The path to approximate N^3LO NNPDF Parton Distributions

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NNPDF

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Why do we need N^{3}LO PDFs?

$$\sigma(x, Q^2) = \sum_i \int_x^1 \frac{dz}{z} \mathscr{L}_{ij}(z, \mu^2)\, \hat{\sigma}_{ij}(\frac{x}{z}, \frac{Q^2}{\mu^2}, \alpha_s) + \mathcal{O}(\frac{1}{Q^2})$$

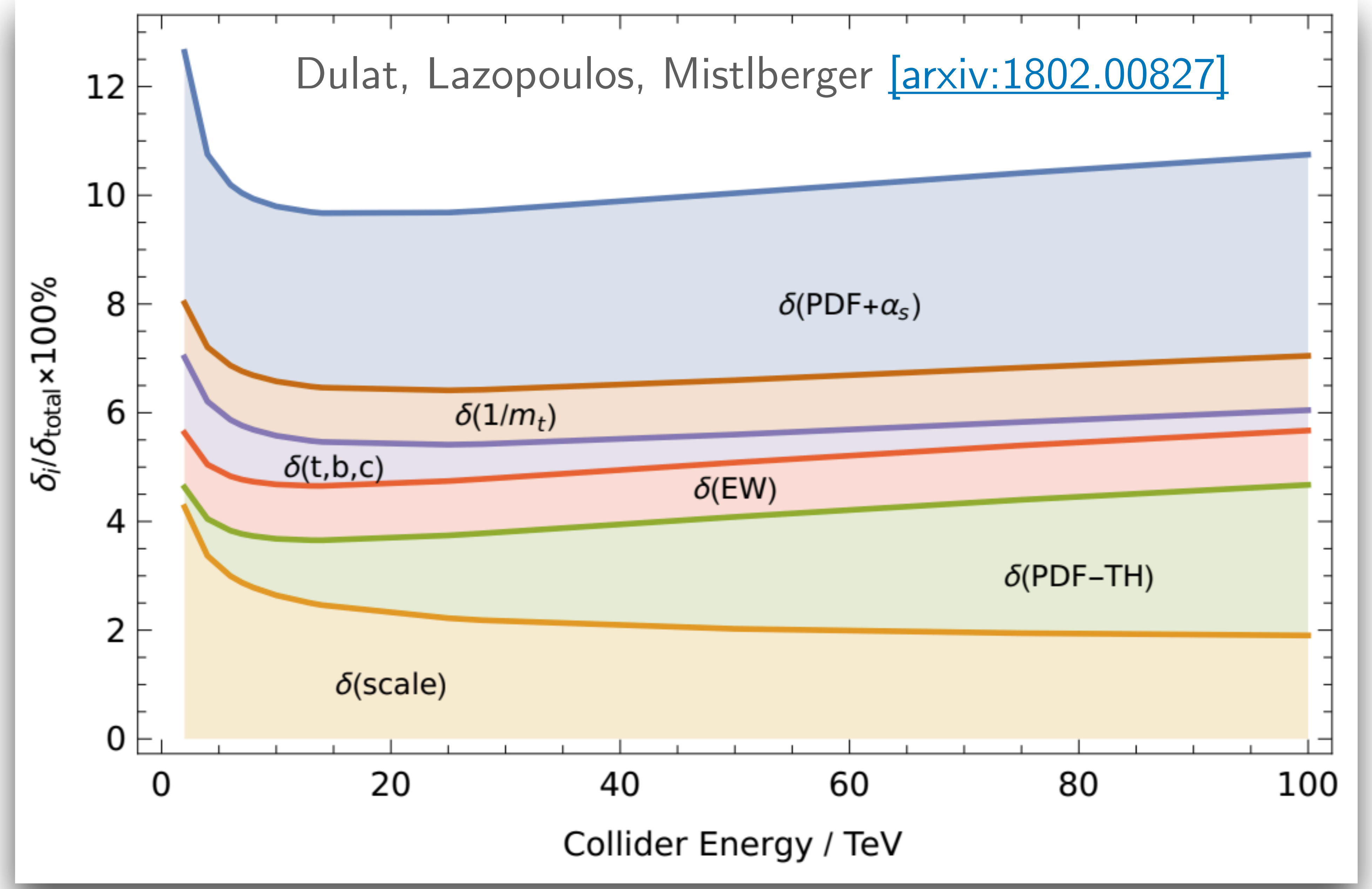
- Predictions for LHC observes relies on two main ingredients: **Parton Distributions Functions (PDFs)** and partonic **Matrix Elements.**

- In the last years many **2 to 1 processes** have been calculated up to QCD at **N^{3}LO**: $gg \to H$ [arxiv:1503.06056] $qq \to H\ (VBF)$ [arxiv:1606.00840], [arxiv:1904.09990], [arxiv:2004.04752] $pp \to W^{\pm}$ [arxiv:2007.13313], [arxiv:2205.11426] $pp \to Z/\gamma$, $pp \to VH$ [arxiv:2209.06138], [arxiv:2107.09085],[arxiv:2207.07056]

- **PDFs uncertainties** are becoming a **bottleneck** for LHC precision calculations.

- **Combining results** with different PDFs sets can be **non trivial** and differences have to be motivated.

% Theory Uncertainties in $pp \to H$ cross section



ATLAS collaboration [arxiv:2309.12986]

PDF set	$\alpha_s(m_Z)$	PDF uncertainty
MSHT20 [37]	0.11839	0.00040
NNPDF4.0 [84]	0.11779	0.00024
CT18A [29]	0.11982	0.00050
HERAPDF2.0 [65]	0.11890	0.00027

$$\delta_{PDF} = 0.3\,\%$$

$$\alpha_s(\text{NNPDF}) - \alpha_s(\text{CT18A}) = 1.7\,\%$$

Why do we need N^{3}LO PDFs?

The **interpretation of LHC measurements depends** on the the **PDF accuracy and precision**.

Most widely used **PDFs are at NNLO** and do not include theory uncertainties.

Why do we need N^{3}LO PDFs?

The **interpretation of LHC measurements depends** on the the **PDF accuracy and precision**.

Most widely used **PDFs are at NNLO** and do not include theory uncertainties.

Approximate N^{3}LO PDFs are now available.

Present results based on:

`NNPDF4.0 aN3LO` [arxiv:2402.1863]

`NNPDF4.0 QED aN3LO` [arxiv:20406:0177]

PDFs determination at aN^{3}LO

NNPDF4.0 timeline

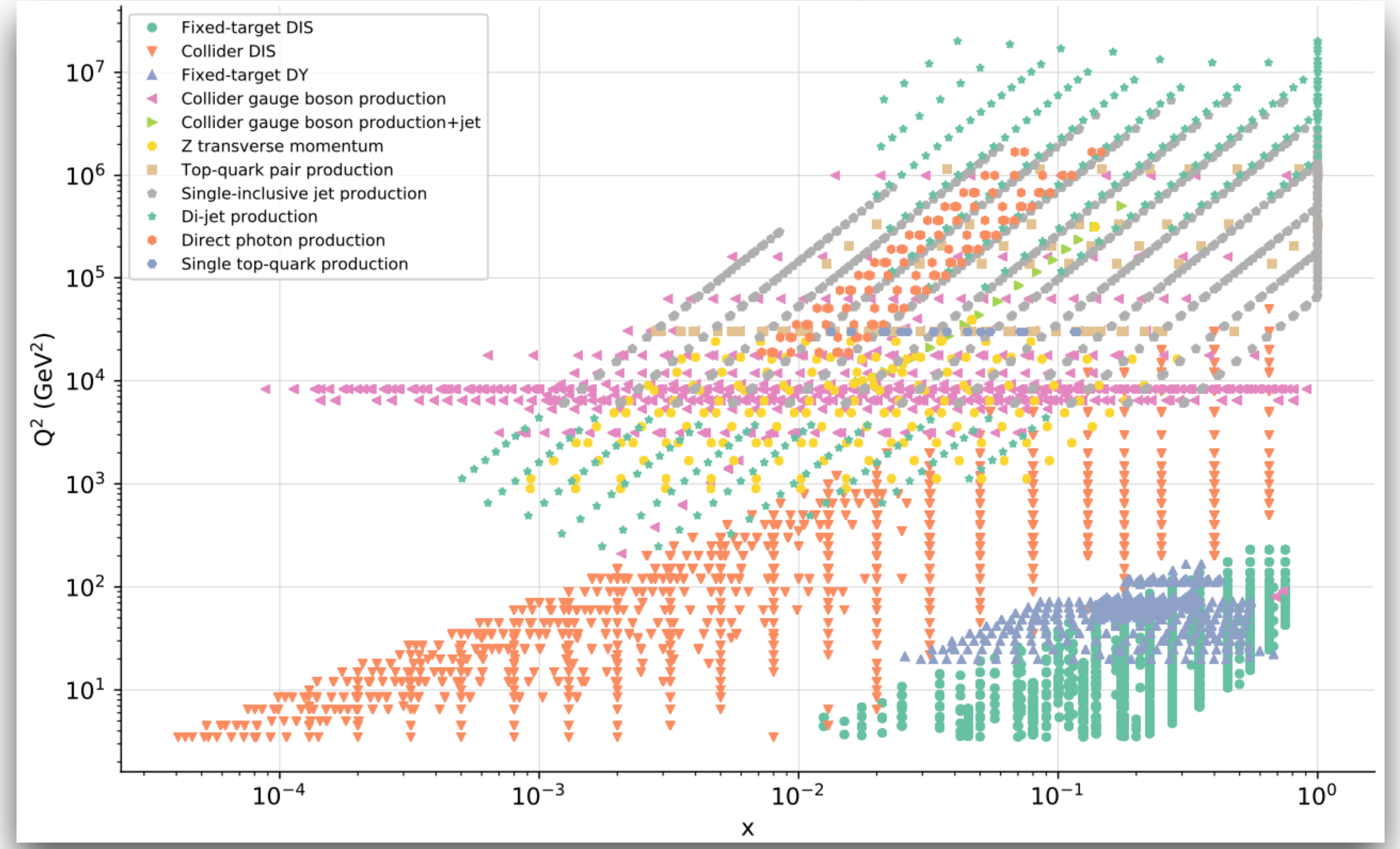
Sep 2021:
NNPDF4.0 [code & paper]

Jan 2024:
NNPDF4.0 MHOU
NNPDF4.0 QED

Feb 2024:
NNPDF4.0 aN3LO

Jun 2024:
NNPDF4.0 QED aN3LO

See also N.Laurenti talk



NNPDF4.0 Kinematic coverage

Several theoretical **inputs are needed in a PDF fit**:

1. QCD **splitting functions** which controls the DGLAP evolution.

$$Q^2 \frac{df_i}{dQ^2} = P_{ij}(x, \alpha_s) \otimes f_i(x, Q^2)$$

2. **VFNS matching conditions** for each running component.

$$f_i^{(n_f+1)}(x, Q^2) = A_{ij}(x, \alpha_s) f_j^{(n_f)}(x, Q^2)$$

3. **Partonic coefficients** functions, accounting for massive corrections when possible.

$$\sigma(x, Q^2) = \sum_{i=0}^{n_f} C_{k,i}(x, \alpha_s) \otimes f_i(x, Q^2)$$

$\mathrm{aN^3LO}$ splitting functions

Analytical calculations of the complete $\mathrm{N^3LO}$ spitting functions are not available.
Approximation can be constructed from the large number of partial results available.

- **Large-n_f:** $\mathcal{O}(n_f^3)$, $P_{NS}^{(n_f^2)}$ [arxiv:1610.07477]; $P_{qq,PS}^{(n_f^2)}$[arxiv:2308.07958]; $P_{gq}^{(n_f^2)}$ [arxiv:2310.01245]
- **NS small-x** [arxiv:2202.10362]: $P_{NS}^{(3)} \supset \sum_{k=0}^{6} \ln^k(x)$
- **Singlet small-x** [arxiv:1805.06460]: $P_{ij}^{(3)} \supset \sum_{k=0}^{3} \frac{\ln^k(x)}{x}$
- **Large-x** [arxiv:2205.04493], [arxiv:1911.10174], [arxiv:0912.0369]:

$$P_{ii}^{(3)} \approx A_{4,i} \frac{1}{(1-x)_+} + B_{4,i}\delta(1-x) + C_{4,i}\ln(1-x) + D_{4,i}$$

$$P_{ij}^{(3)} \approx \sum_{k}^{6} \ln^k(1-x)$$

- 5 or 10 lowest **Mellin Moments** [arxiv:1707.08315] [arxiv:2111.15561], [arxiv:2302.07593],[arxiv:2307.04158] ([arxiv:2404.09701], not included)

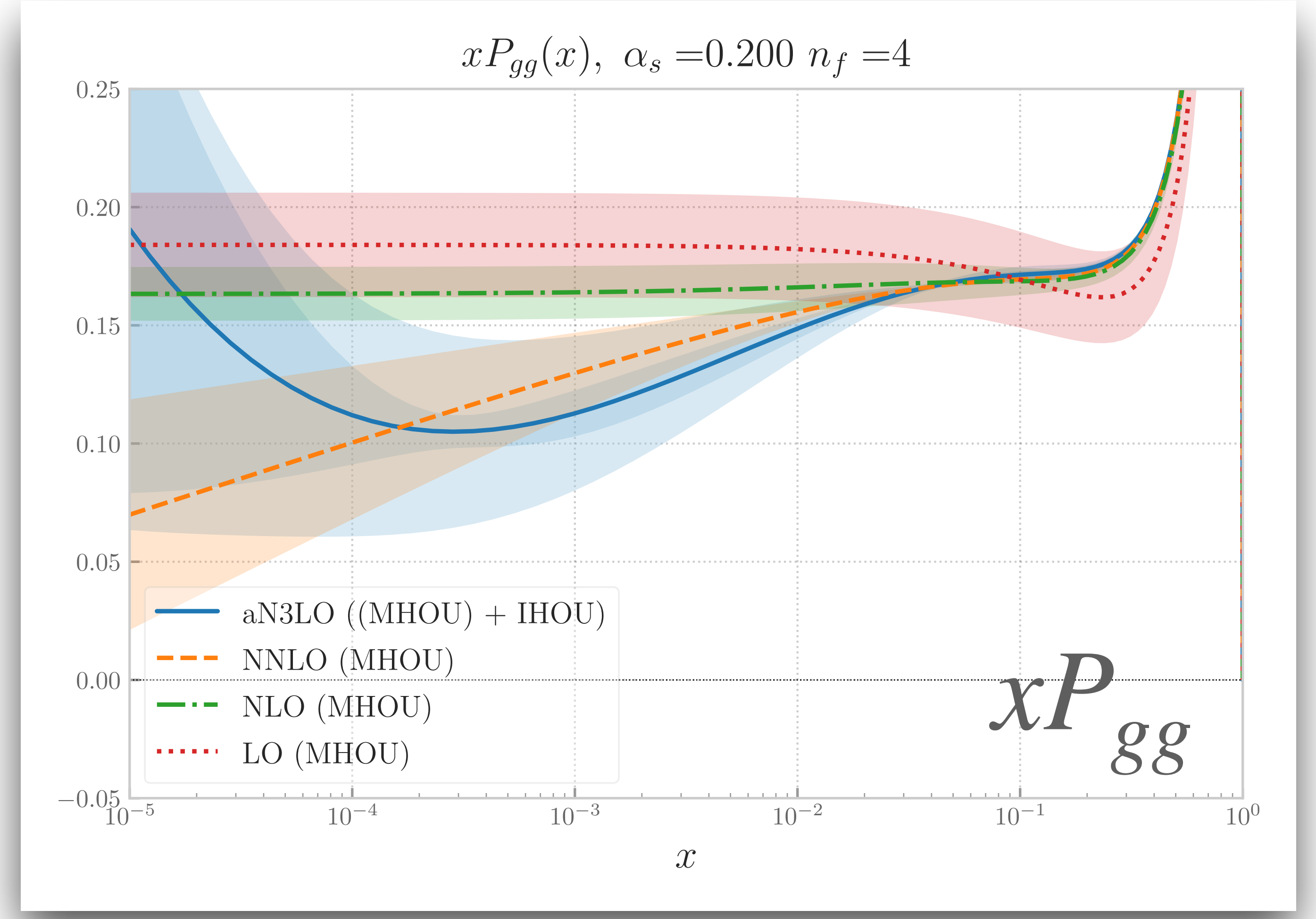
How do we combine the different limits?

