



Prospects for Measuring Quark Polarization and Spin Correlations in $b\bar{b}$ and $c\bar{c}$ samples at the LHC

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arXiv:2311.08226, JHEP03(2024)063

Motivation

CMS and ATLAS measured the polarization and spin correlations in top pair production $pp \rightarrow t\bar{t}$.

The spin density matrix for t and \overline{t} :

$$\rho = \frac{1}{4} \left(\mathbb{1} \otimes \mathbb{1} + \tilde{B}_i^+ \sigma^i \otimes \mathbb{1} + \tilde{B}_i^- \mathbb{1} \otimes \sigma^i + \tilde{C}_{ij} \sigma^i \otimes \sigma^j \right)$$

Can we conduct similar measurements for the **other quarks**?

CMS Collaboration, PRD 100, 072002 (2019) [arxiv:1907.03729]



Simulated Quark Spin Correlations

MadGraph and MadSpin simulation of t, b and c.

$$\tilde{\mathbf{C}} = \begin{pmatrix} c_{kk} & c_{kn} + c_r & c_{rk} - c_n \\ c_{kn} - c_r & c_{nn} & c_{nr} + c_k \\ c_{rk} + c_n & c_{nr} - c_k & c_{rr} \end{pmatrix}$$

 $\{\hat{k}, \hat{n}, \hat{r}\}\$ set of orthonormal axes defined in the $q\bar{q}$ CM frame.

	$t\bar{t}$, no cuts	$b\bar{b}$, no cuts	$c\bar{c}$, no cuts
c_{kk}	0.324 ± 0.006	0.296 ± 0.004	0.284 ± 0.004
c_{rr}	0.009 ± 0.006	0.004 ± 0.004	-0.006 ± 0.004
c_{nn}	0.333 ± 0.006	0.299 ± 0.004	0.298 ± 0.004
$2c_{rk}$	-0.211 ± 0.008	-0.197 ± 0.006	-0.188 ± 0.006

Simulated Quark Spin Correlations

MadGraph and MadSpin simulation of t, b and c.

$$ilde{\mathbf{C}} = egin{pmatrix} c_{kk} & c_{kn} + c_r & c_{rk} - c_n \ c_{kn} - c_r & c_{nn} & c_{nr} + c_k \ c_{rk} + c_n & c_{nr} - c_k & c_{rr} \end{pmatrix}$$

 $\{\hat{k}, \hat{n}, \hat{r}\}\$ set of orthonormal axes defined in the $q\bar{q}$ CM frame.

	$t\bar{t}$, no cuts	$b\bar{b}$, no cuts	$c\bar{c}$, no cuts	$b\bar{b}$ with cuts	$c\bar{c}$ with cuts
c_{kk}	0.324 ± 0.006	0.296 ± 0.004	0.284 ± 0.004	-0.987 ± 0.004	-0.984 ± 0.006
c_{rr}	0.009 ± 0.006	0.004 ± 0.004	-0.006 ± 0.004	-0.603 ± 0.004	-0.609 ± 0.006
c_{nn}	0.333 ± 0.006	0.299 ± 0.004	0.298 ± 0.004	0.591 ± 0.004	0.603 ± 0.006
$2c_{rk}$	-0.211 ± 0.008	-0.197 ± 0.006	-0.188 ± 0.006	-0.038 ± 0.006	-0.008 ± 0.009

Measuring Polarization and Spin Correlations

For non-top quarks, we use baryons (Λ_q) .

$$\mu_q \propto \frac{1}{m_q}, \qquad m_q \gg \Lambda_{\rm QCD}$$



Measuring Polarization and Spin Correlations

For non-top quarks, we use baryons (Λ_q) .

 $\mu_q \propto \frac{1}{m_a}$, $m_q \gg \Lambda_{
m QCD}$

 $r_{T/L} = \frac{\mathcal{P}_{T/L}(\Lambda_q)}{\mathcal{P}_{T/L}(q)} \sim 0.5$

Some loss during hadronization:

Polarization retention factor. Depends on the direction (transverse or longitudinal).

