Bound state basics

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Bound states are central in the SM

QED atoms, QCD hadrons

But they are not found in QFT textbooks.

Cf. the *S*-matrix (the Interaction Picture).

“The notion of the quantum state … seems to fade from view when doing QFT.”

A.S. Blum, Stud. Hist. Phil. Sci. B60 (2017) 46 [2011.05908]

Scattering amplitudes vs. Bound states

The perturbative S-matrix expands around $\mathcal{O}(\alpha^0)$ (free) states.

Atoms are perturbed around initial bound states with $\mathcal{O}(\alpha^\infty)$ wave functions:

Schrödinger equation, Bethe-Salpeter equation, …

The choice is not unique since $\mathcal{O}(\alpha^n)$ may be shuffled from the initial state

Caswell & Lepage (1978)

Feynman diagrams do not have bound state poles at any finite order in α

$\Longrightarrow$ Atoms may be considered "non-perturbative"

Positronium hyperfine splitting $\Delta E = M(^3S_1) - M(^1S_0)$ is given by a power expansion in α

$$\frac{\Delta E}{m_e} = \frac{7}{12}\alpha^4 - \left(\frac{8}{9} + \frac{\ln 2}{2}\right)\frac{\alpha^5}{\pi} - \frac{5}{24}\alpha^6 \ln\alpha + \left[\frac{1367}{648} - \frac{5197}{3456}\pi^2 + \left(\frac{221}{144}\pi^2 + \frac{1}{2}\right)\ln 2 - \frac{53}{32}\zeta(3)\right]\frac{\alpha^6}{\pi^2}$$

$$-\frac{7\alpha^7}{8\pi}\ln^2\alpha + \left(\frac{17}{3}\ln 2 - \frac{217}{90}\right)\frac{\alpha^7}{\pi}\ln\alpha + \mathcal{O}(\alpha^7)$$

$\Longrightarrow$ Atoms may be considered "perturbative"

Hamiltonian eigenstates (QED)

$$H(t)\,|M,\boldsymbol{P},t\rangle = \sqrt{M^2+\boldsymbol{P}^2}\,|M,\boldsymbol{P},t\rangle$$

M: Rest mass $\boldsymbol{P}$: CM momentum

$$|M,\boldsymbol{P},t\rangle = \sum_j \phi_j\,|\{e^-\},\{e^+\},\{\gamma_T\}\rangle_j$$

Fock expansion in terms of e^-, e^+, γ_T constituents

Note: γ_T are transversely polarized photons

Positronium: The $|e^-e^+\rangle$ Fock state dominates at $\boldsymbol{P}=0$,

$$|M,\boldsymbol{P}=0,t=0\rangle \simeq \int \frac{d^3\boldsymbol{k}}{2E_k(2\pi)^3}\,\phi_{e^-e^+}(\boldsymbol{k})\,|e^-(\boldsymbol{k})e^+(-\boldsymbol{k})\rangle$$

and $\phi_{e^-e^+}$ satisfies the Schrödinger eq.

$$\left[-\frac{\nabla^2}{m_e}-\frac{\alpha}{|\boldsymbol{x}|}\right]\phi_{e^-e^+}(\boldsymbol{x}) = E_B\,\phi_{e^-e^+}(\boldsymbol{x})$$

with binding energy $E_B \equiv M - 2m_e$

Questions: Does the potential $-\alpha/|\boldsymbol{x}|$ remain instantaneous when $|\boldsymbol{P}| \gg M$? [Yes]

Do other Fock states than $|e^-e^+\rangle$ contribute when $|\boldsymbol{P}| \gg M$? [Yes]

Instantaneous potential (QED)

No physical particle can move faster than light.

Gauge theory Lagrangians lack $\partial_t A^0$ and $\nabla \cdot \boldsymbol{A}$ terms.