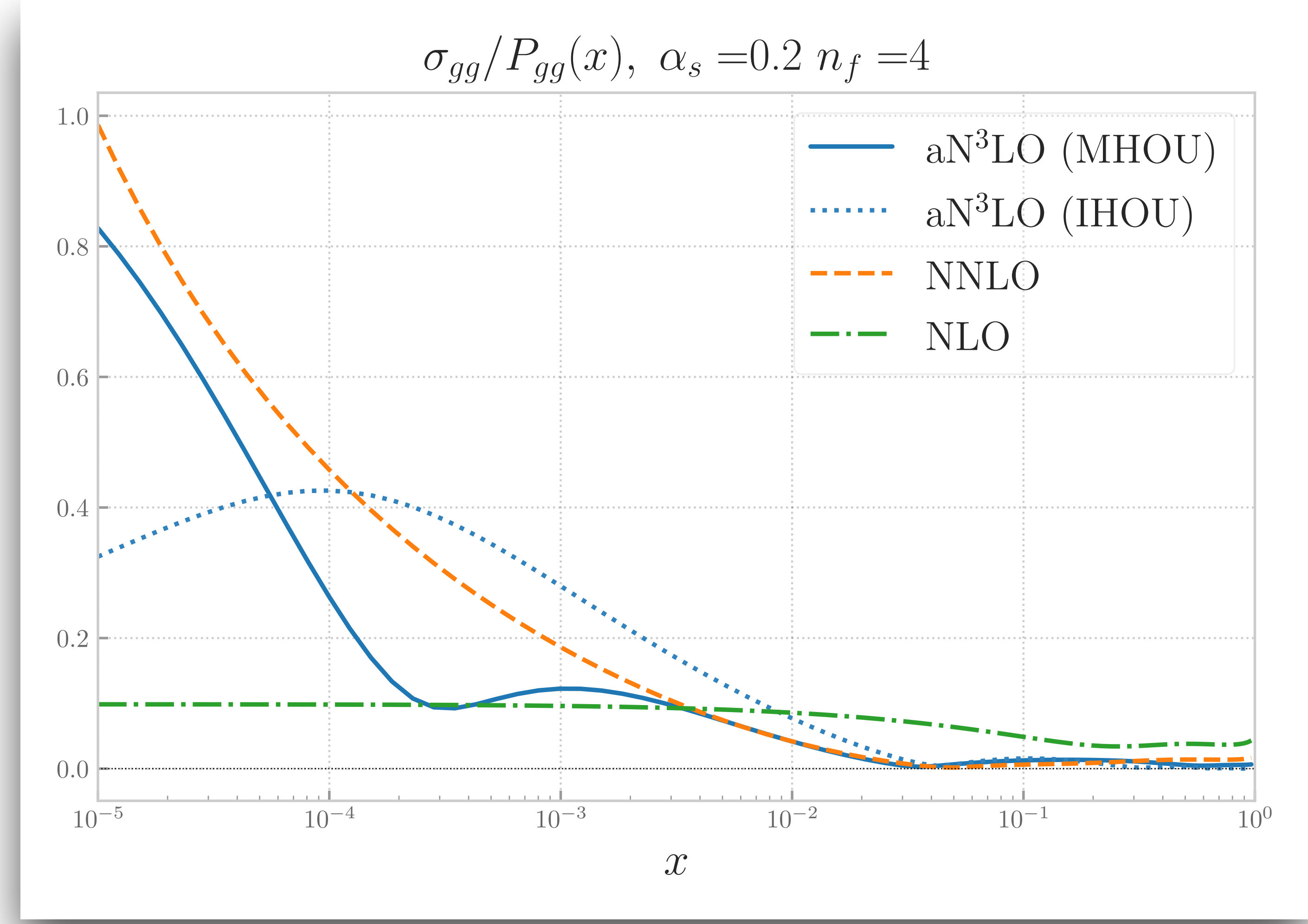
- The approximation procedure is performed in **Mellin space** for each n_f part independently:
- Combine **small-x and large-x** limits to match the Mellin moments, with different possible trial functions.
- Vary the parametrised part to generate a set of approximation and determine **Incomplete Higher Order Uncertainties (IHOU)**
- Determine independently **Missing Higher Order Uncertainties (MHOU)** from scale variation

aN^3LO splitting functions

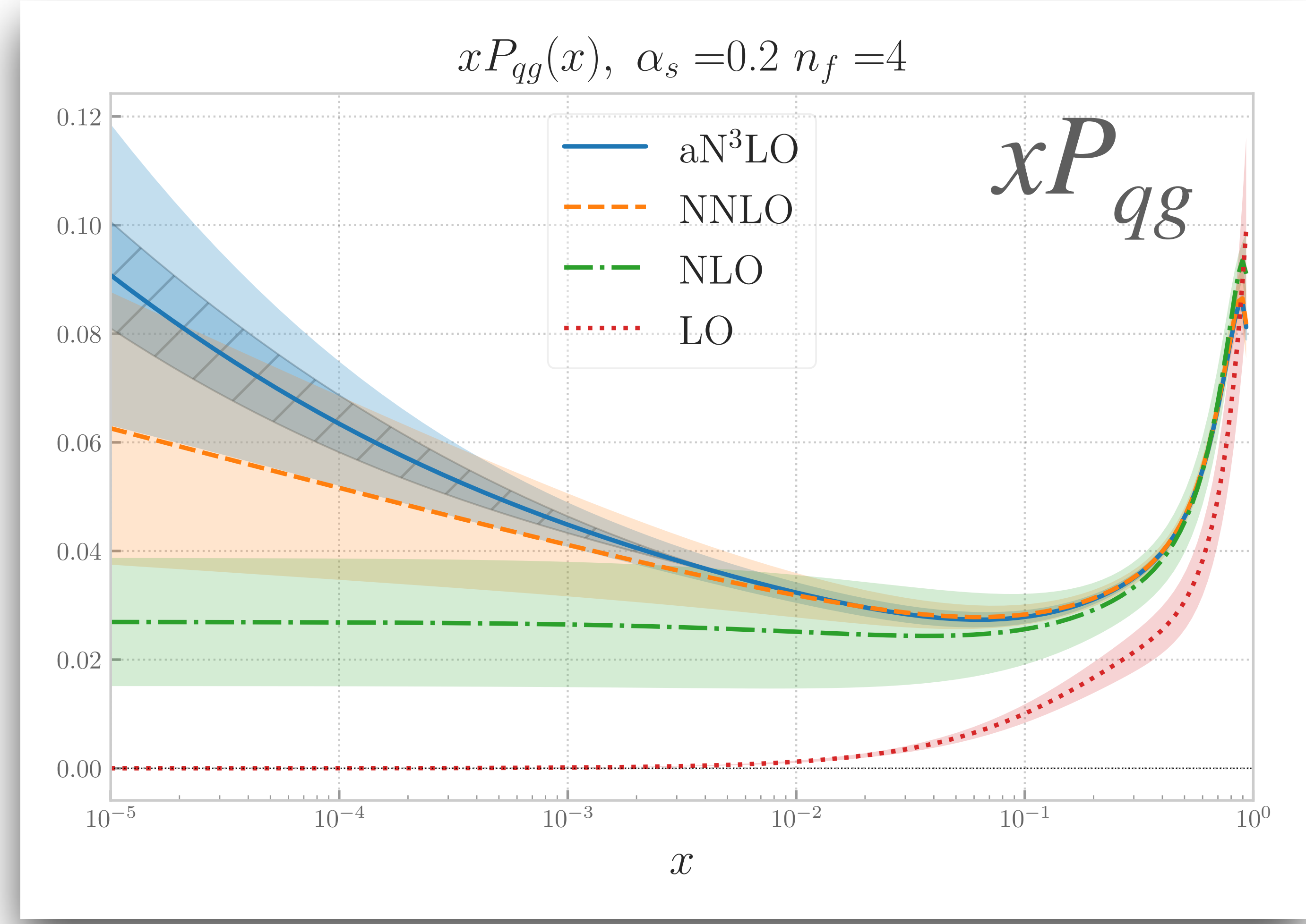
- For P_{qg}, P_{qq}, P_{gq} the N^3LO **approximation uncertainty is negligible** [IHOU < MHOU].
- In P_{gg} the **N^3LO approximation uncertainty is significant** [IHOU > MHOU for $x \geq 10^{-4}$].
- **Large-x**: good perturbative stability,
- **small-x**: effect of BFKL logarithms spoils the convergence.

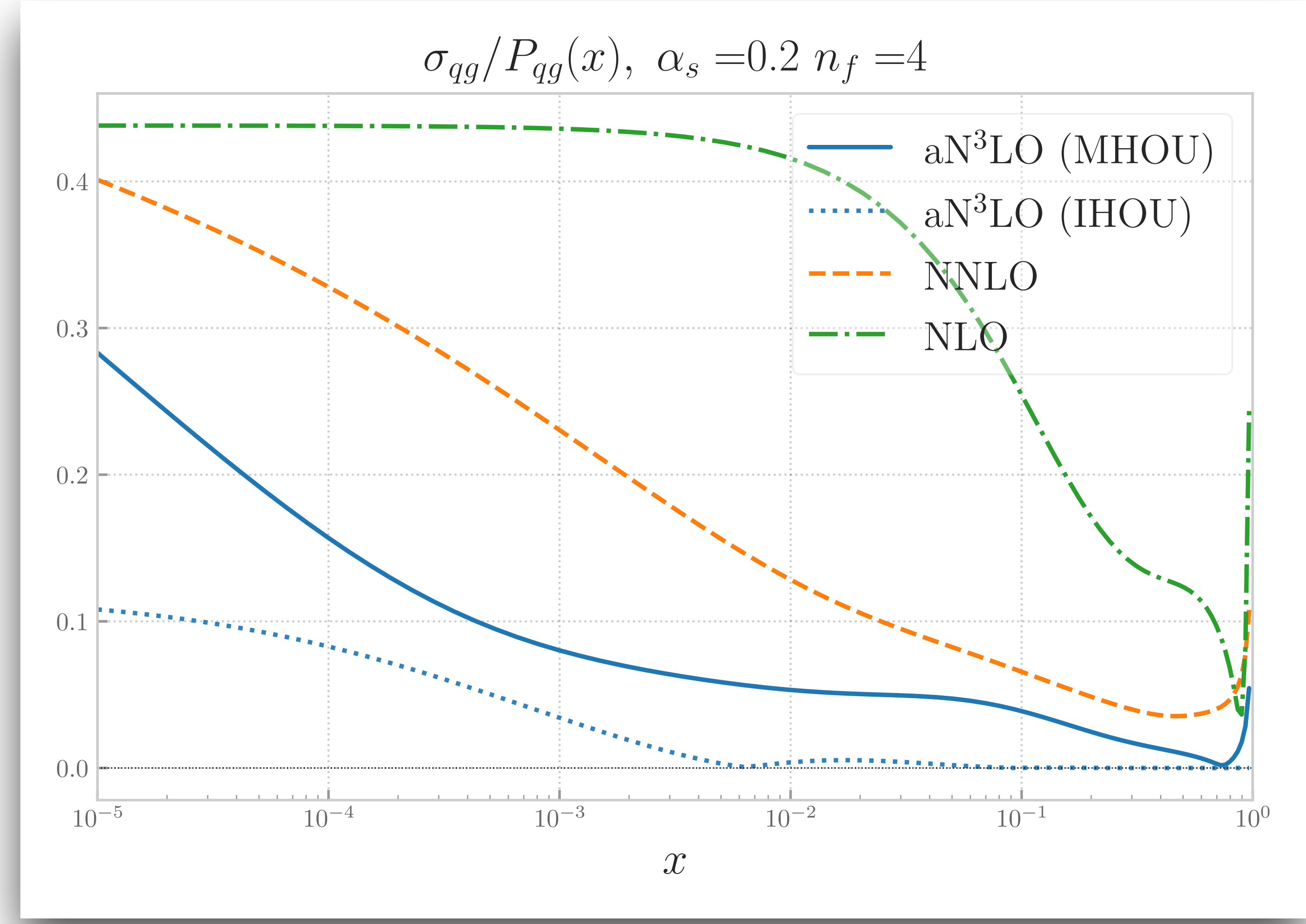
Gluon sector





Quark sector





- **IHOU** = incomplete higher order uncertainties [only for aN^3LO].
- **MHOU** = missing higher order uncertainties.