Falk and Peskin, PRD 49, 3320 (1994) [hep-ph/9308241] Galanti, Giammanco, Grossman, Kats, Stamou, Zupan, JHEP 11 (2015) 067 [1505.02771]



Baryon Decay Angular Distributions

Using θ_i , the decay product angle in relation to one of the axes $\{\hat{k}, \hat{n}, \hat{r}\}$:

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_i^{\pm}} = \frac{1}{2} \left(1 + B_i^{\pm}\cos\theta_i^{\pm} \right)$$

$$B_i^{\pm} = \alpha_{\pm} r_i f b_i^{\pm}$$

$$Polarization factor factor for r_L$$

$$f = \frac{N_{sig}}{N_{sig} + N_{bg}}$$
Spin analyzing retention factor Sample Purity $(r_T \text{ or } r_L)$

$$\frac{1}{\sigma} \frac{d\sigma}{d(\cos\theta_i^{+}\cos\theta_j^{-})} = \frac{1}{2} \left(1 + C_{ij}\cos\theta_i^{+}\cos\theta_j^{-} \right) \ln\left(\frac{1}{|\cos\theta_i^{+}\cos\theta_j^{-}|}\right)$$

$$C_{ij} = \alpha_{+}\alpha_{-}r_i r_j f c_{ij}$$

Bottom Channel $\Lambda_b \rightarrow \mu^- \bar{\nu}_{\nu} X_c$

Selection	Decay Modes	Branching Ratio
Inclusive	$\Lambda_b \to X_c \mu^- \bar{\nu}_\mu$	11%
Somi inclusivo	$\Lambda_c^+ \to \Lambda X$	38%
Semi-menusive	$\Lambda \to p\pi^-$	64%
	$\Lambda_c^+ \to p K^- \pi^+$	6.3%
	$\Lambda_c^+ \to \Lambda \pi^+ \to p \pi^- \pi^+$	0.8%
	$\Lambda_c^+ \to pK_S \to p\pi^-\pi^+$	1.1%
Fyelusiyo	$\Lambda_c^+ \to \Lambda \pi^+ \pi^+ \pi^- \to p \pi^+ \pi^+ \pi^- \pi^-$	2.3%
Exclusive	$\Lambda_c^+ \to p K_S \pi^+ \pi^- \to p \pi^+ \pi^+ \pi^- \pi^-$	1.1%
	$\Lambda_c^+ \to \Sigma^+ \pi^+ \pi^-$	4.5%
	$\Lambda_c^+ \to \Sigma^- \pi^+ \pi^+$	1.9%
	total	18%

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Trigger Motivated Cuts

ATLAS Dimuon Triggers:

Dimuon trigger motivated cuts: $p_T^{\mu} > 15$ GeV without isolation and $|\eta| < 2.4$.

CMS Parked Data:

Recorded CMS data during Run 2 ($\sim 42 \text{ fb}^{-1}$) to be processed later. CMS collaboration, arXiv:2403.16134

- Single muon trigger.
- Lower specialized trigger p_T cuts, between 7 and 12 GeV.
- Transverse impact parameter significance cut.

Expected Statistical Uncertainties for Run 2 Data

${\rm channel} \rightarrow$	inclusive
m_{jj} cut [GeV]	$r_i \Delta b_i^{\pm}$
no cut	0.003
100	0.004
300	0.013
500	0.036
750	0.093
1000	0.19

Polarization and spin correlations range: [-1,1].

Simulated using Pythia8 + MadGraph.

Expected Statistical Uncertainties for Run 2 Data

$\mathrm{channel} \rightarrow$	inclusive	inclusive/inclusive		inclusive/exclusive		
$m_{jj} \operatorname{cut} [\text{GeV}]$	$r_i \Delta b_i^{\pm}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$	
no cut	0.003	0.14	0.10	0.11	0.079	
100	0.004	0.18	0.13	0.15	0.10	
300	0.013					
500	0.036					
750	0.093					
1000	0.19					

Expected Statistical Uncertainties for Run 2 Data

$\mathrm{channel} \rightarrow$	inclusive	inclusiv	inclusive/inclusive		inclusive/exclusive		
$m_{jj} \operatorname{cut} [\text{GeV}]$	$r_i \Delta b_i^{\pm}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$		
no cut	0.003	0.14	0.10	0.11	0.079		
100	0.004	0.18	0.13	0.15	0.10		
300	0.013						
500	0.036						
750	0.093						
1000	0.19						
parked data	0.0003	0.039	0.027	0.031	0.022		

Spin correlations can be measured in Run 2.

