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Theoretical predictions to differential cross sections and decay rates from the loop-tree duality

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Based on:

- 2404.05491
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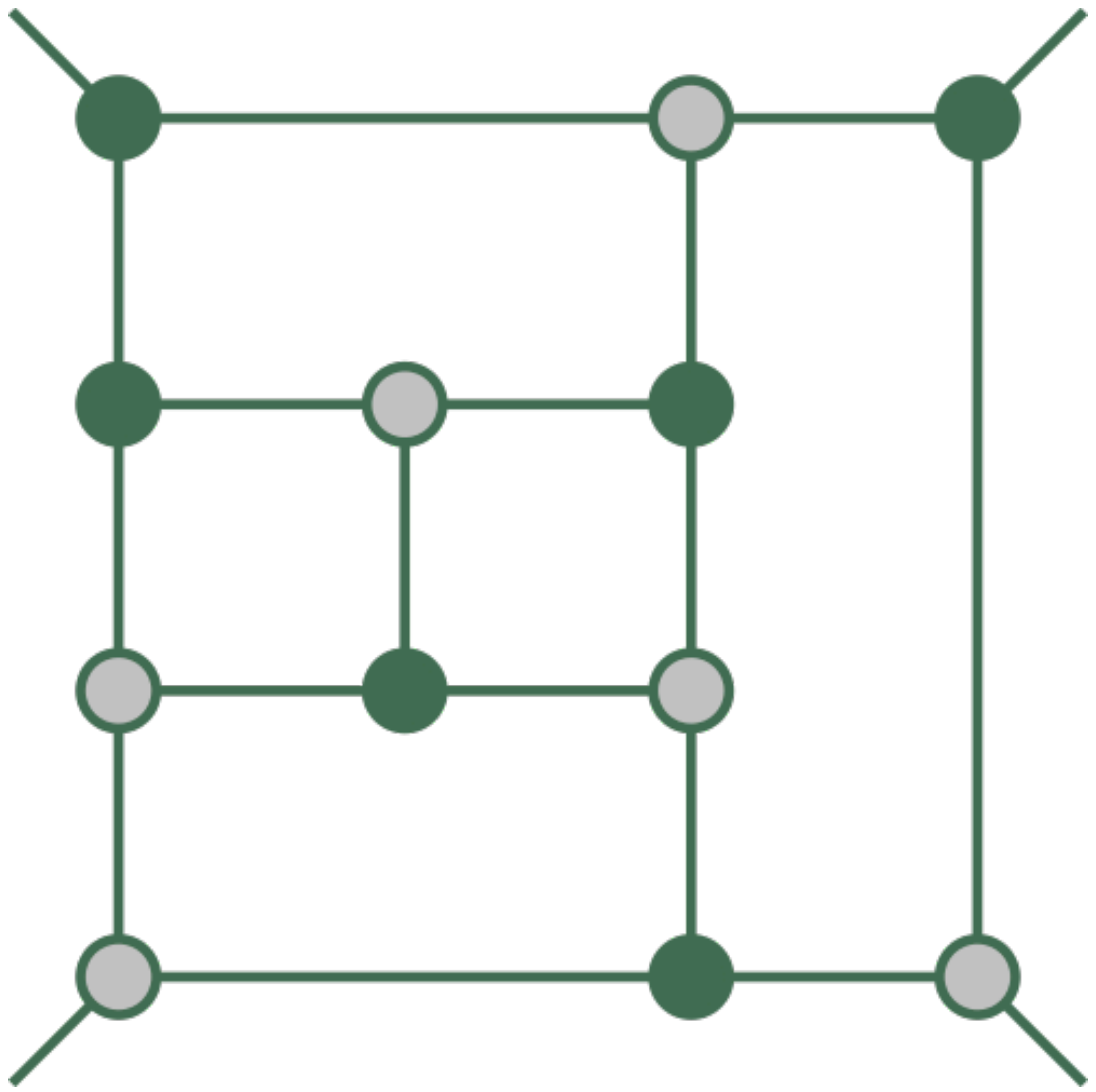
Scattering Amplitudes



are the **building blocks** for theoretical predictions at high-energy colliders (actually, their square)

- Intriguing mathematical properties
- New special functions at higher orders
- Geometric reinterpretation
- Efficient and sophisticated methods for their evaluation (integration by parts IBP, recursive, differential equations, canonical form, …)