$\mathrm{aN^3LO}$ QCD corrections to DIS

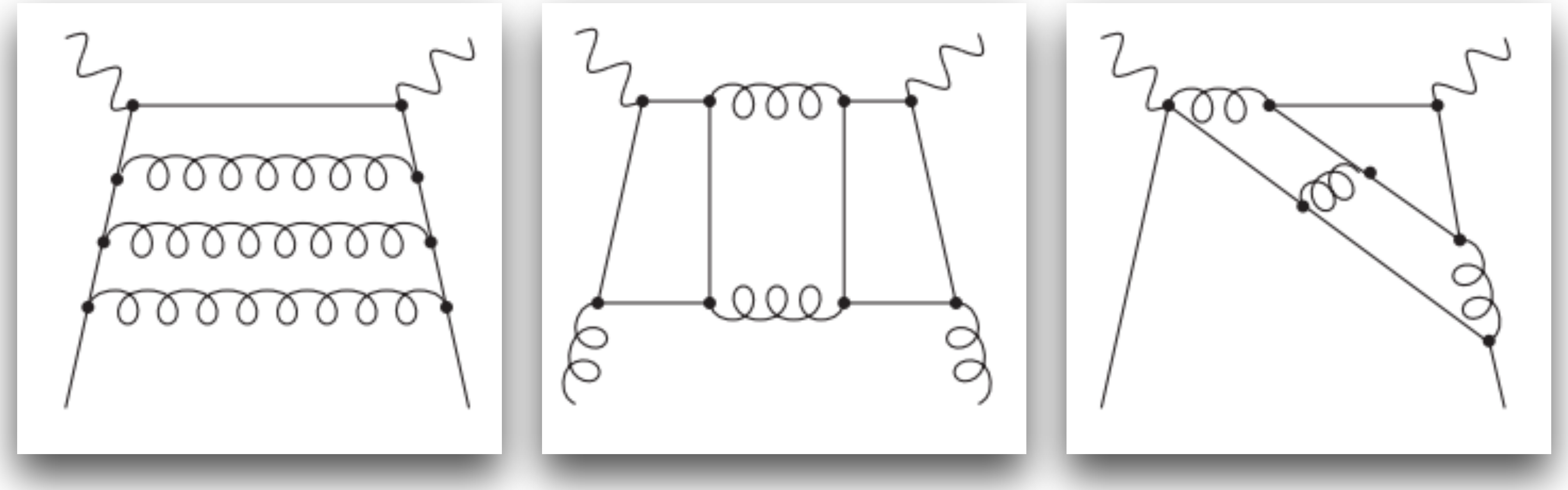
DIS cross sections can be written in terms of coefficient functions $C_{i,g}, C_{i,q}$ $i = 2,\ L,\ 3$, which are computed in pQCD.

$$\sigma_{DIS} \propto \sum_{i=2,L,3} k_i F_i \propto \sum_{i=2,L,3} k_i \left[C_{i,g}(x,\alpha_s) \otimes g(x,Q^2) + \sum_q C_{i,q}(x,\alpha_s) \otimes q(x,Q^2) \right]$$

$$C_{i,j} = \alpha_s^0 C_{i,j}^{(0)} + \alpha_s^1 C_{i,j}^{(1)} + \alpha_s^2 C_{i,j}^{(2)} + \alpha_s^3 C_{i,j}^{(3)}, \quad j = q, g$$

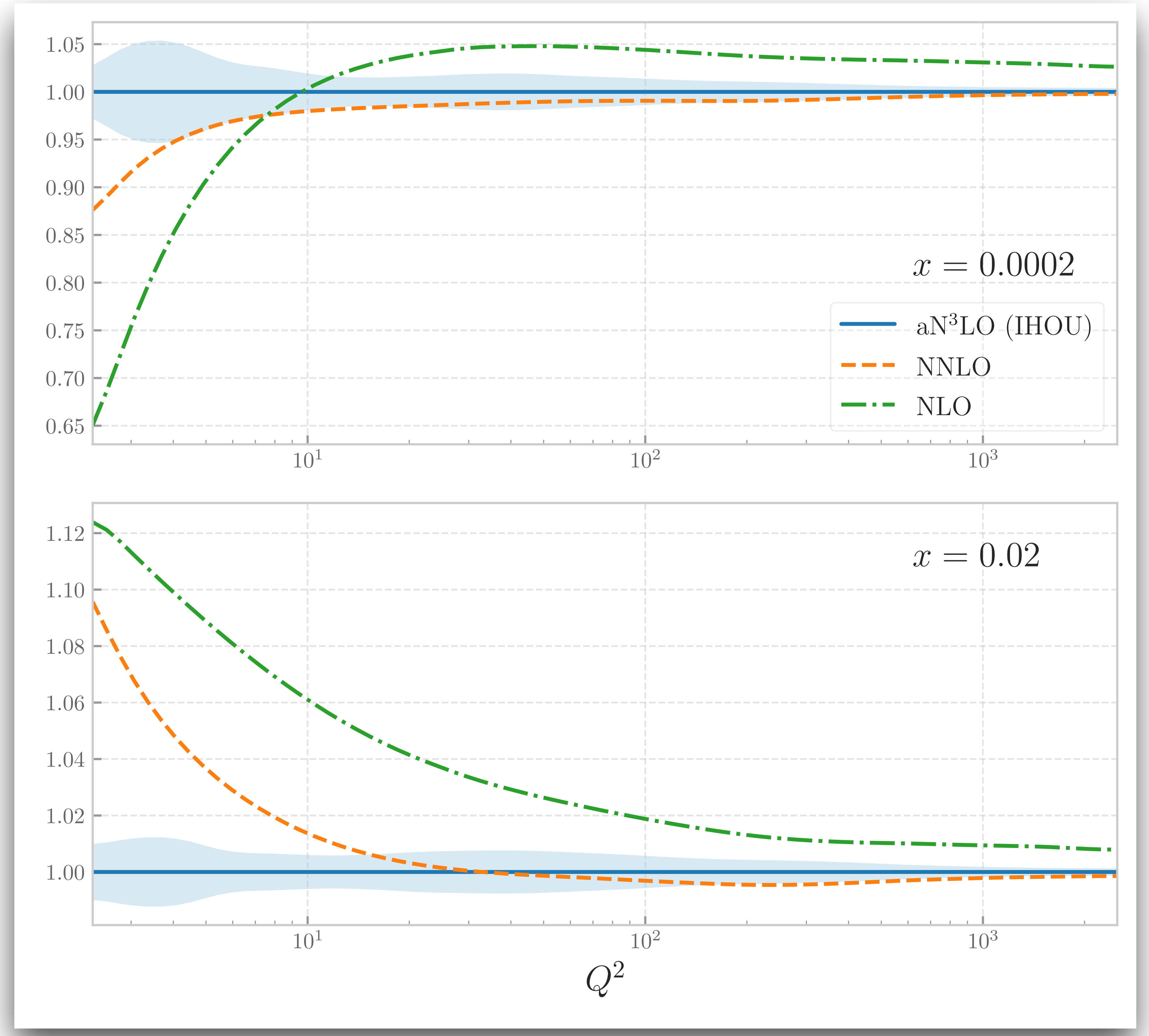
- DIS structure functions are known at $\mathrm{N^3LO}$ in the **massless limit**.
- New color structures can appear, which complicates the flavour decomposition.
- **Massive $\mathrm{N^3LO}$** contributions can be **approximated** joining the known limits ($Q \to m_h^2$ $Q^2 \gg m_h^2$ and $x \to 0$) with proper damping functions. [arxiv:2401.12139]
- Massless and massive scheme combined with **$\mathrm{N^3LO}$ matching.** [arxiv:0904.3563] [arxiv:1008.3347] [arxiv:1402.0359] [arxiv:1409.1135] [arxiv:1406.4654] [arxiv:2211.0546] [arxiv:2311.00644] ([arxiv:2403.00513])

Representative $\mathrm{N^3LO}$ QCD corrections to DIS



γ/Z: [arxiv:9605317] [arxiv:0411112] [arxiv:2208.14325], $W^{\pm}$: [arxiv:1606.08907]

$F_2(Q^2)$ at different pQCD orders



The NNPDF4.0 aN^{3}LO PDF set

To produce our aN^{3}LO PDF fit:

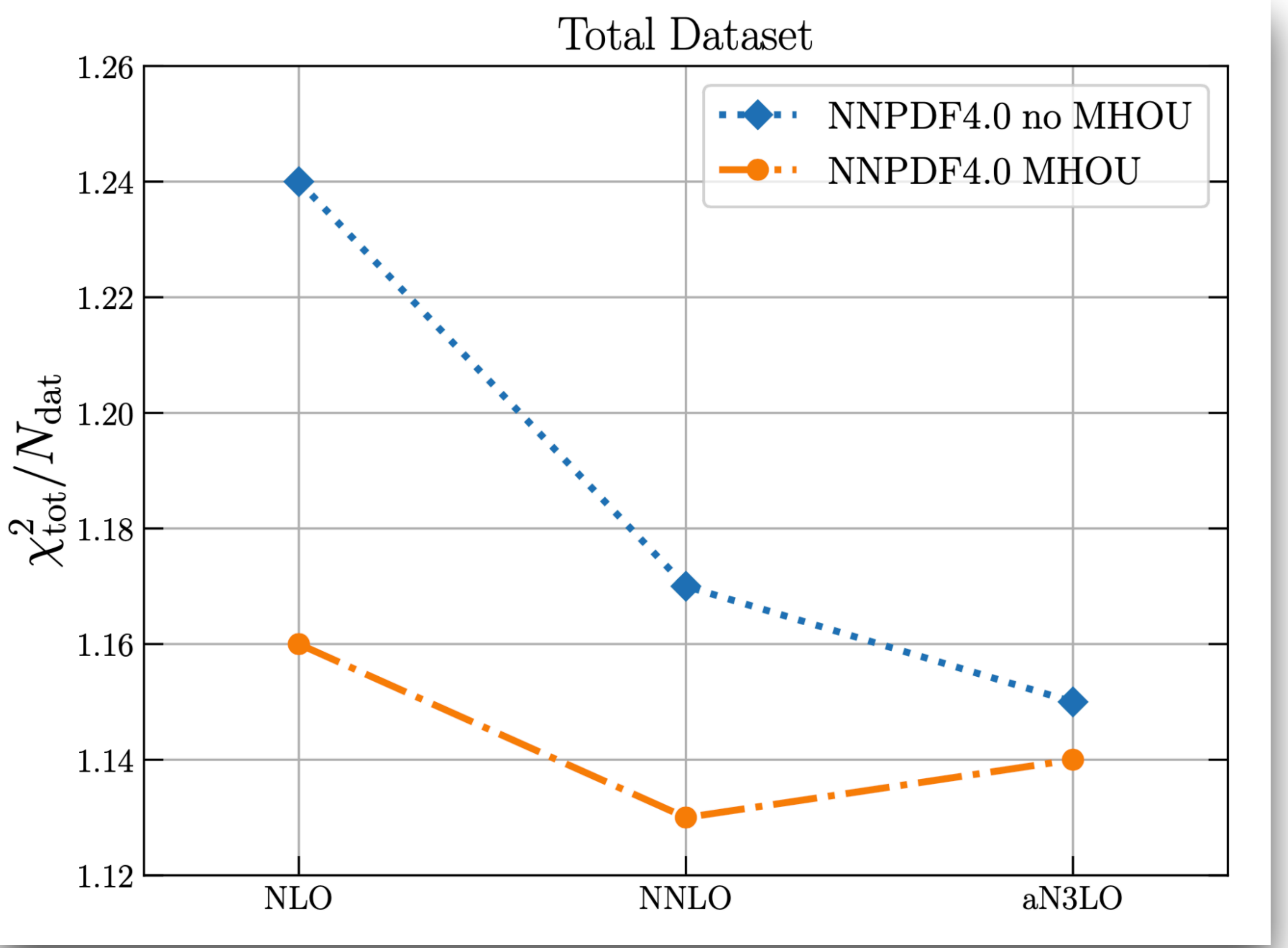
- We include **N^{3}LO corrections in DIS and DGLAP** with their respective IHOU.
- We adopt NNLO scale variation to estimate *unknown* N^{3}LO effects in DY, jets and top data.
- **MHOU and IHOU** are propagated to PDF fit with the **covariance formalism**:

$$\mathrm{Cov}_{tot} = \mathrm{Cov}_{exp} + \mathrm{Cov}_{DGLAP,IHOU} + \mathrm{Cov}_{DIS,IHOU} + \mathrm{Cov}_{HAD,MHOU}\ [+\mathrm{Cov}_{MHOU}]$$

- We fit more than 4000 different experimental datapoints (DIS, Drell Yann, Jets, Top), with the **NNPDF4.0 methodology** parametrising PDFs at initial scale Q_0 with a Neural Network.

