Diagrams and constraints for matter-antimatter asymmetry generation in the early universe

Peter Maták
In collaboration with **T. Blažek**

[Phys. Rev. D 103 (2021) L091302; Eur. Phys. J. C 81 (2021) 1050]

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Introducing two theoretical tools

- Holomorphic cutting rules for CP asymmetry calculations [Phys. Rev. D 103 (2021) L091302]
- Quantum statistics as a consequence of unitarity [Eur. Phys. J. C 81 (2021) 1050]

CP asymmetries and unitarity constraints

$$S = 1 + \mathrm{i}\,T \qquad\qquad T_{fi} = (2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi} \tag{1}$$

$$CPT \text{ symmetry implies } |T_{\overline{fi}}|^2 = |T_{if}|^2 \quad \rightarrow \quad \Delta |T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2 \tag{2}$$

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$$\left.\begin{aligned} T_{fi} &= C_{fi}^{\text{tree}} K_{fi}^{\text{tree}} + C_{fi}^{\text{loop}} K_{fi}^{\text{loop}} \\ T_{if} &= C_{if}^{\text{tree}} K_{if}^{\text{tree}} + C_{if}^{\text{loop}} K_{if}^{\text{loop}} \end{aligned}\right\} \quad \Delta |T_{fi}|^2 = -4\,\mathrm{Im}\left[C_{fi}^{\text{tree}} C_{fi}^{\text{loop}*}\right] \mathrm{Im}\left[K_{fi}^{\text{tree}} K_{fi}^{\text{loop}*}\right]$$

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$$S^\dagger S = S S^\dagger \quad \rightarrow \quad \sum_f \Delta |T_{fi}|^2 = 0 \tag{3}$$

[Dolgov '79; Kolb, Wolfram '80; see also Hook '11; Baldes *et al.* '14]

Holomorphic cutting rules

$$S^{\dagger}S = 1 \quad \to \quad \mathrm{i}\,T^{\dagger} = \mathrm{i}\,T - \mathrm{i}\,T\mathrm{i}\,T^{\dagger} \tag{4}$$

$$|T_{fi}|^2 = -\mathrm{i}\,T^{\dagger}_{if}\mathrm{i}\,T_{fi} = -\mathrm{i}\,T_{if}\mathrm{i}\,T_{fi} + \sum_{n} \mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \sum_{n,k} \mathrm{i}\,T_{in}\mathrm{i}\,T_{nk}\mathrm{i}\,T_{kf}\mathrm{i}\,T_{fi} + \ldots \tag{5}$$

[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21]

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[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21]

$$\begin{aligned} \Delta|T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2 = &\sum_n \left(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fn}\mathrm{i}\,T_{ni}\right) \\ &- \sum_{n,k} \left(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nk}\mathrm{i}\,T_{kf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fk}\mathrm{i}\,T_{kn}\mathrm{i}\,T_{ni}\right) \\ &+ \ldots \end{aligned} \tag{6}$$

[Blažek, Maták '21a; see also Covi, Roulet, Vissani '98]

CP asymmetry in the Boltzmann equation

Change in # of particles $\leftrightarrow$ average # of their interactions

$$\dot{n}_{f_1} + 3Hn_{f_1} = \sum_{\text{all reactions}} \gamma_{fi} - \gamma_{if} \qquad \gamma_{fi} = \frac{1}{V_4} \int \prod_{k=1}^{p} [\mathrm{d}\boldsymbol{p}_k]\, f_{i_k}(\boldsymbol{p}_k) \int \prod_{l=1}^{q} [\mathrm{d}\boldsymbol{p}_l] |\, T_{fi}|^2 \tag{7}$$

$$[\mathrm{d}\boldsymbol{p}_k] = \frac{\mathrm{d}^3 \boldsymbol{p}_k}{(2\pi)^3 2E_{\boldsymbol{p}_k}} \qquad |\, T_{fi}|^2 = V_4 (2\pi)^3 \delta^{(3)}(\boldsymbol{p}_f - \boldsymbol{p}_i) |M_{fi}|^2 \tag{8}$$

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$$f_{i_k}^{\mathrm{eq}}(\boldsymbol{p}_k) = \exp\left\{-\frac{E_{\boldsymbol{p}_k}}{T}\right\} \qquad \gamma_{fi} = \frac{n_{i_1}}{n_{i_1}^{\mathrm{eq}}} \frac{n_{i_2}}{n_{i_2}^{\mathrm{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\mathrm{eq}}} \times \gamma_{fi}^{\mathrm{eq}} \quad (9)$$

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Holomorphic cuts and classical kinetic theory