However, scattering amplitudes are **not physical objects** because they are defined for a fixed number of external particles

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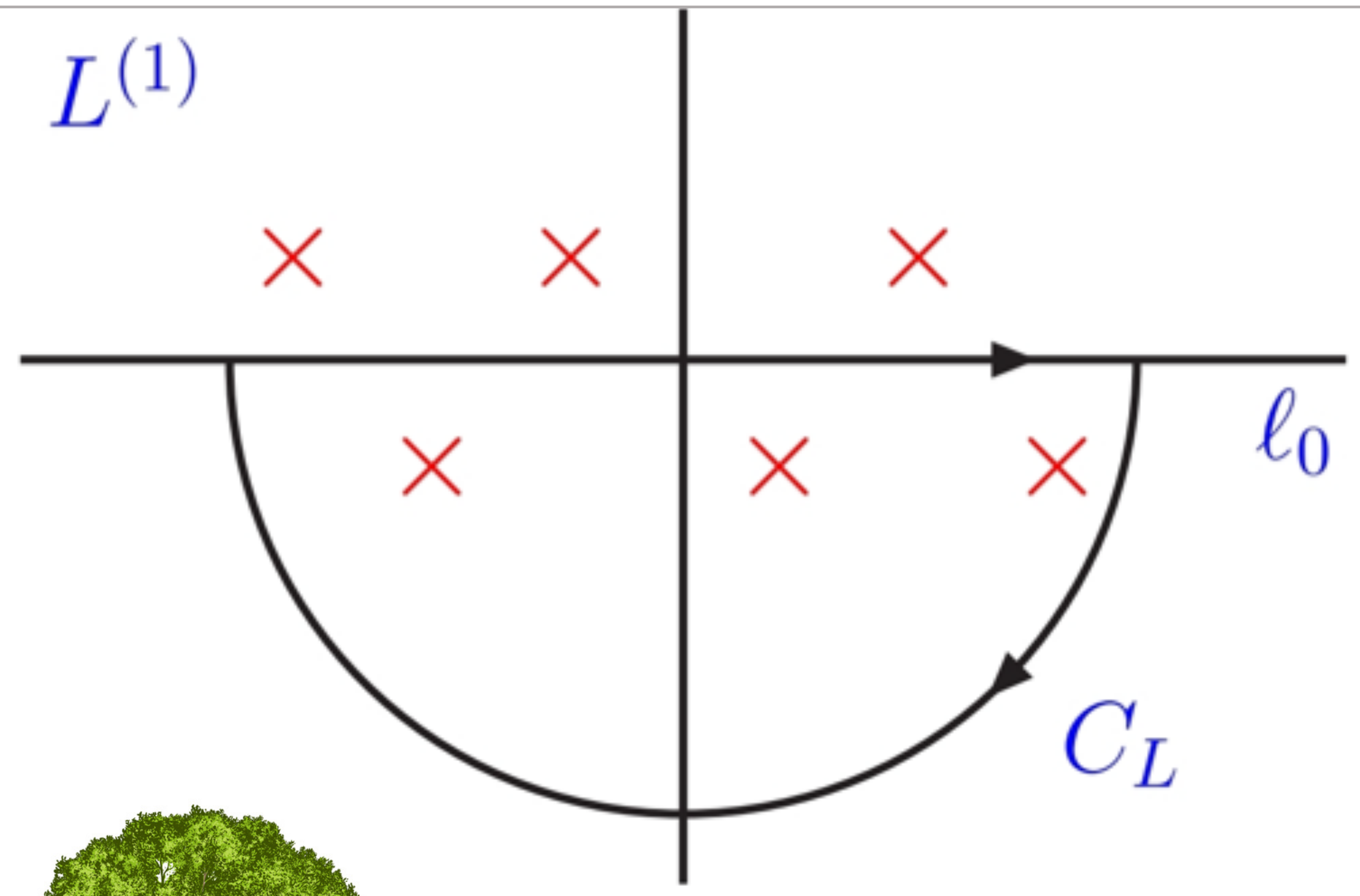
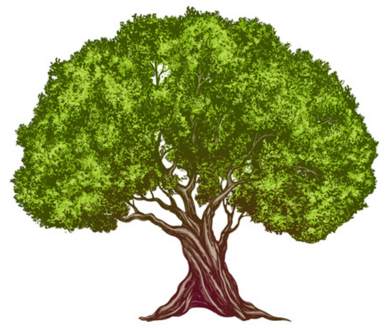
○ Catani, Gleisberg, Krauss, G. Rodrigo and Winter, JHEP 09 (2008) 065

