

Light flavour vector mesons between 2 and 3 GeV at **BESIII**

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On behalf of BESIII collaboration



Why?

- Tool to investigate properties in non-perturbative QCD
 - mass, width, decay pattern, production rate ...

- Contribution to hadron spectroscopy

- Many vector states between 2.0 and 3.0 GeV

- ρ^* , ω^* , ϕ^* , exotic states, mixture?

- ρ^* : $\rho(2000)$, $\rho(2150)$, $\rho(2270)$

- ω^* : $\omega(2220)$, $\omega(2290)$, $\omega(2330)$

- ϕ^* : $\phi(2170)$

Need to be confirmed

- Analogy to heavy flavour vector meson, exotic hadron

- Charmonium, bottomonium, *strangeonium*?

- Exotics states with strange quark

- ✓ intermediate mass, bridge between light and heavy quark

- *Couplings to different channels* may help to reveal their nature

- *Common decay patterns*: is there $\rho^* - \omega^* - \phi^*$ mixing?

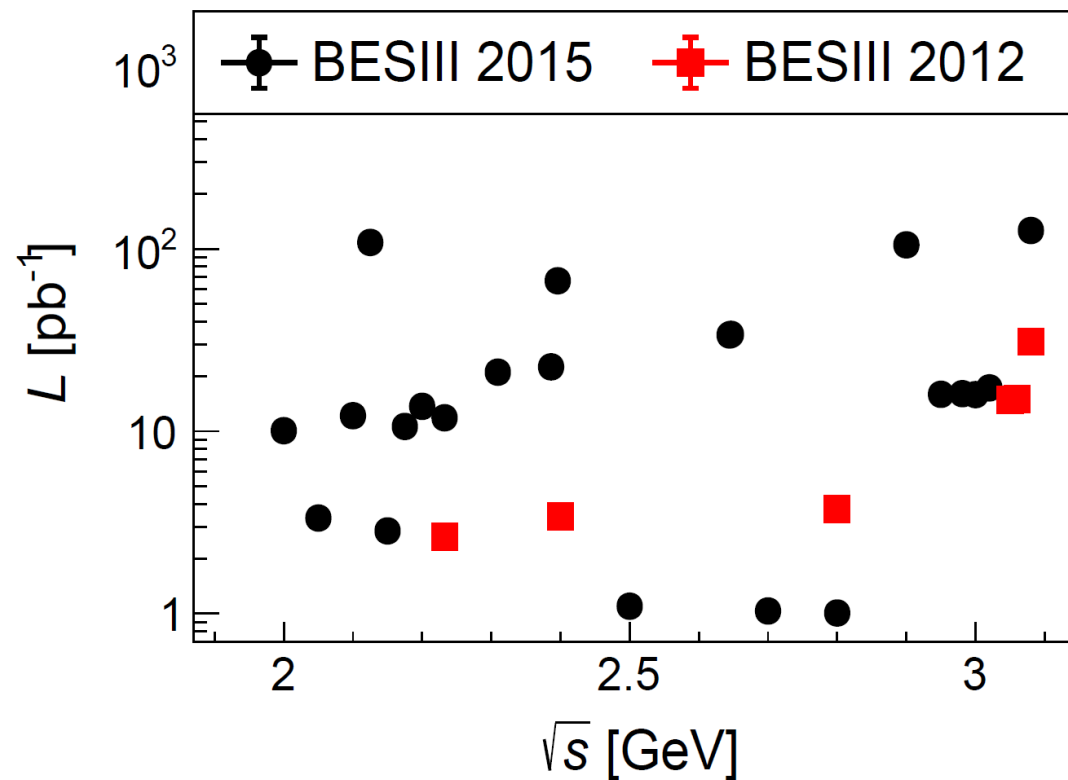
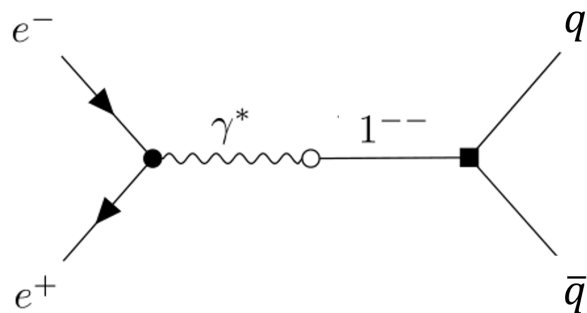
- *Identification of exotic particle* with non exotic quantum numbers



Data samples (for these studies)

1) Data below J/ψ

- \sqrt{s} : 2.0 – 3.08 GeV
- \mathcal{L} : $\sim 650 \text{ pb}^{-1}$
- Excited Light flavour vector meson directly produced in electron-positron annihilation (ρ^* , ω^* , ϕ^*)



• **Line shape scan** to study properties of light flavour vector mesons

2) PWA using charmonia world largest data samples

10 Billions J/ψ

2.7 Billions $\psi(3686)$

Recent (2024) results from

- Study of $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ [JHEP01\(2024\)180](https://arxiv.org/abs/2401.14711)
- Study of $e^+e^- \rightarrow \pi^+ \pi^- \pi^0$ <https://arxiv.org/abs/2401.14711>
Accepted by PRD
- Study of $e^+e^- \rightarrow \omega \eta'$ <https://arxiv.org/abs/2401.14711>
Accepted by JHEP
- Study of $e^+e^- \rightarrow \eta \pi^+ \pi^-$ [PHYSICAL REVIEW D 108, L111101](https://arxiv.org/abs/2401.14711)
- Study of $J/\psi \rightarrow \phi \pi^0 \eta$ *Ask me in QT!!* <https://arxiv.org/abs/2311.07043>
On-going analyses: $J/\psi \rightarrow K^+ K^- \eta$ $\psi(3686) \rightarrow \phi \eta \eta'$



$\phi(2170)$ aka $Y(2175)$

First observed by BaBar in 2006 in ISR process
 $e^+ e^- \rightarrow \phi f_0(980) \gamma$
 Then investigated by Belle, BES, BESIII

Studied in many exclusive channels:

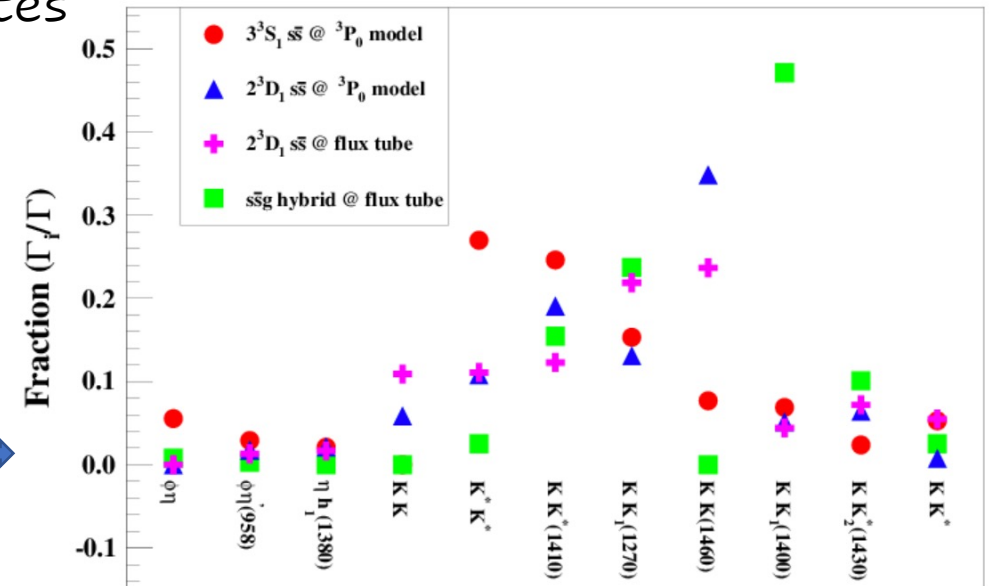
$\phi\eta, \phi\eta', \phi f_0(980), K^+ K^-, K_S^0 K_L^0, K^*(892)^+ K^-, K_2^*(1430)^+ K^-, K^*(892)^+ K^*(892)^- + \text{other excited } K \bar{K} \text{ states}$

None is dominant

Confirmed by BESIII in the
 $K(1400)^+ K^-, K_1(1400)^+ K^-, K_1(1270)^+ K^-,$ and $\phi\eta'$ final states

Its nature still controversial:
 a traditional $S\bar{S}$ state ($3^3S_1, 2^3D_1$), a $S\bar{S}g$ hybrid, a tetraquark state, a $\Lambda\bar{\Lambda}$ bound state, or a $\phi K\bar{K}$ resonance, strangeonium-like state.
 The predicted decay width varies depending on the assumed nature

$\Gamma(\phi\eta) < \Gamma(\phi\eta')$ Disfavour hybrid hypothesis
 Favours strangeonium-like state
 With $\eta - \eta'$ mixing



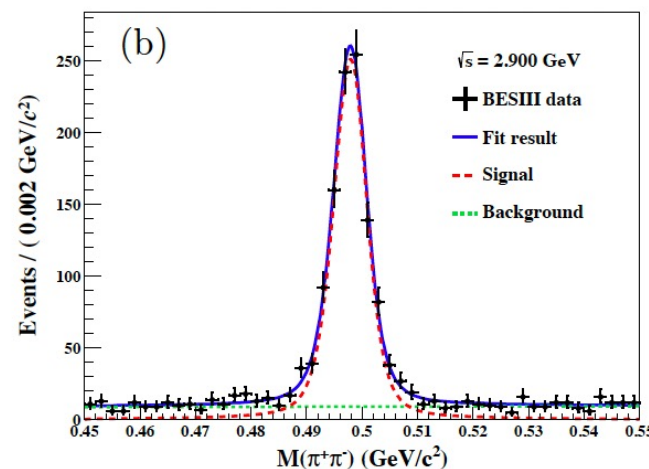
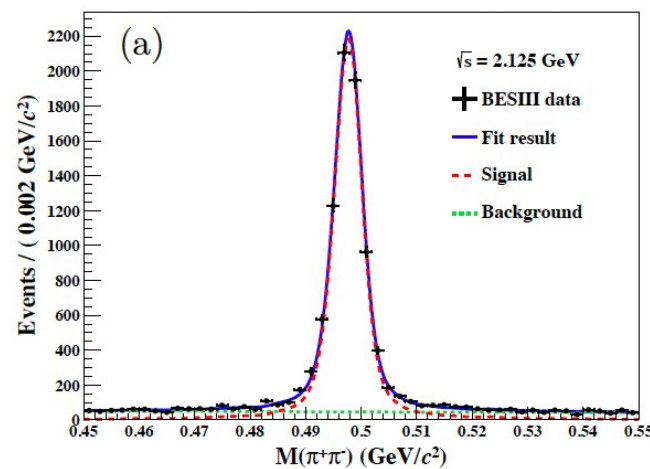
$e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ study

[JHEP01\(2024\)180](#)

$$\sqrt{s} = [2.00-3.08] \text{ GeV}$$

Previously investigated by ISR process by BaBar
And by SND (in lower energy region)

$$e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$$
$$K_S^0 \rightarrow \pi^+ \pi^-$$
$$\pi^0 \rightarrow \gamma\gamma$$



