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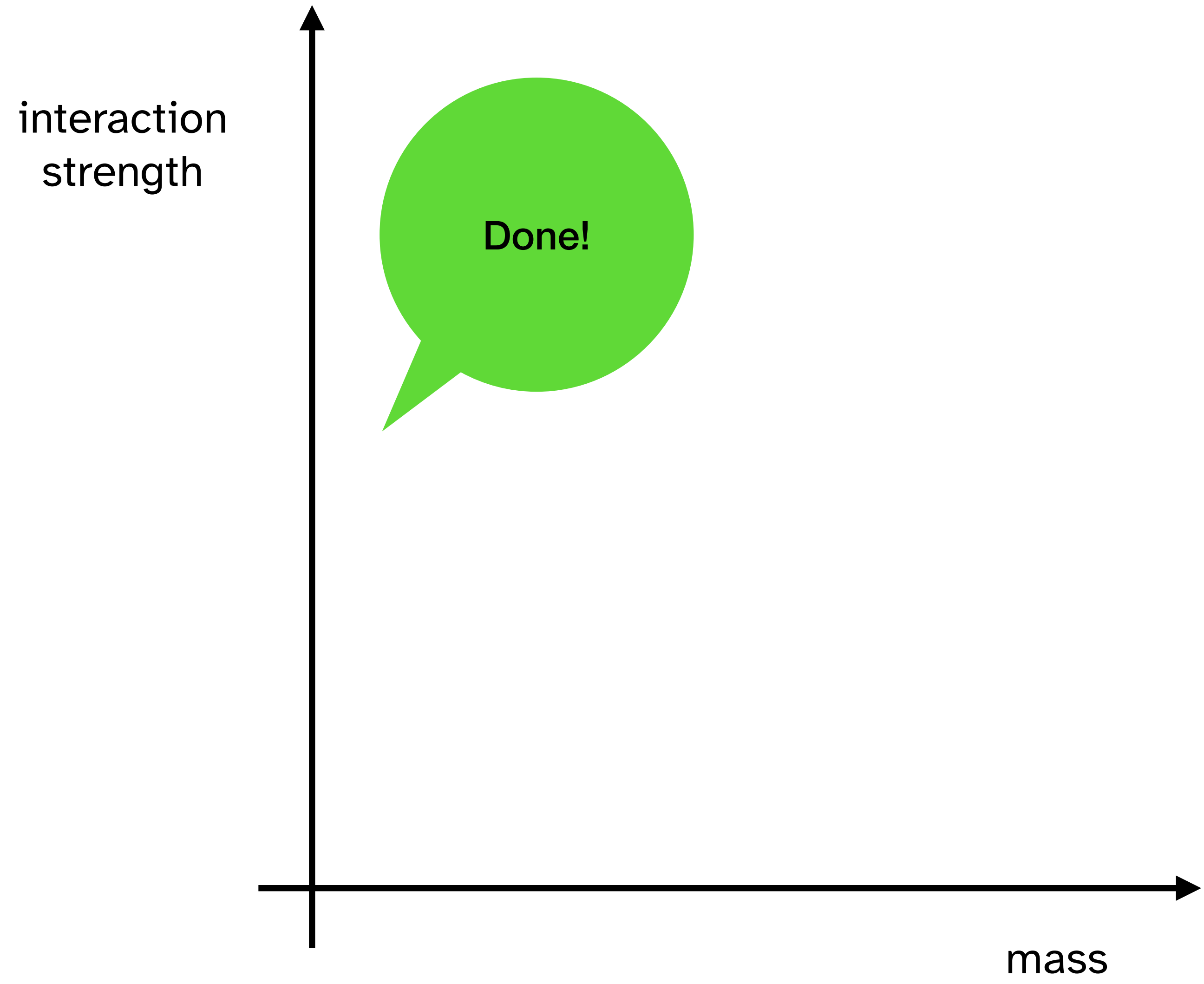
# Recasting scalar-tensor theories of gravity for colliders

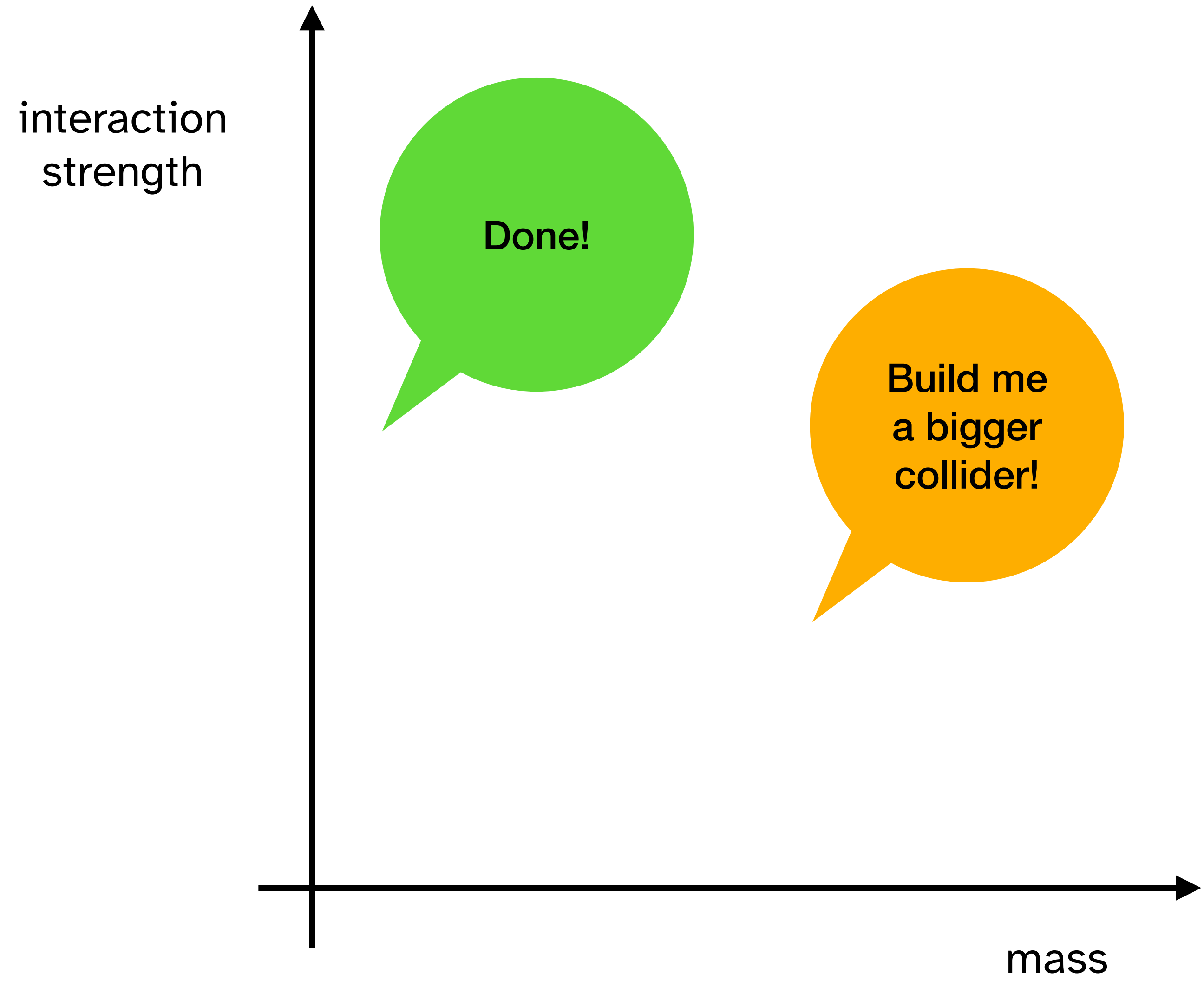
Based on work with: Clare Burrage (Nottingham), Edmund Copeland (Nottingham), Christoph Englert (Glasgow), Andrei Lazanu (Manchester), **Sergio Sevillano Muñoz (IPPP, Durham)**, Michael Spannowsky (IPPP, Durham)

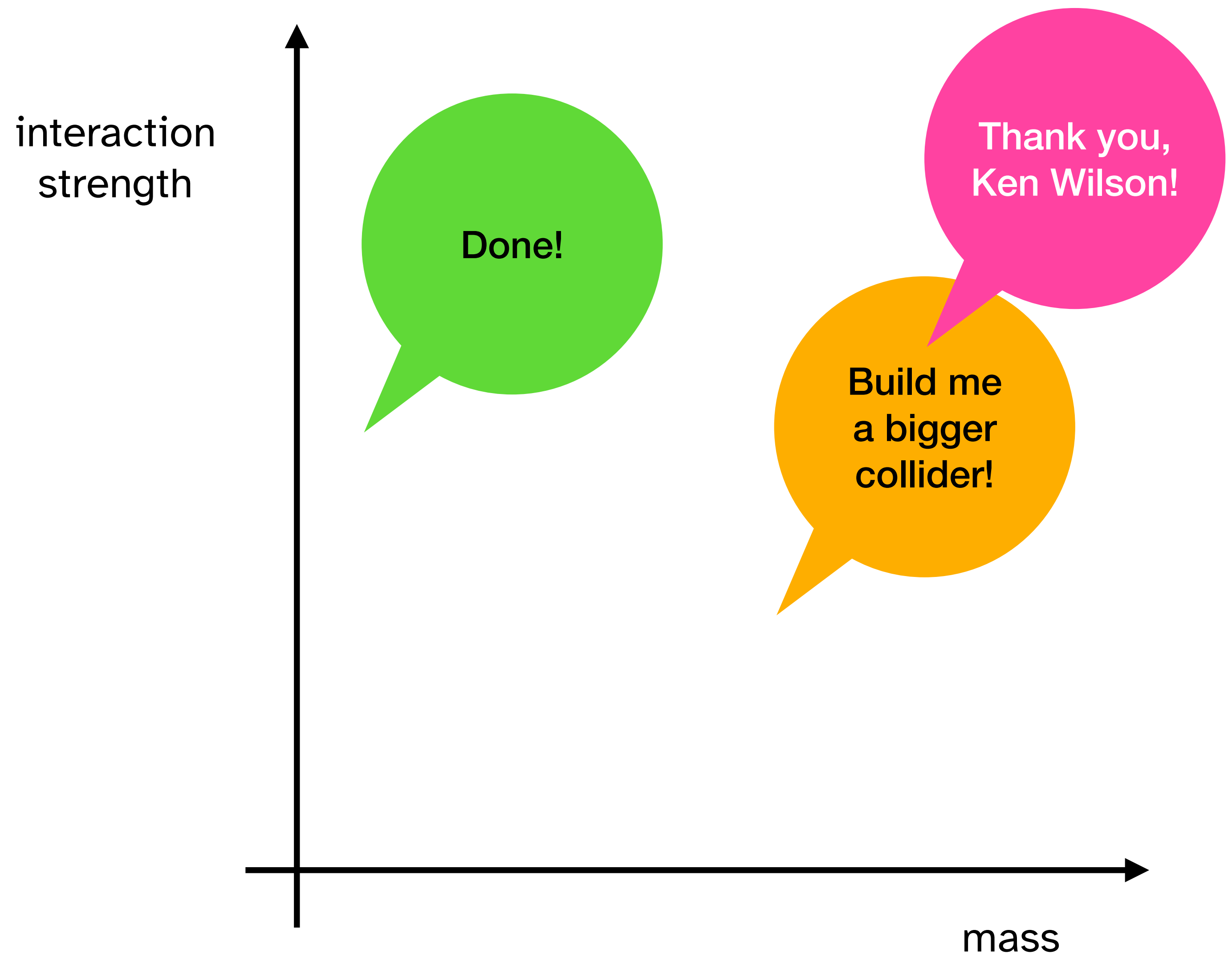
**Peter Millington (UKRI Future Leaders Fellow, Particle Theory Group, University of Manchester)**

