# Path integral treatment of coherence effects in charmonium production in nuclear ultra-peripheral collisions


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# Introduction

- heavy quarkonia are important tool for studying dense matter created in heavy-ion collisions and QCD dynamics
- ultra-peripheral collisions (UPC) are dominant source of heavy quarkonia and provide unique acces to photon-nuclear interactions
- description of coherent quarkonium photoproduction $\gamma^* A \to VA$ within the light-front (LF) color dipole approach

  *B.Z. Kopeliovich et al., PRD 107 (2023) 054005*

  *M. Krelina, J. Nemchik, PRD 102 (2020) 114033*

- this talk - proper treatment of finite coherence length as well as gluon shadowing via Green's function approach in coherent production of $J/\psi$ and $\psi'$

  *J. Obertova, J. Nemchik, arXiv:2407.02219, submitted to PRD (2024)*

# Light-front color dipole approach

- the amplitude of a diffractive process is treated as elastic scattering of a $\bar{Q}Q$ fluctuation of the incident particle
- possible expansion of a projectile into the Fock states, e.g. photon

$$|\gamma\rangle = |\bar{Q}Q\rangle + |\bar{Q}Qg\rangle + |\bar{Q}Q2g\rangle + ...$$

- **coherence length** defined for each Fock state
$l_c^{\bar{Q}Q} = \frac{2k}{M^2_{\bar{Q}Q}+Q^2} \gg l_c^{\bar{Q}Qg} = \frac{2k}{M^2_{\bar{Q}Qg}+Q^2} \gg l_c^{\bar{Q}Q2g} \gg ...$
- **shadowing phenomena** $\Rightarrow$ destructive interference of amplitudes related to interactions on different bound nucleons
- $l_c^{\bar{Q}Q} \Rightarrow$ **quark shadowing**, $l_c^{\bar{Q}Qg} \Rightarrow$ **gluon shadowing**

# Quarkonium photoproduction cross-section in UPC

- cross section for photoproduction of a vector meson in the rest frame of the target nucleus $A$:

$$k\frac{d\sigma}{dk} = \int d^2 b_A \int d^2 b\, n_\gamma(k, \vec{b} - \vec{b}_A, y)\frac{d^2\sigma_A(b, s)}{d^2 b} + \{y \to -y\},$$

  $\vec{b}_A > 2R_A \Rightarrow$ relative impact parameter of the collision
  $\vec{b} \Rightarrow$ impact parameter of photon-nucleon collision relative to the center of one of the nuclei

- photon flux $n(k, \vec{b})$ induced by the projectile nucleus within the one-photon-exchange

$$n_\gamma(k, \vec{b}) = \frac{\alpha_{em} Z^2 k^2}{\pi^2 \gamma^2}\left[K_1^2\left(\frac{bk}{\gamma}\right) + \frac{1}{\gamma^2} K_0^2\left(\frac{bk}{\gamma}\right)\right],$$

  the Lorenz factor gamma $\gamma = 2\gamma_{\rm col}^2 - 1$, where $\gamma_{\rm col} = \frac{\sqrt{s_N}}{2M_N}$

# Cross section for coherent quarkonia production

- In UPC at LHC the photon energy is sufficiently high at $y = 0$ $\Rightarrow l_c^{\bar{Q}Q} \gg R_A$
- coherent (elastic) cross section for quarkonia production $\gamma^* A \to VA$

$$\frac{d^2\sigma_A(b,s)}{d^2b}\Big|_{l_c^{\bar{Q}Q}\gg R_A} = \Big| \int d^2r \int_0^1 d\alpha \Psi_V^*(\vec{r},\alpha)\left(1-\exp\left[-\frac{1}{2}\sigma_{\bar{q}q}(r,s)T_A(b)\right]\right)\Psi_{\gamma^*}(\vec{r},\alpha)\Big|^2$$

where $T_A(b) = \int_{-\infty}^{\infty} dz \rho_A(b,z)$ is the nuclear thickness function and c.m energy $s = M_V\sqrt{s_N}\exp(y)$

- "frozen" (eikonal) approximation - transverse separation of the $|\bar{Q}Q\rangle$ Fock state does not change during propagation through the medium