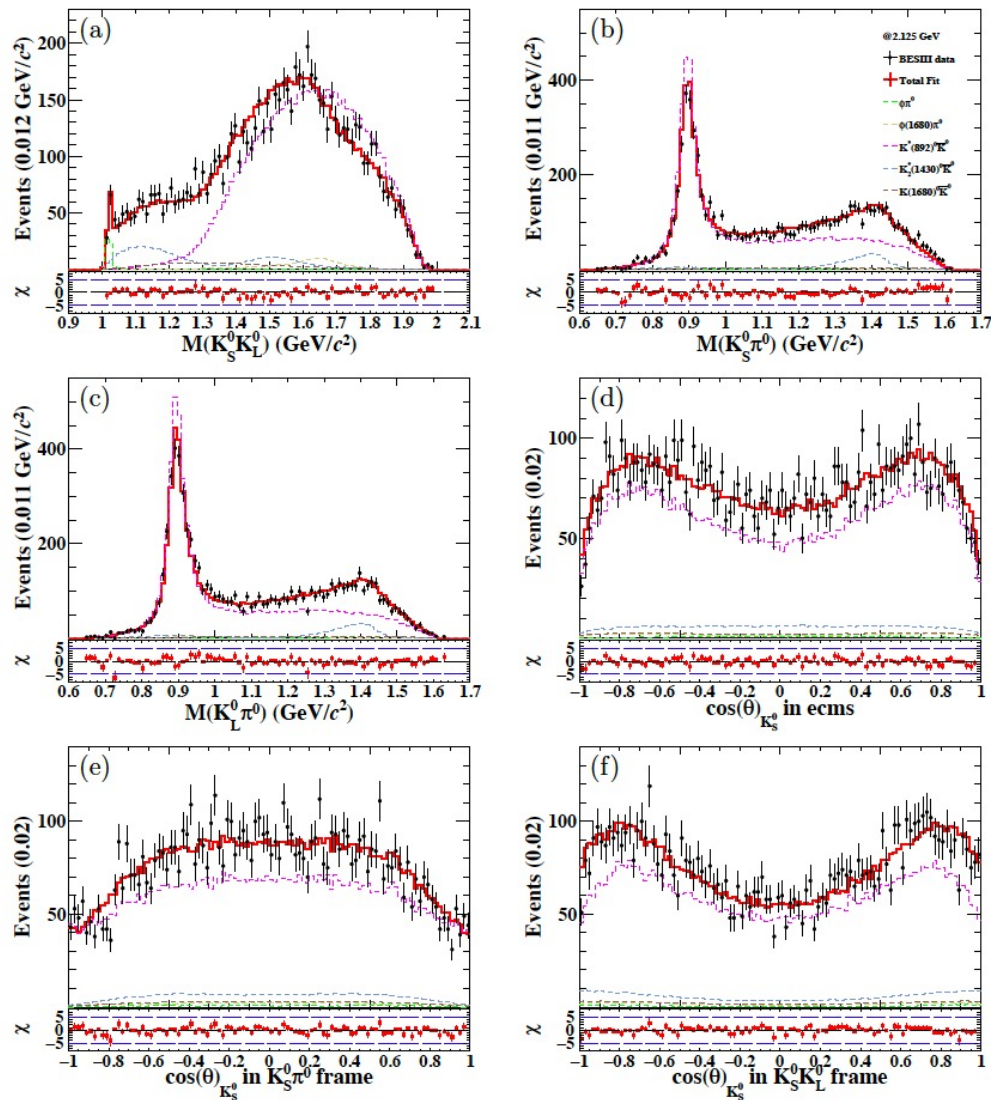
$e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ study

arXiv:2309.13883v2

$\sqrt{s} = 2.125$ GeV

$\sqrt{s} = [2.00-3.08]$ GeV

Previously investigated by ISR process by BaBar
And by SND (in lower energy region)



PWA (in GPUPWA framework),
Masses and widths of intermediate resonances fixed
to PDG (quasi two-body decay amp and covariant tensor
amp formalism)

BASELINE: $K^*(892)^0 \bar{K}^0, K_2^*(1430)^0 \bar{K}^0$

Intermediate states (decaying. $K_S^0 K_L^0$ Or $K_S^0 (K_L^0) \pi^0$) are
included in the solution if the statistical significance is
greater than 5σ ,

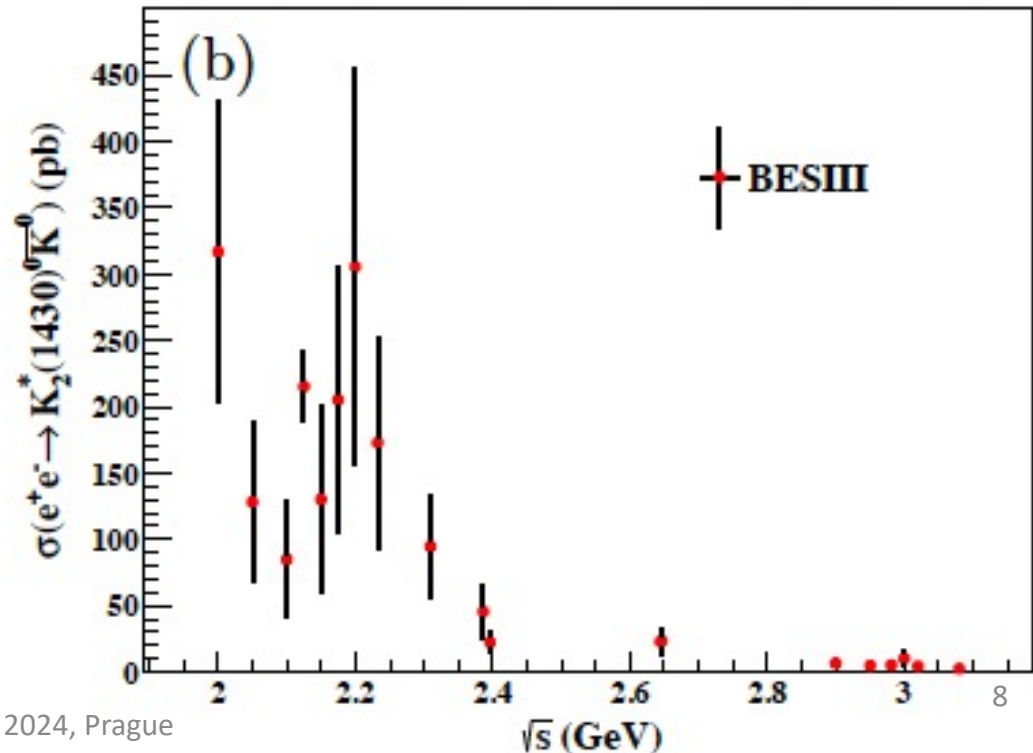
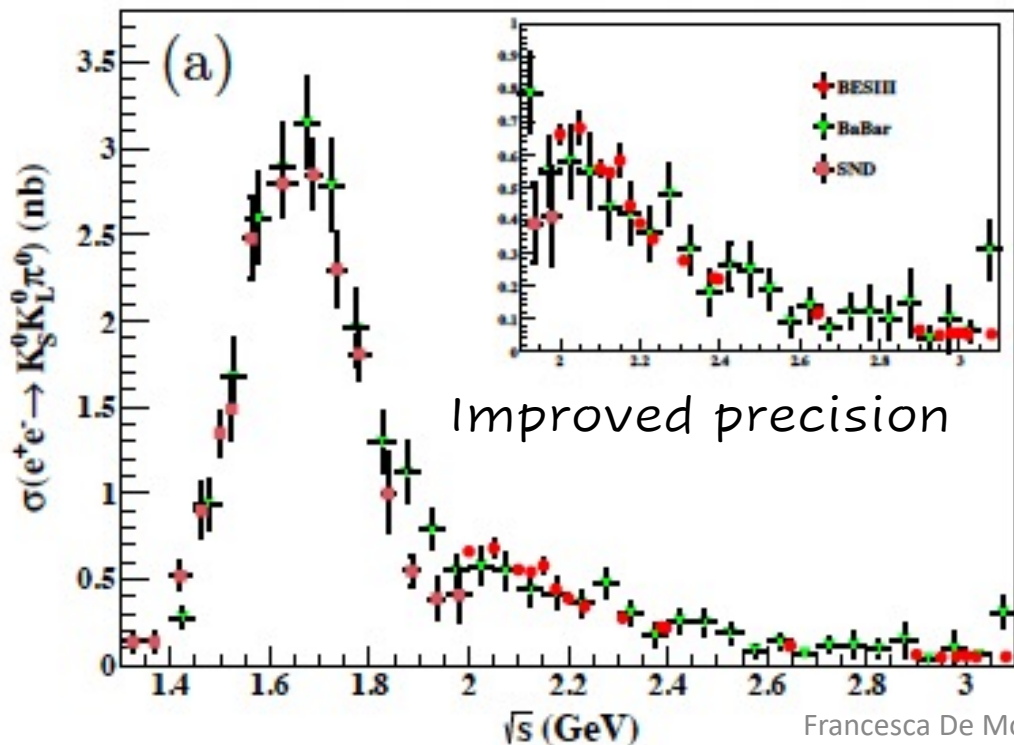
Process	Significance		
	2.125 GeV	2.396 GeV	2.900 GeV
$\phi\pi^0$	13.1σ	8.6σ	9.7σ
$\phi(1680)\pi^0$	11.1σ	12.2σ	8.3σ
$K^*(892)^0 \bar{K}^0$	$>30\sigma$	$>30\sigma$	$>30\sigma$
$K_2^*(1430)^0 \bar{K}^0$	29.2σ	5.7σ	5.1σ
$K(1680)^0 \bar{K}^0$	9.8σ	8.4σ	7.6σ

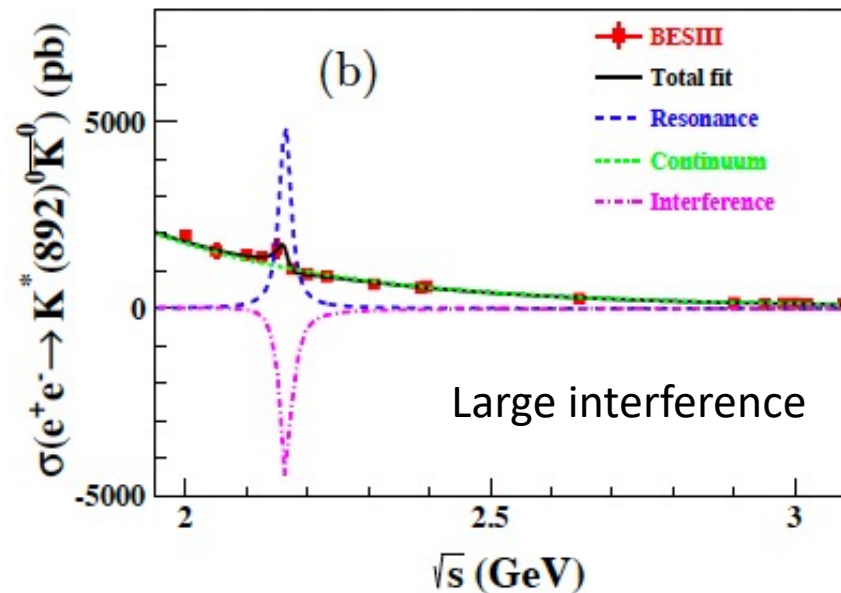
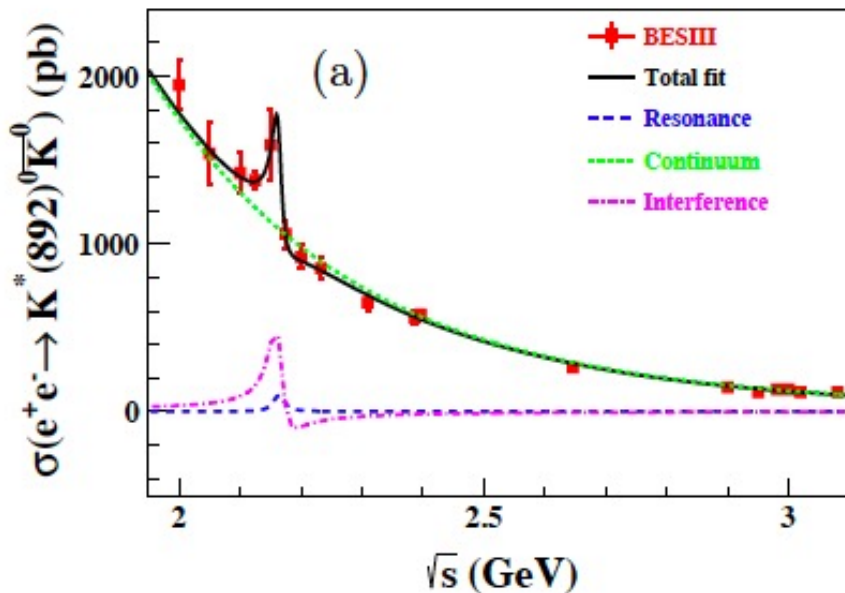
Process	Fraction (%)		
	2.125 GeV	2.396 GeV	2.900 GeV
$\phi\pi^0$	0.78 ± 0.56	0.87 ± 0.61	1.82 ± 1.11
$\phi(1680)\pi^0$	2.39 ± 1.23	5.96 ± 2.10	5.22 ± 1.50
$K^*(892)^0\bar{K}^0$	79.89 ± 1.12	86.01 ± 1.38	72.65 ± 2.11
$K_2^*(1430)^0\bar{K}^0$	7.42 ± 0.83	1.93 ± 0.57	1.85 ± 0.82
$K(1680)^0\bar{K}^0$	3.00 ± 1.11	6.73 ± 1.91	5.82 ± 1.96