$$f_i(x, Q_0) = x^{a_i}(1-x)^{b_i}\mathrm{NN}(\theta, x,)_i,\ i = q_i, g$$

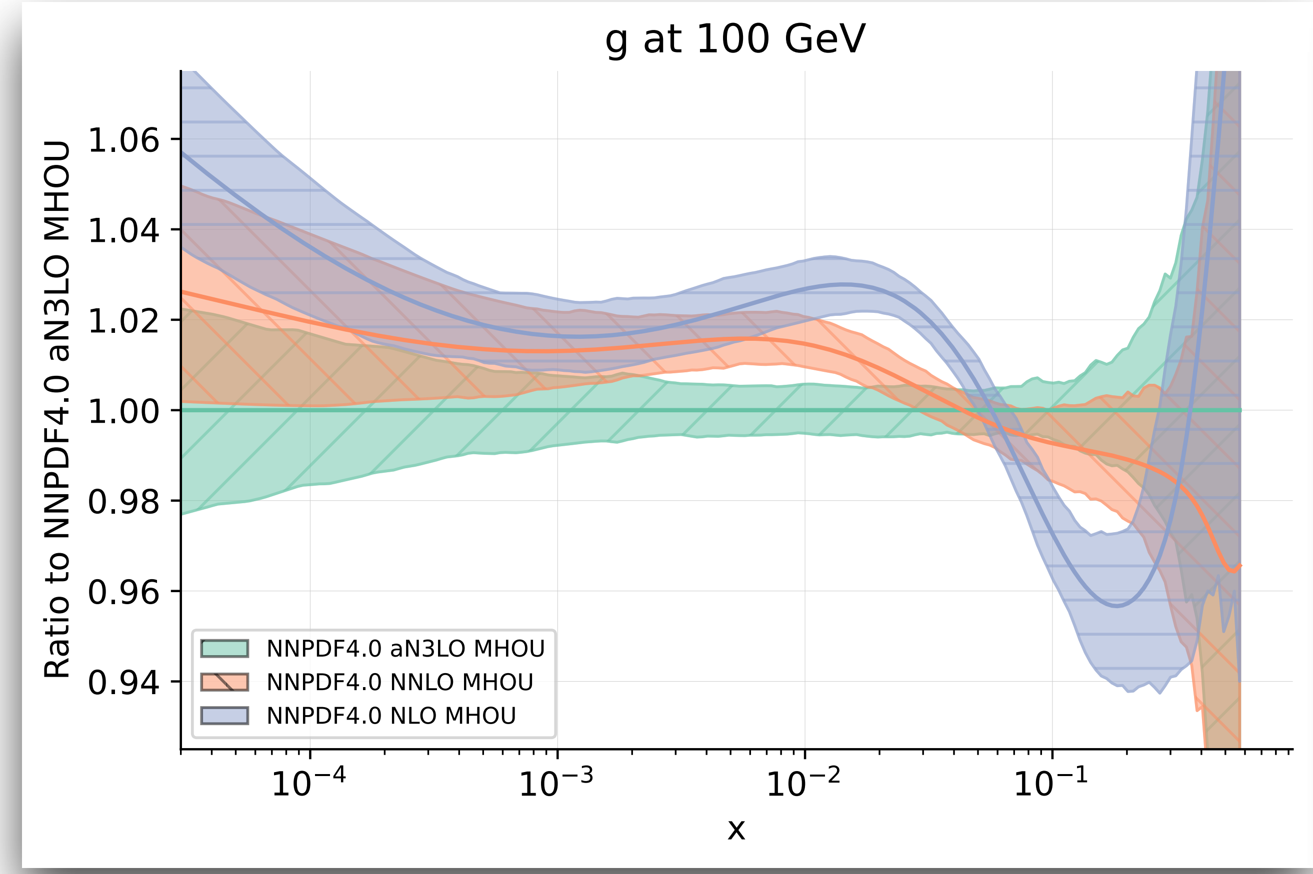
Total χ^2 at different pQCD orders

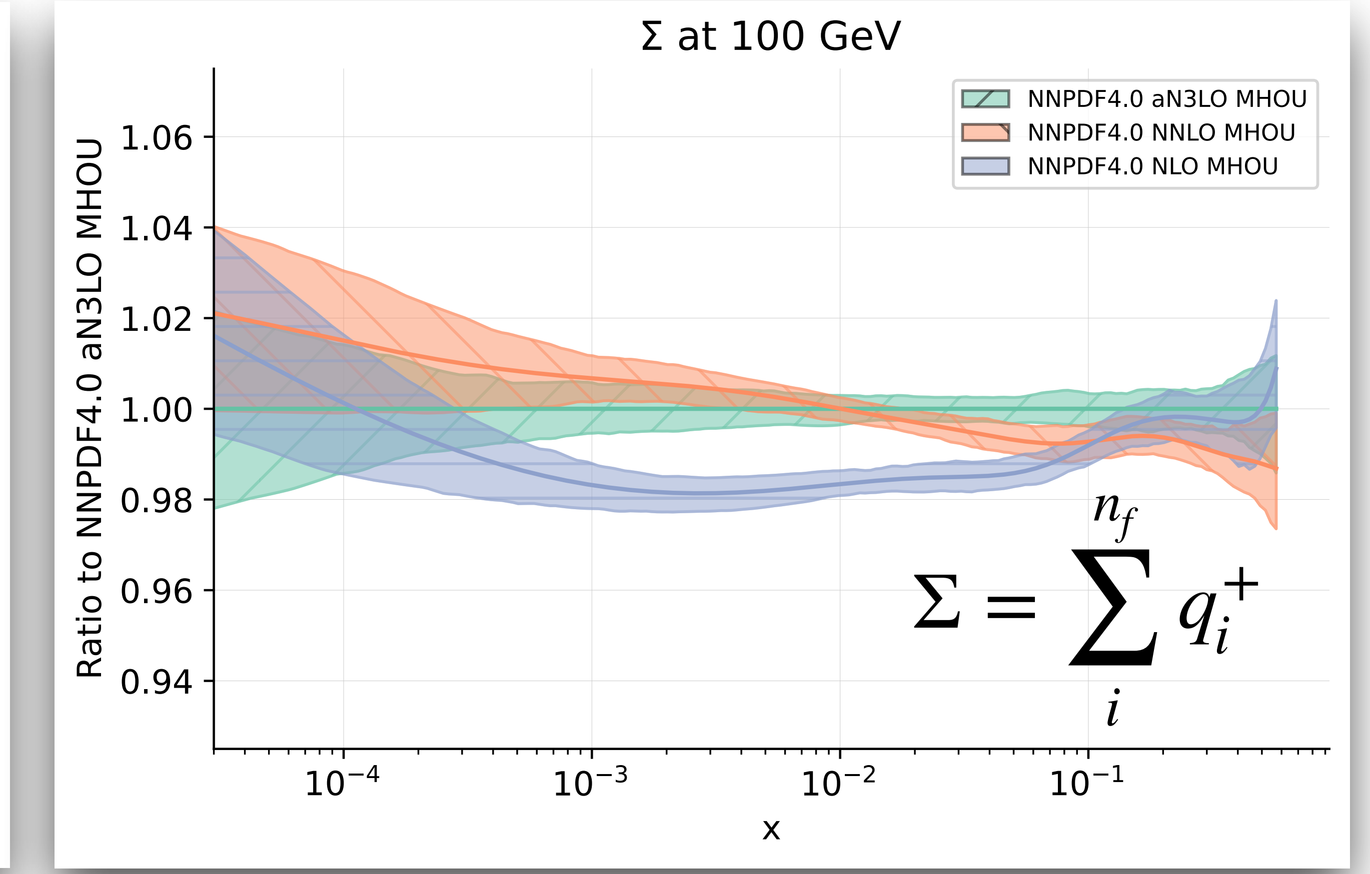


MHOU stabilise the fit: χ^2 is less dependent on QCD order.

The NNPDF4.0 aN^{3}LO PDF set

Perturbative convergence

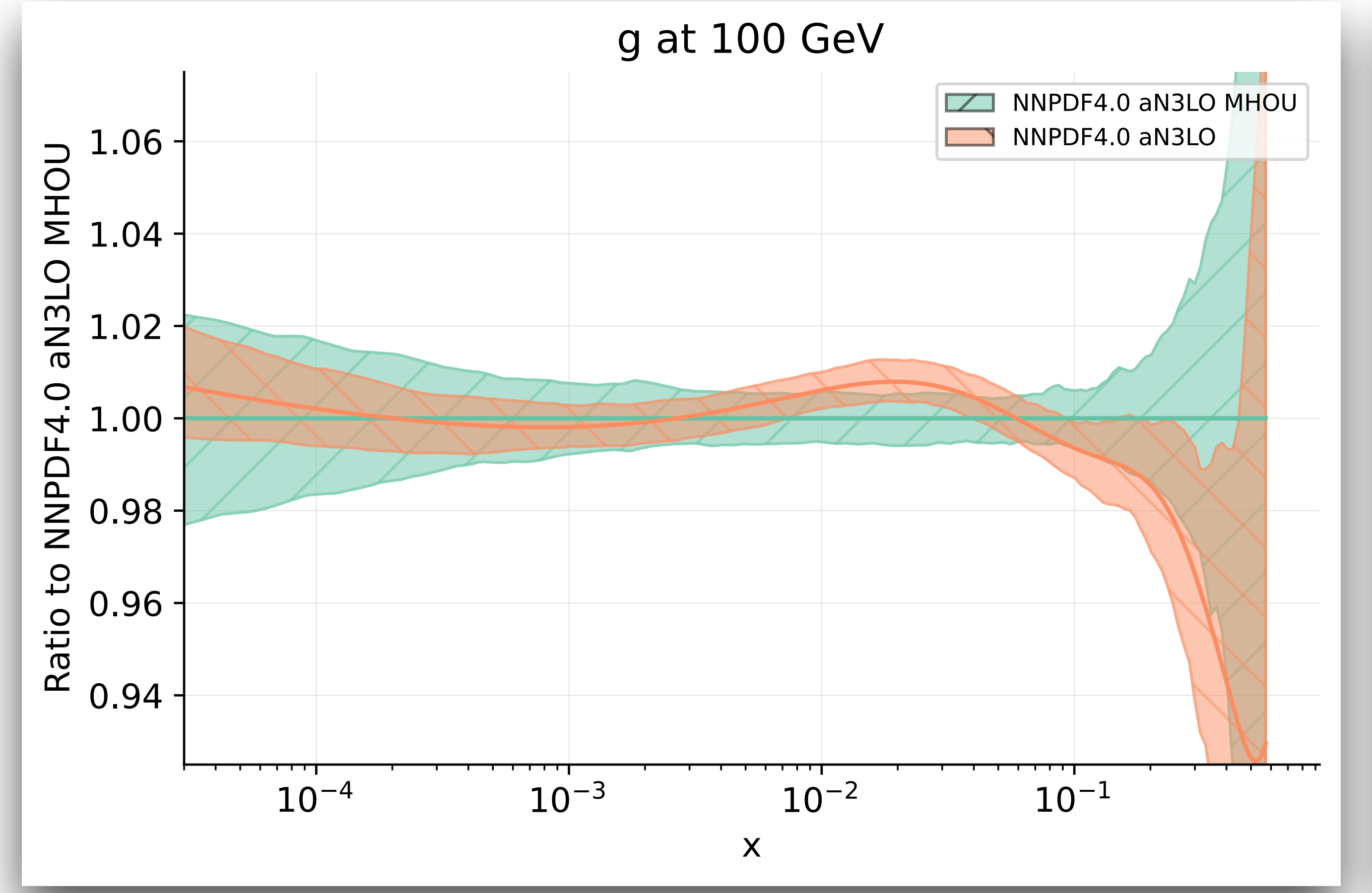


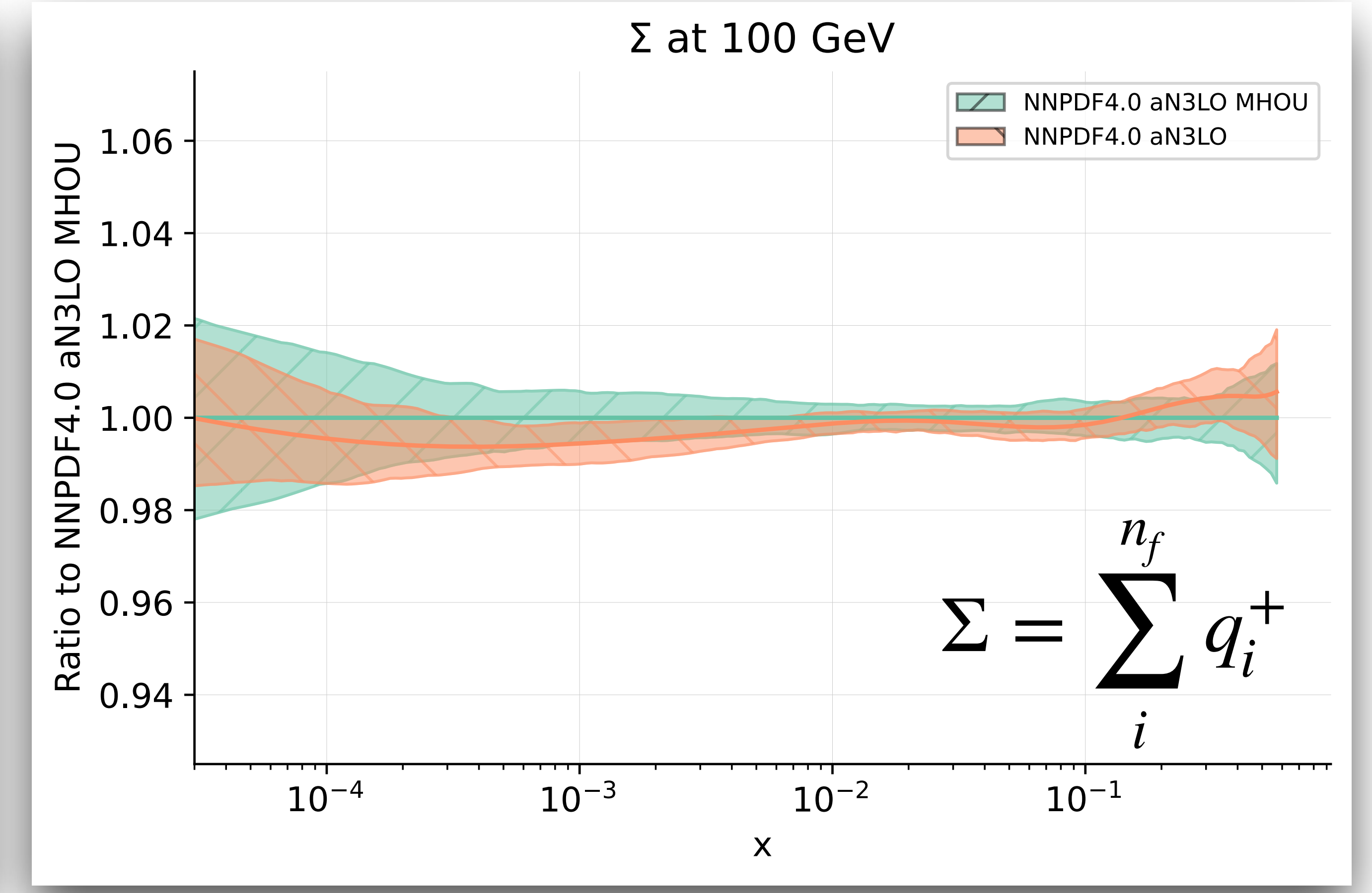


- Good perturbative convergence in the data region $x \in [10^{-4}, 0.7]$.
- **Impact of aN^{3}LO** corrections is **mild on quarks PDFs.**
- $\sim 2\,\%$ depletion of the **gluon around $x \approx 10^{-2}$ wrt NNLO.**

Impact of MHOU

- aN^{3}LO PDFs with/without MHOU are compatible.
- MHOU can shift central value, resolve tensions among datasets. Mainly de-weight jets datasets.
- aN^{3}LO corrections have a larger effect on the small-x, low-Q DIS data.