# Correction for finite coherence length $l_c^{\bar{Q}Q}$

- forward or backward rapidities at LHC or RHIC $\Rightarrow$ coherence length becomes too short ($l_c^{\bar{Q}Q} \sim 3 - 6$ fm $\lesssim R_A$) at least for one of the colliding nuclei
- correction for finite coherence length using form factors

$$\frac{d^2\sigma_A(b,s)}{d^2b} = \frac{d^2\sigma_A(b,s)}{d^2b}\Big|_{l_c^{\bar{Q}Q} \gg R_A} \cdot F^{coh}(s, l_c(s))$$

- form factor calculated using Green's function technique $\Rightarrow$ harmonic-oscillator (HO) VM wave function + quadratic form for the dipole cross section = analytic form of the Green's function

  *J. Nemchik et al., PRD 107 (2023) 054005*

# Gluon shadowing

- gluon shadowing (GS) = another source of nuclear suppression, leading twist effect
- GS corresponds to higher Fock components of the photon $|\bar{Q}Qg\rangle, |\bar{Q}Q2g\rangle, ...$
- at sufficiently high energies $l_c^{\bar{Q}Qg} \gg 2$ fm $\Rightarrow$ intensity of gluon radiation reduced compared to Bethe-Heitler regime $\Rightarrow$ gluon shadowing
- transverse size of $\bar{Q}Q - g$ dipole fluctuates during propagation through the nucleus $\Rightarrow$ GS calculated using the Green's function formalism *B.Z. Kopeliovich et al., PRD 105 (2022) 054023*

# The Green's function approach

- coherent cross section for quarkonia production $\gamma^* A \to VA$

$$\frac{d^2\sigma_A(b,s)}{d^2 b} = |\int_{-\infty}^{\infty} dz_1 \rho_A(b,z_1) H_1(s,b,z_1)|^2,$$

where $\rho_A(b,z_1)$ is the nuclear density distribution and

$$H_1(s,b,z_1) = \int_0^1 d\alpha \int d^2 r_1 \int d^2 r_2 \psi_V^*(\vec{r}_2,\alpha) G_{Q\bar{Q}}(z_2,\vec{r}_2;z_1,\vec{r}_1) \sigma_{Q\bar{Q}}(\vec{r}_1,s) \psi_\gamma(\vec{r}_1,\alpha)|_{z_2\to\infty}.$$

- $G_{Q\bar{Q}}(z_2,\vec{r}_2;z_1,\vec{r}_1)$ - Green's function describes the propagation of $\bar{Q}Q$ fluctuation throught the medium
- $\sigma_{\bar{Q}Q}(r,s)$ - dipole cross section, we used KST *(B. Kopeliovich et al., PRD62 (2000) 054022)* and GBW *(K. Golec-Biernat et al., JHEP 03 (2018) 102)* models
- $\psi_V^*(\vec{r}_2,\alpha)$ - VM wave function, $\psi_\gamma(\vec{r}_1,\alpha)$ - photon wave function

$\vec{r}$ is transverse separation of $Q\bar{Q}$ pair, $\alpha = \frac{p_Q^+}{p_\gamma^+}$ - fraction of photon momentum carried by quark

# Schrödinger equation for the Green's function

- evolution equation for the Green's function in LF frame

$$i\frac{d}{dz_1}G_{Q\bar{Q}}(z_2,\vec{r}_2;z_1,\vec{r}_1) = \left[\frac{\epsilon^2 - \triangle_{r_2}}{2k\alpha(1-\alpha)} + V_{\bar{Q}Q}(z_2;\vec{r}_2,\alpha)\right] G_{Q\bar{Q}}(z_2,\vec{r}_2;z_1,\vec{r}_1) \tag{1}$$

  where $\epsilon = m_Q^2 + \alpha(1-\alpha)Q^2$ and k is the photon energy

- $V_{\bar{Q}Q}(z;\vec{r}_2,\alpha)$ - complex potential in the light-front frame
  - $\mathrm{Im}\,V_{\bar{Q}Q}(z;\vec{r}_2,\alpha) = -i\frac{\sigma_{\bar{q}q}}{2}\rho_A(b,z)$