Fundamental concepts



Cauchy's residue theorem
in the loop energy complex plane

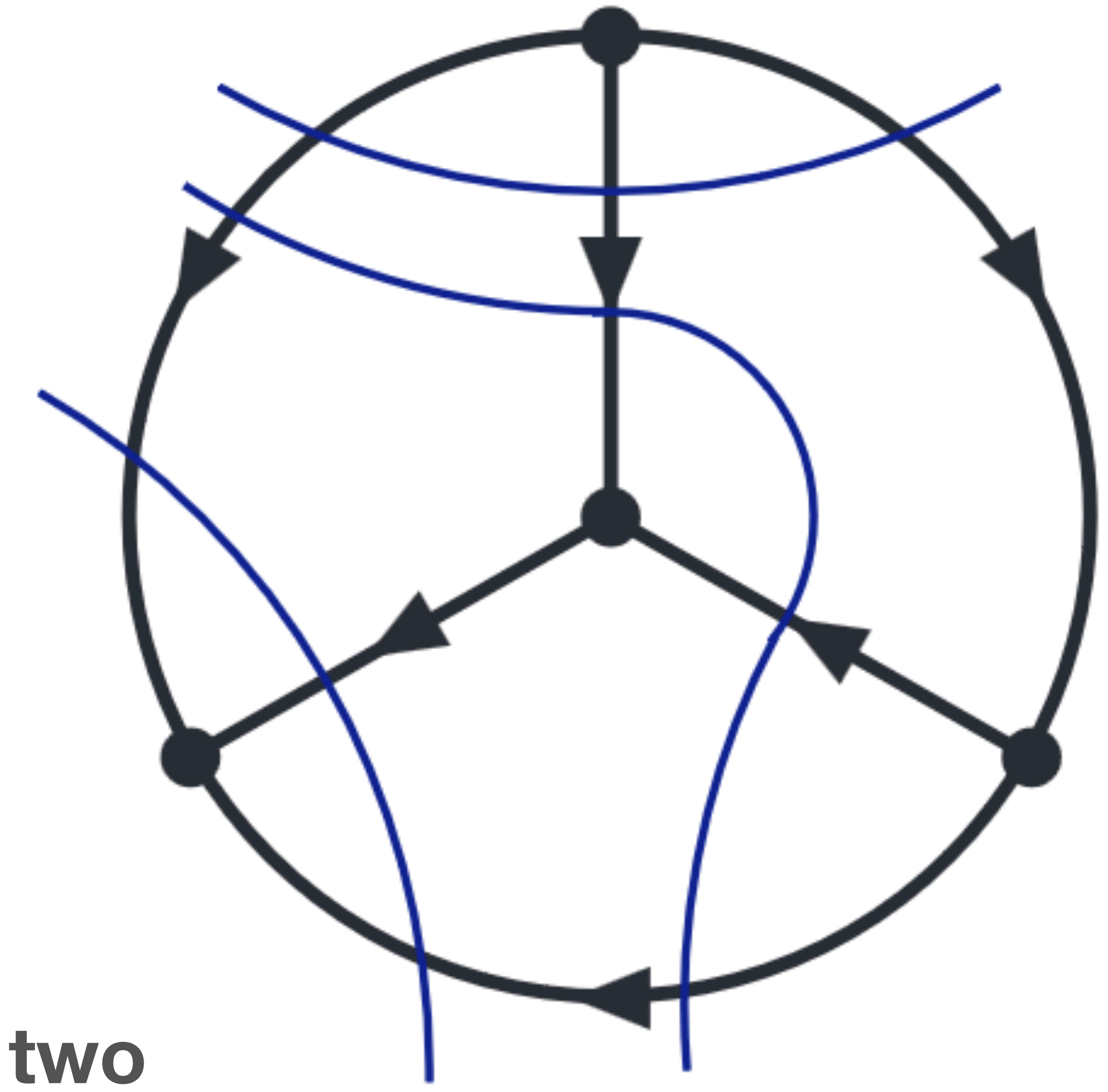
$$\int_{C_L} \prod_{i=1}^{n} G_F(q_i) = -2\pi i \sum \mathrm{Res}\left(\prod_{i=1}^{n} G_F(q_i),\ \mathrm{Im}\,(\eta \bullet q_i) < 0 \right) = \sum$$ 