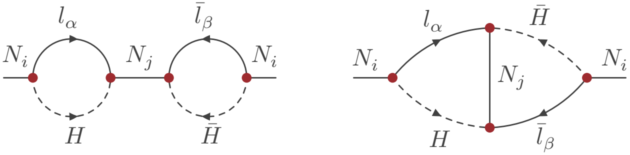
$$\gamma^{\mathrm{eq}}_{fi} = \frac{1}{V_4}\int \prod_{k=1}^{p} [\mathrm{d}\boldsymbol{p}_k]\, f^{\mathrm{eq}}_{i_k}(\boldsymbol{p}_k) \int \prod_{l=1}^{q} [\mathrm{d}\boldsymbol{p}_l] \Big(-\mathrm{i}\,T_{if}\,\mathrm{i}\,T_{fi} + \sum_n \mathrm{i}\,T_{in}\,\mathrm{i}\,T_{nf}\,\mathrm{i}\,T_{fi} + \ldots \Big) \tag{10}$$

- cyclic permutations relate contributions to different process rates
- similarly for $\Delta\gamma^{\mathrm{eq}}_{fi}$ entering asymmetry source term

Example: asymmetries in seesaw type-I leptogenesis

$$L_{\text{int}} = -Y_{\alpha i}\bar{N}_i P_L l_\alpha H + \text{H.c.} \tag{11}$$

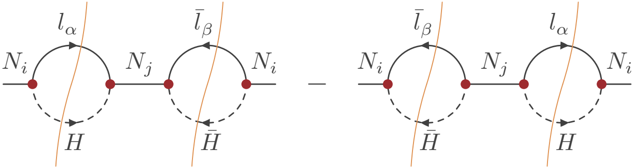
[Fukugita, Yanagida '86]



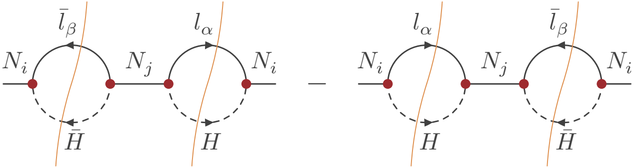
Example: asymmetries in seesaw type-I leptogenesis

$$\Delta |T_{fi}|^2 = \sum_n \left(\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni} \right) - \dots \tag{12}$$

$$\Delta |T_{N\to lH}|^2 \leftarrow \tag{13}$$



$$\Delta |T_{N\to \bar{l}\bar{H}}|^2 \leftarrow \tag{14}$$

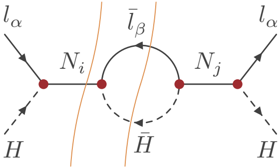


$$\Delta |T_{N\to lH}|^2 + \Delta |T_{N\to \bar{l}\bar{H}}|^2 = 0 \tag{15}$$

Example: asymmetries in seesaw type-I leptogenesis

$$\Delta|T_{fi}|^2 = \sum_n \left(\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni}\right) - \ldots \tag{12}$$

$\Delta|T_{lH\to N}|^2 \leftarrow$



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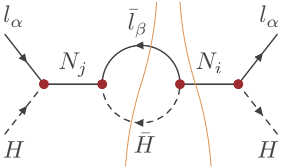
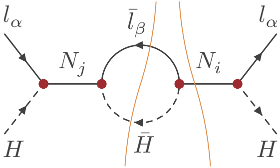


(16)

$\Delta|T_{lH\to\bar{l}\bar{H}}|^2 \leftarrow$



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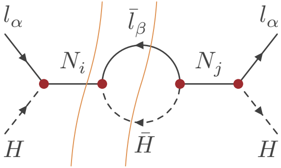
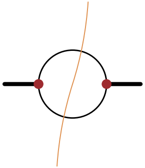


(17)

$$\Delta|T_{lH\to N}|^2 + \Delta|T_{lH\to\bar{l}\bar{H}}|^2 = 0 \tag{18}$$

From unitarity to quantum statistics

$$\gamma_{fi}^{\mathrm{eq}} = \int \prod_{k=1}^{p} [\mathrm{d}\boldsymbol{p}_k]\, f_{i_k}^{\mathrm{eq}}(\boldsymbol{p}_k) \int \prod_{l=1}^{q} [\mathrm{d}\boldsymbol{p}_l]\, V_4 \Big(-\mathrm{i}\,T_{if}\mathrm{i}\,T_{fi} + \sum_n \mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \ldots \Big) \tag{19}$$



$$\int [\mathrm{d}\boldsymbol{p}_1]\mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2](2\pi)^4\delta^{(4)}(p_1 - k_1 - k_2)$$