$K^*(892)^0\bar{K}^0$ dominates over $K_2^*(1430)^0\bar{K}^0$
 Opposite with respect to the charged channel $e^+e^- \rightarrow K^+K^-\pi^0$ JHEP 07 (2022) 04

Lineshape of subprocesses can be measured using relative fraction obtained in PWA

As found from BaBar previously for analogous channels





Fit with coherent sum of continuum and resonant (BW) component as PDF

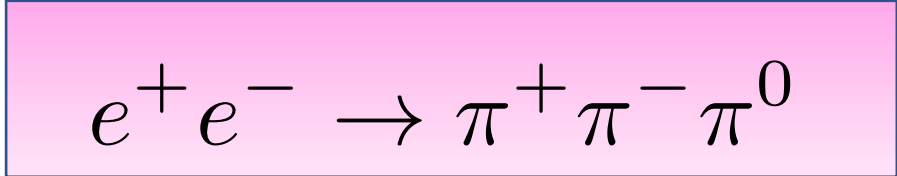
$$\sigma = \left| \frac{\sqrt{12\pi\Gamma_Y\Gamma_Y^{e^+e^-}\mathcal{B}}}{M_Y^2 - s - iM_Y\Gamma(\sqrt{s})} \sqrt{\frac{P(\sqrt{s})}{P(M_Y)}} e^{i\phi_Y} + c_1 \frac{\sqrt{P(\sqrt{s})}}{s^{c_2}} \right|^2,$$

$$M_Y = (2164.7 \pm 9.1 \pm 3.1)\text{MeV}/c^2$$

$$\Gamma_Y = (32.4 \pm 21.0 \pm 1.8)\text{MeV}$$

Hint of a vector meson state around 2.2 GeV with a significance of 3.2σ .

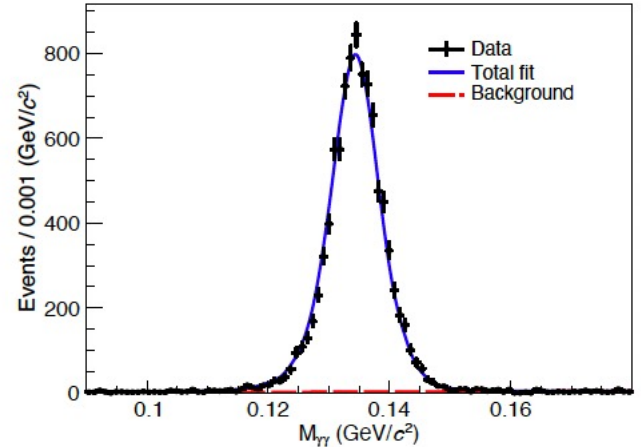
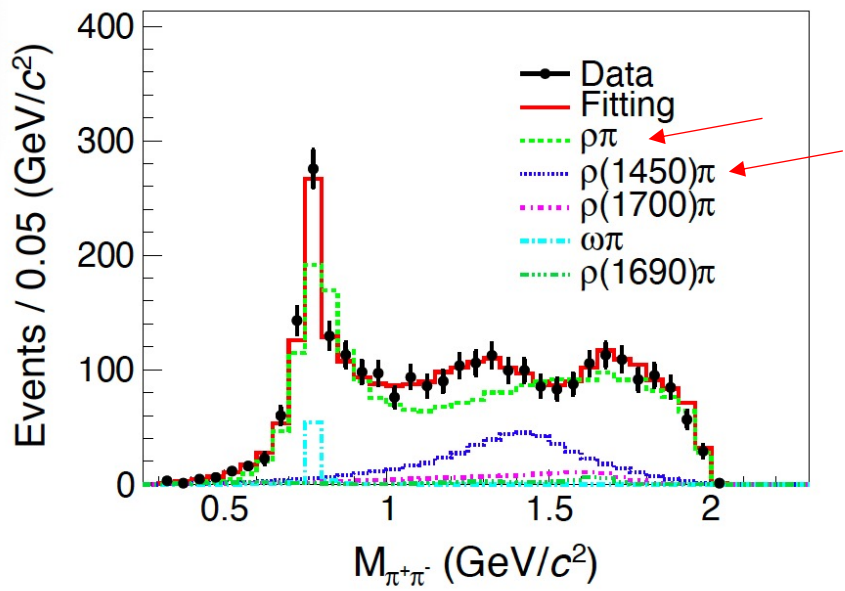
The resonance parameters (very narrow width especially) very close to the BESIII results measured through the $\phi\eta$ channel [1] of the $\phi(2170)$ meson.



<https://arxiv.org/abs/2401.14711>

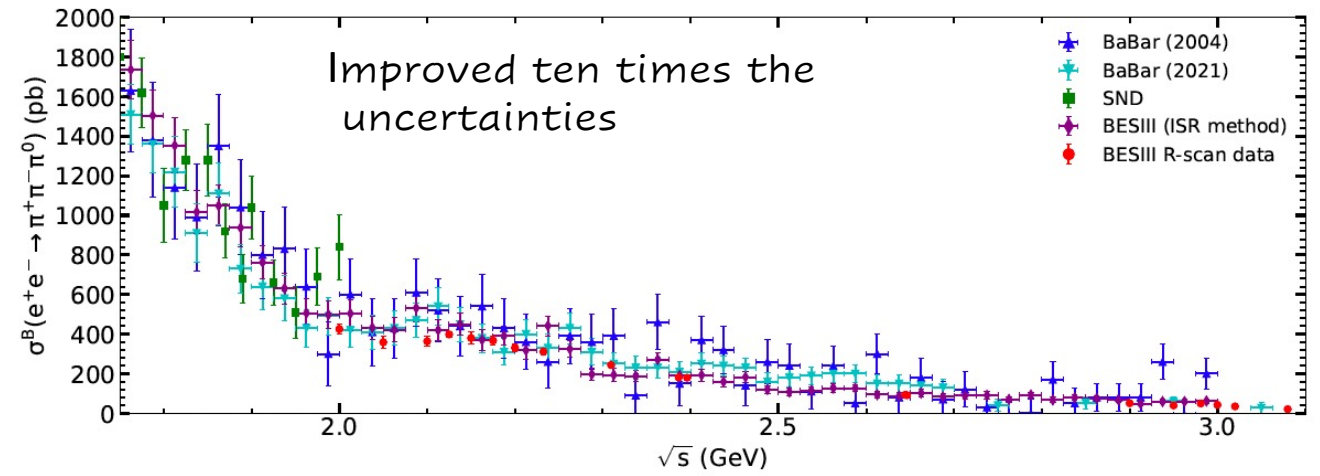
$$\sqrt{s} = [2.00-3.08] \text{ GeV}$$

PWA performed



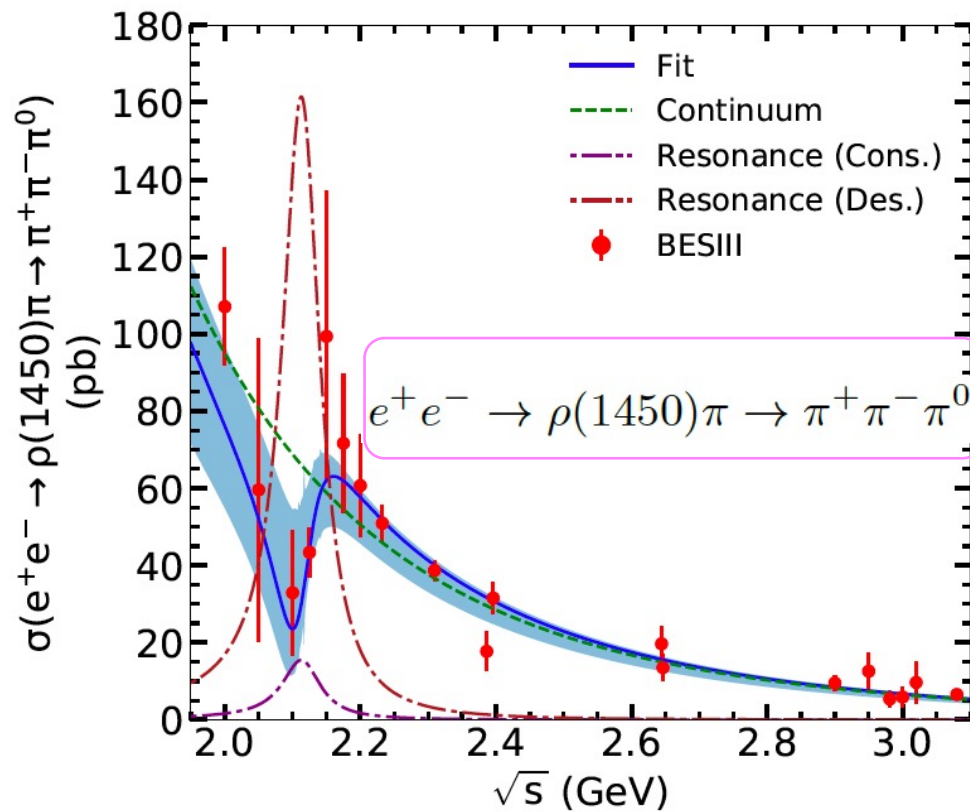
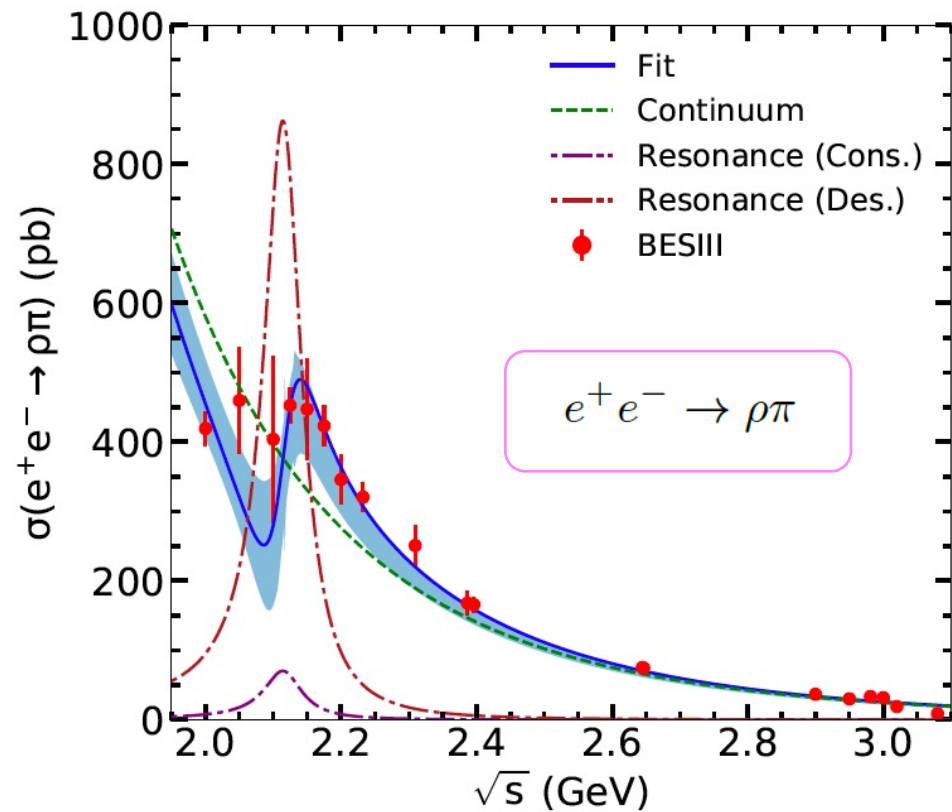
UML fit
For signal yield