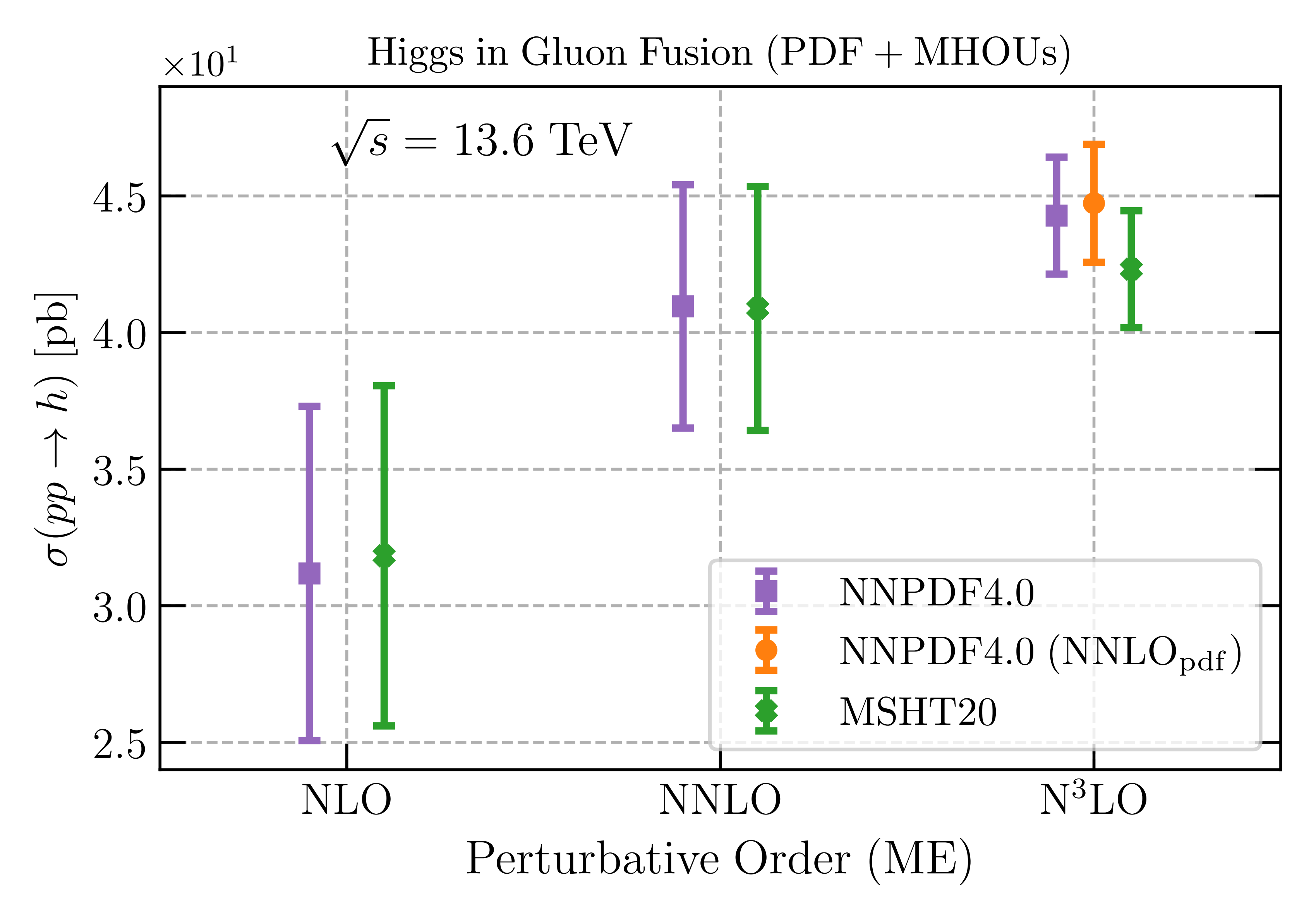


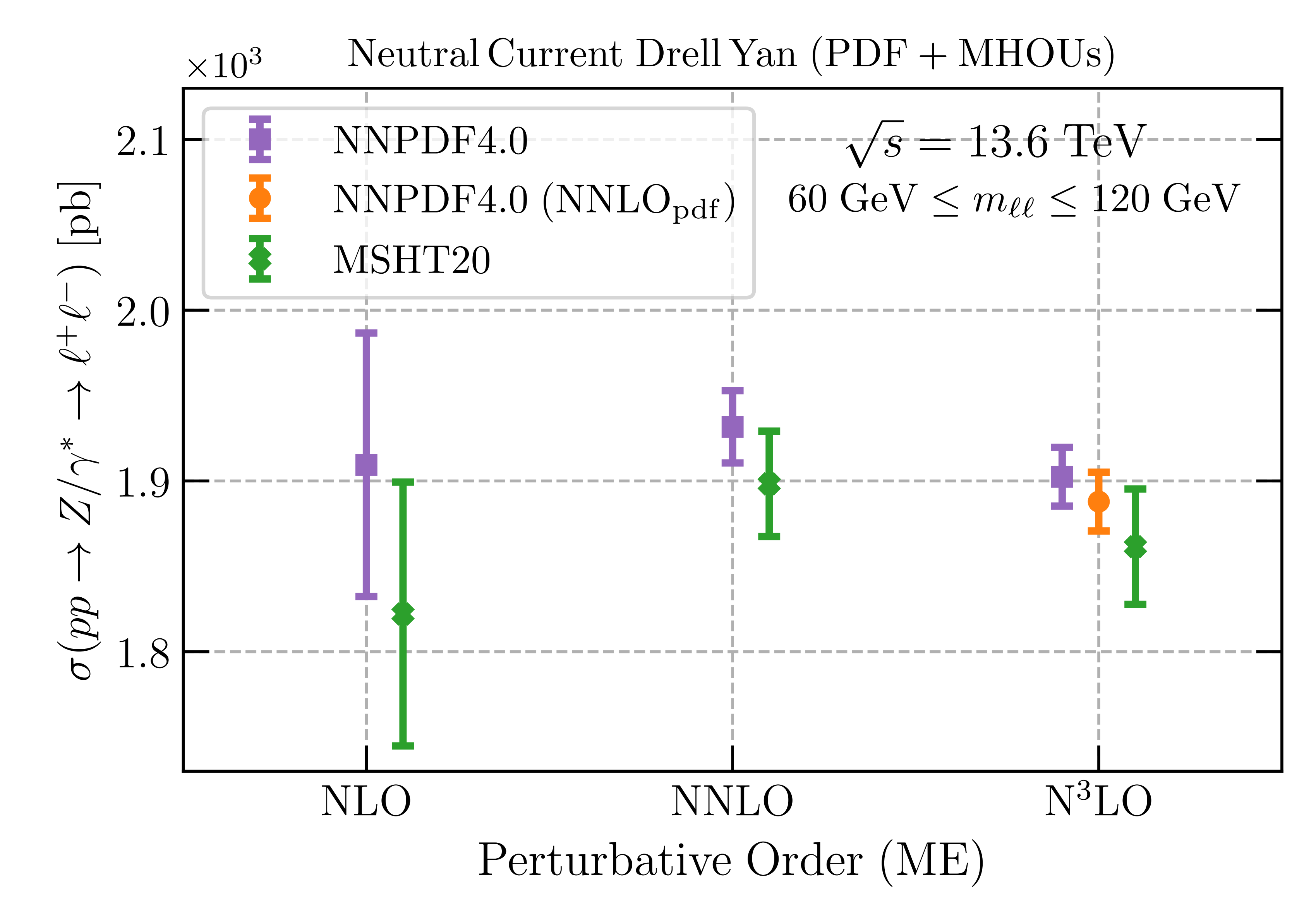


Impact on LHC cross sections

- **aN^{3}LO PDFs** effects are visible in **Higgs gluon fusion**, leading to a $2.1\,\%$ **suppression** w.r.t NNLO PDFs.

- Higgs VBF is more stable at different perturbative orders, although the PDF dependency is not negligible.





- Also for collider **gauge boson production**, usage of aN3LO PDFs seems to **improve the perturbative convergence**.

- **Benchmarking**: similar N3LO/NNLO ratio as in MSHT20 aN3LO. [arxiv:2207.04739]

aN^3LO PDF with QED corrections

See also N.Laurenti talk

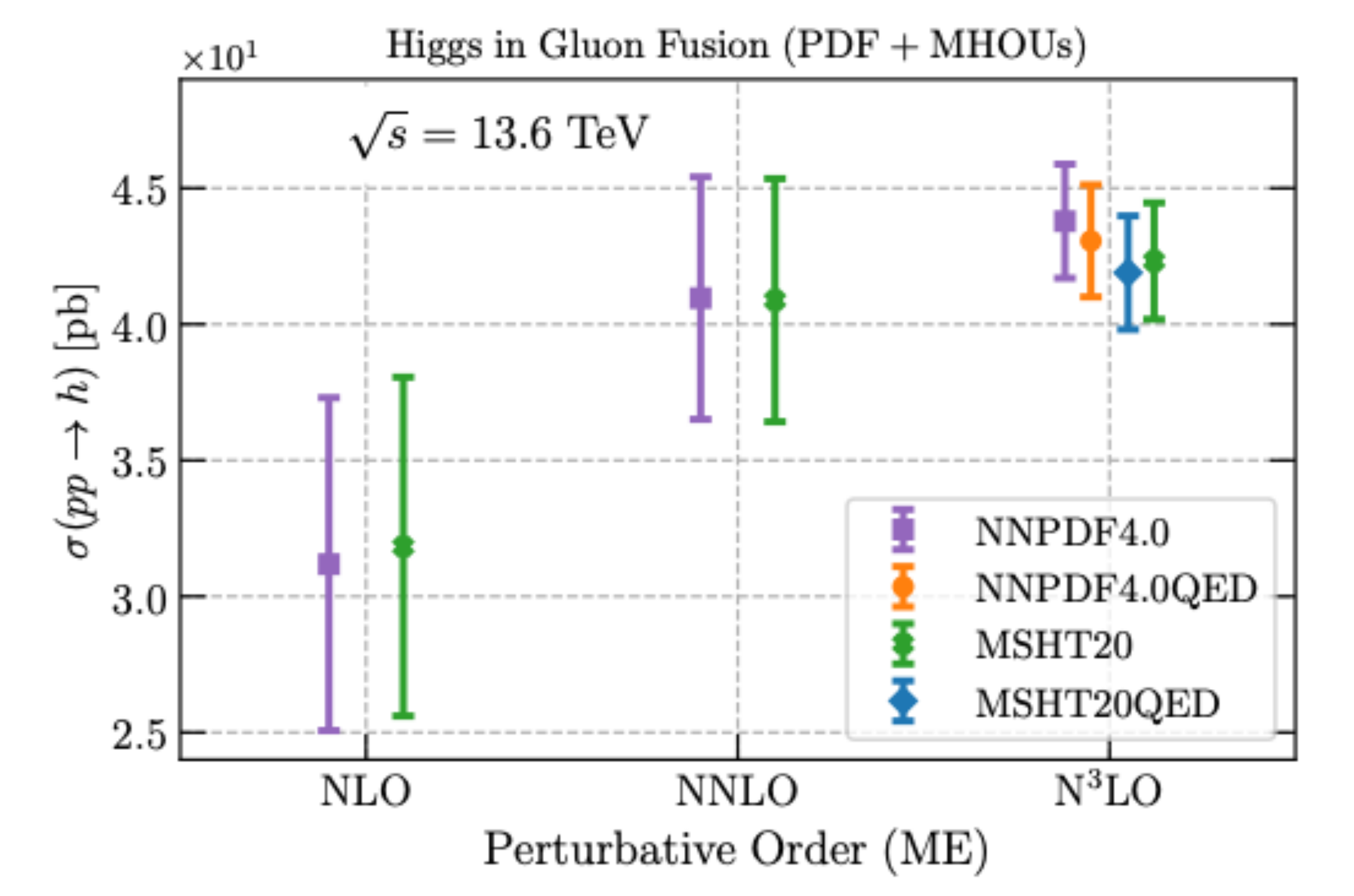
Barontini, Laurenti, Rojo [arxiv:2406.01779]

Recently we have also provided an additional global fit:

✓ **NNPDF40 QED aN3LO**

The photon **PDF is computed from DIS** structure functions at a given **high** Q^2 scale.
[LuxQED [arxiv:1607.04266] [arxiv:1708.01256]]

DGLAP with mixed $QED \otimes QCD$: $\mathcal{O}(\alpha_s \alpha_{em}), \mathcal{O}(\alpha_{em}^2)$

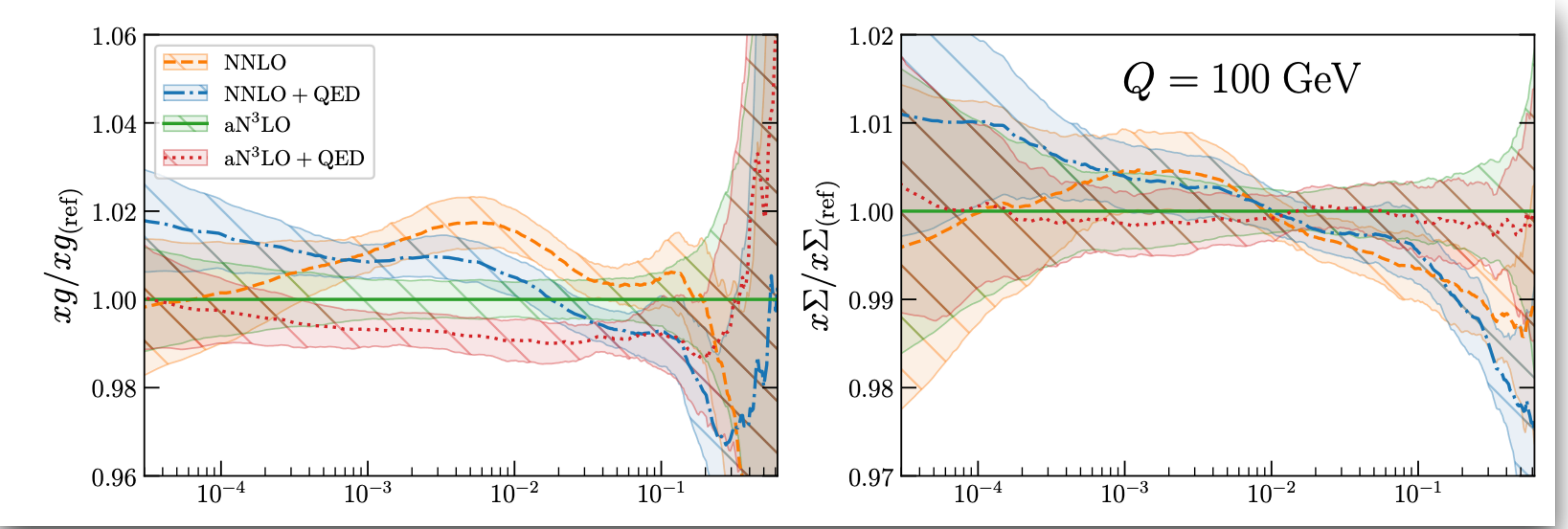


- The photon PDF subtracts some momentum from other partons (especially gluon):

$$\int_0^1 xdx \left(g(x) + \sum_i q_i^+(x) + \gamma(x) \right) = 1$$

- **QED** effects on the PDFs **are comparable to QCD aN^3LO** corrections, **both must be taken into account** to achieve best accuracy.

- Similar effect on the $\gamma(x, Q^2)$ PDF as in MSTH20 aN3LO QED [arxiv:2312.07665]



Summary & outlook

Newest NNPDF4.0 releases:

- ✓ NNLO theory uncertainties through scale variations.
- ✓ aN^{3}LO QCD: state of the art **DGLAP** and **DIS**, along with theory uncertainties.
- ✓ Determination of Photon PDF. [See also N.Laurenti talk]

> NNPDF40 aN^{3}LO PDFs can be used:
>
> - To **compute N^{3}LO cross sections** more precisely.
> - To **evaluate missing higher order** effects on NNLO calculation more accurately.

Ongoing projects (NNPDF4.1):

- Full NNLO: removal of NNLO k-factors.
- EWK corrections through k-factors
- Improved methodology: for ex. extended Hyperoptimization
- Extension of fitted data (LHC 13 TeV): DY, Top, Jets; DIS + Jet

Jan 2024:
NNPDF4.0 MHOU
NNPDF4.0 QED

Feb 2024:
NNPDF4.0 aN3LO

Jun 2024:
NNPDF4.0 QED aN3LO

...