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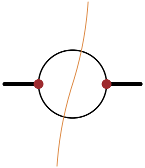

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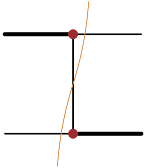

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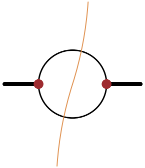

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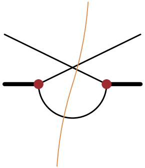
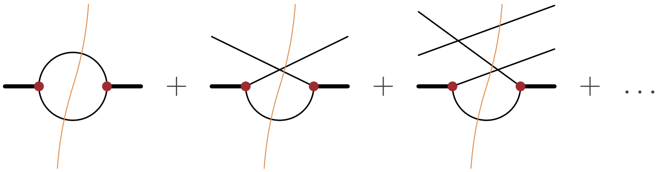


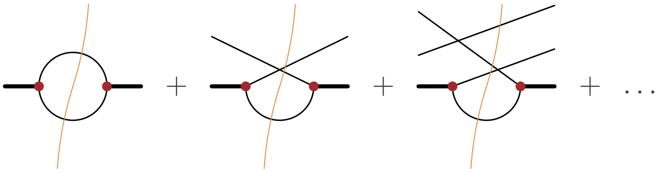
$$\int [\mathrm{d}\boldsymbol{p}_1] \mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2] \mathrm{e}^{-E_{k_1}/T} (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2)$$

From unitarity to quantum statistics



$$\int [\mathrm{d}\boldsymbol{p}_1] \mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2] \left[1 + \frac{1}{\mathrm{e}^{E_{k_1}/T} - 1}\right] (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2) \tag{20}$$

From unitarity to quantum statistics



$$\int [\mathrm{d}\boldsymbol{p}_1] \mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2] \left[1 + \frac{1}{\mathrm{e}^{E_{k_1}/T} - 1}\right] (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2) \tag{20}$$

Application to leptogenesis

$$\dot{n}_{\Delta L} + 3H n_{\Delta L} = \int \ldots \delta f_N \left(1 - f_l^{\mathrm{eq}}\right)\left(1 + f_H^{\mathrm{eq}}\right)\left(1 - f_{\bar{l}}^{\mathrm{eq}} + f_{\bar{H}}^{\mathrm{eq}}\right) + \ldots$$

[Blažek, Maták '21b; agrees with Garny, Hohenegger, Kartavtsev, Lindner '09; '10; Garny, Hohenegger, Kartavtsev '10; Beneke, Garbrecht, Herranen, Schwaller '10]

What else can be done?

- Thermal masses from anomalous thresholds. [Blažek, Maták '22]

- CPT and unitarity constraints generalized to thermal-corrected asymmetries. In the seesaw type-I leptogenesis at $\mathcal{O}(Y^4 Y_t^2)$ they look like

$$\Delta\gamma^{\text{eq}}_{N_i Q\to lt} + \Delta\gamma^{\text{eq}}_{N_i Q\to lHQ} + \Delta\gamma^{\text{eq}}_{N_i Q\to \bar{l}\bar{H}Q} + \Delta\gamma^{\text{eq}}_{N_i Q\to \bar{l}QQ\bar{t}} = 0, \tag{21}$$

$$m^2_{H,Y_t}(T)\frac{\partial}{\partial m_H^2}\bigg|_0 \left(\Delta\gamma^{\text{eq}}_{N_i\to lH} + \Delta\gamma^{\text{eq}}_{N_i\to \bar{l}\bar{H}}\right) = 0. \tag{22}$$

[Blažek, Maták, Zaujec '22; see also poster #236]

- No double-counting from intermediate states. Resonant dark matter annihilation at fixed order. [Maták '24; compare to Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

Summary

- Instead of computing imaginary kinematics from loops, direct use of unitarity implies

$$
\begin{aligned}
\Delta |T_{fi}|^2 = &\sum_n \left(\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni} \right) \\
&- \sum_{n,k} \left(\mathrm{i}T_{in}\mathrm{i}T_{nk}\mathrm{i}T_{kf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fk}\mathrm{i}T_{kn}\mathrm{i}T_{ni} \right) \\
&+ \dots
\end{aligned}
$$

- Completing the Boltzmann equations by all possible unitary diagram cuts includes thermal corrections.