$\sqrt{s}(\text{GeV})$	2.1250	2.3960	2.9000
$\rho\pi$	$> 50\sigma$	$> 50\sigma$	27σ
$\rho(1450)\pi$	18σ	10σ	13σ
$\rho(1700)\pi$	6.5σ	6.4σ	6.6σ
$\rho_3(1690)\pi$	14σ	9.2σ	6.8σ
$\rho(1570)\pi$	1.3σ	1.1σ	0.13σ
$\omega\pi$	31σ	11σ	9.7σ
$\omega(1420)\pi$	2.8σ	0.21σ	1.8σ
$\omega(1650)\pi$	3.5σ	0.29σ	0.53σ
$\omega_3(1670)\pi$	3.4σ	1.6σ	0.56σ



Born cross section also for major intermediate states (for $\omega\pi^0$ more precise BESIII result (Phys. Lett. B 813, 136059 (2021) already available)

Simultaneous fit, assuming the same resonant structure (ω^*)



PDF: $|f_r(s)e^{i\phi_r} + f_c(s)|^2$

$$f_r(s) = \frac{M_r}{\sqrt{s}} \frac{\sqrt{12\pi(\hbar c)^2 \Gamma_{ee}^r Br \Gamma_r}}{s - M_r^2 + iM_r \Gamma_r} \sqrt{\frac{\Phi(\sqrt{s})}{\Phi(M_r)}}, \quad f_c(s) = a \frac{\sqrt{\Phi(\sqrt{s})}}{(\sqrt{s})^b}.$$

Multiple solution: same M and width but different phase and $\Gamma_{ee}^R Br$

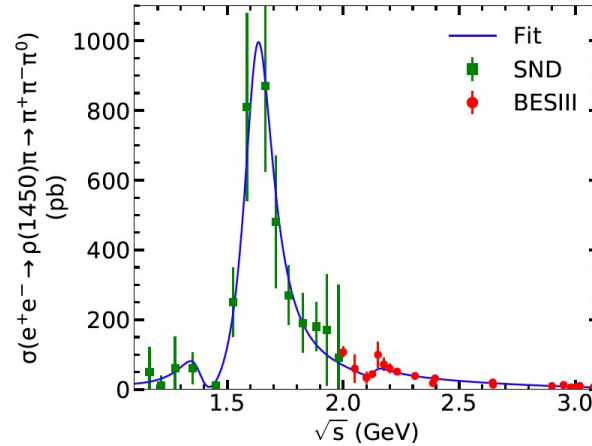
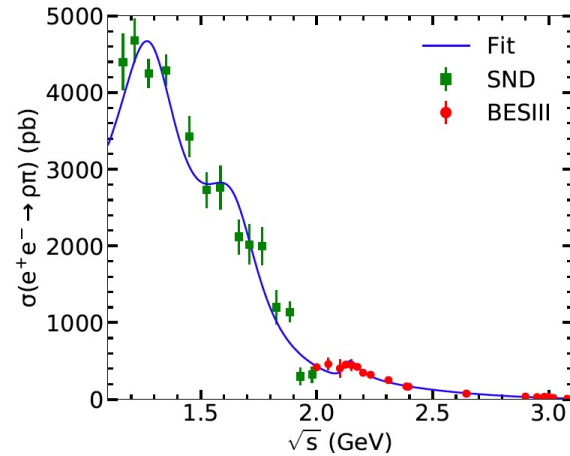


TABLE V. Fit results of the $e^+e^- \rightarrow \rho\pi$ and $e^+e^- \rightarrow \rho(1450)\pi \rightarrow \pi^+\pi^-\pi^0$ processes, where the uncertainties are statistical only.

	Nominal fit	Alternative fit
Mass	$2119 \pm 11 \text{ MeV}/c^2$	$2134 \pm 14 \text{ MeV}/c^2$
Width	$69 \pm 30 \text{ MeV}$	$74 \pm 31 \text{ MeV}$
χ^2	41.9	73.3
Number of free parameters	10	18
Degrees of freedom (ndf)	28	48
χ^2/ndf	1.50	1.53
Significance	5.9σ	5.2σ

Alternative fit with SND data @ low energies
SND-helps to constraint the continuum

To evaluate systematics

$$M = 2119 \pm 11 \pm 15 \text{ MeV}/c^2 \quad \Gamma = 69 \pm 30 \pm 5 \text{ MeV}$$

> 5 σ significance

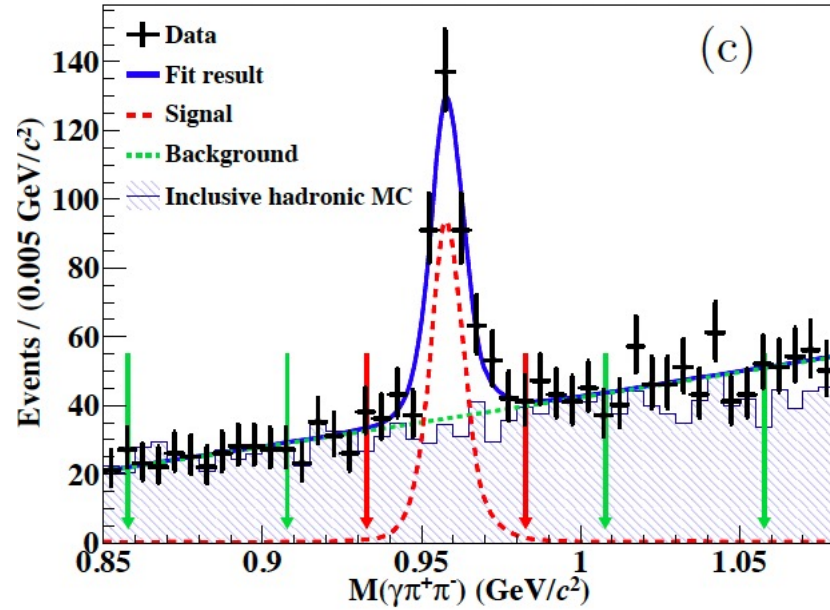
For G-parity conservation it should be an excited ω or ϕ state.
 ϕ State ruled out by OZI suppression in this final state channel.

ω^* candidate

Further studies needed to clarify the situation.

$$e^+e^- \rightarrow \omega\eta'$$

arXiv:2404.07436v1

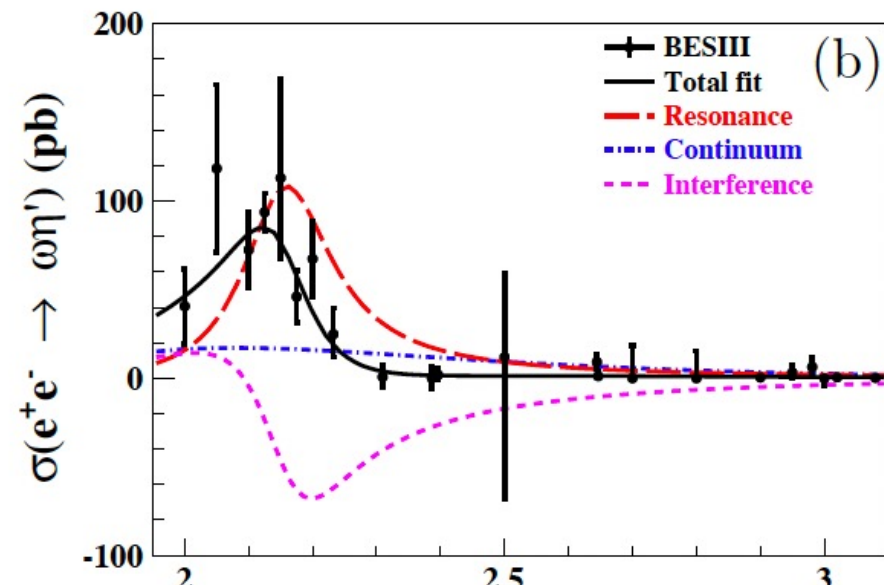
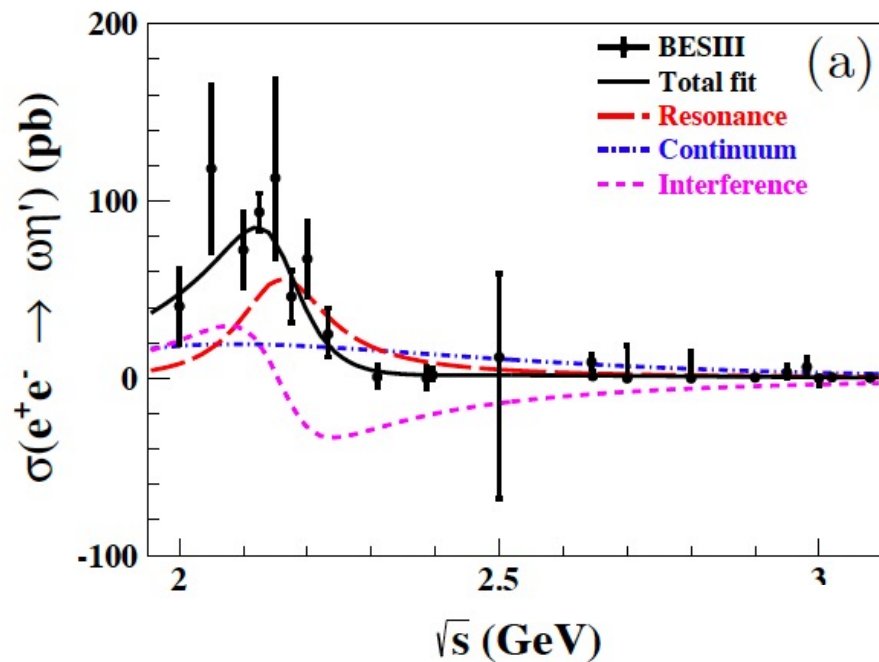