$\Longrightarrow$ A^0 and A_L do not propagate in space-time. They are gauge dependent:

$\nabla \cdot \boldsymbol{A}(t, \boldsymbol{x}) = 0$ Coulomb gauge

$A^0(t, \boldsymbol{x}) = 0$ Temporal gauge

Gauge condition for all $\boldsymbol{x}$ at the same t induces an instantaneous potential

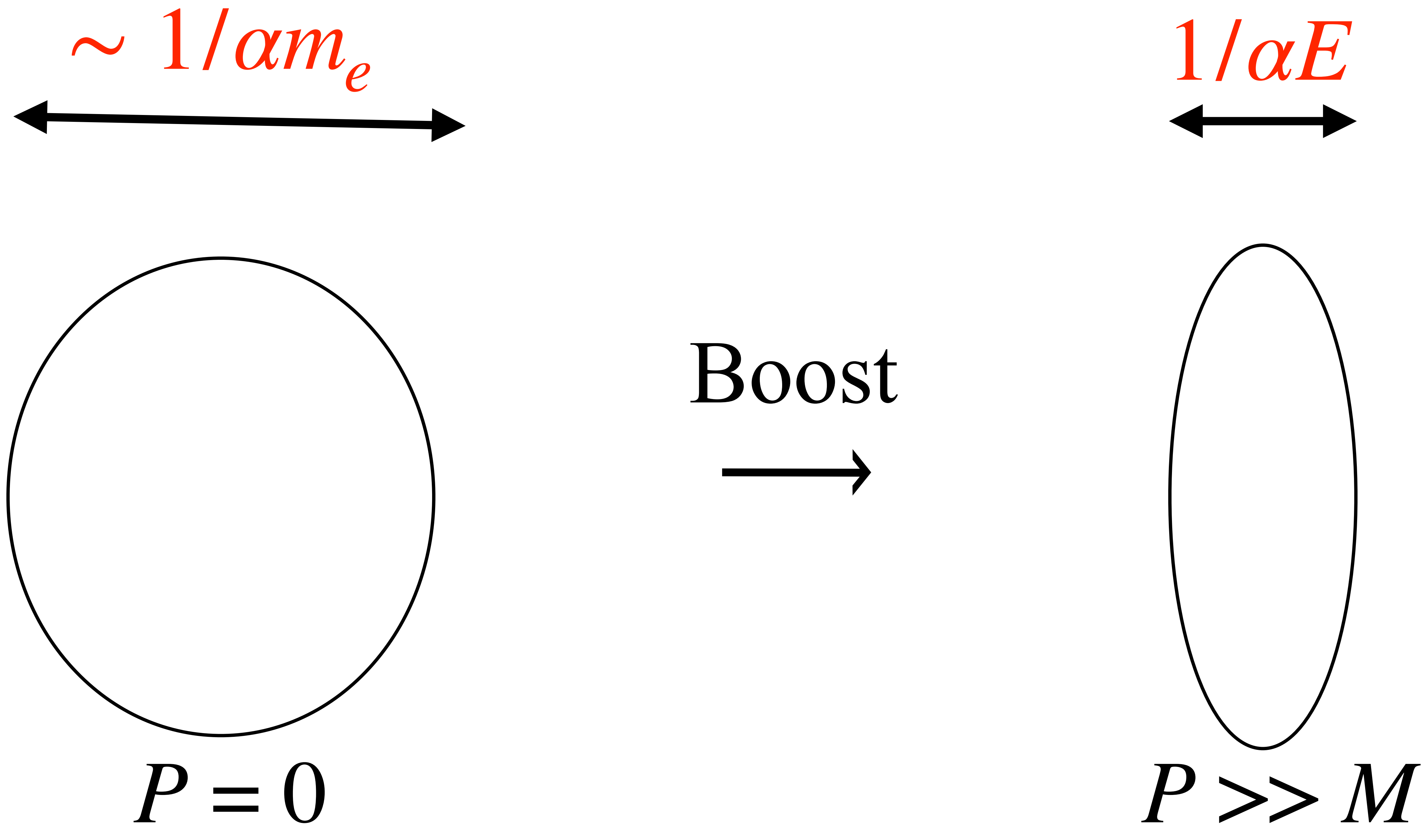
Consider here temporal gauge: $\boldsymbol{E} = -\partial_t \boldsymbol{A}$ (quantization without constraints)

Invariance of physical states under t-independent gauge transformations requires:

$\left(\nabla \cdot \boldsymbol{E} - e\psi^\dagger\psi\right)|phys\rangle = 0$ Determines E_L from instantaneous electron positions:

$$\boldsymbol{E}_L(t, \boldsymbol{x})\,|phys\rangle = -\nabla_x \int d\boldsymbol{y}\, \frac{e}{4\pi|\boldsymbol{x} - \boldsymbol{y}|}\, \psi^\dagger\psi(t, \boldsymbol{y})\,|phys\rangle$$

Higher Fock state (QED)



Lorentz contraction

The Coulomb potential $\sim -\alpha^2 E/4$ grows with P, wheras excitation energies $\propto 1/P$?

