Non-perturbative thermal QCD at very high temperatures

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Collaboration:

Bresciani, Dalla Brida, LG, Harris, Laudicina, Pepe, Rescigno,

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Outline

- Non-perturbative (NP) thermal QCD up to very high T: why ?
- Renormalization and shifted boundary conditions: how ?
- Lattice setup
- Results for mesonic and baryonic screening masses
- Preliminary results for the Equation of State
- Conclusions and Outlook

Thermal QCD: relevant scales and effective theories

[Ginsparg 80; Linde 80; Appelquist, Pisarski 81; Braaten, Nieto 96; . . .]

- The three relevant scales in the problem are:

$M = \pi T + \ldots$	Fermions [3D NRQCD] and non-zero Matsubara gluon modes
$m_{\rm E} \propto gT + \ldots$	A_0 zero Matsubara gluon modes [3D EQCD]
$g_{\rm E}^2 = g^2 T + \ldots$	A_i zero Matsubara gluon modes [3D MQCD]

- Thanks to asymptotic freedom, at asymptotically high T a hierarchy between the three scales is generated

$$\frac{g_{\rm E}^2}{\pi} \ll m_{\rm E} \ll \pi T \qquad \Longleftrightarrow \qquad \left(\frac{g}{\pi}\right)^2 \ll \frac{g}{\pi} \ll 1$$

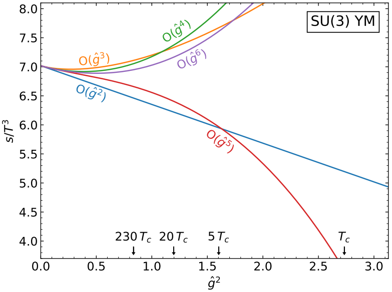
- Perturbation theory developed for high T regime
  [See Laine, Vuorinen 17 for a review]
- Contributions from lowest scale must always be computed NP

Why beyond perturbation theory up to very high T ?

- Perturbative expansion has a very poor convergence rate

- Contributions computable in PT only up to finite order

[Kajantie et al. 02; LG, Pepe 17]



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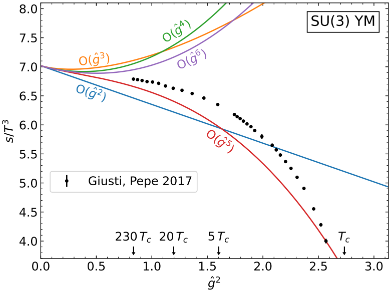

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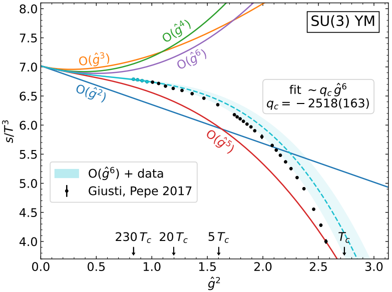

- Perturbative expansion has a very poor convergence rate
- Contributions computable in PT only up to finite order
- For the $SU(3)$ YM theory, if we (assume convergence and) fit the 4 highest temperatures by including an effective (NP) term

$$\frac{s(T)}{T^3} = \frac{32\pi^2}{45}\left\{1+s_2\hat{g}^2+s_3\hat{g}^3+s_4(\hat{g})\hat{g}^4+s_5\hat{g}^5+s_6(\hat{g})\hat{g}^6+\frac{q_c}{(2\pi)^6}\hat{g}^6\right\}$$

the $\mathcal{O}(\hat{g}^6)$ is still $\sim 50\%$ of the total contribution from interactions at $T = 231\,T_c \sim 68$ GeV $(\hat{g}/\pi \sim 0.3)$. More sophisticated PT intensively studied in the literature

Why beyond perturbation theory up to very high T ?