- analytic form of $\mathrm{Re}\,V_{\bar{Q}Q}(z;\vec{r}_2,\alpha)$ known only for HO potential

- other potential models - BT *(W. Buchmuller et al., PRD 24 (1981) 132)*, POW *(A. Martin, PLB 93 (1980) 338)* $\Rightarrow$ boost of the $V_{\bar{Q}Q}(\rho)$ to the light-front frame + numerical solution of Eq.(1) *(J. Nemchik, PRC 68 (2003) 035206)*

# Re$V_{\bar{Q}Q}$ in the light-front frame

- we know the LF VM wave function $\Psi_V(\vec{r},\alpha)$ for any rest frame $\bar{Q}Q$ potential
- derivation of Re$V_{\bar{Q}Q}(z;\vec{r},\alpha)$ from LF Schrödinger equation

$$\left(\frac{A^2(\alpha)-\Delta}{2k\alpha(1-\alpha)} + \mathrm{Re}V_{\bar{Q}Q}(z;\vec{r},\alpha)\right)\Psi_V(\vec{r},\alpha) = E_{LF}\Psi_V(\vec{r},\alpha)$$

$$\Downarrow$$

$$\mathrm{Re}V^*_{\bar{Q}Q}(z;\vec{r},\alpha) = \mathrm{Re}V_{\bar{Q}Q}(z;\vec{r},\alpha)+\frac{A^2(\alpha)}{2k\alpha(1-\alpha)} = E_{LF}+\frac{1}{\Psi_V(r,\alpha)}\frac{\Delta\Psi_V(r,\alpha)}{2k\alpha(1-\alpha)}$$

- we assume that $E_{LF} = E_{rest}\frac{m_Q 4\alpha(1-\alpha)}{2k\alpha(1-\alpha)}$
- shift $A^2(\alpha)$ determined from the non-relativistic limit ($\vec{r}\to\rho, \alpha\to 0.5$)

# Coherent $J/\psi$ and $\psi'$ photoproduction in UPC

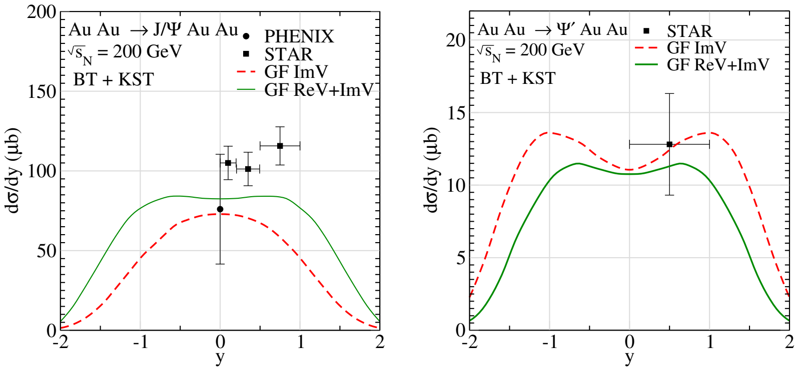


Fig.1: The impact of Re$V_{Q\bar{Q}}$ on rapidity distribution $d\sigma/dy$ for coherent $J/\psi$ (left panel) and $\psi'$ (right panel) production in UPC at c.m. collision energy $\sqrt{s_N} = 200$ GeV. Values of $d\sigma/dy$ are calculated within the Green's function formalism adopting the BT $Q$-$\bar{Q}$ interaction potential and KST model for dipole cross section. The data from STAR and PHENIX collaborations are shown for comparison.