$$\sqrt{s} = [2.00-3.08] \text{ GeV}$$

$$e^+e^- \rightarrow \omega\eta'$$

$$\eta' \rightarrow \gamma\pi^+\pi^-$$

$$\omega \rightarrow \pi^+\pi^-\pi^0$$



Parameter	Solution 1	Solution 2
M_R (MeV/c ²)	2153 ± 30(stat.) ± 31(syst.)	
Γ_R (MeV)	167 ± 77(stat.) ± 7(syst.)	
ϕ (rad)	3.78 ± 0.24(stat.) ± 0.12(syst.)	3.16 ± 0.4(stat.) ± 0.2(syst.)
$\Gamma_R^{e^+e^-} \mathcal{B}_R^{\omega\eta'}$ (eV)	5.72 ± 1.68(stat.) ± 1.5(syst.)	2.99 ± 1.68(stat.) ± 1.2(syst.)
Significance	9.6σ	

$$\sigma = \frac{12\pi}{s^{3/2}} |f_1 + e^{i\phi} f_2|^2 PS(\sqrt{s}),$$

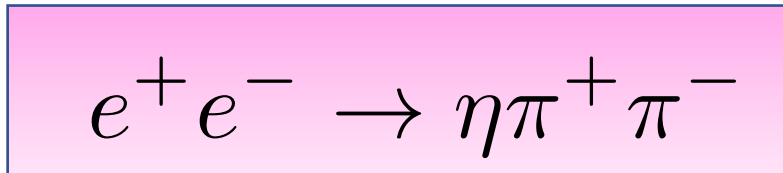
$$f_2 = \sqrt{\frac{\Gamma_R^{e^+e^-} \cdot \mathcal{B}_R^{\omega\eta'}}{PS(m_R)}} \frac{m_R^{3/2} \sqrt{\Gamma_R}}{s - m_R^2 + i\sqrt{s}\Gamma_R},$$

$$f_1 = C_0 \cdot e^{-p_0(\sqrt{s} - M_{th})},$$

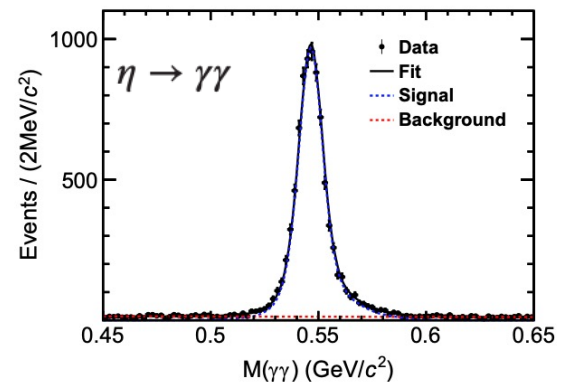
three possible ω excited state candidates around 2.2 GeV, $\omega(2220)$, $\omega(2290)$ and $\omega(2330)$

Mass and width in agreement within 3 σ with structures around 2.2 GeV by BESIII in

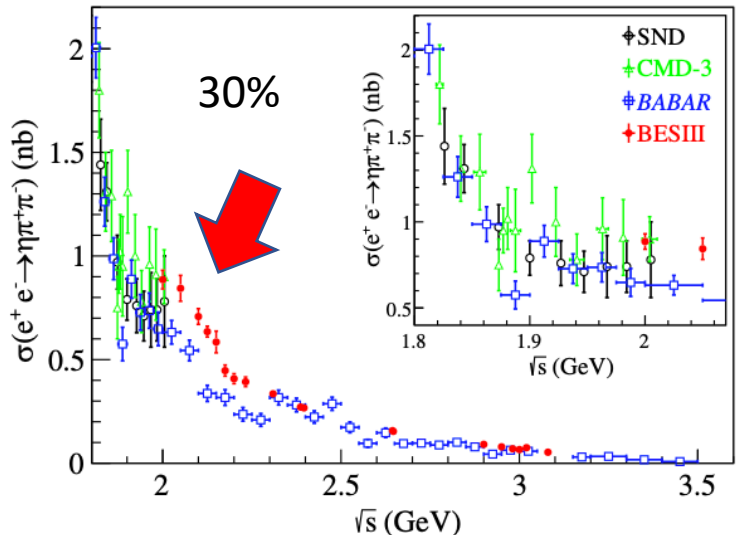
$$\omega\eta, \omega\pi^0\pi^0, \omega\pi^+\pi^- \quad 14$$



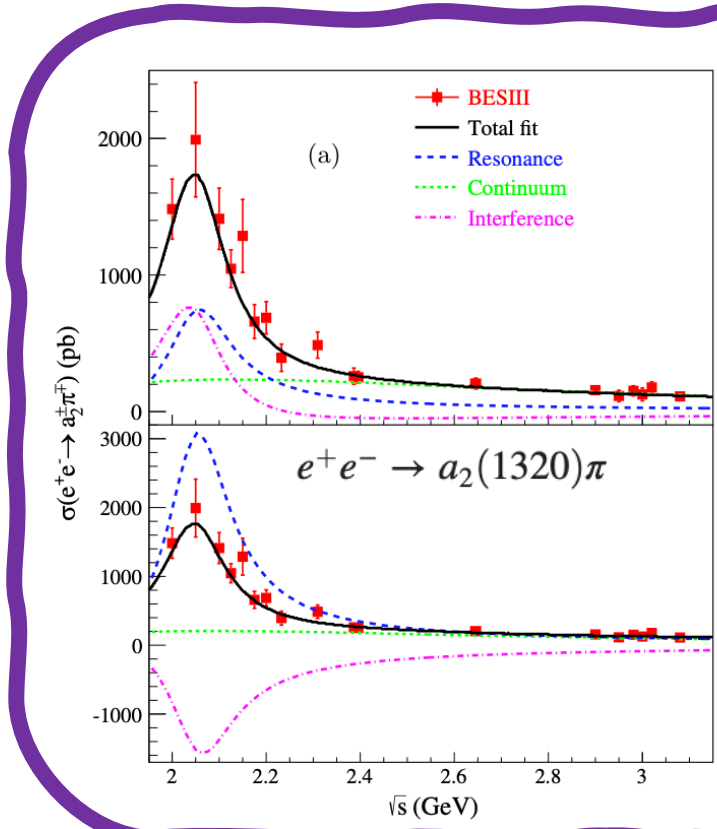
$$\sqrt{s} = [2.00-3.08] \text{ GeV}$$



simultaneous UML fit



PWA based on the GPUPWA framework



$$\sigma(s) = |f_1 + e^{i\phi} f_2|^2,$$

$$f_2 = \frac{\sqrt{12\pi\Gamma_R^{ee}} \cdot \mathcal{B}_R \Gamma_R^{\text{tot}}}{s - M_R^2 + iM_R \cdot \Gamma_R^{\text{tot}}} \sqrt{\frac{\Phi(\sqrt{s})}{\Phi(M_R)}}$$

$$f_1 = C_0 \cdot s^{-n} \sqrt{\Phi(\sqrt{s})},$$

Parameter	Solution 1	Solution 2
M_R (MeV/ c^2)	$2044 \pm 31 \pm 4$	
Γ_{tot}^R (MeV)	$163 \pm 69 \pm 24$	
$\mathcal{B}_R \Gamma_{e^+e^-}^R$ (eV)	$34.6 \pm 17.1 \pm 6.0$	$137.1 \pm 73.3 \pm 2.1$
ϕ (rad)	$1.95 \pm 0.97 \pm 0.06$	$4.35 \pm 0.48 \pm 0.43$

5.5 σ

TABLE I. Statistical significances and fit fractions of possible intermediate processes at $\sqrt{s} = 2.125, 2.396, \text{ and } 2.900$ GeV.

$\sqrt{s} = 2.125$ GeV			$\sqrt{s} = 2.396$ GeV			$\sqrt{s} = 2.900$ GeV		
Process	Significance (σ)	Fraction (%)	Process	Significance (σ)	Fraction (%)	Process	Significance (σ)	Fraction (%)
$\rho(770)\eta$	>20	58.0 ± 1.0	$\rho(770)\eta$	>20	69.5 ± 2.5	$\rho(770)\eta$	>20	66.8 ± 2.2
$a_2(1320)\pi$	>20	24.1 ± 0.8	$a_2(1320)\pi$	>20	13.0 ± 1.1	$a_2(1320)\pi$	>10	21.7 ± 2.1
$\rho(1450)\eta$	>10	1.8 ± 0.3	$\rho(1450)\eta$	5.1	1.0 ± 0.4	$\rho(1450)\eta$	>10	16.5 ± 0.4
$a_2(1700)\pi$	>10	2.0 ± 0.3	$\rho_3(1690)\eta$	9.7	2.5 ± 0.5	$\rho(1700)\eta$	6.5	2.1 ± 0.1
...	$a_2(1700)\pi$	6.8	2.7 ± 0.4
...	$\rho(1700)\eta$	5.8	1.9 ± 0.9

Same properties of $\rho(2000)$ found by BESIII in $e^+e^- \rightarrow \omega\pi^0$ [Phys. Lett. B 813, 136059 (2021)]
 First Observation of $\rho(2000) \rightarrow a_2(1320)\pi$.

Summary

- Relevant contribution of BESIII in the spectroscopy of vector mesons between 2 and 3 GeV
- Recent results presented:

$$e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$$

$$e^+e^- \rightarrow \pi^+ \pi^- \pi^0$$

$$e^+e^- \rightarrow \omega \eta'$$

$$e^+e^- \rightarrow \eta \pi^+ \pi^-$$

- Many others are coming (Stay Tuned!!)



Thank you
For your attention

謝謝



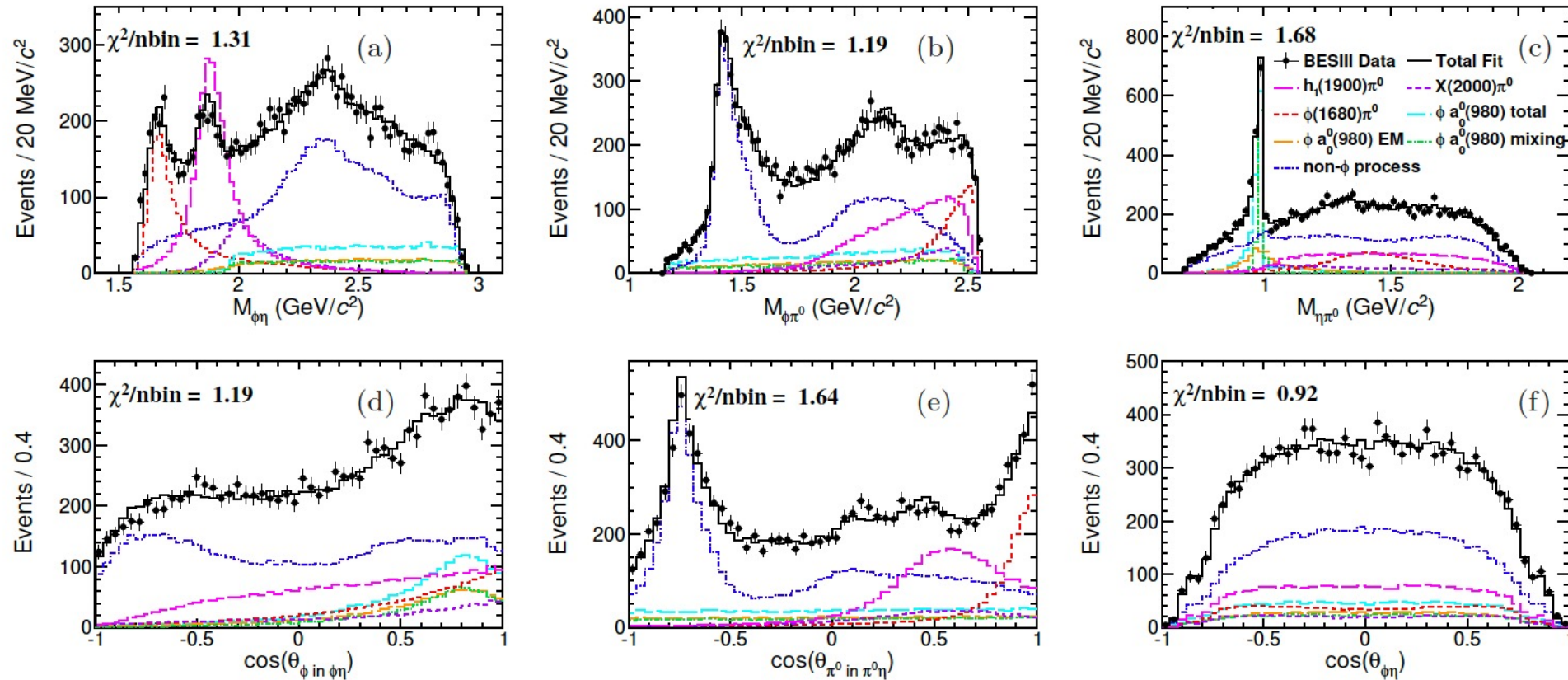
Spares

BESIII Collaboration Meeting in Summer 2024



$$J/\psi \rightarrow \phi \pi^0 \eta$$

PWA covariant tensor amplitude method.



the structure at 1.9 GeV/c² on the $M(\phi\eta)$ distribution can not be described only by a new axial-vector state ($h_1(1900)$), but requires an additional vector state ($X(2000)$).