Charm Channel $\Lambda_{\rm c}^+ \rightarrow \mu^+ \nu_{\mu} \Lambda(\rightarrow p \pi^-)$

HL-LHC (3000 fb^{-1}) results:

$c\bar{c}$, semilep.	polarization	spin co	orrelations
m_{jj} cut [GeV]	$r_i \Delta b_i^{\pm}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$
no cut	0.001	0.060	0.042
100	0.002	0.078	0.054
300	0.005	0.35	0.25
500	0.017		
750	0.043		
1000	0.10		
1500	0.37		

Spin correlations will be measurable at the HL-LHC.

Polarization Retention Factor Measurement

Using the measured values (B_i, C_{ij}) and the partonic predictions (b_i, c_{ij}) :

$$r_i = \frac{B_i}{b_i \alpha f}$$

$$r_i^2 = \frac{C_{ii}}{c_{ii}\alpha^2 f}, \qquad r_i r_j = \frac{C_{ij} \pm C_{ji}}{2c_{k(ij)}\alpha_+\alpha_- f}$$

Can lead to information on QCD polarization transfer in fragmentation.

Entanglement and Bell's Inequality, $pp \rightarrow bb$

$$\Delta = \frac{-C_{nn} + |C_{kk} + C_{rr}| - 1}{2}$$

$$\mathcal{V} = C_{kk}^2 + C_{rr}^2 - 1$$

Entanglement measurement is possible in Run 2 (ATLAS, CMS and LHCb).

Bell's inequality will be possible at the HL-LHC.

Afik, Kats, Muñoz de Nova, Soffer, Uzan, arXiv:2406.04402

Conclusions & Future work

- We showed that there is merit to measuring polarization and spin correlations in dijet events for the heavy quarks.
- There is enough statistics already in Run 2 for polarization measurements in *b* and *c*.
- We can measure spin correlations in Run 2 using parked data for *b*.
- We will be able to measure spin correlations of *c* with HL-LHC.
- We proposed that using the measured spin correlations we can extract $r_{T/L}$.
- The spin correlations can be used as a probe for entanglement (arxiv:2406.04402) and BSM physics.

Thank you!

Supplemental Material

Heavy Quark Decay Chain

We decided to look into four decay chains:

Fragmenta	ation Fraction	Decay Scheme	BR	Spin analyzing power
		$\Lambda_b \to X_c \mu^- \bar{\nu}_\mu$	11%	$\alpha_{\mu^-} \approx -0.26, \ \alpha_{\bar{\nu}_{\mu}} \approx 1$
$b \to \Lambda_b$	7.0%	with $\Lambda \to p\pi^-$	2.7%	
		with Λ_c^+ reco.	2.0%	
		$\Lambda_c^+ \to p K^- \pi^+$	6.3%	$\alpha_{\rm eff} \approx 0.662$
$c \to \Lambda_c$	6.4%	$\Lambda_c^+ \to \Lambda \mu^+ \nu_\mu$	3.5%	$\alpha_{\mu^+} \approx 1$
		with $\Lambda \to p\pi^-$	2.2%	
$s \to \Lambda$	28%	$\Lambda \rightarrow n\pi^{-}$	61%	$\alpha \sim 0.75$
(z > 0.3)	2.070	$\Lambda \rightarrow p\pi$	0470	$\alpha_p \approx 0.15$

Prospects for All Channels Examined

	Polarization					Spin Correlations			
Onerla	Channel	Run	n 2		Quark	Channel	Run	. 2	
Quark	Channel	standard	parked	HL-LHC	Quark	Unanner	$\operatorname{standard}$	parked	
s				(🔨)	s				
	hadronic	(🖍)		\checkmark		hadronic			
c	semileptonic	\checkmark		\checkmark	c	semileptonic			\checkmark
	mixed	(🗸)		\checkmark		mixed			\checkmark
	inclusive	(🗸)	(🗸)	(🗸)		inclusive/inclusive	(\checkmark)	(🖌)	(🖌)
b	semi-inclusive		\checkmark	\checkmark		semi-inclusive/semi-inclusive	\checkmark	\checkmark	\checkmark
	exclusive	\checkmark	\checkmark	\checkmark	h	exclusive/exclusive	\checkmark	\checkmark	\checkmark
					0	inclusive/exclusive	(\checkmark)	(🖌)	(🖌)
						inclusive/semi-inclusive	(\checkmark)	(🖌)	(🖌)
						exclusive/semi-inclusive	\checkmark	\checkmark	\checkmark