# Coherent $J/\psi$ and $\psi'$ photoproduction in UPC

**gluon shadowing**

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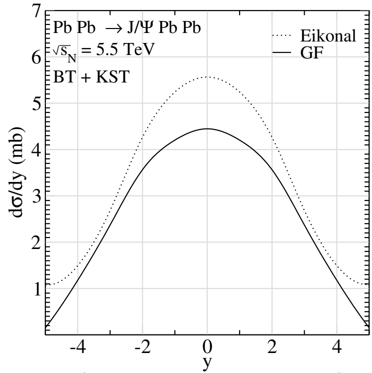


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**reduced coherence length $l_c^{\bar{Q}Q}$ effect**

Fig.2: Rapidity distribution of coherent cross section for photoproduction of $J/\psi$ in UPC at $\sqrt{s_N} = 5.5$ TeV. Comparison of cross section calculated with eikonal formula (dotted line) and with the Green's function formalism + GS (solid line).

# Coherent $J/\psi$ and $\psi'$ photoproduction in UPC

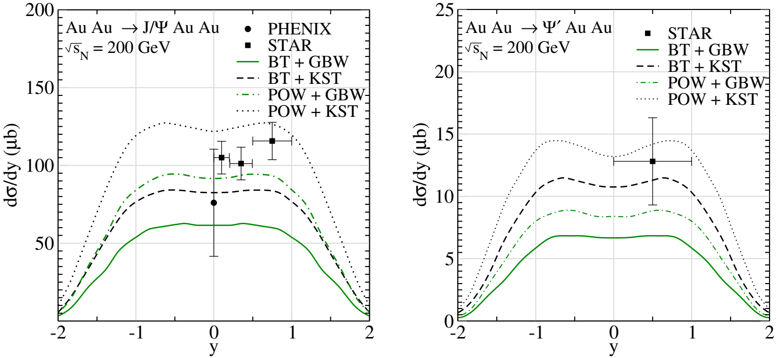


Fig.3: Rapidity distribution of coherent cross section for photoproduction of $J/\psi$ (left) and $\psi'$ (right) in UPC at $\sqrt{s_N} = 200$ GeV. Calculations performed using the POW and BT potentials with GBW and KST models for the dipole cross section. The data from STAR and PHENIX collaborations are shown for comparison.

# Coherent $J/\psi$ and $\psi'$ photoproduction in UPC

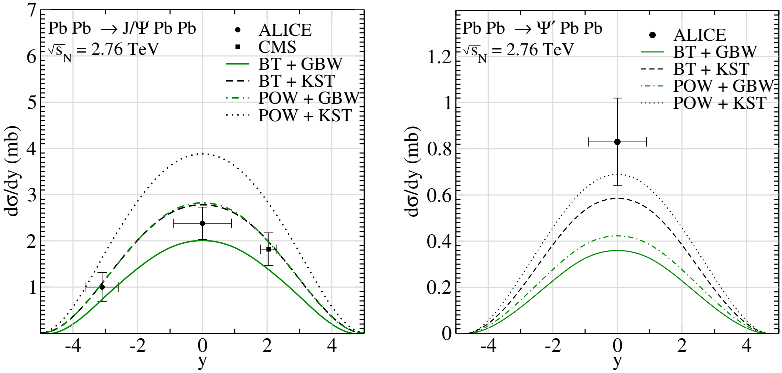


Fig.4: Rapidity distribution of coherent cross section for photoproduction of $J/\psi$ (left) and $\psi'$ (right) in UPC at $\sqrt{s_N} = 2.76$ TeV. Calculations performed using the POW and BT potentials with GBW and KST models for the dipole cross section. The data from ALICE and CMS collaborations are shown for comparison.

# Coherent $J/\psi$ and $\psi'$ photoproduction in UPC

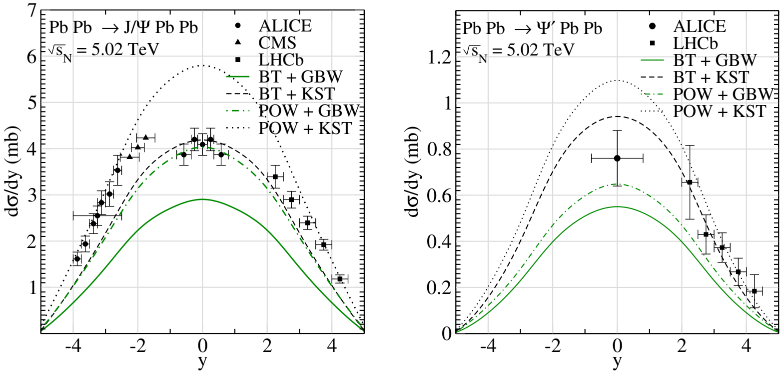


Fig.5: Rapidity distribution of coherent cross section for photoproduction of $J/\psi$ (left) and $\psi'$ (right) in UPC at $\sqrt{s_N} = 5.02$ TeV. Calculations performed using the POW and BT potentials with GBW and KST models for the dipole cross section. The data from ALICE, CMS and LHCb collaborations are shown for comparison.