First observation of two new structures

<https://arxiv.org/abs/2311.07043>

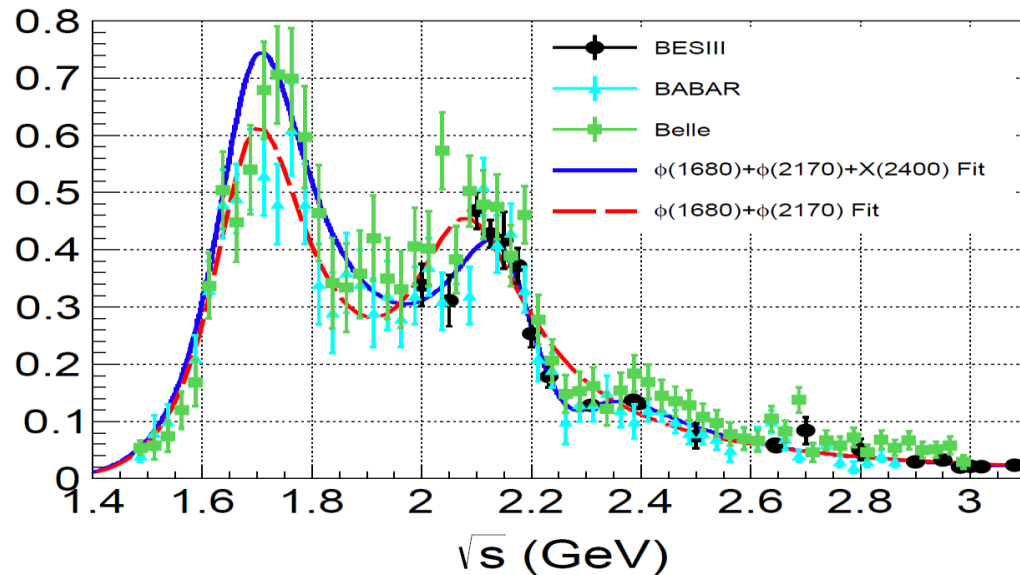
Process	M (MeV/c ²)	Γ (MeV)	\mathcal{B} (10 ⁻⁶)	σ
$\phi(1680)\pi^0$	1668 \pm 7 \pm 25	147 \pm 14 \pm 35	5.34 \pm 0.25 \pm 0.88	36
X(2000) π^0	1996 \pm 11 \pm 30	148 \pm 16 \pm 66	2.76 \pm 0.22 \pm 0.67	17
$h_1(1900)\pi^0$	1911 \pm 6 \pm 14	149 \pm 12 \pm 23	9.29 \pm 0.33 \pm 0.72	24
$\phi a_0(980)_{EM}$	—	—	2.81 \pm 0.24 \pm 0.55	19
$\phi a_0(980)_{mix}$	—	—	2.73 \pm 0.14 \pm 0.19	32

The $h_1(1900)$ axial vector meson can be assigned as the $h_1(2P)$ strangeonium state \rightarrow mass and width agree well with the theoretical predictions

The X(2000) state can be associated to the $\phi(3S)$ or $\phi(2D)$ states, but its mass and width have large discrepancies with the theoretical predictions

Important input to establish the strangeonium spectrum

- $\phi\pi^+\pi^-$
 - Resonances at 2.1, 2.4 GeV
 - Possible contribution from $\phi(2170)$

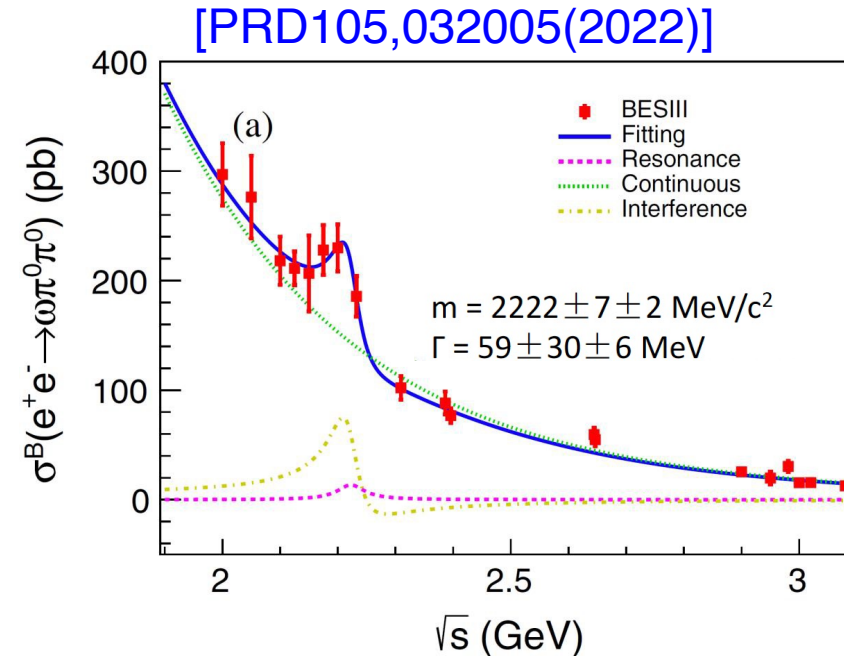
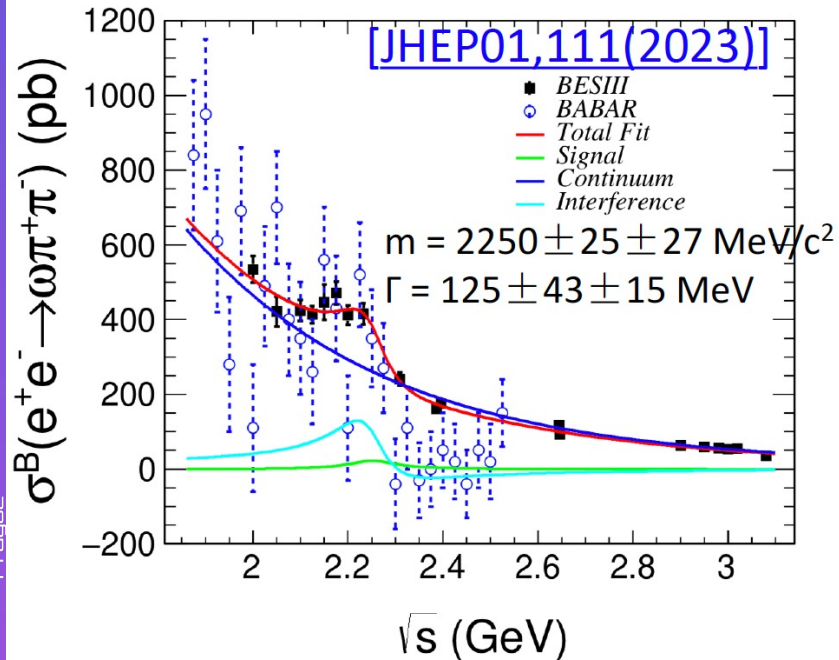


Parameters	Solution I	Solution II	Solution III	Solution IV
$M_r(\phi(1680))$			1675 ± 7	
$\Gamma_r(\phi(1680))$			238 ± 29	
$Br\Gamma_{ee}(\phi(1680))$	35.3 ± 1.5	39.0 ± 1.3	20.7 ± 1.3	18.8 ± 0.9
$M_r(\phi(2170))$			2174 ± 23	
$\Gamma_r(\phi(2170))$			207 ± 49	
$Br\Gamma_{ee}(\phi(2170))$	60.2 ± 34.5	329 ± 211	123 ± 83	22.9 ± 12.6
$\phi_P(\phi(2170)/\phi(1680))$	0.75 ± 0.14	-0.99 ± 0.60	2.90 ± 0.87	2.46 ± 0.68
$M_r(R(2400))$			2276 ± 42	
$\Gamma_r(R(2400))$			320 ± 112	
$Br\Gamma_{ee}(R(2400))$	50.7 ± 35.7	350 ± 247	112 ± 82	15.6 ± 9.0
$\phi_P(R(2400)/\phi(1680))$	-2.92 ± 0.65	2.91 ± 0.52	0.03 ± 0.27	0.56 ± 0.84

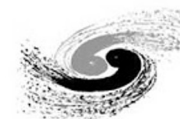
[arXiv:2112.13219](https://arxiv.org/abs/2112.13219)

$\omega\pi\pi$

- $\omega\pi^+\pi^-$ and $\omega\pi^0\pi^0$
 - Clear structure around 2.2 GeV in both channels
 - Combined $\omega\pi\pi$: $m = 2232 \pm 19 \pm 27 \text{ MeV}/c^2$, $\Gamma = 93 \pm 53 \pm 20 \text{ MeV}$
 - Structure also in subprocesses $\omega f_0(500)$, $\omega f_0(800)$, $\omega f_0(1370)$, $\omega f_2(1270)$, $b_1(1235)\pi \rightarrow \omega^*$