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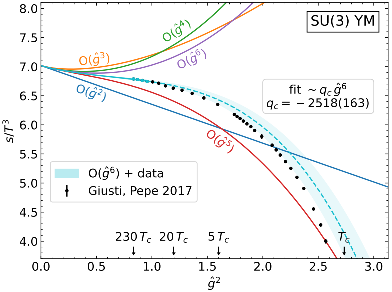
- Contributions computable in PT only up to finite order

[Kajantie et al. 02; LG, Pepe 17]



- All these facts call for a non-perturbative study of thermal QCD up to very high T to identify the origin and the magnitude of the various contributions with controlled and improvable errors

Renormalization

- Hadronic renormalization scheme is not a viable option because

$$M_{\mathrm{hadron}} \ll T$$

  Accommodating 2 very different scales on a lattice too expensive

- Way to go is the NP renormalization of the coupling:

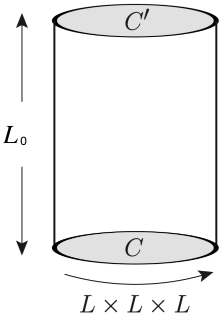
  ⋆ Define the renormalized g^2 NP, e.g. SF (GF) couplings ($L = L_0$)

$$\left.\frac{\partial\Gamma}{\partial\eta}\right|_{\eta=0} \equiv \frac{12\pi}{\bar{g}^2_{\mathrm{SF}}(\mu)}\,, \quad \mu = \frac{1}{L_0}$$

  where C and C' depend on η, and $\Gamma = -\ln[Z]$

  ⋆ Define quark masses NP by WIs

[Lüscher et al 91]



- Avoid zero-temperature subtraction in renormalization of fields by adopting shifted boundary conditions, e.g. Equation of State

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- Way to go is the NP renormalization of the coupling:

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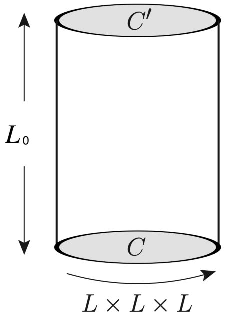
⋆ For each value of T, renormalize thermal QCD by requiring

$$g^2_{\mathrm{SF}}(g_0^2, a\mu) = \bar{g}^2_{\mathrm{SF}}(\mu)$$

with $a\mu \ll 1$ and $\mu = T\sqrt{2}$

⋆ Last condition fixes the dependence of g_0^2 on a, for values of a at which μ and T are easily accommodated



Lattice setup

- Wilson (T_0–T_8) and Lüscher–Weisz (T_9–T_{11}) actions for gluons
- NP $O(a)$-improved Wilson quarks
- Four lattice spacings for each T, $L_0/a = 4, 6, 8$ and 10
- Shifted boundary conditions
- Restriction to zero topology

T	$\bar{g}^2_{\mathrm{SF}}(\mu = T\sqrt{2})$	T (GeV)
T_0	1.01640	164.6(5.6)
T_1	1.11000	82.3(2.8)
T_2	1.18446	51.4(1.7)
T_3	1.26569	32.8(1.0)
T_4	1.3627	20.63(63)
T_5	1.4808	12.77(37)
T_6	1.6173	8.03(22)
T_7	1.7943	4.91(13)
T_8	2.0120	3.040(78)

T	$\bar{g}^2_{\mathrm{GF}}(\mu = T/\sqrt{2})$	T (GeV)
T_9	2.7359	2.833(68)
T_{10}	3.2029	1.821(39)
T_{11}	3.8643	1.167(23)

- The linear extension of spatial directions is $L/a = 288$, i.e. $20 < LT < 50$. Finite volume effects negligible given the mass gap. Explicitly checked at the highest and lowest temperature

Results for mesonic screening masses

- Effective theory + NLO matching predict

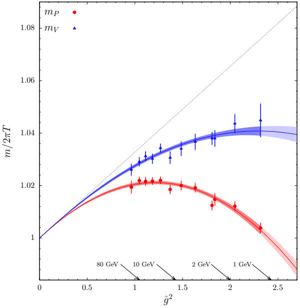
$$m_{\mathcal{O}}^{\mathrm{PT}} = 2\pi T \left(1 + p_2^{\mathrm{PT}} g^2\right)$$

where $p_2^{\mathrm{PT}} = 0.03274$. In particular m_P and m_v are degenerate

- NP Results can be fitted by a quartic polynomial in

$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2}\ln\frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}} + \frac{4}{9\pi^2}\ln\left(2\ln\frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}}\right)$$

where for our purpose this is a funct. of T designed to coincide with the $\overline{\mathrm{MS}}$ inverse coupling squared



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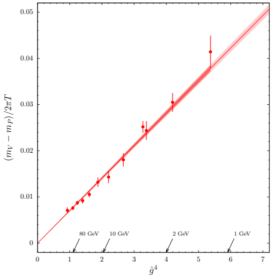

- Masses non-degenerate even at electroweak scale!