The Poincaré covariance of atoms is realised dynamically.

QED: $|e^- e^+ \gamma_T\rangle$ Fock state contributes for $P > 0$, and subtracts the large Coulomb energy, ensuring $E = \sqrt{M^2 + P^2}$

M. Järvinen, Phys. Rev. D71 (2005) 085006 [hep-ph/0411208]

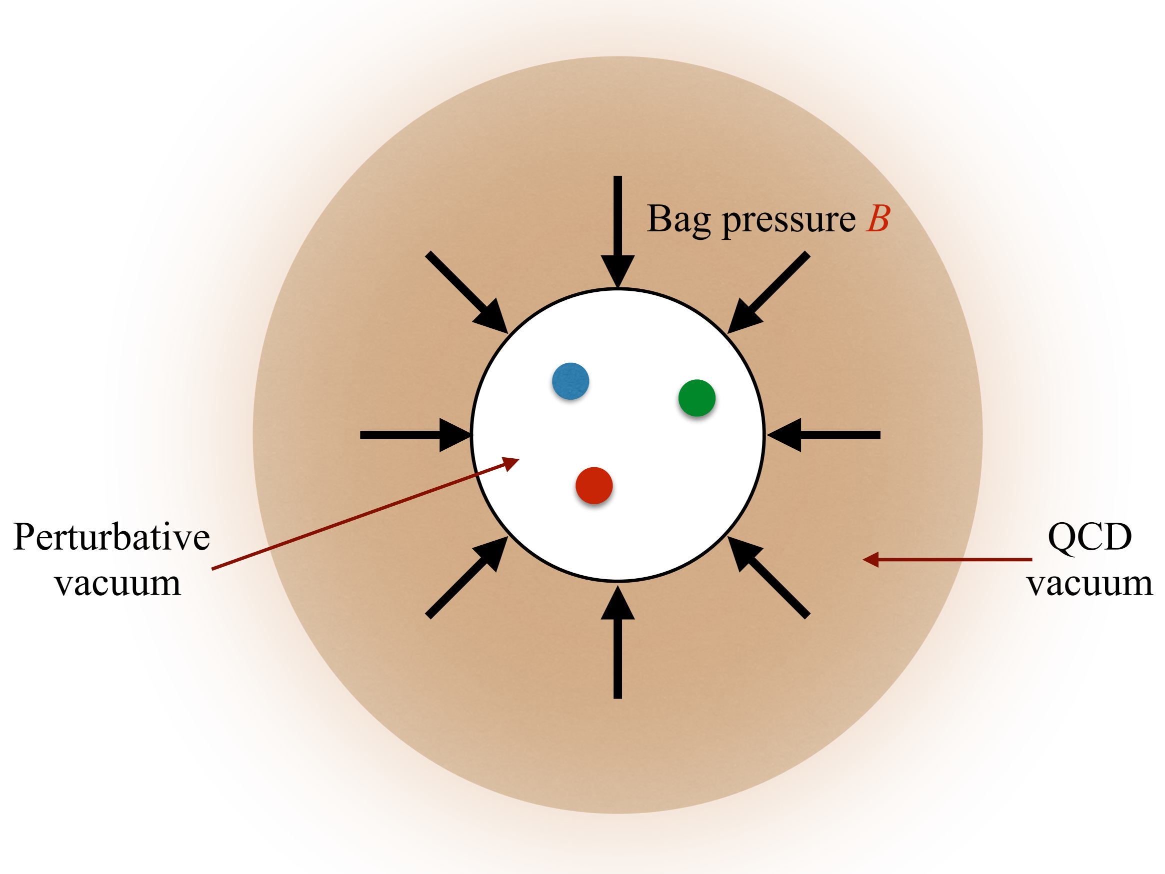
Λ_{QCD} from a boundary condition (I)

The QCD Lagrangian $\mathscr{L}_{QCD}$ lacks the confinement scale $\Lambda_{QCD} \sim 1\,\mathrm{fm}^{-1}$