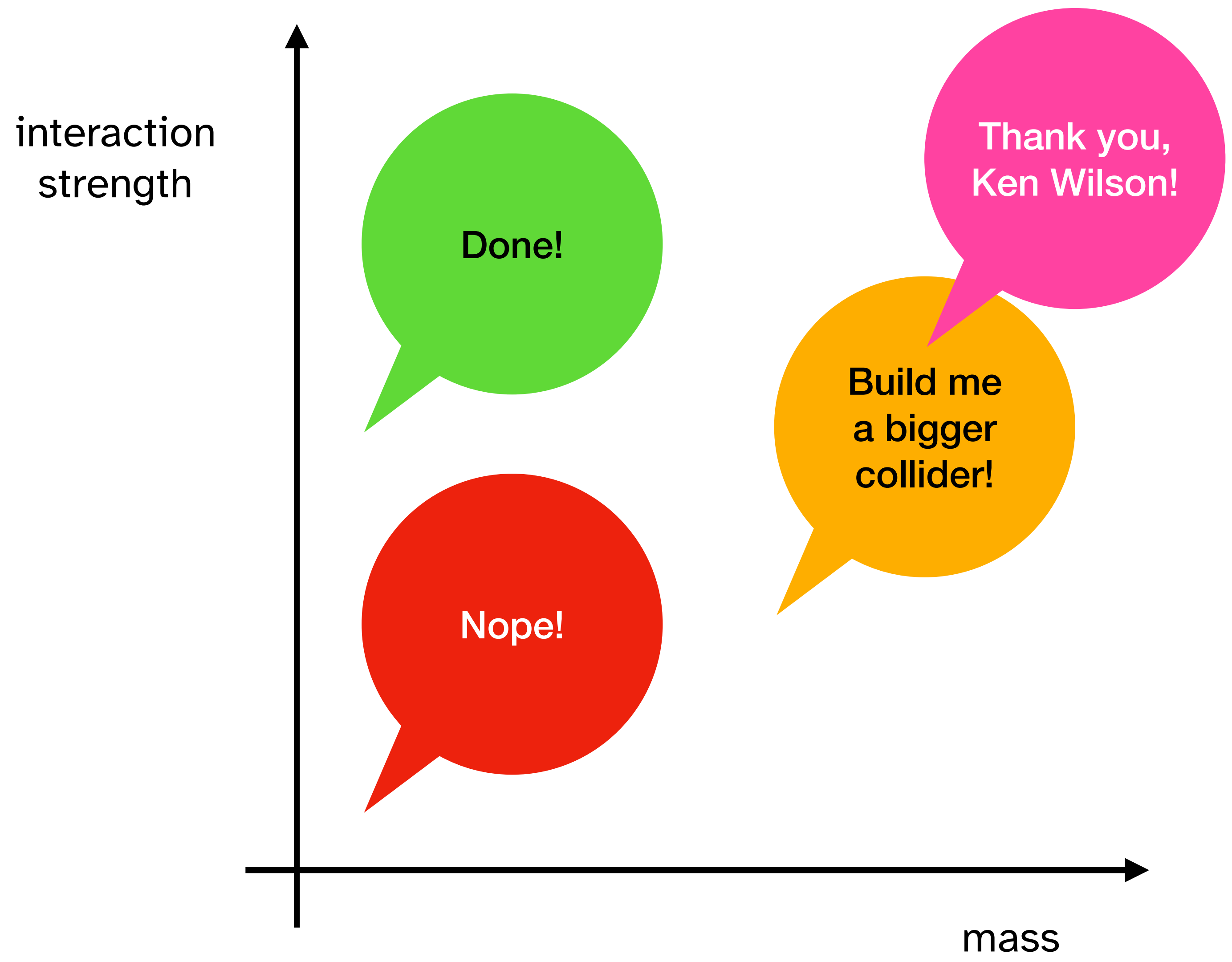
**[peter.millington@manchester.ac.uk](mailto:peter.millington@manchester.ac.uk)**