Results for baryonic (nucleon) screening mass

- Effective theory + NLO matching predict

$$m_{N^+}^{\mathrm{PT}} = 3\pi T \left(1 + q_2^{\mathrm{PT}} g^2\right)$$

where $q_2^{\mathrm{PT}} = 0.046$.

- NP Results can be fitted by a quartic polynomial in

$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2} \ln\frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}} + \frac{4}{9\pi^2} \ln\left(2\ln\frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}}\right)$$

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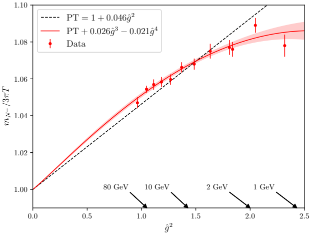
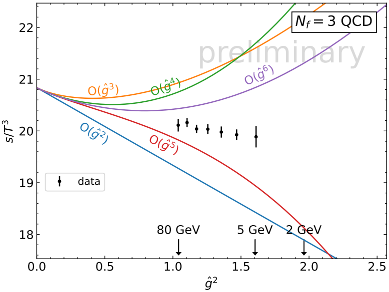


- PT within 0.5% down to $T \sim 5\,\mathrm{GeV}$, but curvature needed!

$N_f = 3$ QCD Equation of State up to very high T

- EoS can be obtained up to very hight T NP with controlled and improvable errors

- In this first computation an accuracy of $\lesssim 1\%$ has been reached up to $T \sim 82$ GeV



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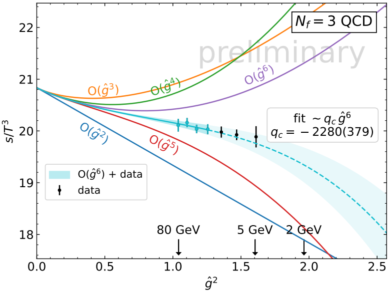

- To compare with PT, if we (assume convergence and) fit the 4 highest temperatures by including an effective (NP) term

$$\frac{s(T)}{T^3} = \frac{95\pi^2}{45}\left\{1 + s_2\hat{g}^2 + s_3\hat{g}^3 + s_4(\hat{g})\hat{g}^4 + s_5\hat{g}^5 + s_6(\hat{g})\hat{g}^6 + \frac{32 q_c}{95(2\pi)^6}\hat{g}^6\right\}$$

the $\mathcal{O}(\hat{g}^6)$ is still $\sim 45\%$ of the total contribution from interactions at $T = T_1 \sim 82$ GeV ($\hat{g}/\pi \sim 1/3$). PT clearly still not converging

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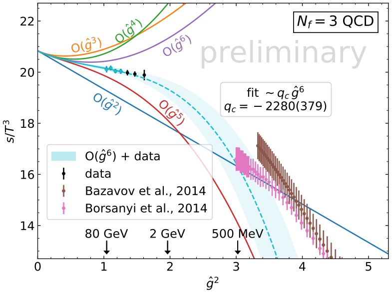

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Conclusions and Outlook

- With today HPC technology and known algorithms is possible to simulate thermal QCD up to very high temperatures

- Systematics due to the use of perturbation theory can be fully removed up to the electroweak scale

- The strategy proposed here opens the way to study many properties of thermal QCD in the high temperature regime:

  ⋆ Screening masses of mesons and baryons

  ⋆ Equation of State

  ⋆ Transport coefficients

  ⋆ .....