WIP:
Towards NNPDF4.1

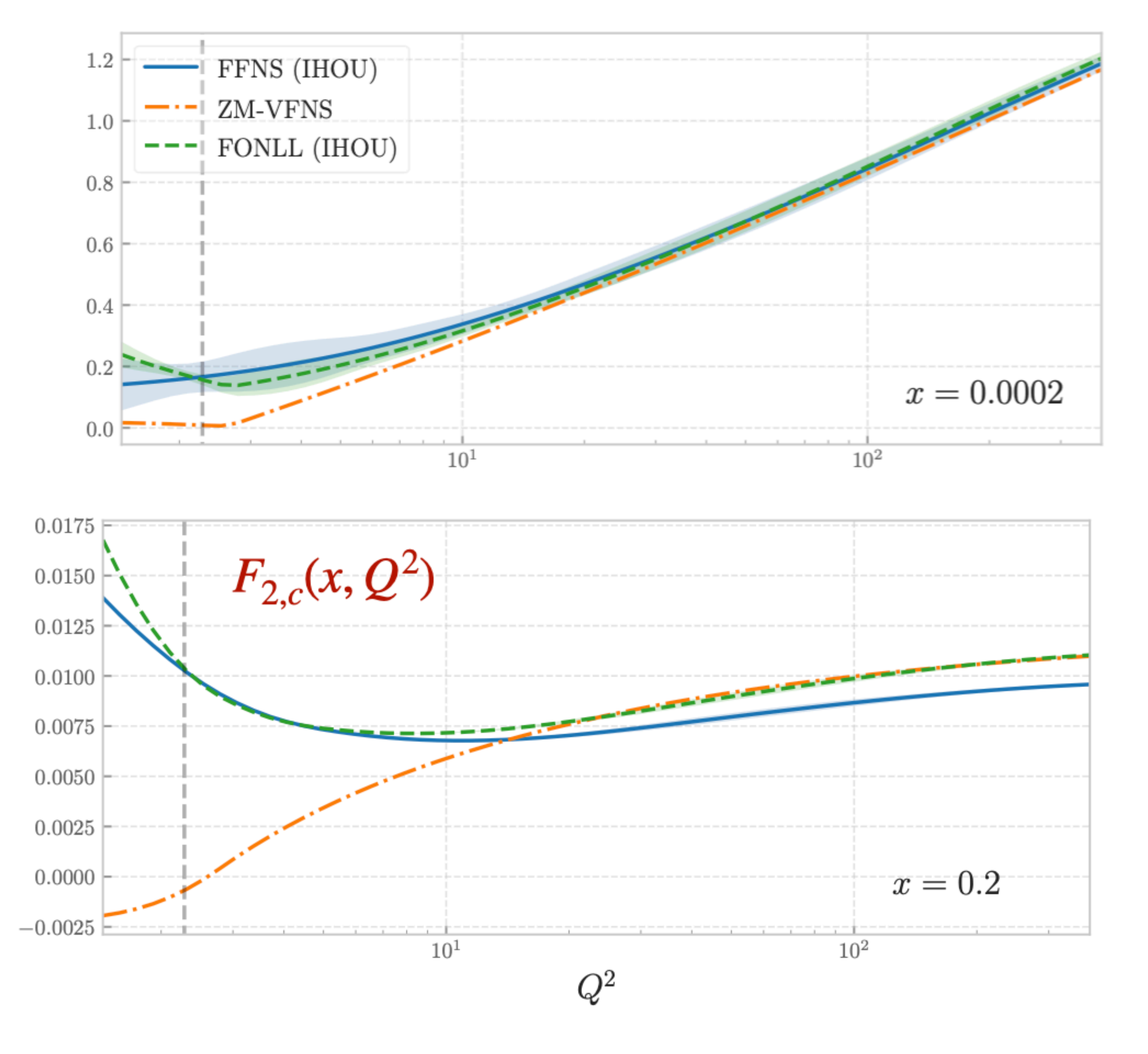
Back up slides

VFNS at aN^{3}LO

To treat heavy quarks consistently during a PDF fit we must adopt a **Variable Flavor Number Scheme.**

$$\begin{pmatrix} g \\ \Sigma \\ h^+ \end{pmatrix}^{n_f+1} (\mu_h^2) = \mathbf{A}^{(n_f)}_{S,h^+}(\mu_h^2) \cdot \begin{pmatrix} g \\ \Sigma \\ h^+ \end{pmatrix}^{n_f} (\mu_h^2)$$

PDFs matching conditions included at N^{3}LO almost completely [arxiv:0904.3563] [arxiv:1008.3347] [arxiv:1402.0359] [arxiv:1409.1135] [arxiv:1406.4654] [arxiv:2211.0546] [arxiv:2311.00644] exception of $a^{(3)}_{H,g}$, computed in [arxiv:2403.00513]



DIS structure functions are computed in the **FONLL** procedure: [arxiv:1001.2312]

$$F_{FONLL} = F^{(n_f+1)}_{ZM} + F^{(n_f)}_{FFNS} - \lim_{m_h \to 0} F^{(n_f)}_{FFNS}$$

- Extended up to N^{3}LO for the Heavy structure functions F_{heavy}
- Extended up to NNLO for light F_{light} + massless N^{3}LO contributions.

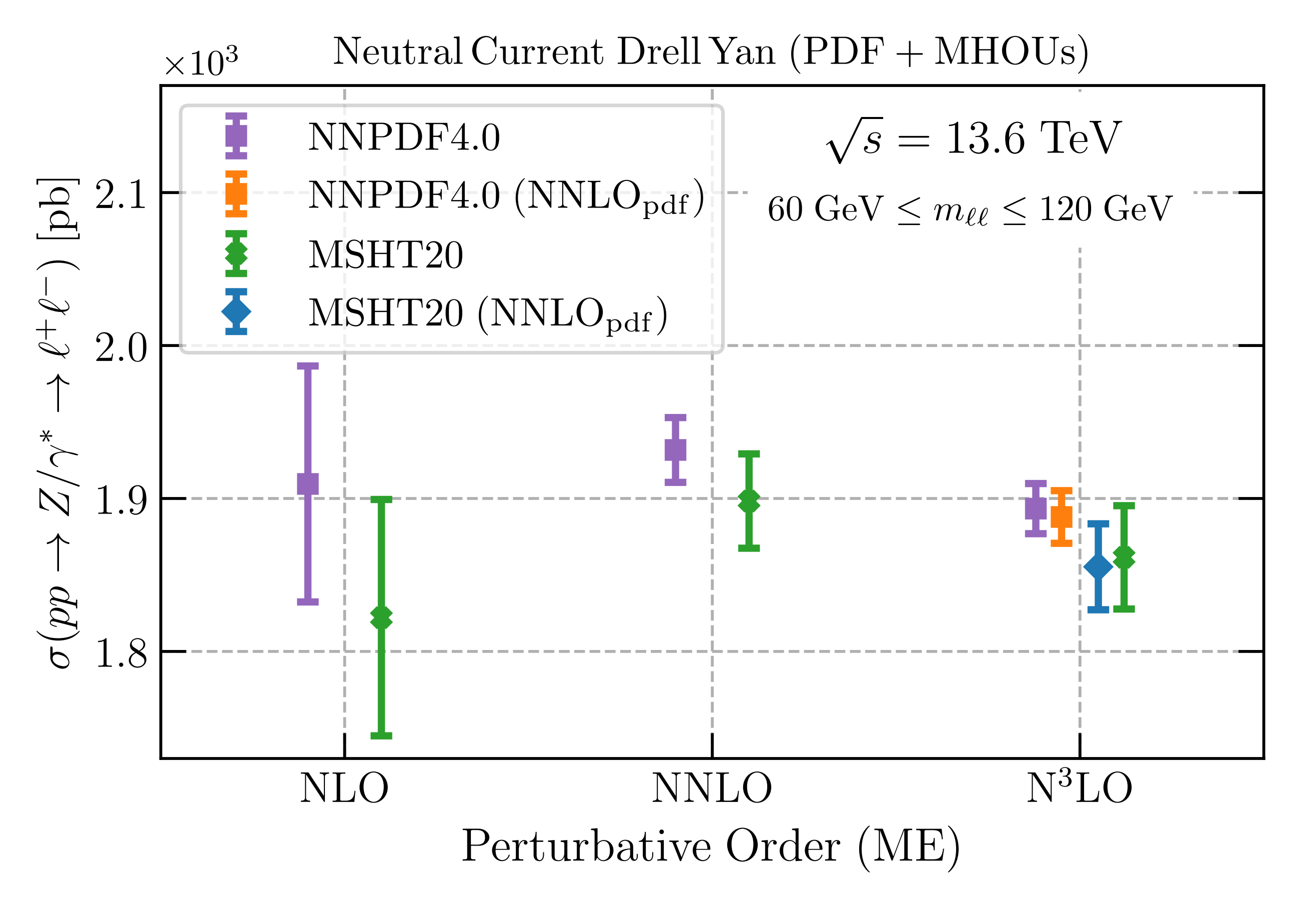
- **ZM** = massless scheme.
- **FFNS** = massive scheme.

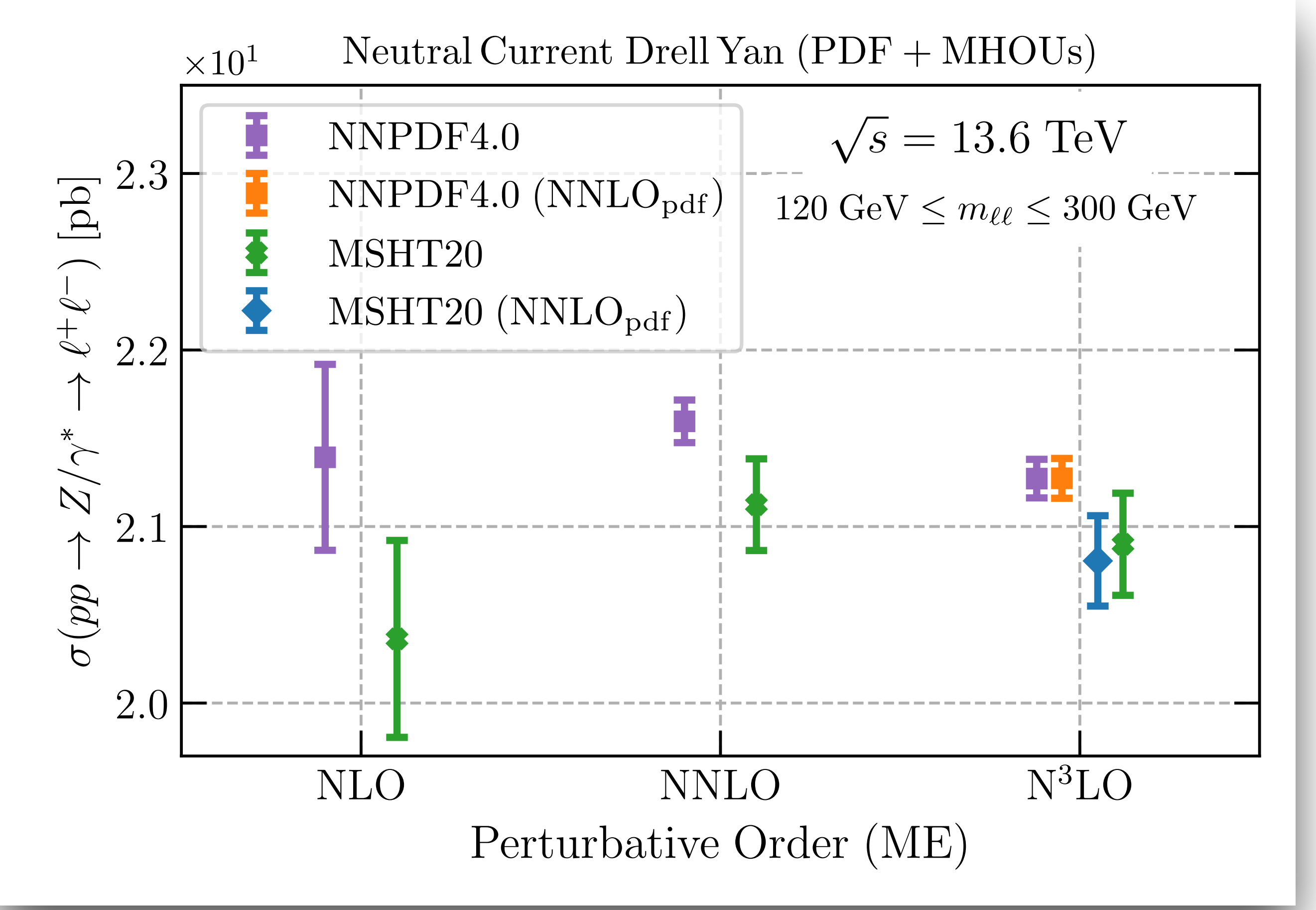
LHC phenomenology: Drell-Yan

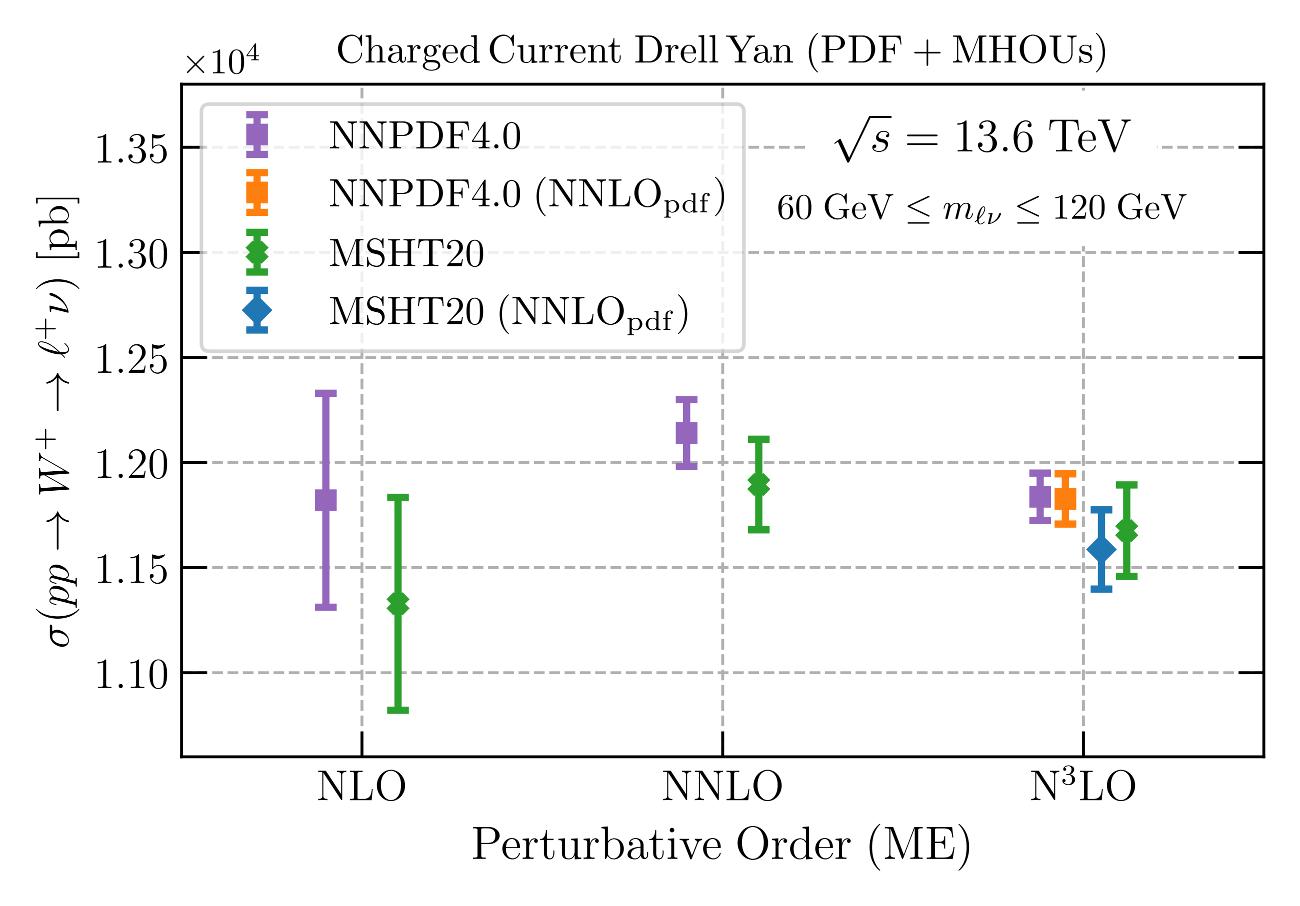
- For gauge boson production (depending on quark luminosities), the usage of aN^{3}LO PDFs **improve the perturbative convergence**.