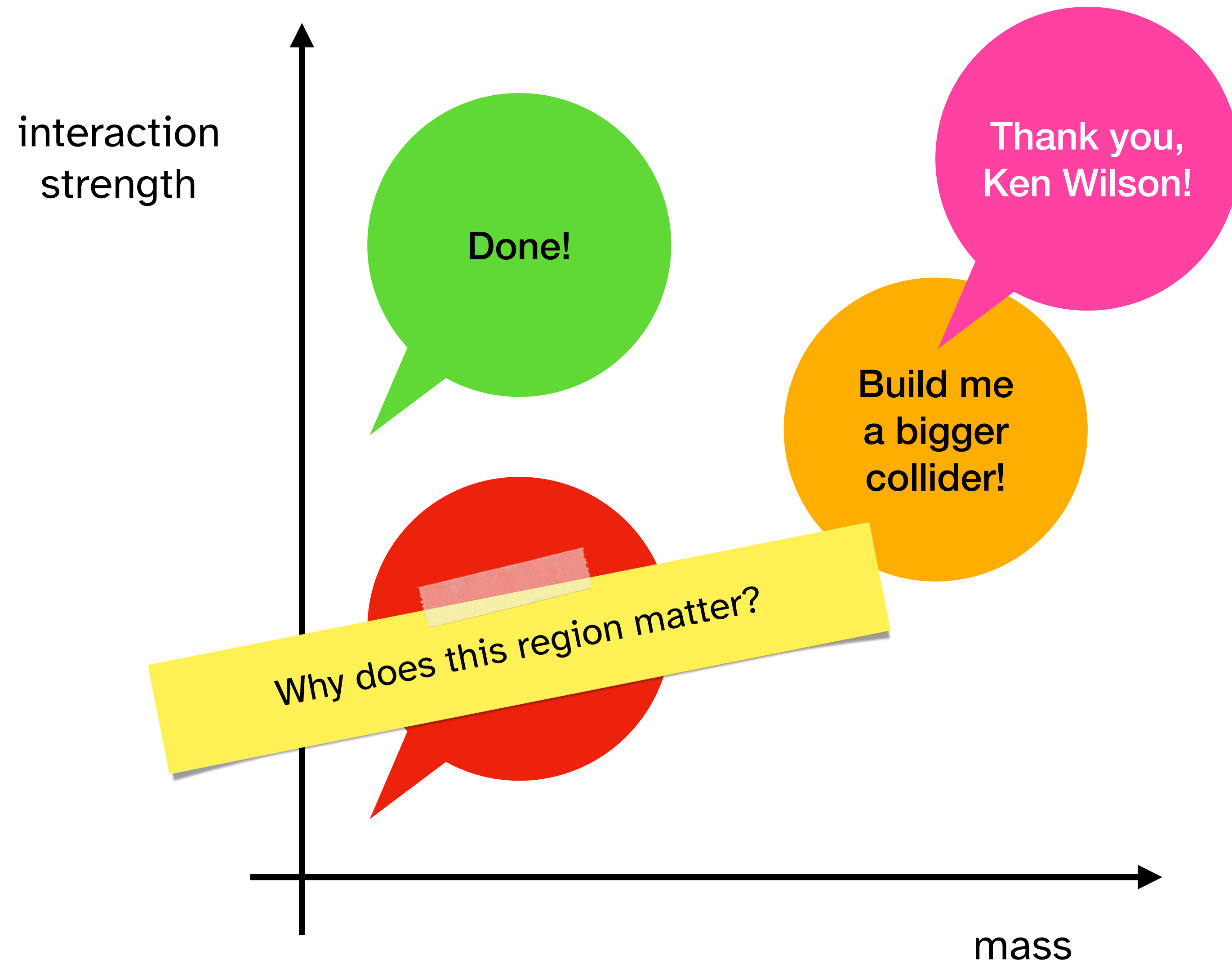
ICHEP 2024, Prague, 18 July 2024

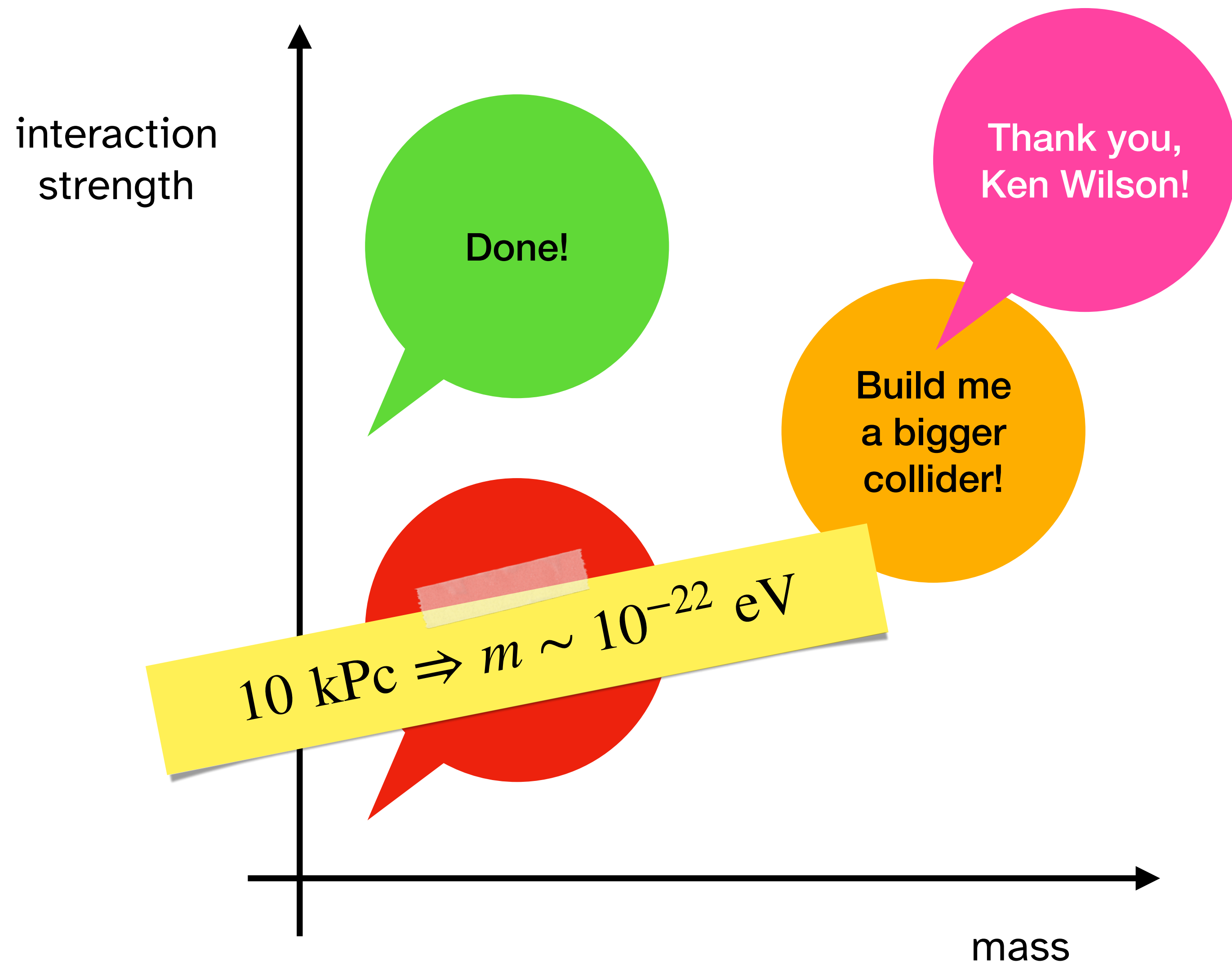


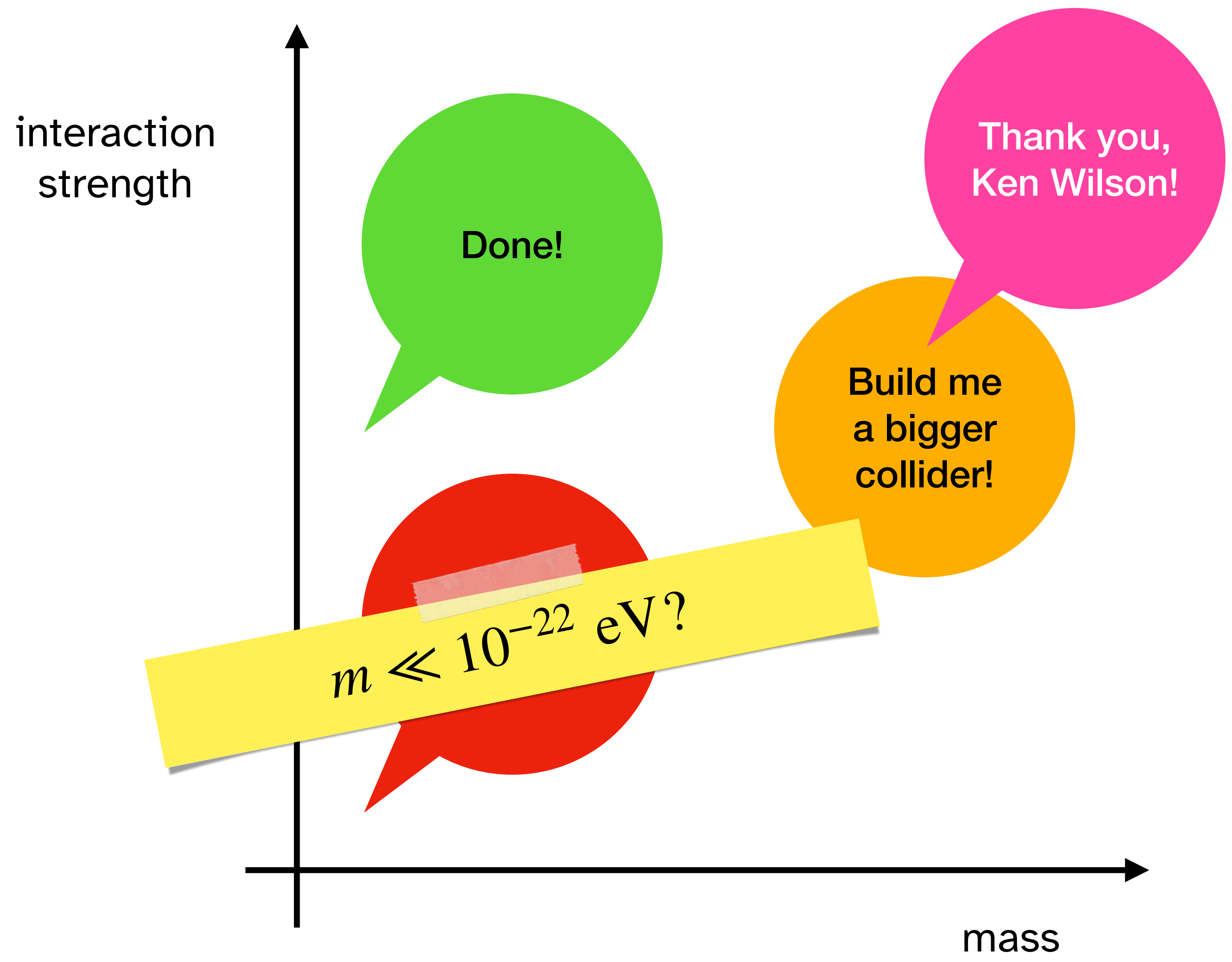














# Lovelock's theorem

a local gravity action in (3+1)D containing only 2nd-order derivatives of the metric  $g_{\mu\nu}$  necessarily leads to the **Einstein field equations**

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \kappa T_{\mu\nu}$$

Einstein Tensor

CC

SM

# The Einstein-Hilbert action + CC + SM

$$S = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} (R - 2\Lambda) + S_{\text{SM}} = S^\dagger$$

# The Einstein–Hilbert action + CC + SM

gives  
Newton's coupling



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spacetime  
volume element

# The Einstein-Hilbert action + CC + SM

gives  
Newton's coupling

↓

$$S = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} (R - 2\Lambda) + S_{\text{SM}} = S^\dagger$$

spacetime volume element      Ricci scalar

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gives  
Newton's coupling

CC

spacetime  
volume element

Ricci scalar

SM action

# Going beyond ...

$$S = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} (R - 2\Lambda) + S_{\text{SM}} = S^\dagger$$



# Going beyond ...

make the Planck mass dynamical



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make the Planck mass dynamical



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add extra  
dimensions

# Going beyond ...

make the Planck mass dynamical

$$S = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} (R - 2\Lambda) + S_{\text{SM}} = S^\dagger$$

The diagram shows the action  $S$  with three highlighted components in colored boxes: a blue box under the integral  $\int d^4x \sqrt{-g}$ , a green box around the term  $-2\Lambda$ , and an orange box around  $S_{\text{SM}}$ . An arrow points from the text 'make the Planck mass dynamical' to the blue box, and another arrow labeled 'CC' points to the green box.

add extra  
dimensions

# Going beyond ...

change the underlying symmetries



make the Planck mass dynamical



CC



$$S = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} (R - 2\Lambda) + S_{\text{SM}} = S^\dagger$$

add extra  
dimensions

# Going beyond ...

change the underlying symmetries

make the Planck mass dynamical

$$S = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} (R - 2\Lambda) + S_{\text{SM}} = S^\dagger$$