Density Matrix

The most general case for spin-1/2:

$R \propto \tilde{A} \mathbb{1} \otimes \mathbb{1} + \tilde{B}_i^+ \sigma^i \otimes \mathbb{1} + \tilde{B}_i^- \mathbb{1} \otimes \sigma^i + \tilde{C}_{ij} \sigma^i \otimes \sigma^j$

where \tilde{A} determines the total cross section and kinematics, \tilde{B}^{\pm} are three-dimensional vectors characterizing the polarization, and \tilde{C} is a 3×3 matrix characterizing the spin correlations.

Statitical Uncertainty of Polarization and Spin Correlations

$$\Delta b_{i} = \frac{A_{B}}{r_{i}\alpha\sqrt{fN_{sig}}}$$
$$\Delta c_{ii} = \frac{A_{C_{ii}}}{r_{i}^{2}\alpha^{2}\sqrt{fN_{sig}}}$$
$$\Delta c_{k(ij)} = \frac{A_{C_{ij}}}{r_{i}r_{j}\alpha^{2}\sqrt{fN_{sig}}}$$



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Spin Measurement through Mesons

Mesons can't be used for spin measurements.

 $|\downarrow\rangle_b|\downarrow\rangle_{\bar{q}}, \qquad |\downarrow\rangle_b|\uparrow\rangle_{\bar{q}}$

For B^* meson, where $\frac{1}{2} \bigotimes \frac{1}{2} = 0 \bigoplus 1 = B \bigoplus B^*$:

$$P(\overline{B}^*(-1)) = \frac{1}{2}, \qquad P(\overline{B}^*(0)) = \frac{1}{4}, \qquad P(\overline{B}^*(1)) = 0, \qquad P(B(0)) = \frac{1}{4}$$
$$B^* \to B\gamma$$

We get uniform angular distribution, no information of the spin of the b quark.

Falk and Peskin, PRD 49 , 3320 (1994) [hep-ph/9308241]

19/07/2024

Spin Measurement through Baryons

For the baryons the spin is retained:

$$\begin{split} \downarrow \rangle_{b} |\downarrow \rangle_{q} |\downarrow \rangle_{q'}, \qquad |\downarrow \rangle_{b} |\downarrow \rangle_{q} |\uparrow \rangle_{q'}, \qquad |\downarrow \rangle_{b} |\uparrow \rangle_{q} |\downarrow \rangle_{q'}, \qquad |\downarrow \rangle_{b} |\uparrow \rangle_{q} |\uparrow \rangle_{q'} \\ \left(\frac{1}{2}\right)_{b} \otimes \left(\frac{1}{2}\right)_{q} \otimes \left(\frac{1}{2}\right)_{q'} = \frac{1}{2} \otimes (1 \oplus 0) = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2} \\ \Lambda_{b} \quad \Sigma_{b} \quad \Sigma_{b}^{*} \end{split}$$

We lose polarization information in hadronization through indirect Λ_b production:

$$b \to \Sigma_b, \Sigma_b^*, \Lambda_b, \qquad \Sigma_b^{(*)} \to \Lambda_b \pi$$

Falk and Peskin, PRD 49, 3320 (1994) [hep-ph/9308241]

19/07/2024

Energy and Momentum Reconstruction

Energy and momentum reconstruction in the b channel:

$$E_{\Lambda_b} = \langle z \rangle E_b = E_{X_c \mu} + E_{\nu}$$

The term E'_{jet} is the jet energy with subtraction from all tracks originating in the primary vertex:

$$E_{X_{c}\mu} = \frac{3\langle z \rangle E_{jet}' - (1 - \langle z \rangle) E_{\nu}}{1 + 2\langle z \rangle} \approx \frac{3\langle z \rangle}{2\langle z \rangle + 1} E_{jet}'$$

The second term in the numerator is an order of magnitude lower than that of the jet.