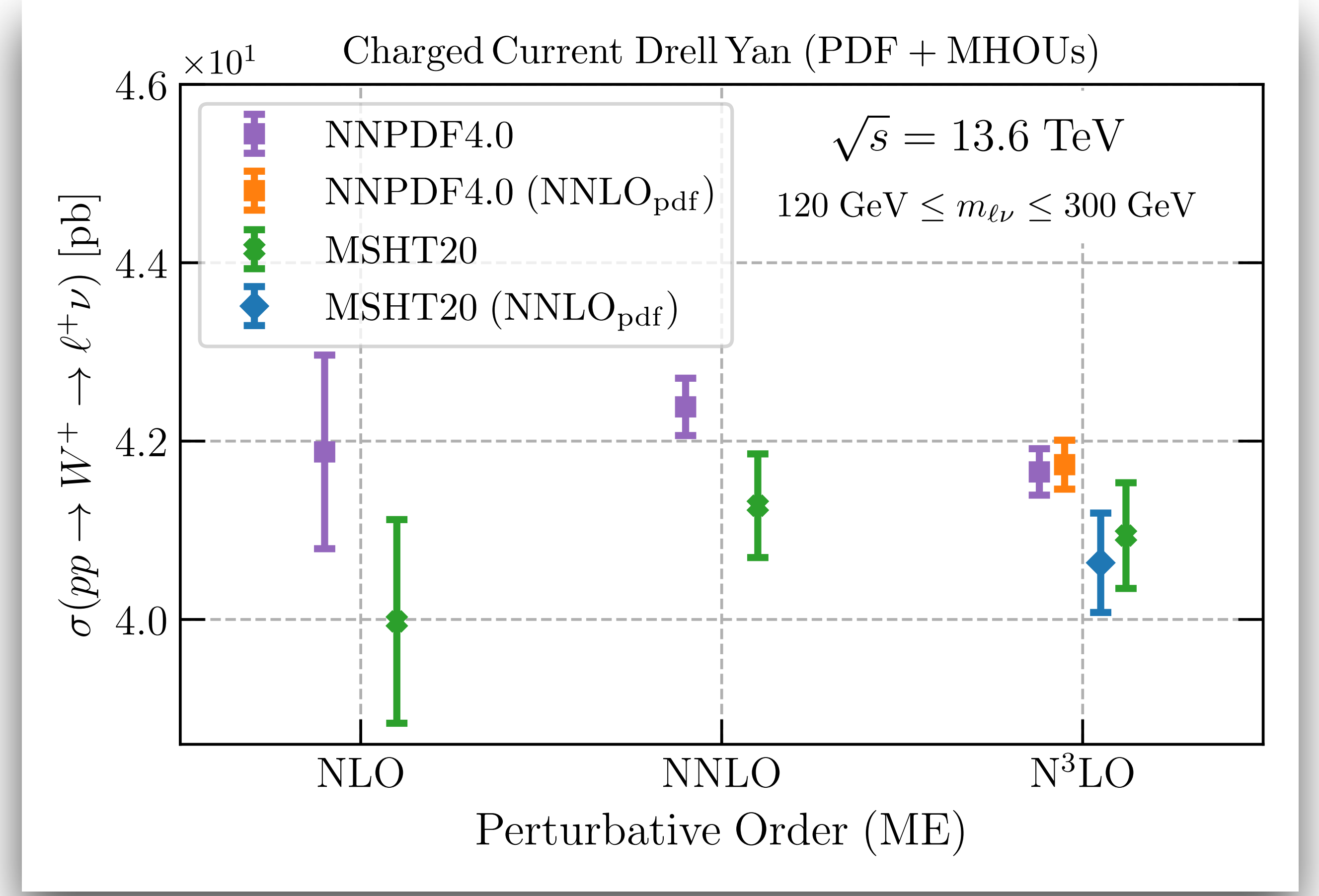
- **Similar N^{3}LO/NNLO ratio** to MSHT20aN3LO.

Process	NNPDF4.0 σ (pb)	$\delta_{\rm th}$	$\delta_{\rm PDF}^{\rm noMHOU}$	$\delta_{\rm PDF}^{\rm MHOU}$	$\Delta_{\rm NNLO}^{\rm app}$	$\Delta_{\rm NNLO}^{\rm exact}$
W^+ (p)	1.2×10^4	1.0	0.5	0.5	1.1	0.1
W^- (p)	8.8×10^3	1.0	0.5	0.5	1.1	0.1
Z (p)	1.9×10^3	0.9	0.4	0.5	1.1	0.3
W^+ (hm)	4.7×10^{-4}	2.8	2.8	3.3	3.2	1.1
W^- (hm)	1.4×10^{-4}	2.9	2.9	3.3	3.3	0.1
Z (hm)	2.1×10^{-4}	2.3	2.3	2.5	3.4	0.3





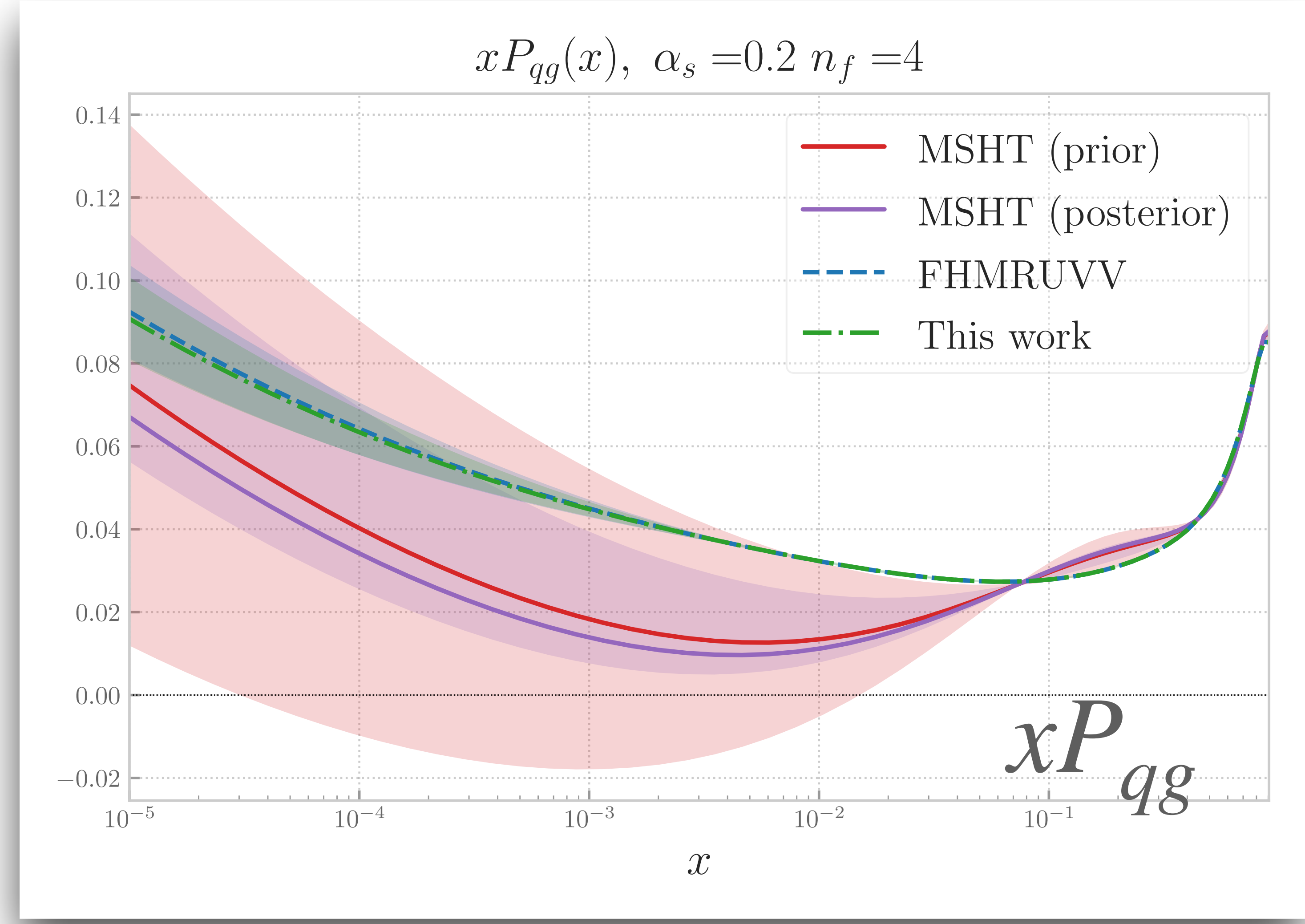


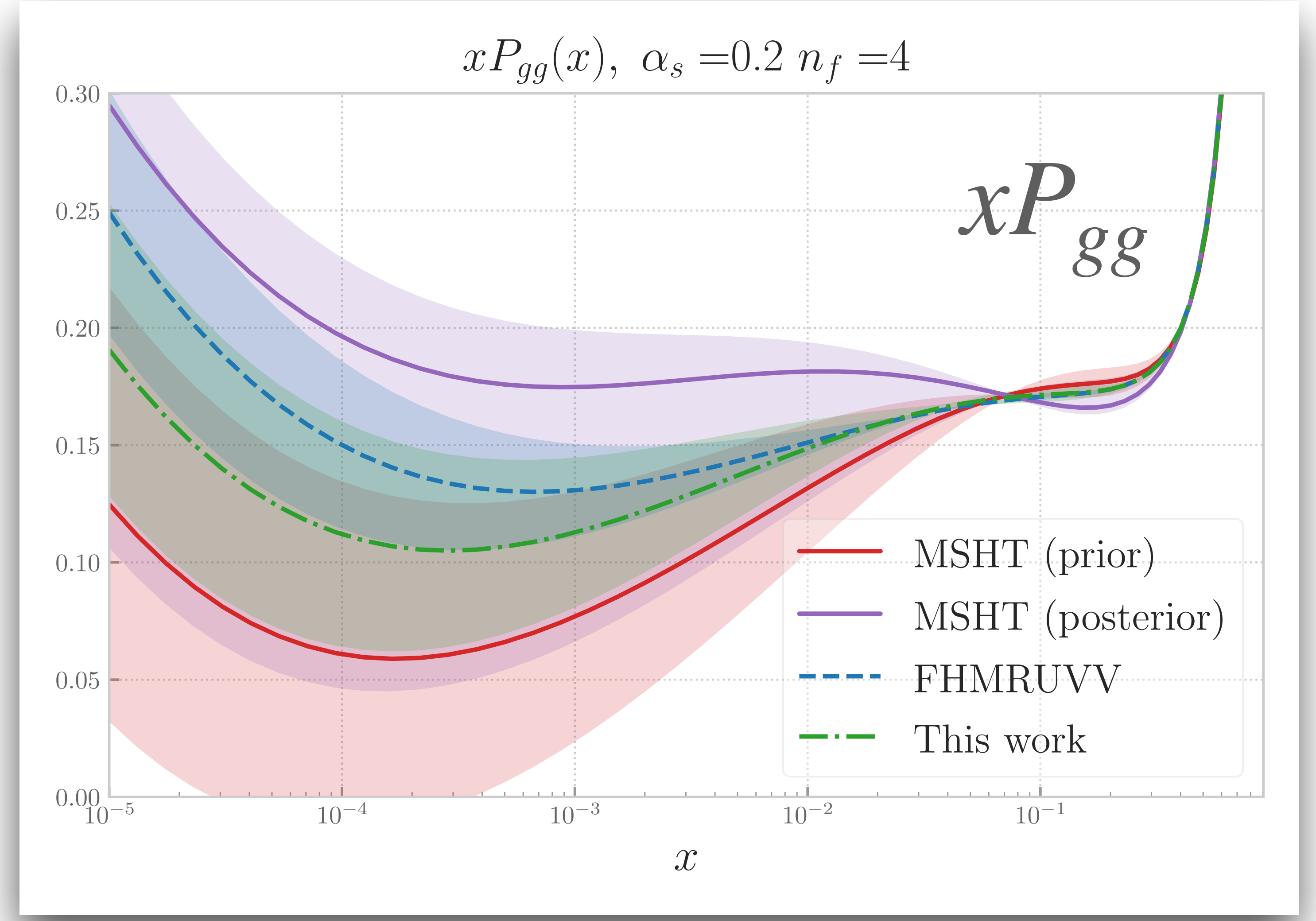


Comparison to MSHT20 aN3LO

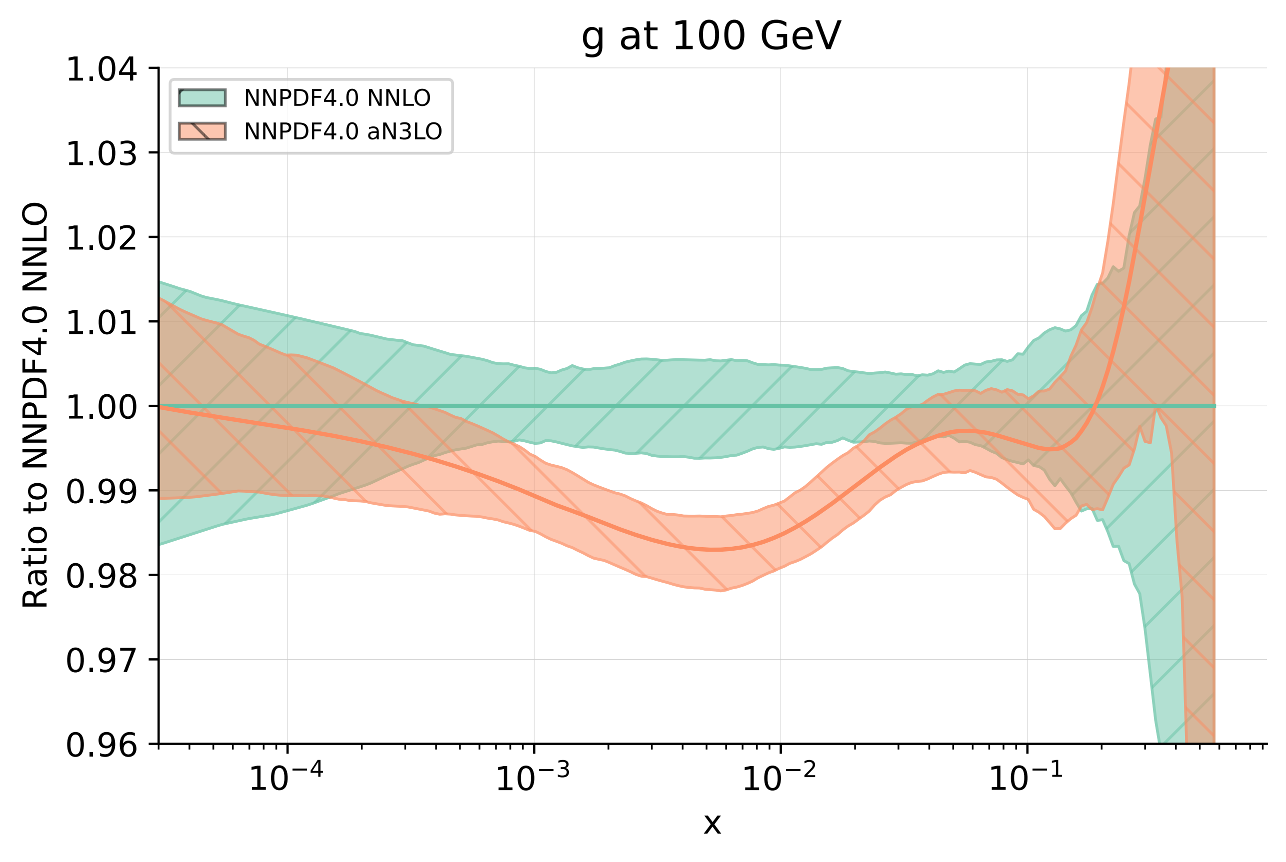
McGowan, Cridge, Harland-Lang, Thorne [arxiv:2207.04739]