add extra dimensions

add other curvature terms

CC

# Going beyond ...

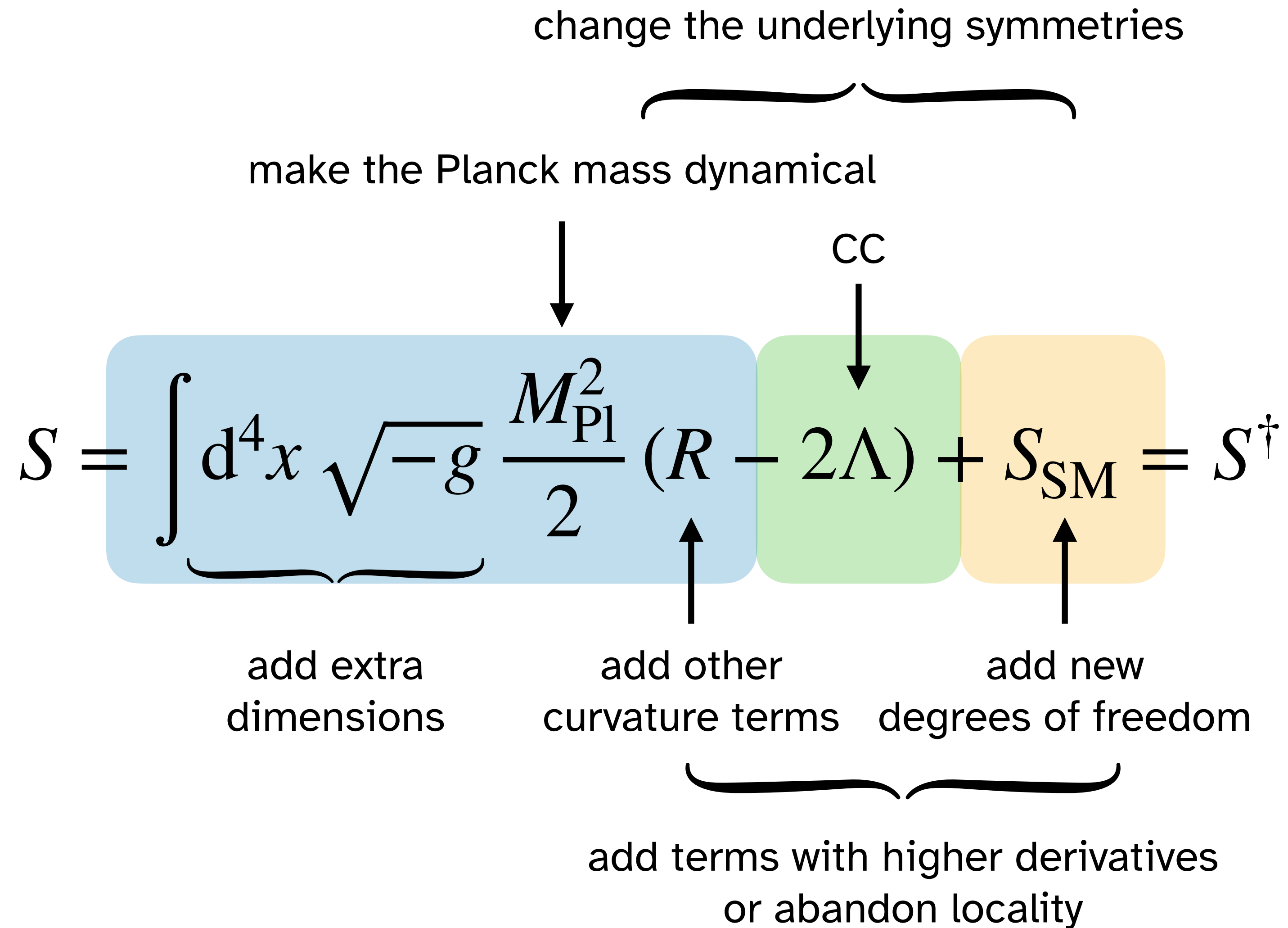
change the underlying symmetries

make the Planck mass dynamical

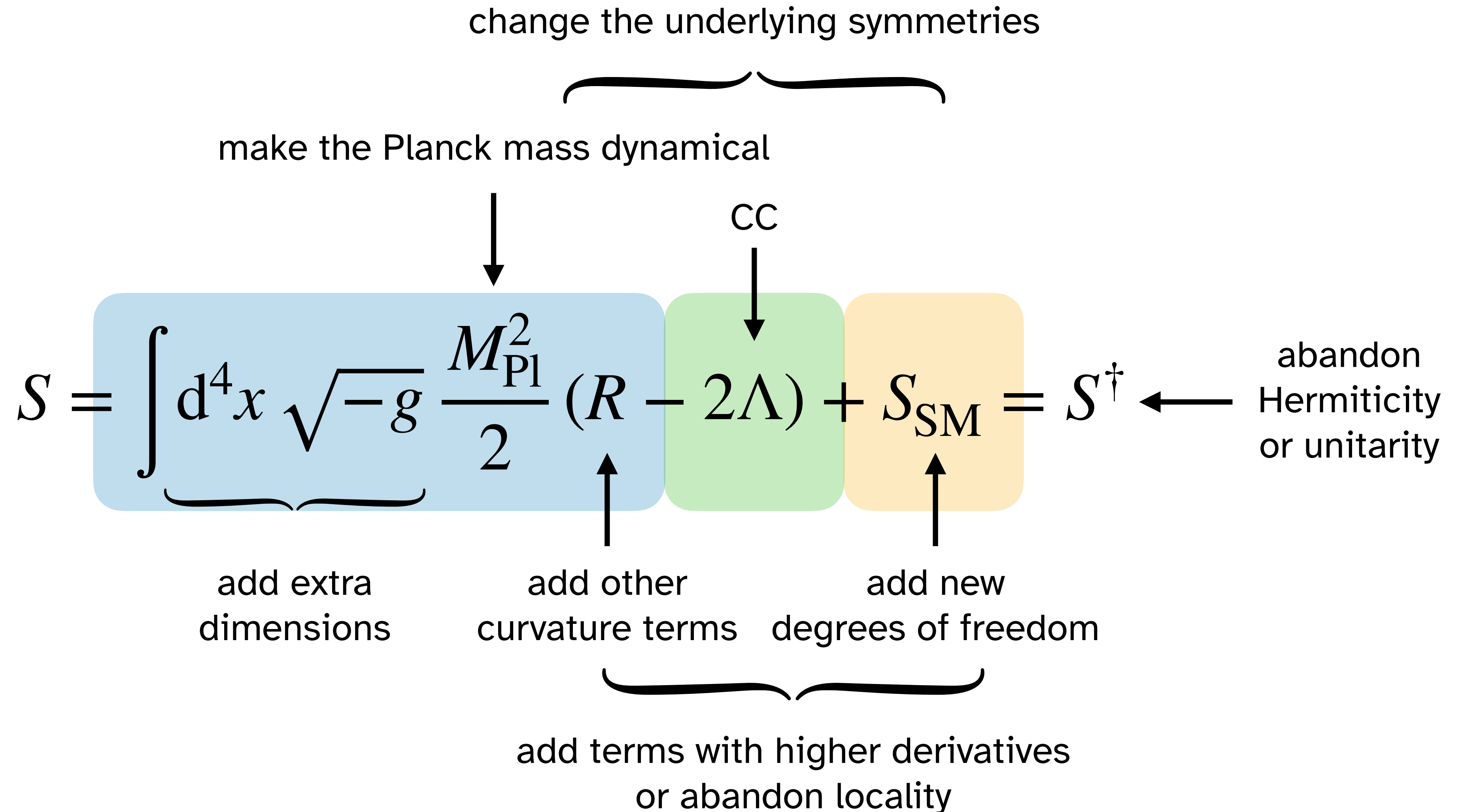
$$S = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} (R - 2\Lambda) + S_{\text{SM}} = S^\dagger$$

add extra dimensions      add other curvature terms      add new degrees of freedom

# Going beyond ...

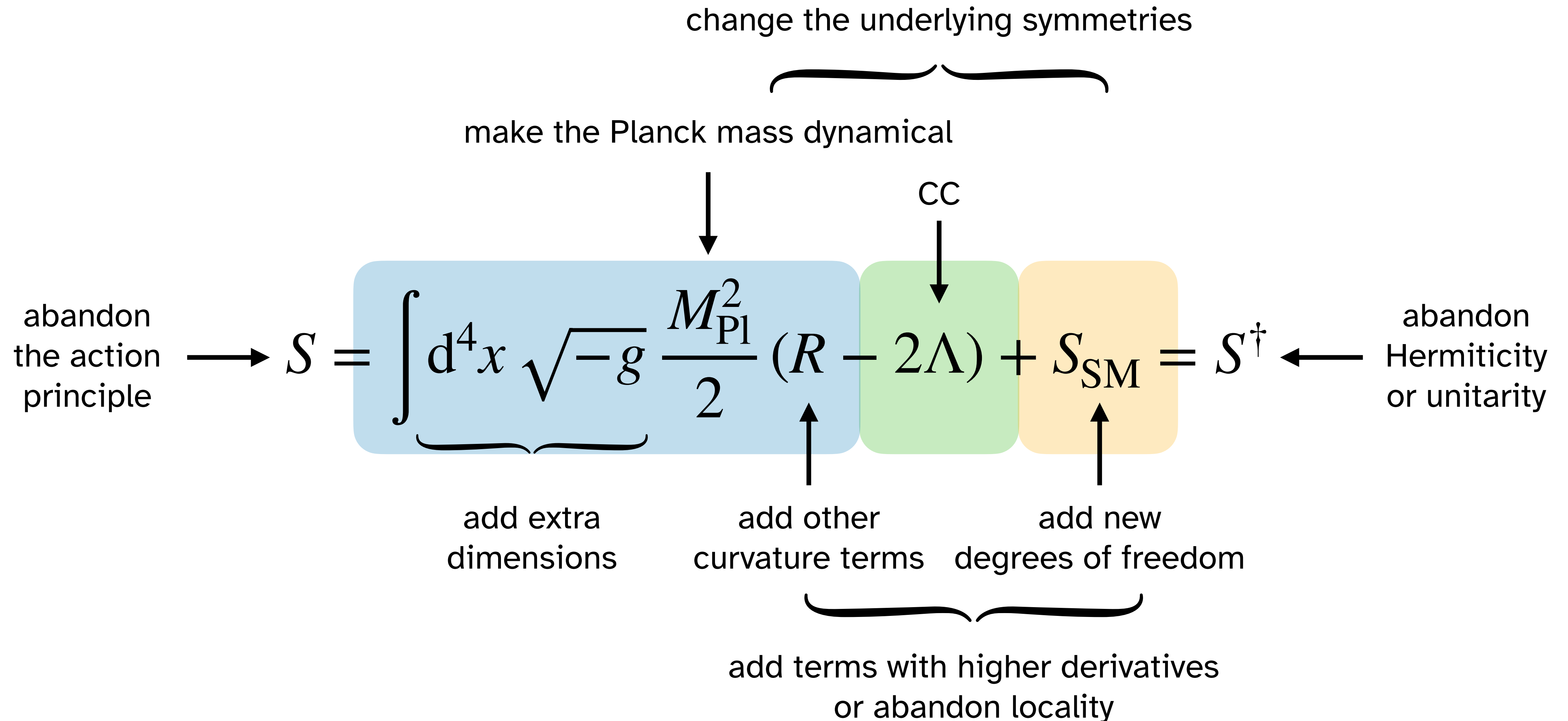


# Going beyond ...

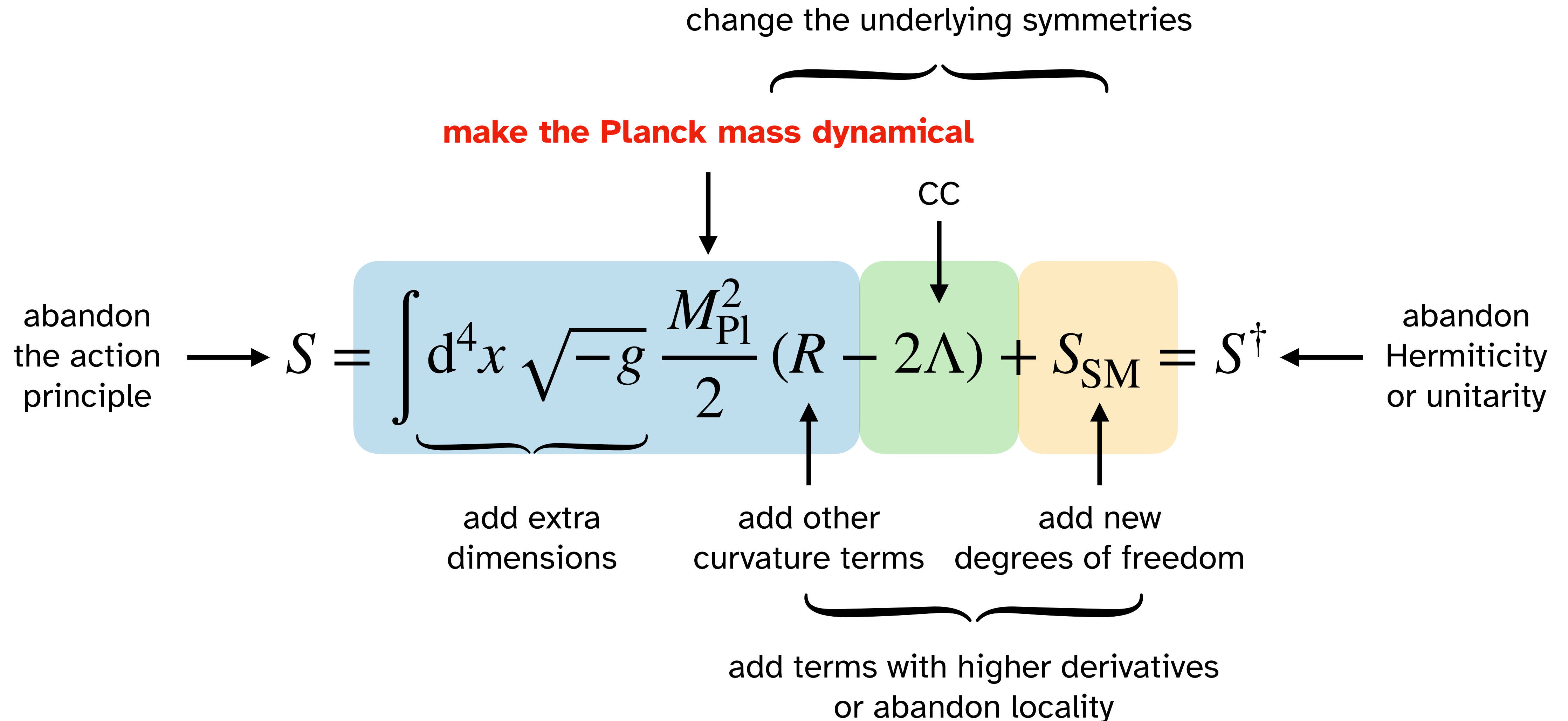




# Going beyond ...



# Going beyond ...



# Geodesic equation

**connection** encodes the geometry

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = 0$$

$g_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}$  can be **timelike**, **spacelike** or **null** (i.e., zero)

# Weyl rescaling

$$g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu} \quad \text{and} \quad \frac{d}{d\lambda} \rightarrow \Omega^{-2}(x)\frac{d}{d\lambda}$$

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} - g_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} \frac{\partial}{\partial x_\mu} \ln \Omega = 0$$

geodesic motion

“fifth force”

# Jordan versus Einstein frame

## Jordan frame

$$S = \int d^4x \sqrt{-g} \left[ \frac{F(\phi)}{2} R - \frac{Z^{\mu\nu}(\phi, \partial\phi, \dots)}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \mathcal{L}_{\text{SM}}(g_{\alpha\beta}) \right]$$

**Einstein frame**, via  $g_{\mu\nu} = M_{\text{Pl}}^2 F^{-1}(\phi) \tilde{g}_{\mu\nu} = M_{\text{Pl}}^2 A^2(\tilde{\phi}) \tilde{g}_{\mu\nu}$

$$S = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{M_{\text{Pl}}^2}{2} \tilde{R} - \frac{\tilde{Z}^{\mu\nu}(\tilde{\phi}, \partial\tilde{\phi}, \dots)}{2} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \tilde{V}(\tilde{\phi}) + \mathcal{L}_{\text{SM}}(A^2(\tilde{\phi}) \tilde{g}_{\alpha\beta}) \right]$$

# Einstein frame

$$S_{\text{SM}}[A^2(\tilde{\phi})\tilde{g}_{\mu\nu}, \{\psi\}] = S_{\text{SM}}[\tilde{g}_{\mu\nu}, \{\psi\}] + \frac{1}{2}[A^2(\tilde{\phi}) - 1]\tilde{T}_{\text{SM}} + \dots$$

↑  
“universal”  
coupling to SM

Test particle experiences a **fifth force**:  $\vec{F}/m = -\vec{\nabla} \ln A(\tilde{\phi})$