Energy and Momentum Reconstruction

We can get the momentum of the neutrino through:

$$P_{\nu}^{\perp} = -P_{\perp}, \qquad P_{\nu}^{\parallel} = -a \pm \sqrt{b}$$

where:

$$a = \frac{\left(m_{\Lambda_b}^2 - m^2 - 2P_{\perp}^2\right)P_{\parallel}}{2(P_{\parallel}^2 - E^2)}, \qquad b = \frac{\left(m_{\Lambda_b}^2 - m^2 - 2P_{\perp}^2\right)E^2}{4(P_{\parallel}^2 - E^2)} + \frac{E^2P_{\perp}^2}{P_{\parallel}^2 - E^2}$$

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Expected Number of Events for $b\overline{b}$ **in Run 2**

	m_{jj} c	ut [GeV]	$\sigma_{b\bar{b}}$ [pb]	N_b^i	N_b^s	N_b^e	
	n	o cut	9400	4.0×10^6	2.0×10^5	3.6×10^5	
		100	7200	3.1×10^6	1.5×10^5	2.8×10^5	
		300	560	2.3×10^5	1.0×10^4	2.1×10^4	
		500	82	3.1×10^4	$1.0 imes 10^3$	$2.8 imes 10^3$	
		750	14	4.8×10^3	120	430	
	-	1000	3.4	1.1×10^3	22	100	
	-	1500	0.34	100		9.2	
	park	ed data		$3.5 imes 10^8$	$3.6 imes 10^7$	3.1×10^7	
	puri	ty f [%]		7.4	57	66	
$\frac{m_{jj} \text{ cut}}{[\text{GeV}]}$	$\sigma_{bar{b}}\ [{ m pb}]$	$N^{ii}_{bar{b}}$	$N^{ss}_{b\bar{b}}$	$N^{ee}_{b\bar{b}}$	$N^{is}_{bar{b}}$	$N^{ie}_{bar{b}}$	$N^{se}_{b\bar{b}}$
no cut	10000	8.0×10^{4}	200	640	8.1×10^3	1.4×10^4	730
100	5900	4.7×10^4	121	380	4.8×10^3	$8.5 imes 10^3$	430
300	340	2.7×10^3	5.0	21	230	490	20
500	46	360		2.9	20	65	1.8
parked data		1.1×10^{6}	1.1×1	0^4 8700	2.2×10^5	$1.9 imes 10^5$	2.0×10^4
purity f [%]		0.55	32	44	4.2	4.9	38

Expected Number of Polarization Events for $b\overline{b}$ **in HL-LHC**

m_{jj} cut [GeV]	$\sigma_{b\bar{b}} \ [\mathrm{pb}]$	N_b^i	N_b^s	N^e_b
no cut	33000	$3.0 imes 10^8$	$3.3 imes 10^7$	$2.7 imes 10^7$
100	18300	1.7×10^8	1.8×10^7	$1.5 imes 10^7$
300	990	8.2×10^6	$7.3 imes 10^5$	$7.4 imes 10^5$
500	126	$9.3 imes 10^5$	$6.3 imes 10^4$	8.4×10^4
750	21	$1.5 imes 10^5$	$7.6 imes 10^3$	1.4×10^4
1000	5.6	$3.7 imes 10^4$	$1.6 imes 10^3$	$3.3 imes 10^3$
1500	0.49	$2.9 imes 10^3$	94	260
2000	0.088	490	12	44
purity f [%]		7.4	57	66

Expected Number of Spin Correlations Events for $b\overline{b}$ **in HL-LHC**

$\frac{m_{jj} \text{ cut}}{[\text{GeV}]}$	$\sigma_{bar{b}}\ [{ m pb}]$	$N^{ii}_{bar{b}}$	$N^{ss}_{bar{b}}$	$N^{ee}_{bar{b}}$	$N^{is}_{bar{b}}$	$N^{ie}_{bar{b}}$	$N^{se}_{bar{b}}$
no cut	39000	$6.7 imes 10^6$	$8.1 imes 10^4$	5.4×10^4	$1.5 imes 10^6$	1.2×10^6	1.3×10^5
100	15000	$2.6 imes 10^6$	3.1×10^4	2.1×10^4	$5.7 imes 10^5$	4.7×10^5	$5.1 imes 10^4$
300	570	$9.6 imes 10^4$	610	780	$1.5 imes 10^4$	1.7×10^4	1.4×10^3
500	74	1.2×10^4	35	98	1.3×10^3	2.2×10^3	120
750	12	$2.0 imes 10^3$	3.0	16	150	360	13
1000	2.9	460		3.7	27	82	2.5
purity f [%]		0.55	32	44	4.2	4.9	38