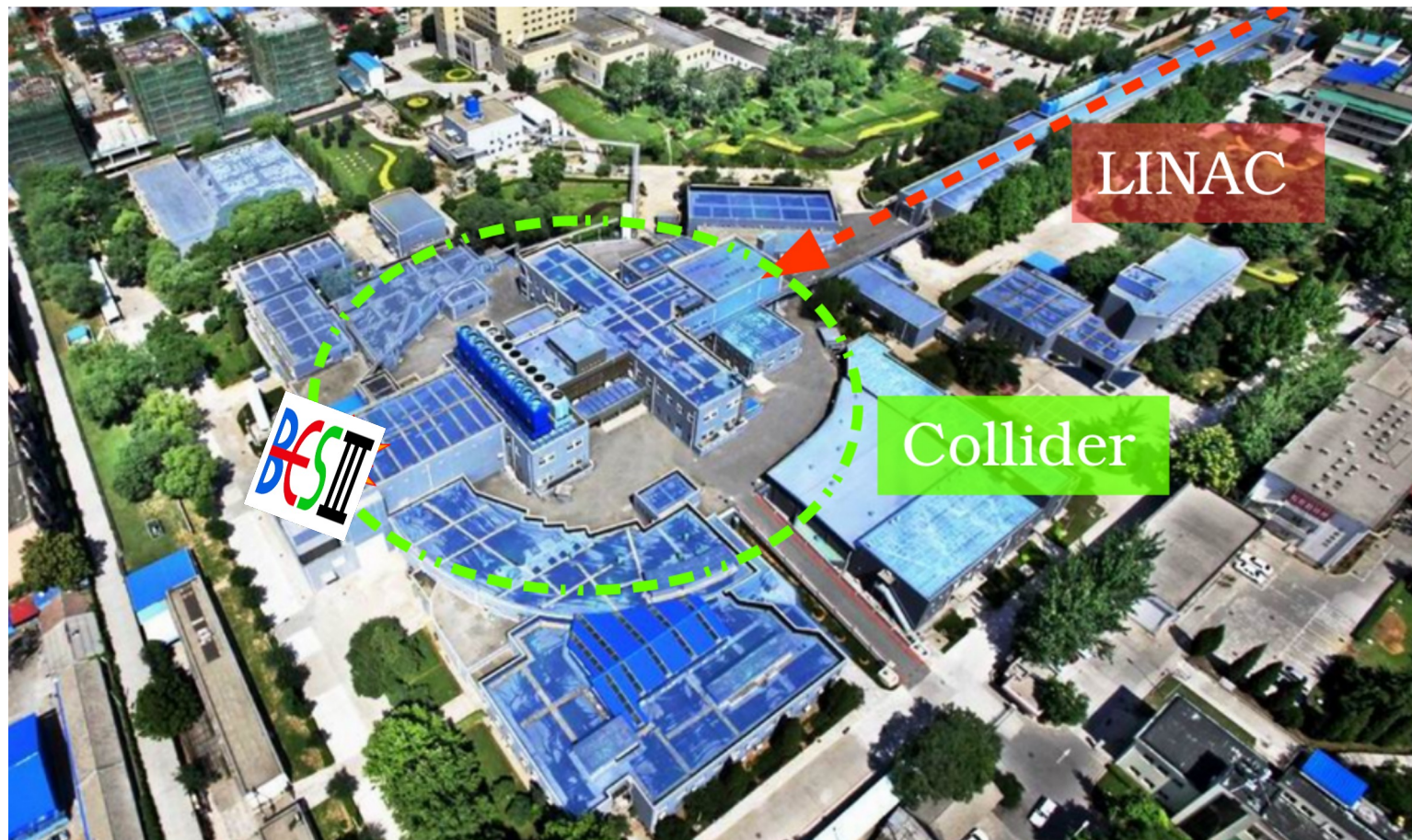


BEP-CII @ IHEP



中国科学院高能物理研究所

Institute of High Energy Physics
Chinese Academy of Sciences



Electron-positron collider

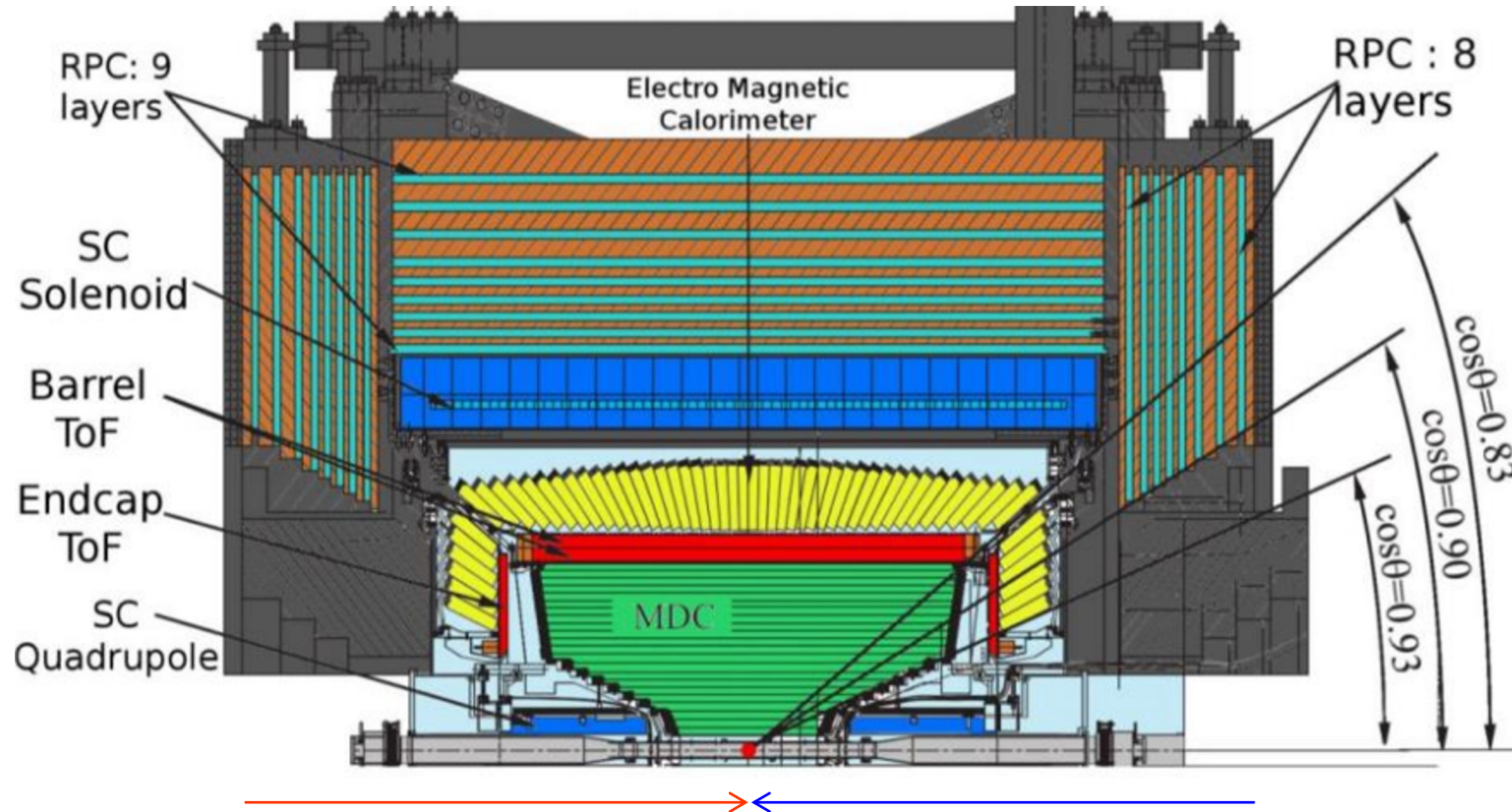
$$E_{cm} = 2 - 4.95 \text{ GeV}$$

$$\text{Luminosity} = 10^{33} \text{ cm}^{-2}\text{s}^{-1}$$

Being upgraded
to increase
luminosity at high
energy and extend
the cme energy range

BESIII @ BEPCII

1 Tesla Magnetic field



Muon counters:
 $\delta_{r\phi} = 1.4 \text{ cm} - 1.7 \text{ cm}$

Electromagnetic Calorimeter:
 $dE/\sqrt{E} (1 \text{ GeV}) = 2.5 \%$

Time Of Flight:
 $\sigma_t (\text{barrel}) = 70 \text{ ps}$
 $\sigma_t (\text{endcap}) = 60 \text{ ps}$

Main Drift Chamber:
 $\sigma_x (1 \text{ GeV}/c) \sim 130 \mu\text{m}$
 $dp/p (1 \text{ GeV}/c) = 0.5 \%$

Mass and width of the structure are in agreement within 3σ $e^+e^- \rightarrow \omega\eta'$ structures around 2.2 GeV by BESIII in $\omega\eta, \omega\pi^0\pi^0, \omega\pi^+\pi^-$

three possible ω excited state candidates around 2.2 GeV, $\omega(2220), \omega(2290)$ and $\omega(2330)$

Assuming same R for BESIII result in $\omega\eta$ [[Phys. Lett. B 813 \(2021\) 136059](#)]

:

$$R = \frac{B_R^{\omega\eta'}}{B_R^{\omega\eta}} = \left| \frac{\cos(\Theta_0 - \Theta)}{\sin(\Theta_0 - \Theta)} \right|^2 \frac{\Omega_{\eta'}}{\Omega_{\eta}} = 0.34 \pm 0.07$$

By lattice QCD, based on the $\eta - \eta'$ mixing angle

ideal mixing angle between pure $s\bar{s}$ and $u\bar{u} - d\bar{d}$,

$$R = 13.30 \pm 6.06 \pm 3.61, 2.39 \pm 1.63 \pm 1.01, 4.58 \pm 2.21 \pm 1.34, 6.95 \pm 4.60 \pm 2.83$$

Combining previous BESIII result with the multiple solutions for the resonance parameters

Many vector states between $s = [2.0, 3.0]$ GeV
 – ρ^* , ω^* , ϕ^* , exotic states, mixture?

- **Couplings to different channels** may help to reveal their nature

- Common decay patterns:
 is there $\rho^* - \omega^* - \phi^*$ mixing?

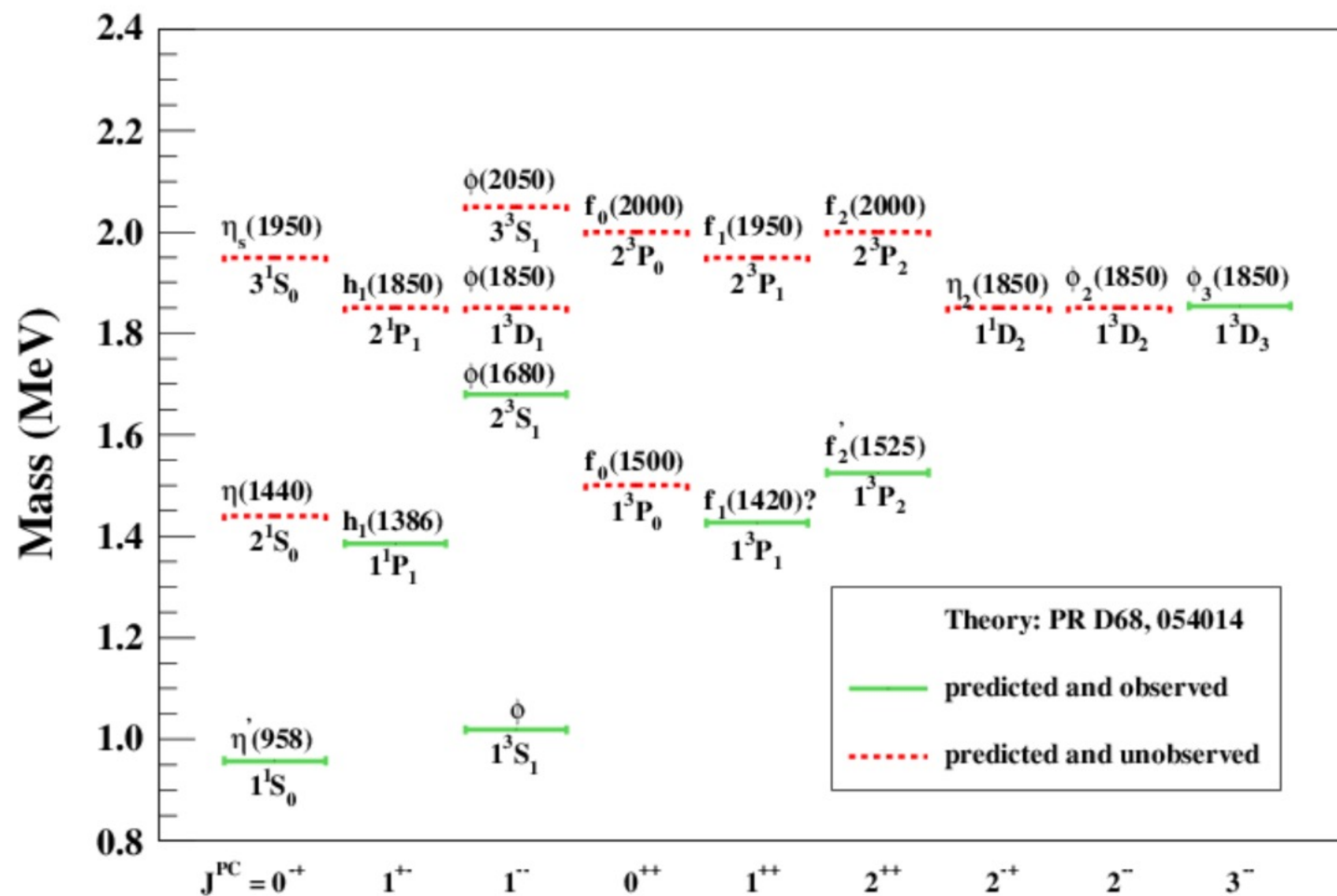
- **Identification of exotic particle**
 with non exotic quantum numbers