# Screening

$$\tilde{U}(r) \supset - \frac{1}{\tilde{Z}(\tilde{\varphi})c_s^2(\tilde{\varphi})} \left[ \frac{dA(\tilde{\varphi})}{d\tilde{\varphi}} \right]^2 \frac{1}{4\pi r} \exp \left[ - \frac{m(\tilde{\varphi})r}{\tilde{Z}^{1/2}(\tilde{\varphi})c_s(\tilde{\varphi})} \right] \mathcal{M}$$

# Screening

$$\tilde{U}(r) \supset -\frac{1}{\tilde{Z}(\tilde{\varphi})c_s^2(\tilde{\varphi})} \left[ \frac{dA(\tilde{\varphi})}{d\tilde{\varphi}} \right]^2 \frac{1}{4\pi r} \exp \left[ -\frac{m(\tilde{\varphi})r}{\tilde{Z}^{1/2}(\tilde{\varphi})c_s(\tilde{\varphi})} \right] \mathcal{M}$$

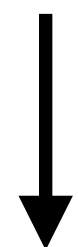
increase the  
effective mass  
↓



# Screening



increase the effective mass



$$\tilde{U}(r) \supset - \frac{1}{\tilde{Z}(\tilde{\varphi})c_s^2(\tilde{\varphi})} \left[ \frac{dA(\tilde{\varphi})}{d\tilde{\varphi}} \right]^2 \frac{1}{4\pi r} \exp \left[ - \frac{m(\tilde{\varphi})r}{\tilde{Z}^{1/2}(\tilde{\varphi})c_s(\tilde{\varphi})} \right] \mathcal{M}$$

# Screening



decrease the coupling strength

increase the effective mass

$$\tilde{U}(r) \supset - \frac{1}{\tilde{Z}(\tilde{\varphi})c_s^2(\tilde{\varphi})} \left[ \frac{dA(\tilde{\varphi})}{d\tilde{\varphi}} \right]^2 \frac{1}{4\pi r} \exp \left[ - \frac{m(\tilde{\varphi})r}{\tilde{Z}^{1/2}(\tilde{\varphi})c_s(\tilde{\varphi})} \right] \mathcal{M}$$

# Screening

see e.g., Gessner 1992, Astrophys. Space Sci. 196, 1 [link]; Damour, Polyakov 1994, Nucl. Phys. B 423, 532 [link]; Hinterbichler, Khoury 2010, Phys. Rev. Lett. 104, 213301 [link]

symmetron/Damour-Polyakov

decrease the coupling strength

chameleon

increase the effective mass

$$\tilde{U}(r) \supset - \frac{1}{\tilde{Z}(\tilde{\varphi})c_s^2(\tilde{\varphi})} \left[ \frac{dA(\tilde{\varphi})}{d\tilde{\varphi}} \right]^2 \frac{1}{4\pi r} \exp \left[ - \frac{m(\tilde{\varphi})r}{\tilde{Z}^{1/2}(\tilde{\varphi})c_s(\tilde{\varphi})} \right] \mathcal{M}$$

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$$\tilde{U}(r) \supset - \frac{1}{\tilde{Z}(\tilde{\varphi})c_s^2(\tilde{\varphi})} \left[ \frac{dA(\tilde{\varphi})}{d\tilde{\varphi}} \right]^2 \frac{1}{4\pi r} \exp \left[ - \frac{m(\tilde{\varphi})r}{\tilde{Z}^{1/2}(\tilde{\varphi})c_s(\tilde{\varphi})} \right] \mathcal{M}$$

increase the effective sound speed

decrease the effective sound speed

# Screening

see e.g., Gessner 1992, Astrophys. Space Sci. 196, 1 [link]; Damour, Polyakov 1994, Nucl. Phys. B 423, 532 [link]; Hinterbichler, Khoury 2010, Phys. Rev. Lett. 104, 213301 [link]

symmetron/Damour-Polyakov

decrease the coupling strength

chameleon

increase the effective mass

$$\tilde{U}(r) \supset - \frac{1}{\tilde{Z}(\tilde{\varphi})c_s^2(\tilde{\varphi})} \left[ \frac{dA(\tilde{\varphi})}{d\tilde{\varphi}} \right]^2 \frac{1}{4\pi r} \exp \left[ - \frac{m(\tilde{\varphi})r}{\tilde{Z}^{1/2}(\tilde{\varphi})c_s(\tilde{\varphi})} \right] \mathcal{M}$$

increase the effective sound speed

decrease the effective sound speed

Vainshtein-like

# Weyl invariance

This is not the full story ...

The SM Lagrangian is **scale invariant** with the **exception** of the **quadratic term** in the **Higgs potential**.

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$$\mathcal{L}_{\text{SM}} \supset -\mu^2 |H|^2$$