Expected Statistical Uncertainty for $b\overline{b}$ **in Run 2**

$\mathrm{channel} \rightarrow$	semi-inclusive	semi-incl	lusive/semi-inclusive	semi-inc	lusive/inclusive
m_{jj} cut [GeV]	$r_i \Delta b_i^{\pm}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$
$\operatorname{no} \operatorname{cut}$	0.005	0.36	0.25	0.16	0.11
100	0.006	0.47	0.33	0.21	0.15
300	0.022				
500	0.072				
750	0.21				
parked data	0.0004	0.050	0.035	0.031	0.022

$\mathrm{channel} \rightarrow$	exclusive	exclusiv	ve/exclusive	exclusive/semi-inclusive				
m_{jj} cut [GeV]	$r_i \Delta b_i^{\pm}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$			
no cut	0.003	0.18	0.11	0.18	0.13			
100	0.004	0.23	0.16	0.23	0.16			
300	0.015							
500	0.040							
750	0.10							
1000	0.21							
parked data	0.0004	0.049	0.034	0.034	0.024			

Expected Statistical Uncertainty for $b\overline{b}$ **in HL-LHC**

$\mathrm{channel} \rightarrow$	inclusive	inclusiv	ve/inclusive	inclusive/exclusive			
$m_{jj} \operatorname{cut} [\text{GeV}]$	$r_i \Delta b_i^{\pm}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$		
no cut	0.0004	0.015	0.011	0.012	0.0086		
100	0.0005	0.025	0.017	0.020	0.014		
300	0.0022	0.13	0.091	0.10	0.071		
500	0.0063	0.36	0.26	0.29	0.20		
750	0.016						
1000	0.032						
1500	0.11						
2000	0.27						

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Expected Statistical Uncertainty for $b\overline{b}$ **in HL-LHC**

$\mathrm{channel} \rightarrow$	semi-inclusive	semi-inclusive/semi-inclusive		sem	semi-inclusive/inclusive		_			
$m_{jj} \operatorname{cut} [\operatorname{GeV}]$	$r_i \Delta b_i^{\pm}$	$r_i^2 \Delta c_{ii}$	$r_i r$	$c_j \Delta c_{ij(\ell)}$	$r_i^2 \Delta$	$\Delta c_{ii} = r_i r_j \Delta c_{ij(\ell)}$				
no cut	0.0004	0.018		0.013	0.0	12	0.0084			
100	0.0005	0.029	(0.021	0.0	19	0.013			
300	0.0027	0.21		0.15	0.1	12	0.082			
500	0.0091									
750	0.026			channe	$1 \rightarrow$	exclusive exclusiv		ve/exclusive	exclusive	e/semi-inclusive
1000	0.058			mai cut [G	eV]	$r_i \Delta b_i^{\pm}$	$r^2 \Delta c_{ii}$	$r_i r_i \Delta c_{ii}(\ell)$	$r^2 \Delta c_{ii}$	$r_i r_i \Delta c_{ii}(\ell)$
1500	0.24			- no cut		0.0004	$\frac{1}{0.019}$	$\frac{0.013}{0.013}$	$\frac{1}{0.013}$	$\frac{0.0093}{0.0093}$
				100		0.0005	0.031	0.022	0.021	0.015
				300		0.0025	0.16	0.11	0.13	0.091
				500		0.0070				
				750 1000 1500 2000		0.018				
						0.037				
						0.13				
						0.32				

Expected Statistical Uncertainty and Number of Events for $c\overline{c}$ **in HL-LHC**