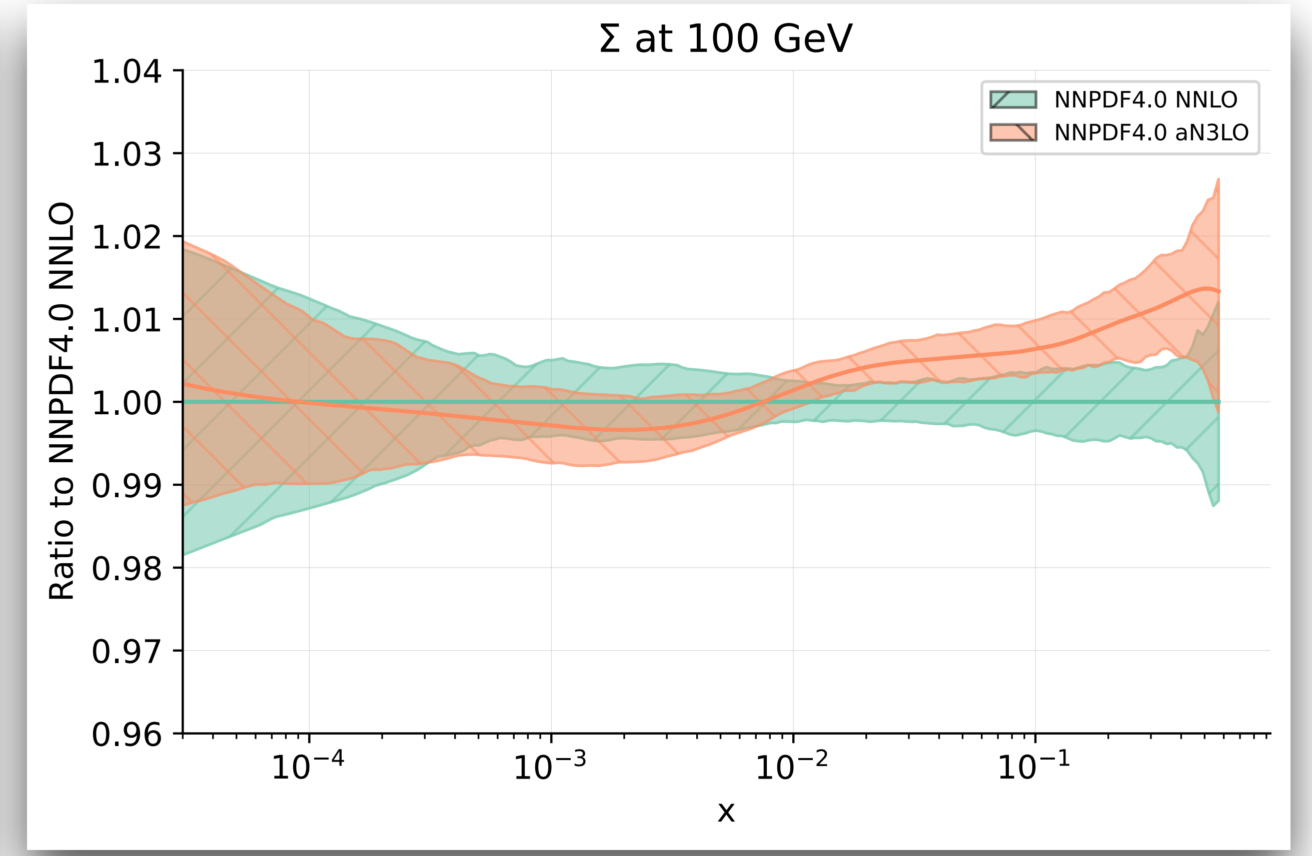
N^3LO Splitting functions



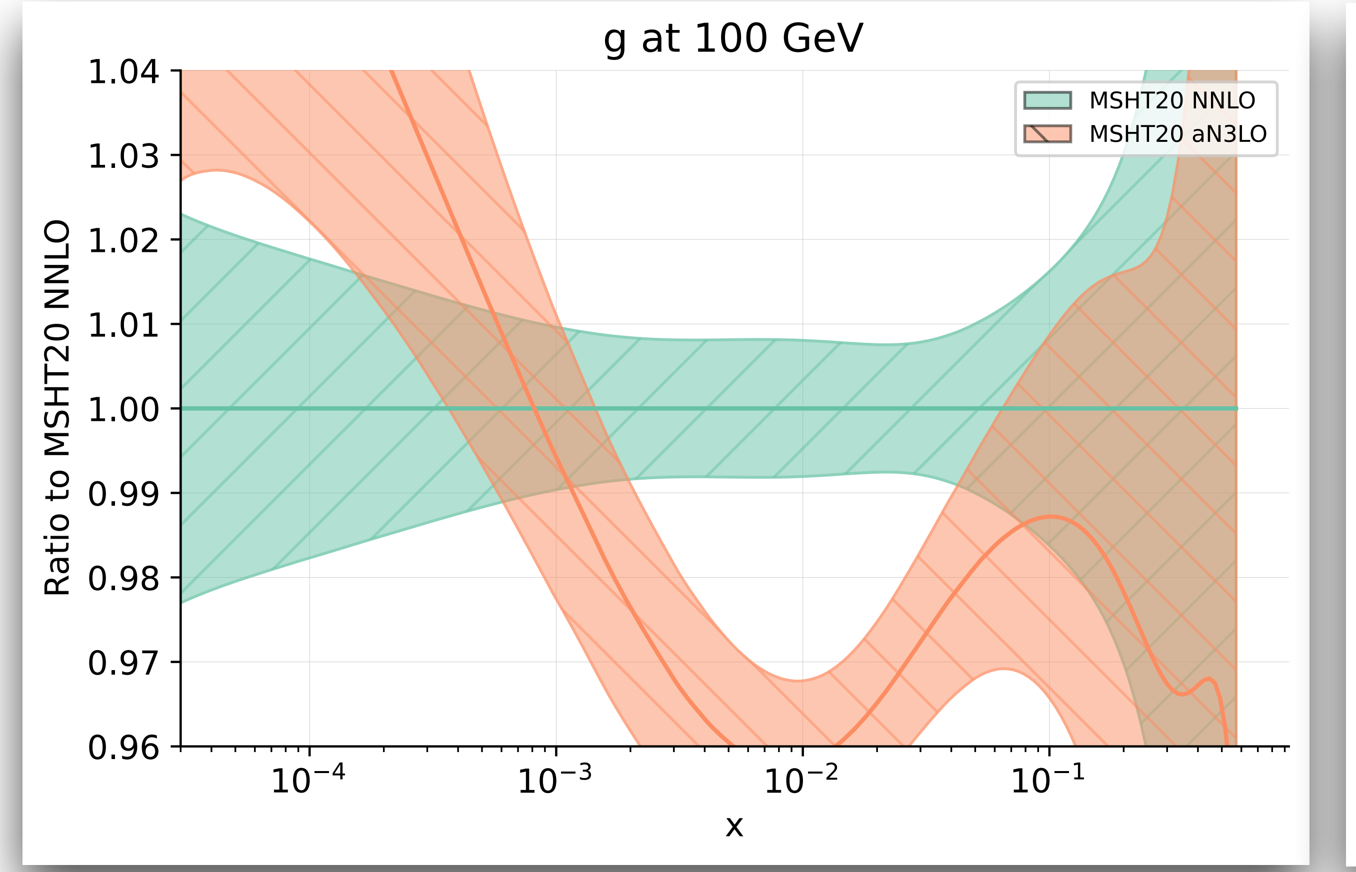


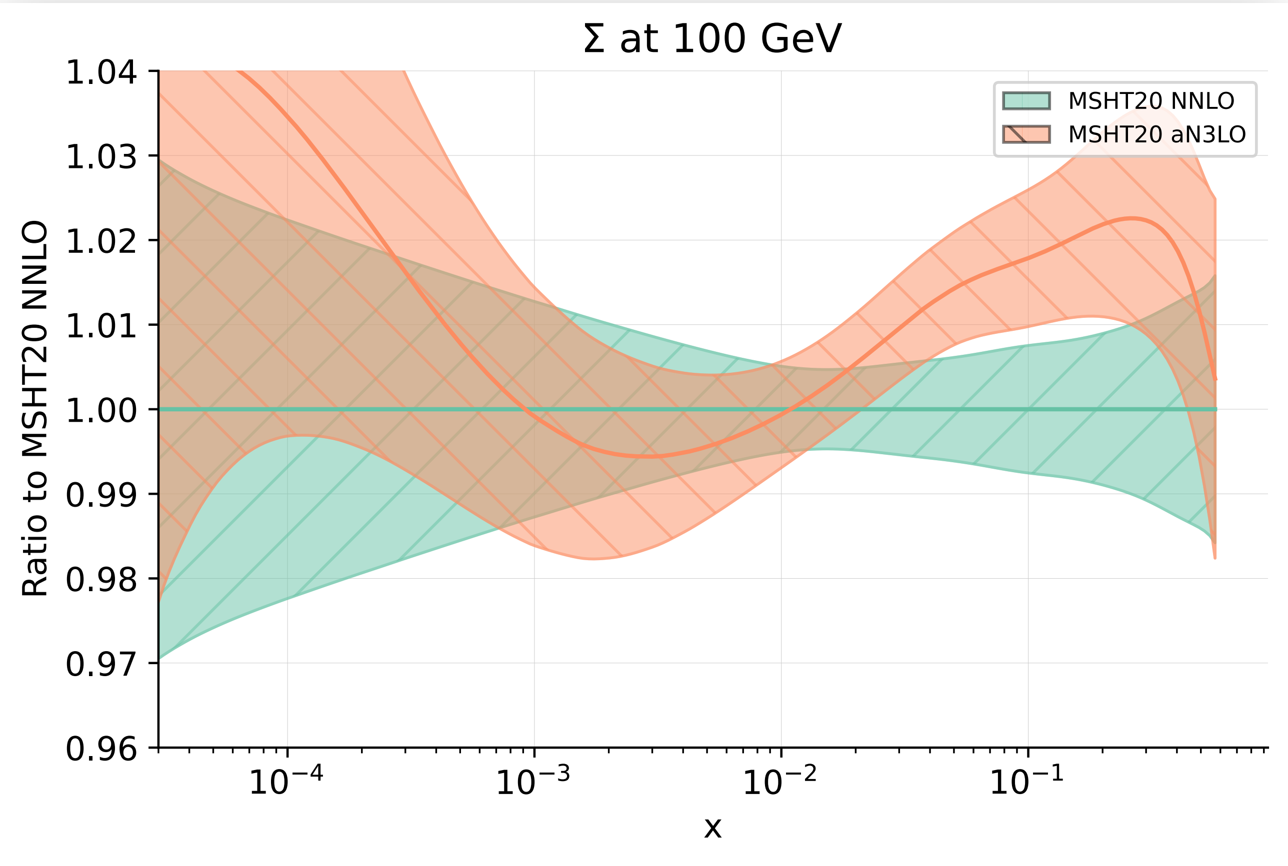
NNPDF4.0 aN^3LO / NNLO





MSHT20 aN^3LO / NNLO





Comparison to MSHT20 aN3LO [pheno]

McGowan, Cridge, Harland-Lang, Thorne [arxiv:2207.04739]

$$\Delta^{\rm exact}_{\rm NNLO} \equiv \left| \frac{\sigma^{\rm N^3LO}_{\rm N^3LO-PDF} - \sigma^{\rm N^3LO}_{\rm NNLO-PDF}}{\sigma^{\rm N^3LO}_{\rm N^3LO-PDF}} \right| \qquad \Delta^{\rm app}_{\rm NNLO} \equiv \frac{1}{2}\left| \frac{\sigma^{\rm NNLO}_{\rm NNLO-PDF} - \sigma^{\rm NNLO}_{\rm NLO-PDF}}{\sigma^{\rm NNLO}_{\rm NNLO-PDF}} \right|$$

Relative uncertainty (%)

Process	NNPDF4.0						MSHT20				
	σ (pb)	$\delta_{\rm th}$	$\delta^{\rm noMHOU}_{\rm PDF}$	$\delta^{\rm MHOU}_{\rm PDF}$	$\Delta^{\rm app}_{\rm NNLO}$	$\Delta^{\rm exact}_{\rm NNLO}$	σ (pb)	$\delta_{\rm th}\sigma$	$\delta_{\rm PDF}$	$\Delta^{\rm app}_{\rm NNLO}$	$\Delta^{\rm exact}_{\rm NNLO}$
$gg \to h$	43.8	4.8	0.6	0.7	0.2	2.2	42.3	5.1	1.7	1.4	5.3
h VBF	4.44	0.6	0.5	0.6	0.2	1.3	4.46	2.1	2.0	1.3	2.9
hW^+	0.97	0.6	0.5	0.6	0.2	0.5	0.95	1.5	1.4	0.8	0.9
hW^-	0.61	0.6	0.6	0.6	0.2	0.3	0.60	1.6	1.5	0.9	1.0
hZ	0.87	0.5	0.4	0.5	0.1	0.3	0.85	1.4	1.4	1.1	0.8

Process	NNPDF4.0						MSHT20				
	σ (pb)	$\delta_{\rm th}$	$\delta^{\rm noMHOU}_{\rm PDF}$	$\delta^{\rm MHOU}_{\rm PDF}$	$\Delta^{\rm app}_{\rm NNLO}$	$\Delta^{\rm exact}_{\rm NNLO}$	σ (pb)	$\delta_{\rm th}\sigma$	$\delta_{\rm PDF}$	$\Delta^{\rm app}_{\rm NNLO}$	$\Delta^{\rm exact}_{\rm NNLO}$
W^+ (p)	1.2×10^4	1.0	0.5	0.5	1.1	0.1	1.2×10^4	1.9	1.7	2.3	0.8
W^- (p)	8.8×10^3	1.0	0.5	0.5	1.1	0.1	8.7×10^3	1.9	1.6	2.1	0.0
Z (p)	1.9×10^3	0.9	0.4	0.5	1.1	0.3	1.9×10^3	1.8	1.6	2.6	0.3
W^+ (hm)	4.7×10^{-4}	2.8	2.8	3.3	3.2	1.1	4.6×10^{-4}	4.0	3.9	2.0	1.3
W^- (hm)	1.4×10^{-4}	2.9	2.9	3.3	3.3	0.1	1.5×10^{-4}	4.2	4.2	2.0	0.6
Z (hm)	2.1×10^{-4}	2.3	2.3	2.5	3.4	0.3	2.2×10^{-4}	3.6	3.6	2.7	0.2