# Beyond GR or Beyond SM?

**scalar-tensor** extension of GR



$$\phi^2 R$$

versus

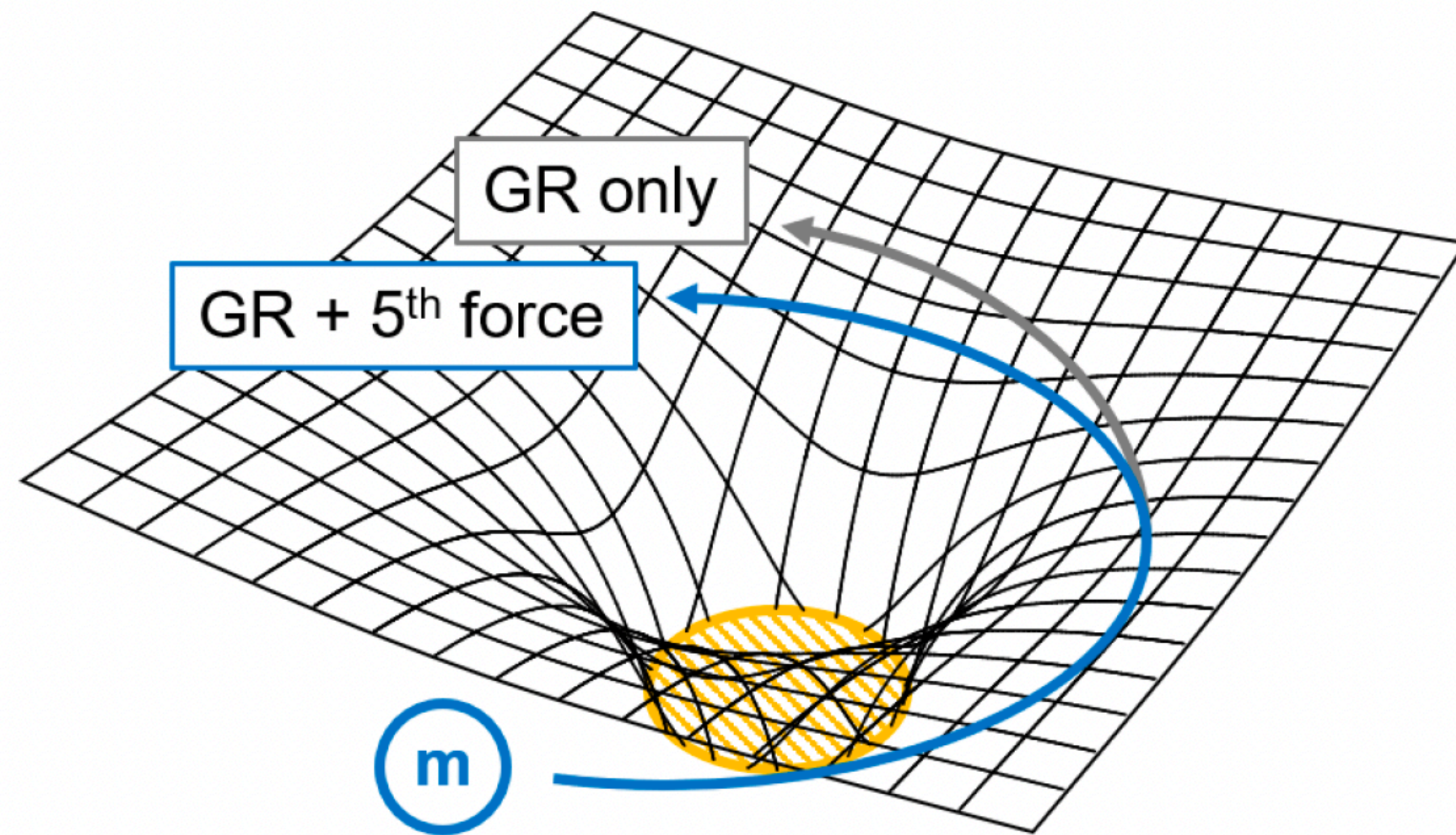
$$\phi^2 |H|^2$$



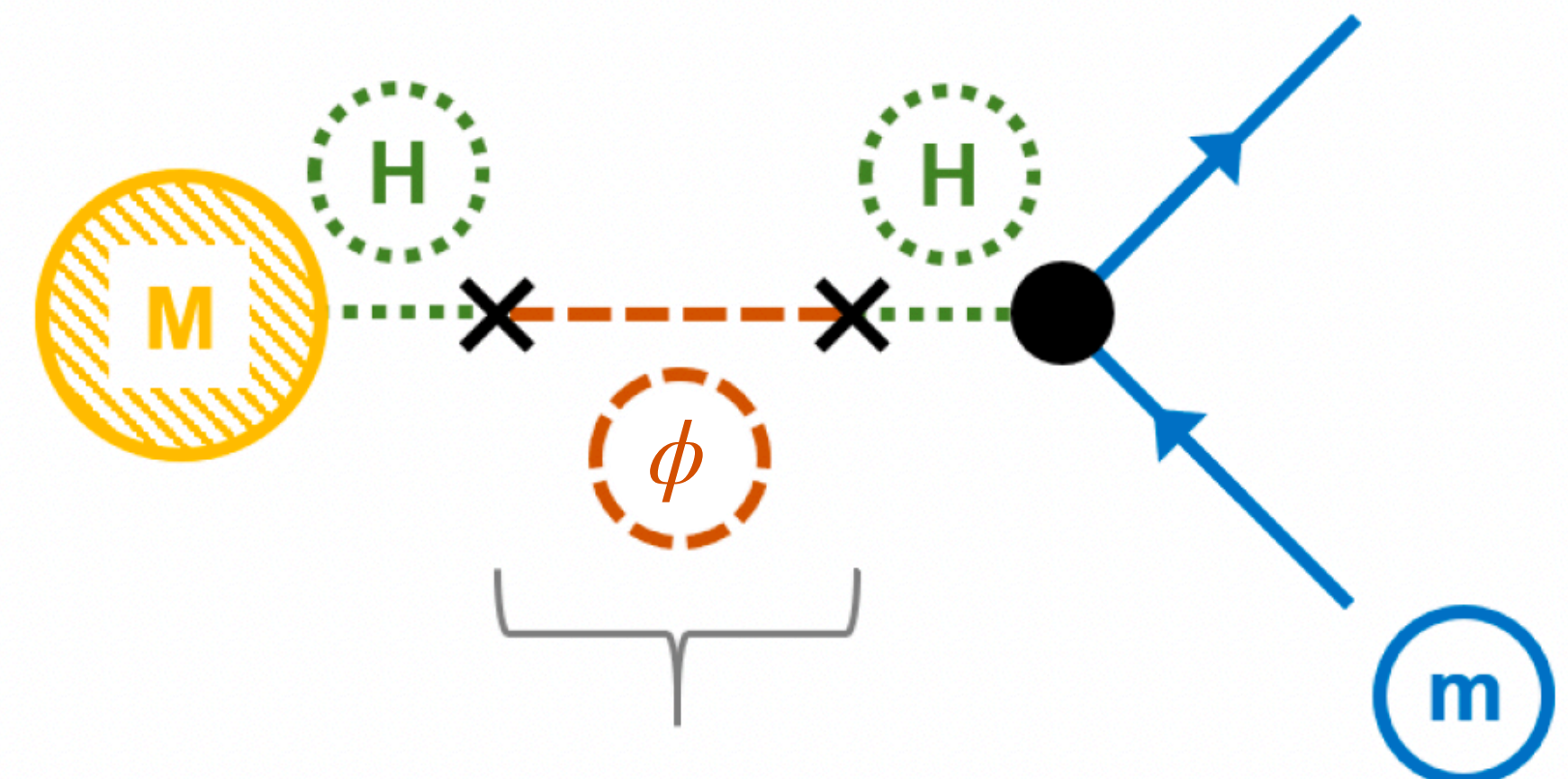
**Higgs-portal** extension of the SM

# Beyond GR or Beyond SM?

as modified gravitational dynamics



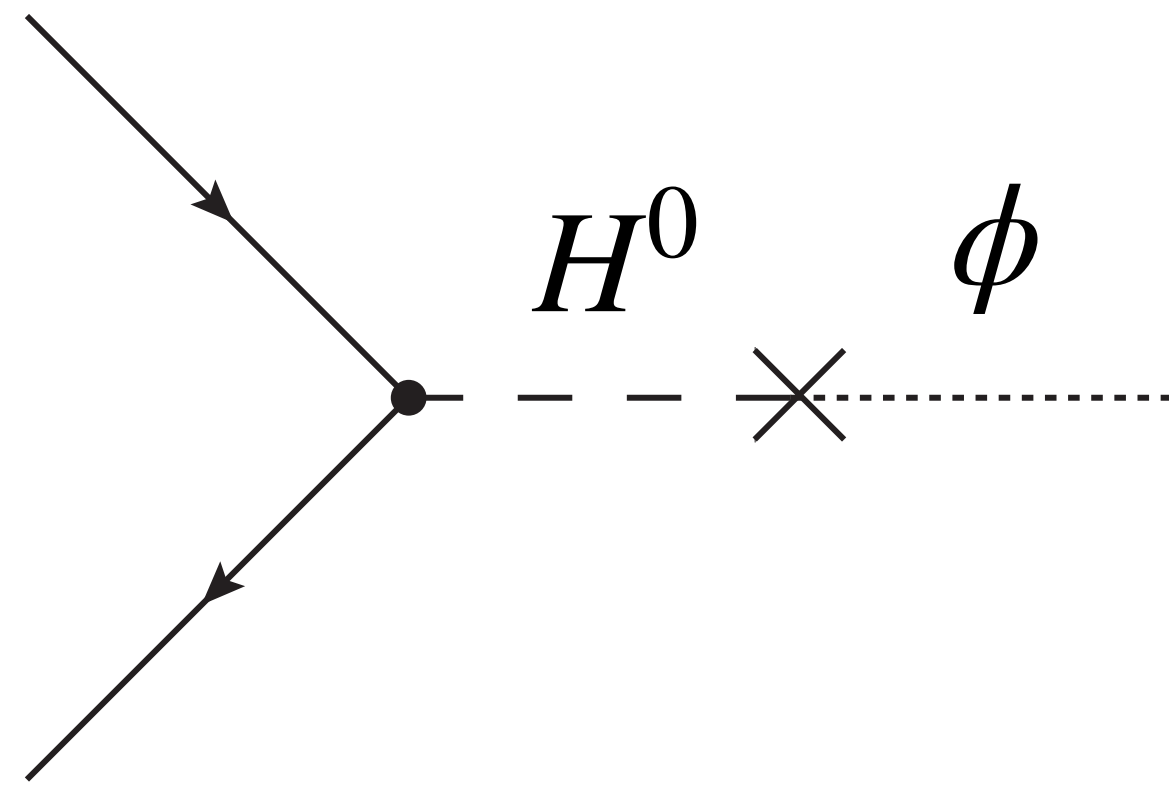
as a Higgs portal



controlled by screening and explicit scale breaking



# Coupling to leptons



$$H^0 = \frac{h}{\sqrt{2}} + \frac{v}{\sqrt{2}} \frac{\zeta}{M}$$

$$\mathcal{L}_{\text{eff}} \supset -\frac{2\mu^2 m_L}{m_H^2 M} \bar{\psi}_L \zeta \psi_L$$

where  $\frac{2\mu^2}{m_H^2} = 1$  for the SM

# Scale symmetry breaking

Fifth forces from  $F(\phi)R$  terms depend on the origin of **scale breaking** in the electroweak and QCD sectors (and the origin of neutrino masses).

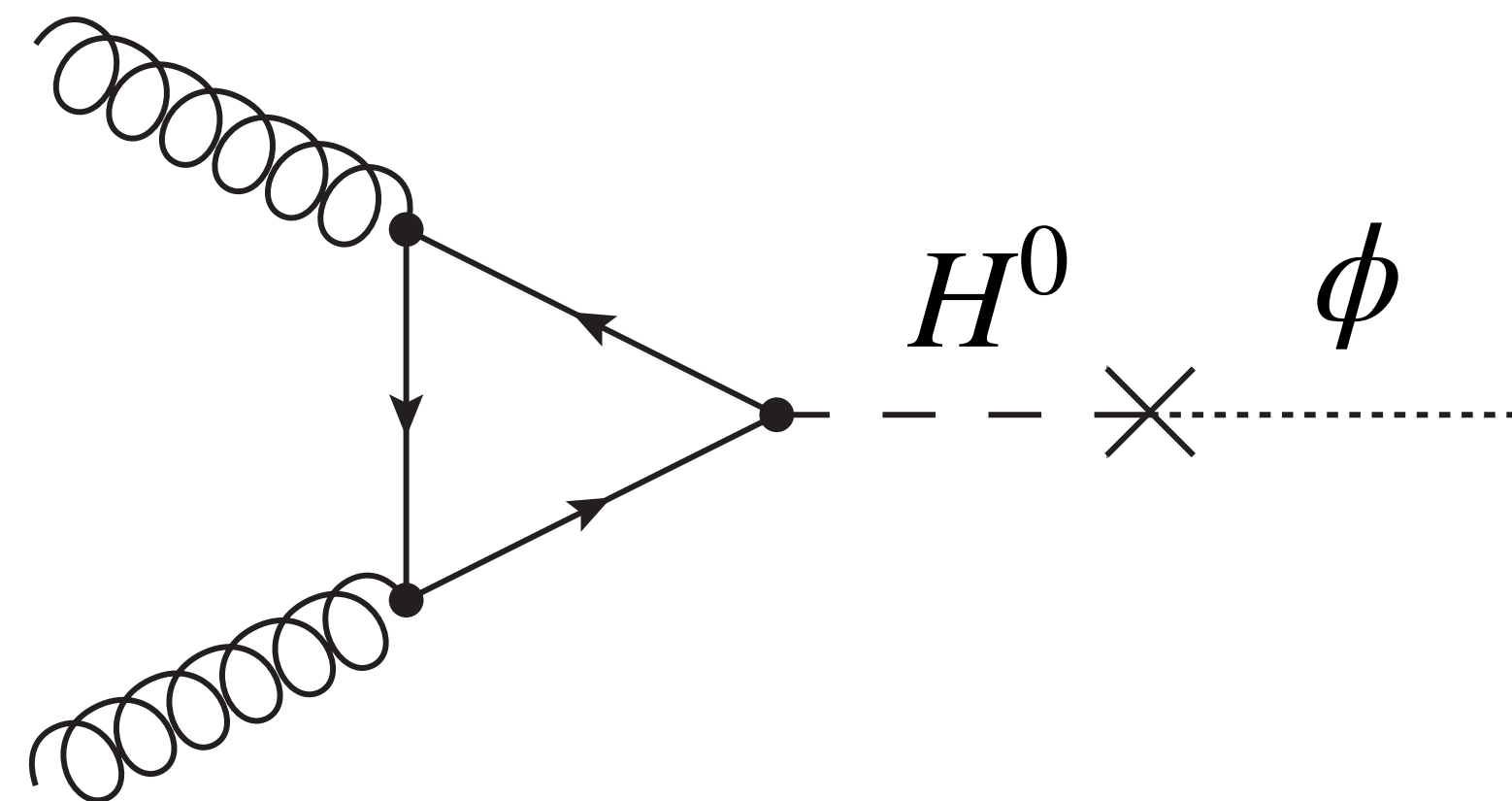
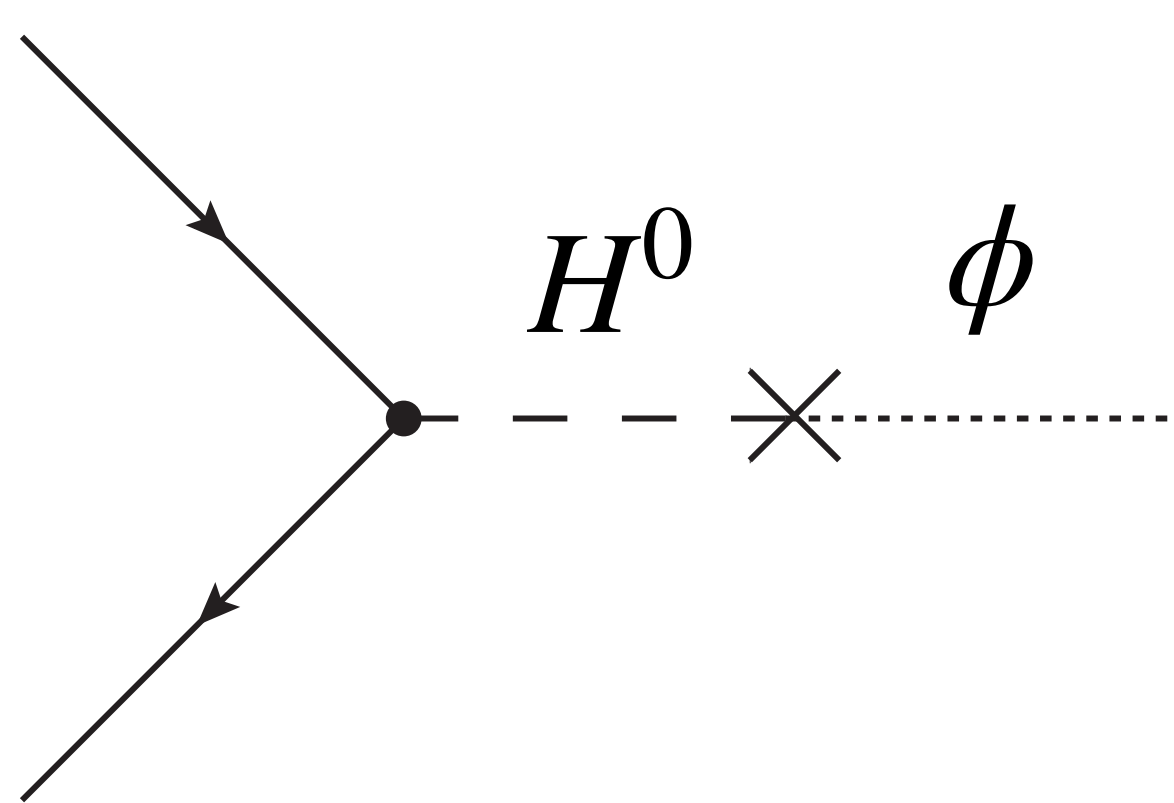
**explicit** scale breaking  $\Rightarrow$  fifth force coupling