$car{c}$, semilep.	l t	olarization		spin correlations						
$m_{jj} \operatorname{cut} [\operatorname{GeV}]$	$\sigma_{c\bar{c}} \; [\mathrm{pb}] \qquad N_c$		$r_i \Delta b_i^{\pm}$	$\sigma_{c\bar{c}} \; [\mathrm{pb}]$	$N_{c\bar{c}}$	$r_i^2 \Delta c_{ii}$	$r_i r_j \Delta c_{ij(\ell)}$			
no cut	9400	$1.7 imes 10^6$	0.001	13000	2.4×10^3	0.060	0.042			
100	6700	$1.2 imes 10^6$	0.002	7700	$1.5 imes 10^3$	0.078	0.054			
300	620	$9.8 imes 10^4$	0.005	540	70	0.35	0.25			
500	110	1.1×10^4	0.017	89	4.8					
750	23	$1.6 imes 10^3$	0.043	16						
1000	5.4	280	0.10	4.1						
1500	0.62	22	0.37	0.47						

Entanglement and Bell's Inequality, $pp \rightarrow b\overline{b}$

	$\sigma \epsilon_{\mu\mu}$ [pb] \mathcal{L} [fb ⁻¹]	N	C_{kk}	C_{rr}	C_{nn}	Δ	v	r_L	$\sigma^{ m stat}_\Delta$	$\sigma_{\mathcal{V}}^{\mathrm{stat}}$	$\frac{\Delta}{\sigma^{\rm stat}_{\Delta}}$	$rac{\mathcal{V}}{\sigma_\mathcal{V}^{\mathrm{stat}}}$	$\frac{\Delta}{\sigma_{\Delta}^{\rm tot}}$	$\frac{\mathcal{V}}{\sigma_{\mathcal{V}}^{\rm tot}}$
	Run 2 , $\sqrt{s} = 13$ TeV														
ATLAS	0.6×10^3	3 140	1.4×10^4	0.06	0.62	2 - 0.61	0.60	0.31	0.75	0.19	0.48	3.1	0.6	2.6	0.6
ATLAS	9.0 × 10	140		0.90				0.51	0.45	0.32	1.11	1.8	0.3	1.7	0.3
LHCb $\Lambda > 0.4$	2.6×10^{6}	57	1.2×10^{4}	0.62	0.76	6 -0.66	0.52	-0.04	0.75	0.11	0.25	4.6	-0.1	3.4	-0.1
LITOD, $\Delta > 0.4$	2.0 × 10	0.1	4.2 ~ 10	0.02	0.10				0.45	0.19	0.46	2.7	-0.1	2.4	-0.1
CMS B parking	1.1×10^5	41 6	$3.7 imes 10^5$	0.88	0.61	-0.58	0.53	0.14	0.75	0.038	0.089	> 10	1.6	4.7	1.5
ONIO D parking		41.0							0.45	0.064	0.20	8.4	0.7	4.3	0.7
					н	L-LHO	C, \sqrt{s}	$\bar{s} = 14$	TeV						
ATLAS V > 0.3	3.7×10^4 30	3000	$6.2 imes 10^5$	0.94	0.86	6 - 0.85	0.82	0.63	0.75	0.03	0.08	> 10	7.5	4.9	4.2
ATEAD, V > 0.5		5000							0.45	0.05	0.17	> 10	3.7	4.8	3.0
LHCb $V > 0.3$	3.0×10^{6}	300	$2.2 \times 10^5 - 0.83$	0.83	$33 \ 0.88 \ -0$	-0.83	-0.83 0.77	7 0.48	0.75	0.040	0.11	> 10	4.3	4.8	3.3
hitob, V > 0.5	J. 0 × 10	000	0.0×10	0.00		-0.00			0.45	0.067	0.21	> 10	2.2	4.6	2.0
CMS B parking, $\mathcal{V} > 0.2$	1.2×10^{5}	⁵ 800	3.2×10^{6}	0.84	0.85	-0.80	0.75	0.43	0.75	0.013	0.035	> 10	> 10	5.0	4.6
	1.2 ~ 10	1.2 × 10 800 3.	0.2 \ 10	5.2 × 10 0.04	0.85	5 – <u>0.8</u> 0	0 0.75	0.10	0.45	0.022	0.068	> 10	6.3	4.9	3.9