$\mathrm{Im}\,(\eta \bullet q_i) < 0$ — Selects the poles with negative imaginary components

$\eta^\mu = (1,\mathbf{0})$ — Euclidean space instead of Minkowski space

Two and three loops — ○ Bierenbaum, Catani, Draggiotis and G. Rodrigo, JHEP 10 (2010) 073

Multiple poles — ○ Bierenbaum, Buchta, Draggiotis, Malamos and G. Rodrigo, JHEP 03 (2013) 025



Vacuum amplitudes in LTD

- Feynman propagators are substituted by **causal propagators** of the form

$$\frac{1}{\lambda_{i_1 i_2 \cdots i_n}} = \frac{1}{\sum_{s=1}^{n} q_{i_s,0}^{(+)}}\,, \qquad q_{i_s,0}^{(+)} = \sqrt{\boldsymbol{q}_{i_s}^2 + m_{i_s}^2 - \imath 0}\,.$$

- Each causal propagator involves a set of internal particles that divide the amplitude in **two subamplitudes**, with the momentum flow of all particles in the set aligned in the same direction
- Two causal propagators are compatible if the common particles are aligned in the same direction $\equiv$ **DAG (Directed acyclic graph) configurations**
- If $\lambda_{i_1 i_2 \cdots i_n} = 0$, all particles in the set would become on shell, but $\lambda_{i_1 i_2 \cdots i_n}$ cannot vanish !!!
- Consequently, the vacuum amplitude cannot generate any soft, collinear or threshold singularities (only UV singularities allowed)

Generate all final states from residues on the on-shell energies after analytic continuation to negative values of those in the initial state

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Differential observables

- From a **vacuum amplitude** $\mathcal{A}_{\rm D}^{(\Lambda)}$ in the LTD representation that depends on Λ loop momenta

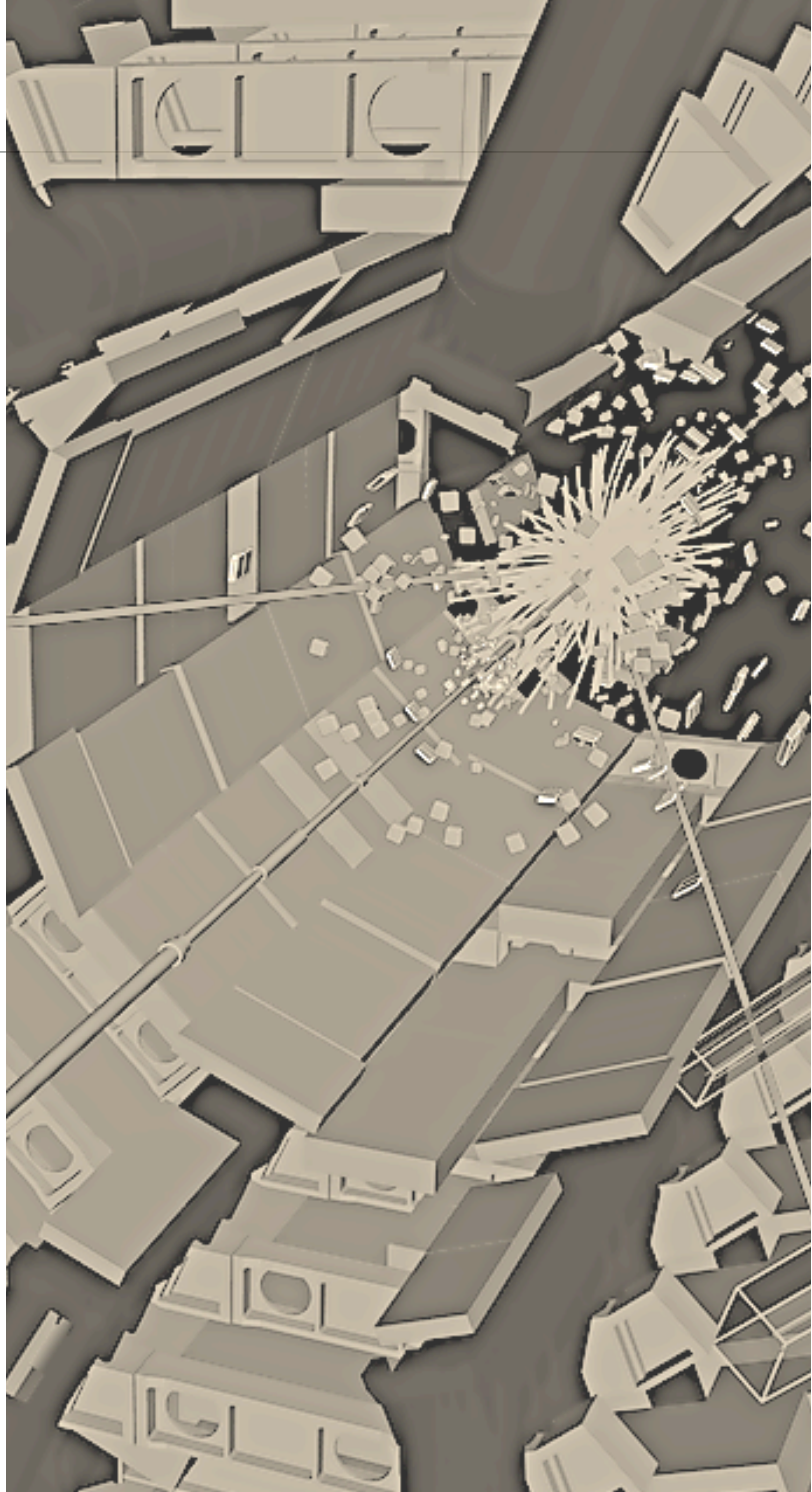
$$d\sigma_{{\rm N}^k{\rm LO}} = \frac{d\Lambda}{2s} \sum_{(i_1\cdots i_n ab)\in\Sigma} \mathcal{A}_{\rm D}^{(\Lambda,{\rm R})}(i_1\cdots i_n ab)\, \mathcal{O}_{i_1\cdots i_n}\, \tilde{\Delta}_{i_1\cdots i_n \bar{a}\bar{b}} \ ,$$