**dynamical** scale breaking  $\Rightarrow$  **no** fifth force coupling

**But beware, fine tuning.**

see e.g. Higgs-Dilaton theories: Shaposhnikov, Zenhäusern 2009, Phys. Lett. B 671, 187 [\[link\]](#); see also Brax, Davis 2014, JCAP05(2014)019 [\[link\]](#); Ferreira, Hill, Ross 2017, Phys. Rev. D 95, 064038 [\[link\]](#); Burrage, Copeland, PM, Spannowsky 2018, JHEP11(2018)036 [\[link\]](#)

# Coupling to hadrons (for the SM)



$$H^0 = \frac{h}{\sqrt{2}} + \frac{v}{\sqrt{2}} \frac{\zeta}{M}$$

heavy  
quarks

$$\mathcal{L}_{\text{eff}} \supset -\eta \frac{m_N}{M} \bar{\psi}_N \zeta \psi_N \quad \text{with} \quad \eta \equiv \frac{2N_H}{3B} + \sum_{q \in \{u,d,s\}} f_{Tq}^N \left( 1 - \frac{2N_H}{3B} \right) < 1$$

light quarks

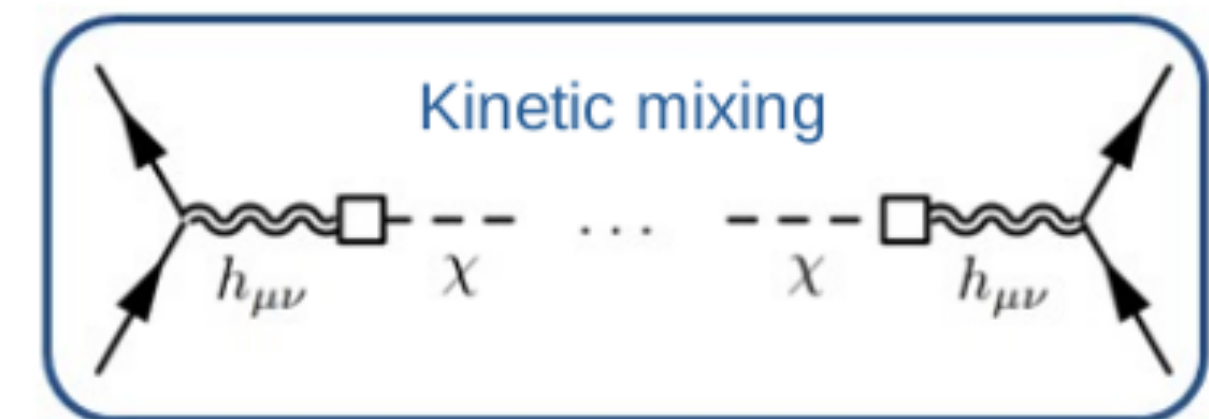
# Fifth forces in the Jordan frame

Fifth forces in the Jordan frame arise via mixing with the graviton ( $g \rightarrow \eta + g$ ):

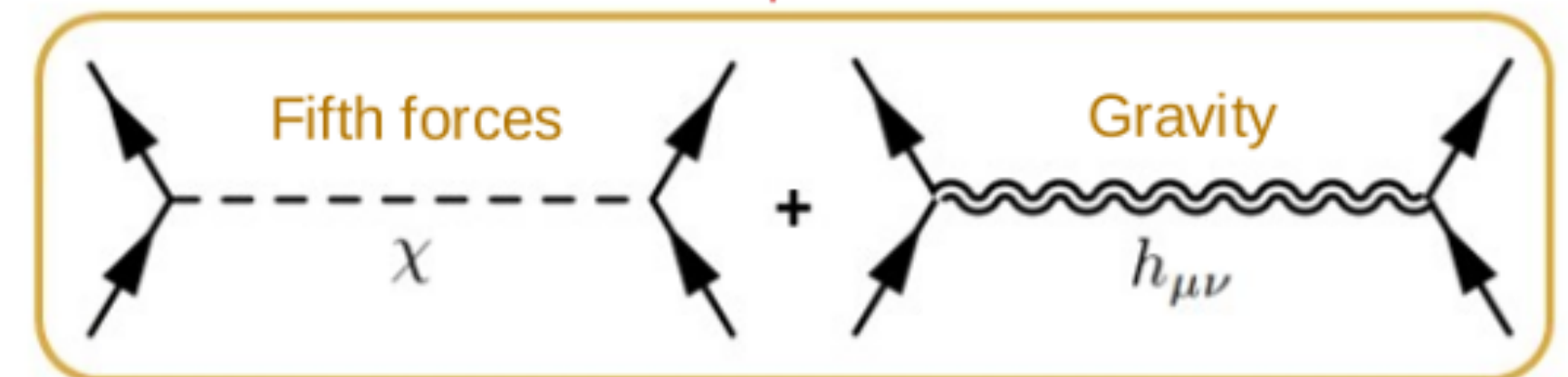
$$\mathcal{L} = \frac{F(\phi)}{4} \left[ \frac{1}{4} \partial_\mu g \partial^\mu g - \frac{1}{2} \partial_\rho g_{\mu\nu} \partial^\rho g^{\mu\nu} \right] + \frac{F'(\phi)}{4} \eta^{\mu\nu} \partial_\mu g \partial_\nu \phi$$

$$- \frac{1}{2} \left[ Z(\phi) + \frac{[F'(\phi)]^2}{2F(\phi)} \right] \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

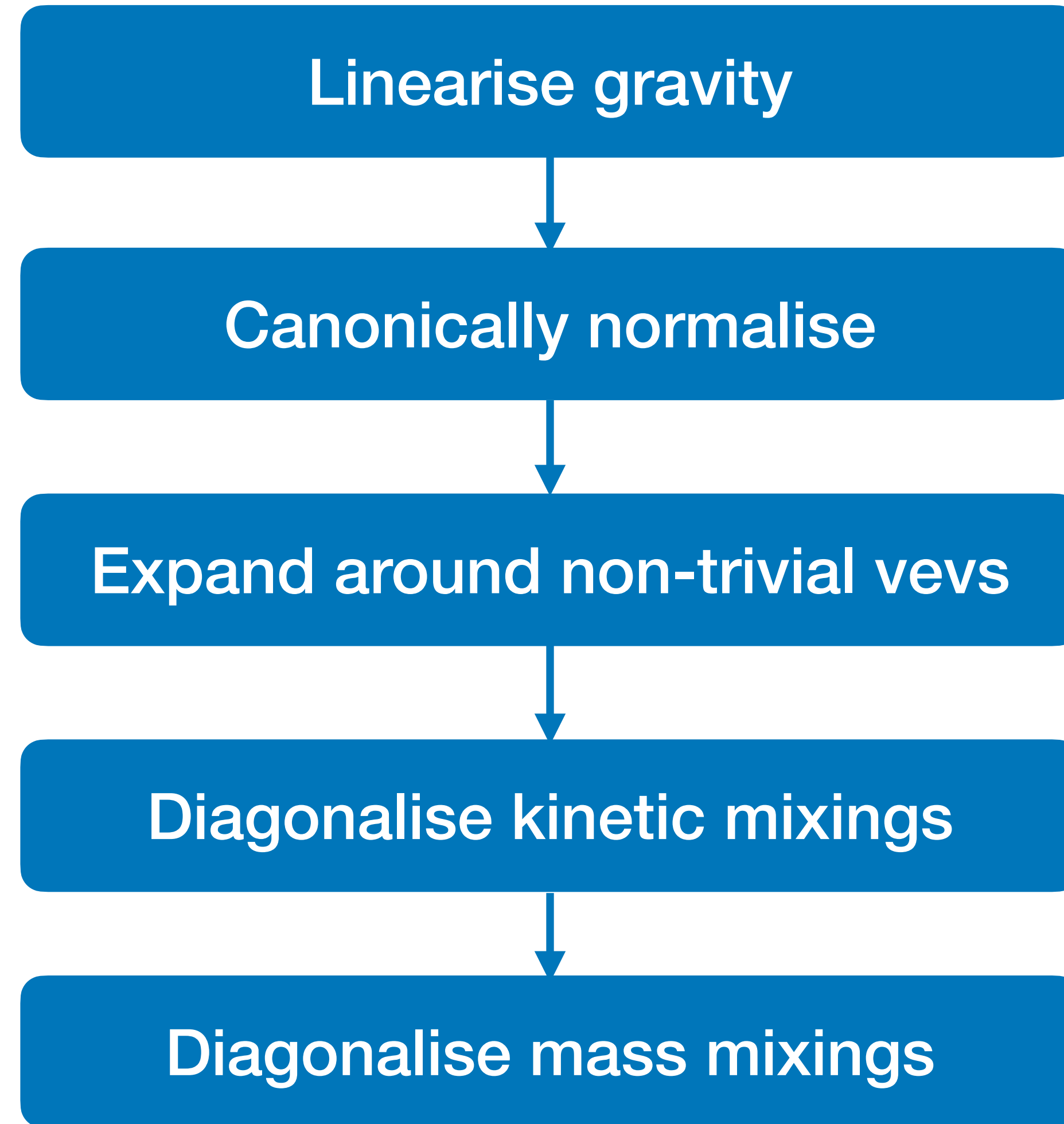
$$+ \frac{1}{2} g^{\mu\nu} T_{\mu\nu} + \mathcal{L}_{\text{SM}}(\eta, \{\psi\})$$



Diagonalization ↓



# Procedure



# Automation – FeynMG



FeynMG: a FeynRules extension for scalar-tensor theories of gravity

Sergio Sevillano Muñoz<sup>a,\*</sup>, Edmund J. Copeland<sup>a</sup>,  
Peter Millington<sup>b</sup>, Michael Spannowsky<sup>c</sup>

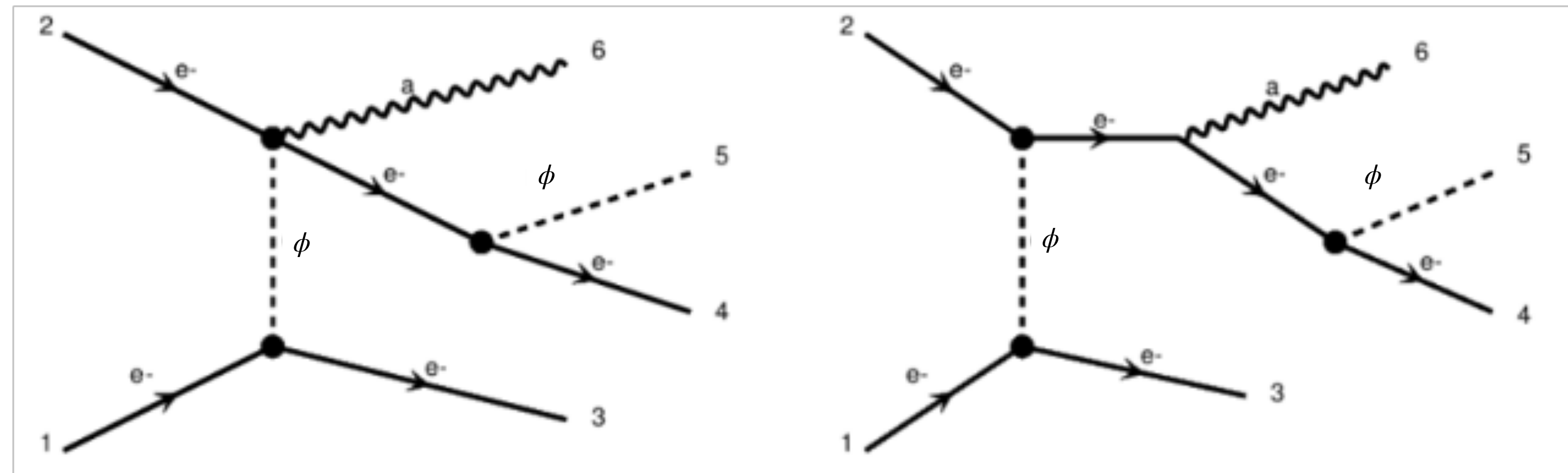
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<sup>b</sup>*Department of Physics and Astronomy, University of Manchester,  
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<sup>c</sup>*Institute for Particle Physics Phenomenology, Department of Physics, Durham University,  
Durham, DH1 3LE, UK*



# FeynMG



**MadGraph  
Output**

1. Load **FeynRules** and **FeynMG** into **Mathematica**.
2. Load a FeynRules Model File, say for the full SM.
3. Use FeynMG to append a (modified) gravitational sector and condition the Lagrangian into a form for FeynRules to process.
4. Generate Feynman rules or export to existing analysis pipelines (e.g., **MadGraph**).