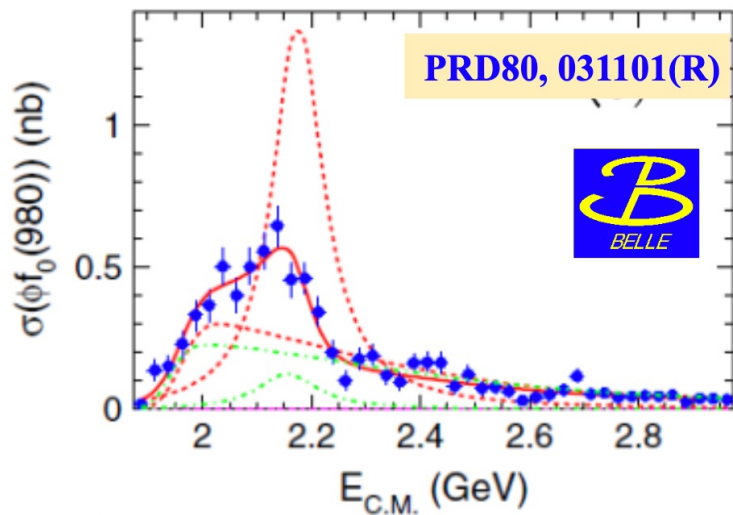
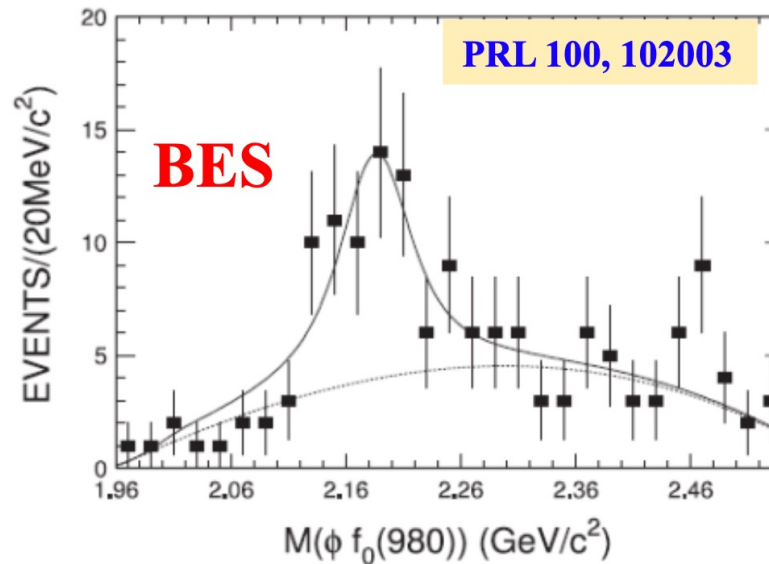
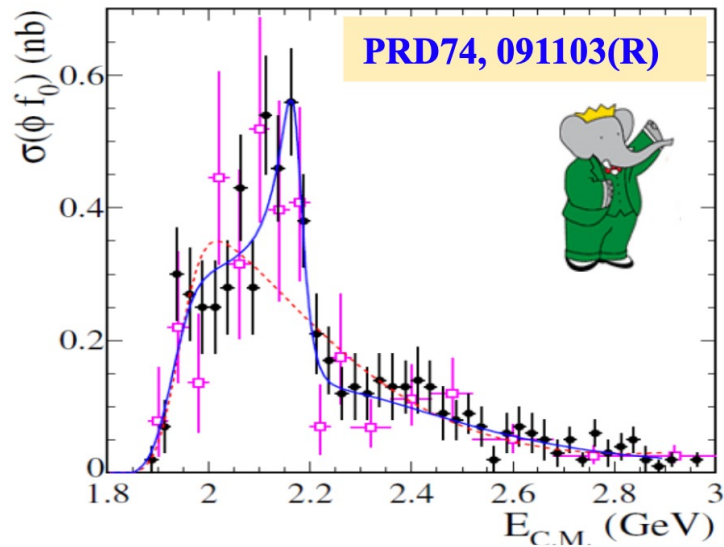
From the experimental POV
 → established meson spectrum required

- to rule out conventional quarkonia
- to study mixing between exotica
 and conventional mesons

TABLE XII. Decay amplitudes squared (excluding the decay strength constants).

Decay	Products	Squared amplitude
$\rho^* \rightarrow PP$	$K\bar{K}$	4
	$\pi\pi$	8
$\rho^* \rightarrow VP$	\bar{K}^*K	2
	$\rho\eta$	$2\cos^2\phi_P$
	$\rho\eta'$	$2\sin^2\phi_P$
	$\omega\pi$	$2\sin^2\phi_V$
	$\phi\pi$	$2\cos^2\phi_V$
$\omega^* \rightarrow PP$	$K\bar{K}$	$12\sin^2\theta_V^*$
$\omega^* \rightarrow VP$	$\phi\eta$	$2(\sqrt{2}\cos\phi_V^*\sin\phi_V\sin\phi_P + \sin\phi_V^*\cos\phi_V\cos\phi_P)^2$
	$\phi\eta'$	$2(\sqrt{2}\cos\phi_V^*\sin\phi_V\cos\phi_P - \sin\phi_V^*\cos\phi_V\sin\phi_P)^2$
	$\omega\eta$	$2(\sqrt{2}\cos\phi_V^*\cos\phi_V\sin\phi_P - \sin\phi_V^*\sin\phi_V\cos\phi_P)^2$
	$\omega\eta'$	$2(\sqrt{2}\cos\phi_V^*\cos\phi_V\cos\phi_P + \sin\phi_V^*\sin\phi_V\sin\phi_P)^2$
	$\rho\pi$	$6\sin^2\phi_V^*$
	$K^*\bar{K}$	$2(\sin\phi_V^* + \sqrt{2}\cos\phi_V^*)^2$
$\phi^* \rightarrow PP$	$K\bar{K}$	$12\cos^2\theta_V^*$
	$\phi\eta$	$2(-\sqrt{2}\sin\phi_V^*\sin\phi_V\sin\phi_P + \cos\phi_V^*\cos\phi_V\cos\phi_P)^2$
$\phi^* \rightarrow VP$	$\phi\eta'$	$2(\sqrt{2}\sin\phi_V^*\sin\phi_V\cos\phi_P + \cos\phi_V^*\cos\phi_V\sin\phi_P)^2$
	$\omega\eta$	$2(\sqrt{2}\sin\phi_V^*\cos\phi_V\sin\phi_P + \cos\phi_V^*\sin\phi_V\cos\phi_P)^2$
	$\omega\eta^*$	$2(-\sqrt{2}\sin\phi_V^*\cos\phi_V\cos\phi_P + \cos\phi_V^*\sin\phi_V\sin\phi_P)^2$
	$\rho\pi$	$6\cos^2\phi_V^*$
	$K^*\bar{K}$	$2(\cos\phi_V^* - \sqrt{2}\sin\phi_V^*)^2$





$$e^+e^- \Rightarrow \begin{cases} Y(2175) \rightarrow \phi(1020)\pi^+\pi^- & \text{strange,} \\ Y(4260) \rightarrow J/\psi\pi^+\pi^- & \text{charm,} \\ \Upsilon(10860) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^- & \text{bottom,} \end{cases}$$

$\phi(2170)$ as strange analogue of $Y(4220)$

ρ^* and ω^* vector mesons

- Members above 2 GeV, their nature controversial
- Pure state or mixture?
- Decay patterns can be used to identify them

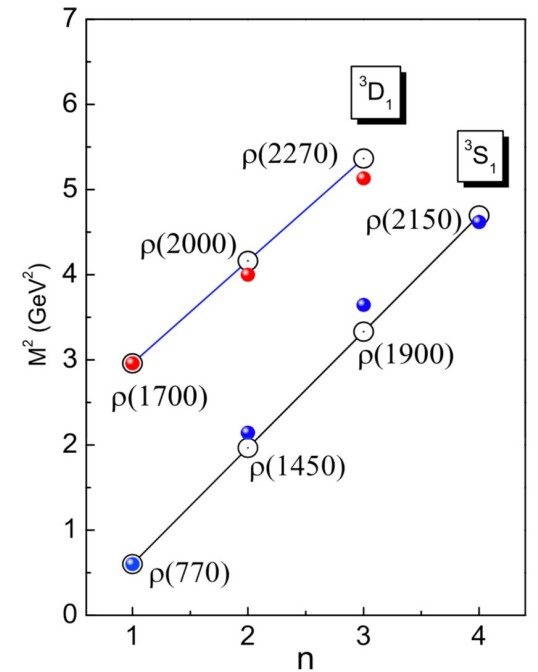
Recently we investigated

ω^* :

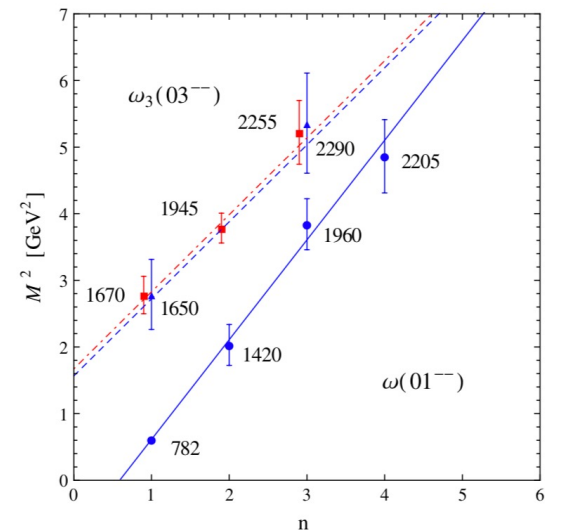
$$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$$

$$e^+ e^- \rightarrow \omega \eta$$

$$e^+ e^- \rightarrow \eta \pi^+ \pi^-$$



PRD88,034008(2013)



PRD85,094006(2012)

In comparison with $e^+e^- \rightarrow K^+K^-\pi^0$ [JHEP 07 (2022) 045]

- $\frac{\sigma(e^+e^- \rightarrow K^*(892)^+K^-)}{\sigma(e^+e^- \rightarrow K^*(892)^0K^0)} < 0.2$
- $\frac{\sigma(e^+e^- \rightarrow K_2^*(1430)^+K^-)}{\sigma(e^+e^- \rightarrow K_2^*(1430)^0K^0)} \sim 0 - 40$

