E_X m_J (m_X^2 + m_J^2) + m_X^2 m_J^2) \right. \\ \left. + 4 E_-^2 (2 E_X^2 m_J^2 + m_X^2 m_J (m_J - 2 E_X) + m_X^4) \right. \\ \left. + 4 E_- (2 E_X^3 m_J^2 - 2 E_X^2 m_J (m_X^2 + m_J^2) + E_X (m_X^4 + 3 m_X^2 m_J^2) - m_X^2 m_J (m_X^2 + m_J^2)) \right. \\ \left. - 16 E_X^2 m_\mu^4 + m_J (-4 E_X^3 m_J^2 + 2 E_X^2 (3 m_X^2 m_J + m_J^3) - 2 E_X (m_X^4 + 2 m_X^2 m_J^2) + m_X^2 m_J (m_X^2 + m_J^2)) \right),$$

where E_- , the energy of μ^- and E_X , the energy of X_0 are measured in the rest frame of J/ψ .

$$\Gamma_{\mu\mu X_{0,1}} = \int_{E_X^{min}}^{E_X^{max}} \int_{E_-^{min}}^{E_-^{max}} \frac{|\mathcal{M}_{\mu\mu X_{0,1}}|^2}{64\pi^3 m_J} dE_- dE_X,$$

$$|\mathcal{M}_{\mu\mu X_1}|^2 = \left(\frac{2}{3}e^2 g_1 \frac{f_J}{m_J}\right)^2 \frac{-16}{3 m_J^2 (m_J - 2 E_-)^2 (-2 E_- - 2 E_X + m_J)^2} \left(16 E_-^4 m_J^2 + 32 E_-^3 m_J^2 (E_X - m_J) \right. \\ \left. + 2 m_\mu^2 (4 E_-^2 (m_J (m_J - 2 E_X) + m_X^2) - 4 E_- (2 E_X^2 m_J - E_X (m_X^2 + 3 m_J^2) + m_J (m_X^2 + m_J^2)) \right. \\ \left. + 2 E_X^2 (m_X^2 + 3 m_J^2) - 2 E_X m_J (m_X^2 + 2 m_J^2) + m_J^2 (m_X^2 + m_J^2)) \right. \\ \left. + 4 E_-^2 (m_J^2 (6 E_X^2 - 14 E_X m_J + 7 m_J^2) + m_X^2 m_J (3 m_J - 2 E_X) + m_X^4) \right. \\ \left. + 4 E_- (2 E_X^3 m_J^2 - 2 E_X^2 m_J (m_X^2 + 4 m_J^2) + E_X (m_X^4 + 5 m_X^2 m_J^2 + 9 m_J^4) - m_J (m_X^4 + 3 m_X^2 m_J^2 + 3 m_J^4)) \right. \\ \left. + 8 E_X^2 m_\mu^4 + m_J (-4 E_X^3 m_J^2 + 2 E_X^2 (3 m_X^2 m_J + 5 m_J^3) - 2 E_X (m_X^2 + 2 m_J^2)^2 + m_J (m_X^4 + 3 m_X^2 m_J^2 + 2 m_J^4)) \right),$$

where E_- , the energy of μ^- and E_X , the energy of X_0 are measured in the rest frame of J/ψ .