# An example: off-shell coupling

Consider the coupling

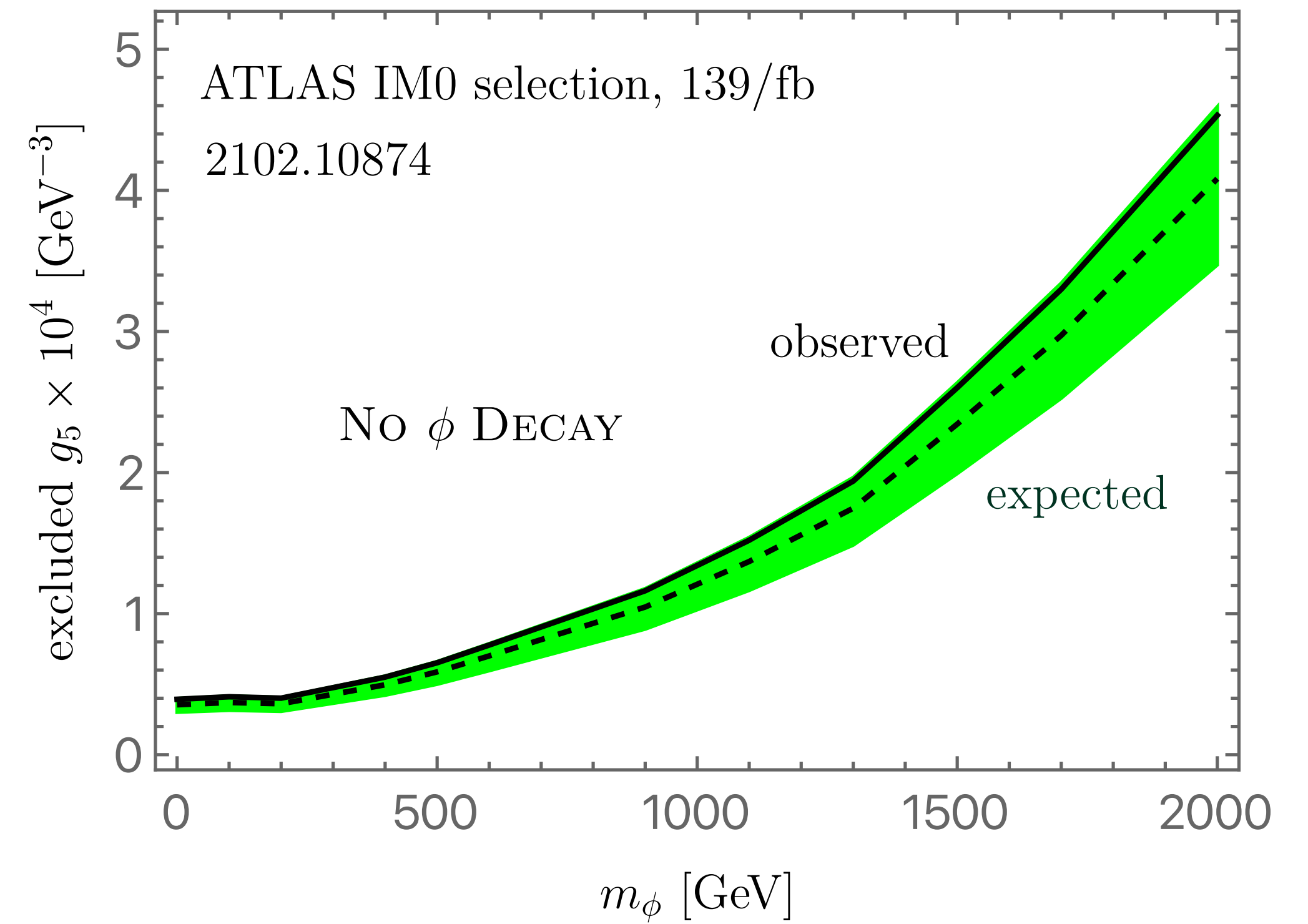
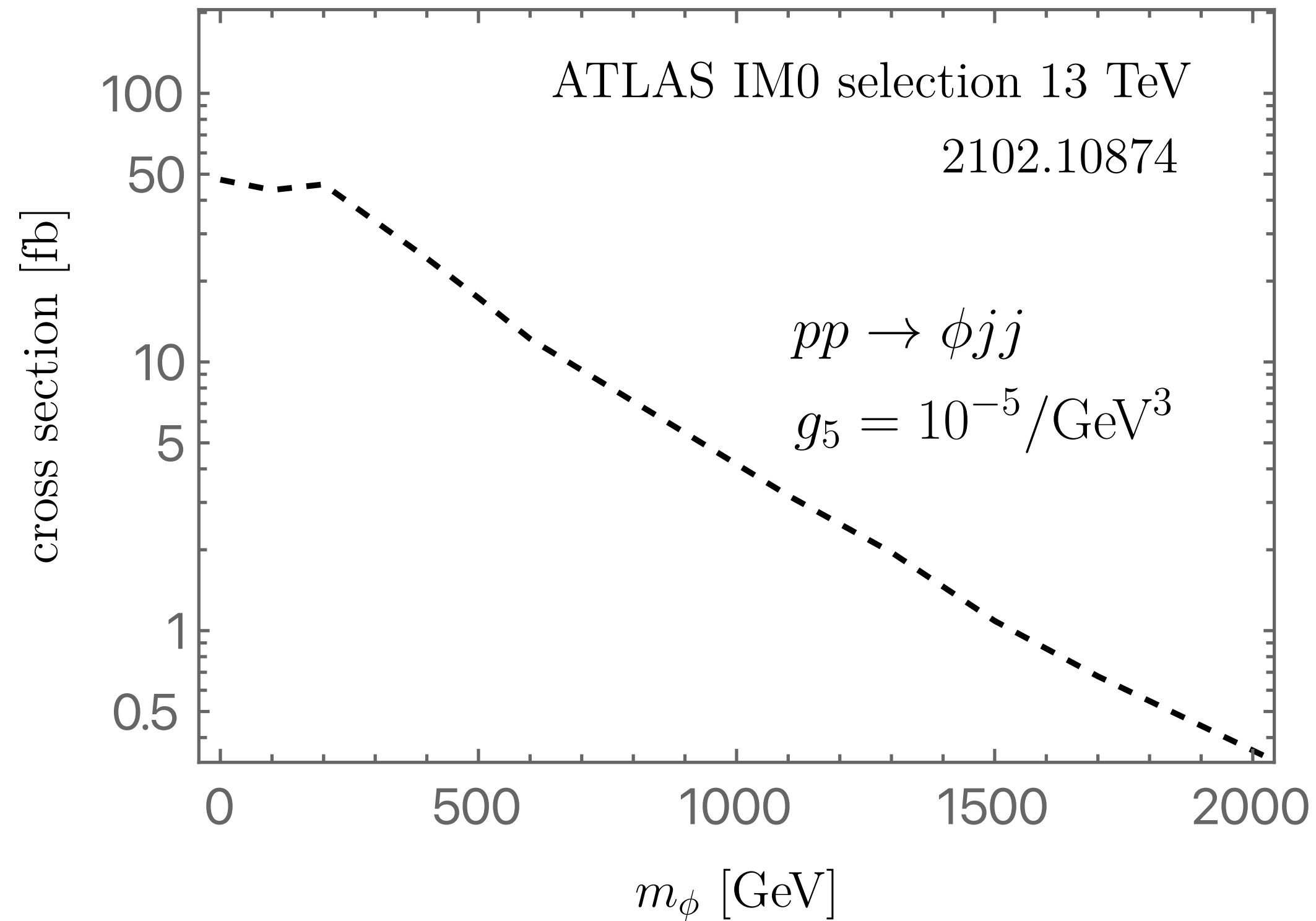
$$\mathcal{L} \supset \frac{C}{M^3} T_{\mu\nu} \partial^\mu \partial^\nu \phi$$

The energy-momentum tensor  $T_{\mu\nu}$  of an on-shell state has vanishing divergence, so  $\phi$  will couple only to off-shell states.

First deviations for 2 to 3 processes or at the loop level ...



# An example: off-shell coupling



Left: Cross-section for  $\phi$  plus two jets for the ATLAS IM0 selection before the  $\phi$  decay.

Right: Translation of this into exclusion contours assuming  $\phi$  is stable.

# Summary

**QFT** prevents us from disentangling **scalar extensions of GR** from **scalar extensions of the SM**.

**Screened fifth forces** have rich phenomenology due to their environmental dependence.

Parallels between scalar-tensor theories of gravity and: **axion-like models, ultra-light dark matter models, Higgs-portal theories, ...**

**Particle physics** of a broad class of models can be automated ... with **FeynMG** ... with novel collider signatures.

# Fifth force

**Classical equation of motion** for perturbations  $\delta\tilde{\varphi} = \langle \tilde{\phi} \rangle - \tilde{\varphi}$ :

$$\tilde{Z}(\tilde{\varphi})(\delta\ddot{\tilde{\varphi}} - c_s^2(\tilde{\varphi})\nabla^2\delta\tilde{\varphi}) + m^2(\tilde{\varphi})\delta\tilde{\varphi} = -\frac{1}{2}\frac{dA^2(\tilde{\varphi})}{d\tilde{\varphi}}\tilde{T}_{\text{SM}}$$

**Yukawa potential** around a point source  $\tilde{T} = -A^{-1}(\tilde{\varphi})\mathcal{M}\delta^3(\mathbf{x})$ :

$$\tilde{U}(r) \supset -\frac{1}{\tilde{Z}(\tilde{\varphi})c_s^2(\tilde{\varphi})}\left[\frac{dA(\tilde{\varphi})}{d\tilde{\varphi}}\right]^2\frac{1}{4\pi r}\exp\left[-\frac{m(\tilde{\varphi})r}{\tilde{Z}^{1/2}(\tilde{\varphi})c_s(\tilde{\varphi})}\right]\mathcal{M}$$

# Higgs-Dilaton

$$M_{\text{Pl}}^2 R \rightarrow (\xi_H H^2 + \xi_\phi \phi^2) R \quad \text{and} \quad -\mu^2 H^2 + \frac{\lambda}{4} H^4 \rightarrow \frac{\lambda}{4} \left( H^2 - \frac{\beta}{\lambda} \phi^2 \right)^2$$

There is a **conserved dilatation current**, and a **massless Goldstone mode**

$$\sigma \propto \ln \left[ (6\xi_\phi + 1)\phi^2 + (6\xi_H + 1)H^2 \right]$$

with at most derivative couplings to the Higgs boson  $\Rightarrow$  **no fifth forces.**