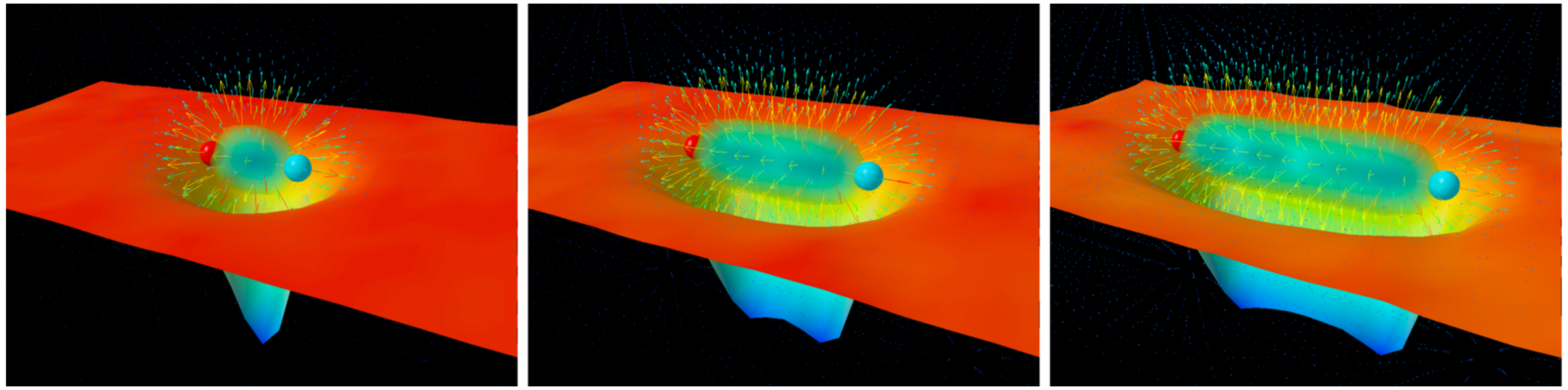
The scale may be introduced, preserving $\mathscr{L}_{QCD}$, through a boundary condition

C.f. the "Bag model": $\mathscr{L}_{bag} = (\mathscr{L}_{QCD} - B)\,\theta(bag)$

A. Chodos, et al., Phys. Rev. **D9** (1974) 3471



Lattice QCD indicates the emergence of a color string between quarks.



F. Gross, et al., Eur.Phys.J.C 83 (2023) 1125 [2212.11107]

Λ_{QCD} from a boundary condition (II)

In $A_a^0 = 0$ gauge the longitudinal color electric field is constrained by

$$\left(\nabla \cdot \boldsymbol{E}_L^a + g f_{abc} \boldsymbol{A}_b \cdot \boldsymbol{E}_c - g\psi^\dagger T^a \psi\right) |phys\rangle = 0$$

For color singlet states $|phys\rangle$ the (total) color octet field $\boldsymbol{E}_L^a(t,\boldsymbol{x})$ cancels in the sum over the quark and gluon colors (N/A in QED).

Hence $\boldsymbol{E}_L^a(t,\boldsymbol{x} \to \infty)$ need not vanish separately for each color component in $|phys\rangle$, allowing a (unique) homogeneous solution (boundary condition) $\propto \kappa$,

$$\boldsymbol{E}_L^a(\boldsymbol{x})|phys\rangle = \nabla_x \int d\boldsymbol{y} \left[\kappa\, \boldsymbol{x}\cdot\boldsymbol{y} + \frac{g}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\right]\left[f_{abc}\boldsymbol{A}_b \cdot \boldsymbol{E}_c(\boldsymbol{y}) - \psi^\dagger T^a \psi(\boldsymbol{y})\right]|phys\rangle$$

$\kappa \neq 0$ provides a confining, instantaneous potential,

while preserving Poincaré invariance.

Summary

1. A systematic method for QED and QCD bound states, based on the action, is required for bound states to become a QFT textbook topic.

Cf. the derivation of the perturbative S-matrix using the Interaction Picture.

2. The Poincaré covariance of equal-time bound states merits attention.

3. The confinement scale Λ_{QCD} can be introduced, maintaining $\mathscr{L}_{QCD}$, through a boundary condition on the gauge field in $A_a^0 = 0$ gauge.

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