- Integration measure $d\Lambda = \prod_{j=1}^{\Lambda-2} d\Phi_{\ell_j} = \prod_{j=1}^{\Lambda-2} \mu^{4-d} \frac{d^{d-1}\ell_j}{(2\pi)^{d-1}}$,
- Energy conservation $\tilde{\Delta}_{i_1\cdots i_n \bar{a}\bar{b}} = 2\pi\, \delta(\lambda_{i_1\cdots i_n \bar{a}\bar{b}})$,
- If $\mathcal{O}_{i_1\cdots i_n} = \mathbf{1}$, total cross section
- Renormalised **phase-space residues**

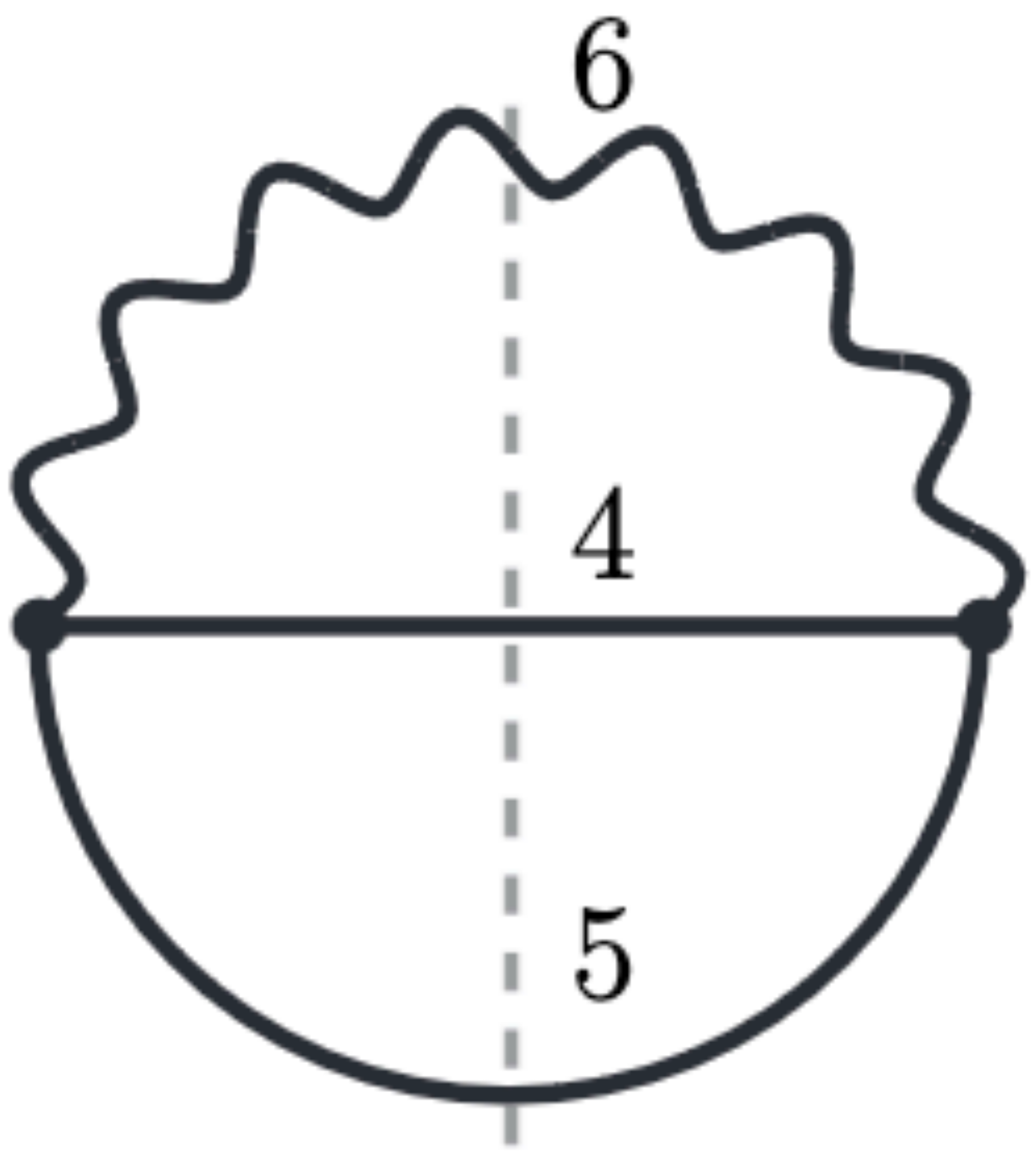
$$\mathcal{A}_{\rm D}^{(\Lambda,{\rm R})}(i_1\cdots i_n ab) = {\rm Res}\left(\frac{x_{ab}}{2}\mathcal{A}_{\rm D}^{(\Lambda)}, \lambda_{i_1\cdots i_n ab}\right) - \mathcal{A}_{\rm UV/C}^{(\Lambda)}(i_1\cdots i_n ab)$$

$x_{ab} = 4 q_{a,0}^{(+)} q_{b,0}^{(+)}$

local UV renormalisation and local subtraction of initial-state collinear



Proof of concept LO



- The two-loop vacuum amplitude