BACKUP SLIDES

Effective field theories at large T: MQCD

- For Physics at energies $E = O(g_{\mathrm{E}}^2)$, the scalar field can be integrated out, and one is left with Magnetostatic QCD (MQCD)

$$S_{\mathrm{MQCD}} = \frac{1}{g_{\mathrm{E}}^2} \int d^3x \left\{ \frac{1}{2} \operatorname{Tr}\left[F_{ij}F_{ij}\right] \right\} + \ldots$$

- This is a 3D Yang–Mills theory which needs to be solved NP. All dimensionful quantities proportional to appropriate power of g_{E}^2

- As a result, at asymptotically high T the mass gap developed by thermal QCD is proportional to $g_{\mathrm{E}}^2 = g^2 T + \ldots$

- Quarks have very heavy masses $M = \pi T(1 + \frac{g^2}{6\pi^2} + \ldots)$, and can be considered, in first approximation, as static fields

Effective field theories at large T: EQCD

- Physics at energies $E \ll \pi T$ is described by a 3-dimensional effective gauge theory dubbed Electrostatic QCD (EQCD)

$$S_{\rm EQCD} = \frac{1}{g_{\rm E}^2} \int d^3x \left\{ \frac{1}{2} \,{\rm Tr}\,[F_{ij}F_{ij}] + {\rm Tr}\,[(D_jA_0)(D_jA_0)] + m_{\rm E}^2 \,{\rm Tr}\left[A_0^2\right] \right\} + \ldots$$

  where the fields are the Matsubara zero-modes of 4D gauge field

- The 4D temporal component A_0 behaves as a 3D scalar field of mass $m_{\rm E}$ in the adjoint representation of the gauge group

- When the QCD coupling g^2 is small, perturbative matching gives

$$m_{\rm E}^2 = \frac{3}{2} g^2 T^2 + \ldots \qquad \text{and} \qquad g_{\rm E}^2 = g^2 T + \ldots$$

  and at asymptotically hight T, three energy scales develop

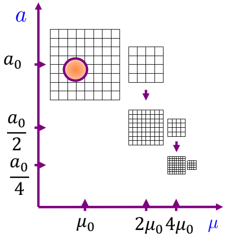
$$\frac{g_{\rm E}^2}{\pi} \ll m_{\rm E} \ll \pi T$$

Renormalization: extra material

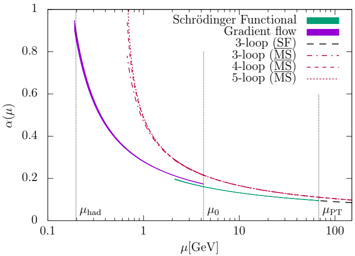
- Relate $g^2(\mu_{\rm hadron})$ to $M_{\rm hadron}$ NP
- Determine running of $g^2(\mu)$ NP
- Compute $g^2(\mu)$ for values of μ up to the electroweak scale
- For each value of T, renormalize thermal QCD by requiring

$$g^2_{\rm SF}(g_0^2, a\mu) = \bar{g}^2_{\rm SF}(\mu)$$

with $a\mu \ll 1$ and $\mu = T\sqrt{2}$



[Bruno et al. 17]



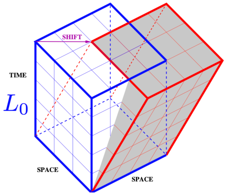
- Last condition fixes the dependence of the bare g_0^2 on a, for values of a at which μ and T can be easily accommodated

Shifted boundary conditions

- By adopting shifted boundary conditions

[Meyer, LG 11-13]

$$
\begin{aligned}
U_\mu(x_0 + L_0, \boldsymbol{x}) &= U_\mu(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi}) \\
\psi(x_0 + L_0, \boldsymbol{x}) &= -\psi(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi}) \\
\overline{\psi}(x_0 + L_0, \boldsymbol{x}) &= -\overline{\psi}(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi})
\end{aligned}
$$



the entropy density can be computed as

$$
s = -\frac{L_0\,(1+\vec{\xi}^2)^{3/2}}{\xi_k}\,\langle T_{0k}\rangle_{\vec{\xi}}
$$

and the zero-temperature subtraction is avoided in the EoS

Systematics: topology and finite-size effects

- At very high temperature the topological charge distribution is expected to be highly peaked at zero [$b \sim 9$ for $N_f = 3$]

$$P_\nu = \frac{1}{\sqrt{2\pi <\nu^2>}} e^{-\frac{\nu^2}{2<\nu^2>}} + \ldots\,, \quad <\nu^2> \propto L^3 m^3 T^{-b}$$