# Coherent electroproduction of $J/\psi$ and $\psi'$ at EIC energies

$$R_A^{coh} = \frac{\sigma_{\gamma^* A \to J/\psi A}}{A\ \sigma_{\gamma^* N \to J/\psi N}}$$

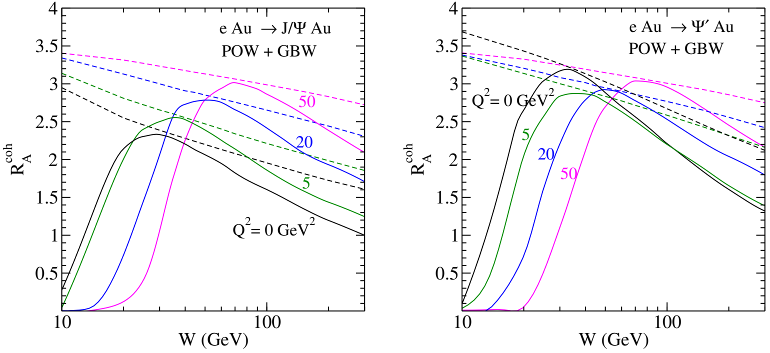


Fig.6: Ratios $R_A^{coh}$ for the $J/\psi$ (left) and $\psi'$ (right) coherent electroproduction on the gold target as a function of c.m. energy $W$ at several fixed values of the photon virtuality $Q^2 = 0$, 5, 20 and 50 GeV$^2$.

# Conclusions

- study of coherent photoproduction of heavy quarkonia within LF color dipole model using the Green's function approach

  *J. Obertova, J. Nemchik, arXiv:2407.02219, submitted to PRD*

- proposed procedure for obtaining Re$V_{\bar{Q}Q}(z, \vec{r}, \alpha)$ in the LF frame for any $\bar{Q} - Q$ potential model
- effects of quantum coherence included in the calculations:
  - reduction of coherence length for $|\bar{Q}Q\rangle$ Fock state
  - gluon shadowing for $|\bar{Q}Qg\rangle$ state
- predictions for rapidity distributions for coherent $J/\psi$ and $\psi'$ photoproduction are in good agreement with data
- predictions for $R_A^{coh}$ for electroproduction of $J/\psi$ and $\psi'$ manifest the effects of reduced coherence length and gluon shadowing and can be verified by future measurements at EIC

**Thank you for your attention!**

# Back-up slides

# Back-up

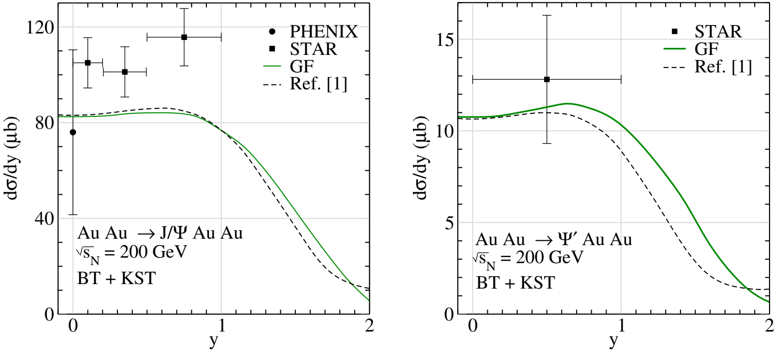


Fig.7: Present calculations based on path integral technique (solid lines) compared with our previous results from Ref. [1] (dashed lines).


[1] *J. Nemchik et al., PRD 107 (2023) 054005*