Summary

- Instead of computing imaginary kinematics from loops, direct use of unitarity implies

$$\Delta|T_{fi}|^2 = \sum_n \left(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fn}\mathrm{i}\,T_{ni} \right) - \sum_{n,k} \left(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nk}\mathrm{i}\,T_{kf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fk}\mathrm{i}\,T_{kn}\mathrm{i}\,T_{ni} \right) + \ldots$$

- Completing the Boltzmann equations by all possible unitary diagram cuts includes thermal corrections.

Thank you for your attention!

Backup

Density matrix evolution

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp\left\{ -\sum_p F_p a_p^\dagger a_p \right\} \quad \leftarrow \quad Z = \prod_p Z_p = \prod_p \frac{\exp F_p}{\exp F_p - 1} \tag{23}$$

$$\exp\{-E_p/T\} \quad \to \quad \exp\{-F_p\} = \frac{f_p}{1 \pm f_p} \qquad\qquad f_p = \mathrm{Tr}\left[a_p^\dagger a_p \rho\right] \tag{24}$$

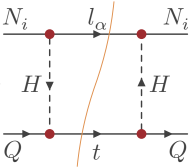
[Wagner ’91]

$$\rho' = S\rho S^\dagger \quad \to \quad (1 + \mathrm{i}\,T)\rho(1 - \mathrm{i}\,T + \mathrm{i}\,T\mathrm{i}\,T - \ldots) \tag{25}$$

[Blažek, Maták ’21b]

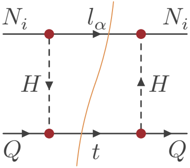
The collision term for the Boltzmann equation is obtained as $\mathrm{Tr}\left[a_p^\dagger a_p(\rho - \rho')\right]/V_4$.

Anomalous thresholds and IR finiteness

$\gamma^{\text{eq}}_{NQ\to lt} \quad \leftarrow \quad -$  $\quad \propto \quad \left[\frac{1}{(p_Q - p_t)^2}\right]^2 \qquad (26)$

Anomalous thresholds and IR finiteness

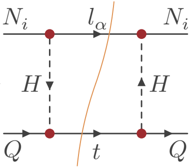
$$\gamma^{\text{eq}}_{NQ\to lt} \quad \leftarrow \quad -$$



$$\propto \quad \int_{-1}^{1} \mathrm{d}\cos\theta \left[\frac{1}{1-\cos\theta}\right]^2 \qquad (26)$$

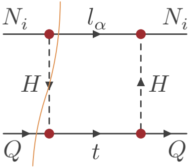
Anomalous thresholds and IR finiteness

$\gamma^{\text{eq}}_{NQ\to lt} \quad \leftarrow \quad -$



$$\propto \int_{-1}^{1} \mathrm{d}\cos\theta \left[\frac{1}{1-\cos\theta}\right]^2 \tag{26}$$

$\gamma^{\text{eq}}_{NQ\to lHQ} \quad \leftarrow \quad -$



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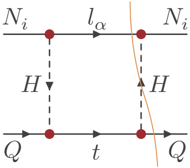

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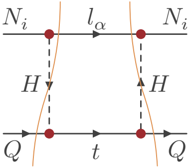
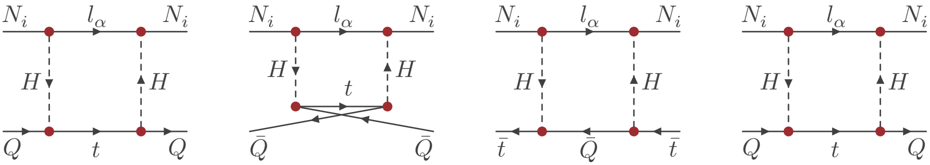


(27)

$\gamma^{\text{eq}}_{NQ\to lt} + \gamma^{\text{eq}}_{NQ\to lHQ}$ is IR finite

[Racker '19; Frye, *et al.* '19]

Anomalous thresholds and IR finiteness



$$\gamma^{\mathrm{eq}}_{NQ\to lHQ} + \gamma^{\mathrm{eq}}_{N\bar{Q}\to lH\bar{Q}} + \gamma^{\mathrm{eq}}_{Nt\to lHt} + \gamma^{\mathrm{eq}}_{N\bar{t}\to lH\bar{t}} = m_H^2(T)\,\frac{\partial}{\partial m_H^2}\bigg|_{m_H^2=0} \gamma^{\mathrm{eq}}_{N\to lH} \tag{28}$$

$$m_H^2(T) = 12\,Y_t^2 \int [d\boldsymbol{p}]\,\exp\left\{-E_{\boldsymbol{p}}/T\right\} \tag{29}$$

[Blažek, Maták ’22; see also Fujimoto, *et al.* ’84; Salvio, Lodone, Strumia ’11]