- The contributions from non-zero topological sectors to observables

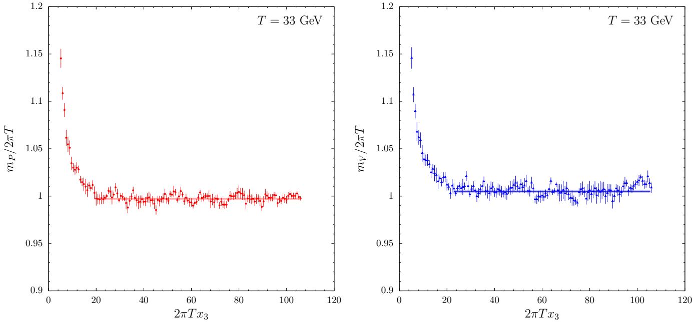
$$\langle \mathcal{O} \rangle = \sum_\nu P_\nu \langle \mathcal{O} \rangle_\nu$$

  are negligible within statistical errors for the volumes considered. Simulations can be safely restricted to the zero topology sector.

- At asymptotically high T thermal QCD has a mass gap proportional to $g_{\mathrm{E}}^2 = g^2 T + \ldots$

- Finite size effects are exponentially small in $g^2 TL$, and can be made negligible within errors in large enough volumes

Screening mass definition



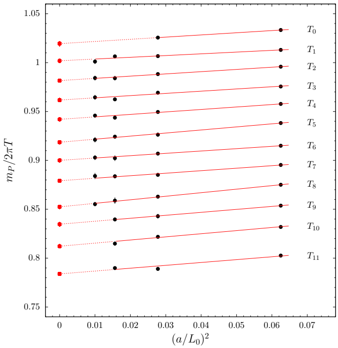
From the two-point correlators $[\mathcal{O} = \{S, P, V_\mu, A_\mu\}]$

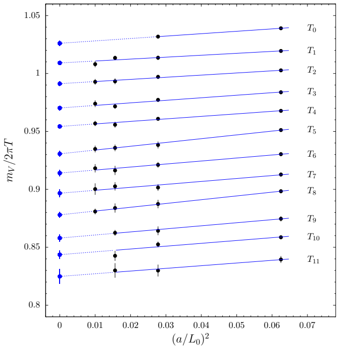
$$C_{\mathcal{O}}(x_3) = a^3 \sum_{x_0, x_1, x_2} \langle \mathcal{O}^a(x) \mathcal{O}^a(0) \rangle$$

screening masses are defined as

$$a m_{\mathcal{O}}(x_3) = \mathrm{arcosh}\left[\frac{C_{\mathcal{O}}(x_3 + a) + C_{\mathcal{O}}(x_3 - a)}{2\, C_{\mathcal{O}}(x_3)}\right]$$

Meson masses: continuum limit





The tree-level improved definitions

$$m_{\mathcal{O}} \to m_{\mathcal{O}} - \left[m_{\mathcal{O}}^{\text{free}} - 2\pi T\right]$$

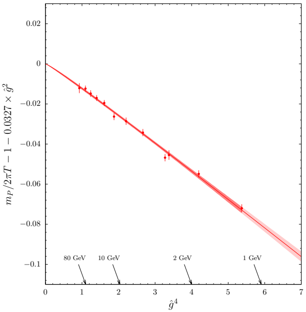
have been extrapolated to the continuum linearly in $(a/L_0)^2$

Meson masses: discussion and interpretation

Pseudoscalar mass:

$$\frac{m_P}{2\pi T} = 1 + p_2^{\mathrm{PT}}\,\hat{g}^2 + p_3\,\hat{g}^3 + p_4\,\hat{g}^4$$

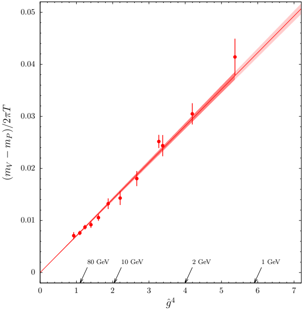
p_3=0.0038(22) and p_4=−0.0161(17)



Pseudoscalar-vector mass difference:

$$\frac{(m_V - m_P)}{2\pi T} = s_4\,\hat{g}^4$$

$s_4 = 0.00704(14)$



An effective $\hat{g}^4$ term explain the difference with PT in both cases over 2 orders of magnitude in T!

Comparison with the literature for mesons