$$\mathcal{A}_{\rm D}^{(2,f)} = \frac{2\, g_f^{(0)}}{x_{456}} \left(\frac{\overline{|\mathcal{M}^{(0)}_{f\to q\bar{q}}|^2}}{\lambda_{456}} + 2\lambda_{45\bar{6}} \right) , \qquad f = H, \gamma^* .$$

- The phase-space residue

$$\mathcal{A}_{\rm D}^{(2,f)}(456) \equiv \text{Res}\left(\frac{x_6}{2} \mathcal{A}_{\rm D}^{(2,f)}, \lambda_{456} \right) = \frac{g_f^{(0)}}{x_{45}} \overline{|\mathcal{M}^{(0)}_{f\to q\bar{q}}|^2} ,$$

- The decay rate

$$d\Gamma^{\rm LO}_{f\to q\bar{q}} = \frac{d\Phi_{\ell_2}}{2\sqrt{s}} \mathcal{A}_{\rm D}^{(2,f)}(456)\, \tilde{\Delta}_{45\bar{6}} \, , \quad ✓$$

$$\tilde{\Delta}_{45\bar{6}} = \frac{\pi}{\beta}\, \delta\left(\ell_2 - \frac{\beta\sqrt{s}}{2} \right) .$$

Double-collinear configuration (NLO)

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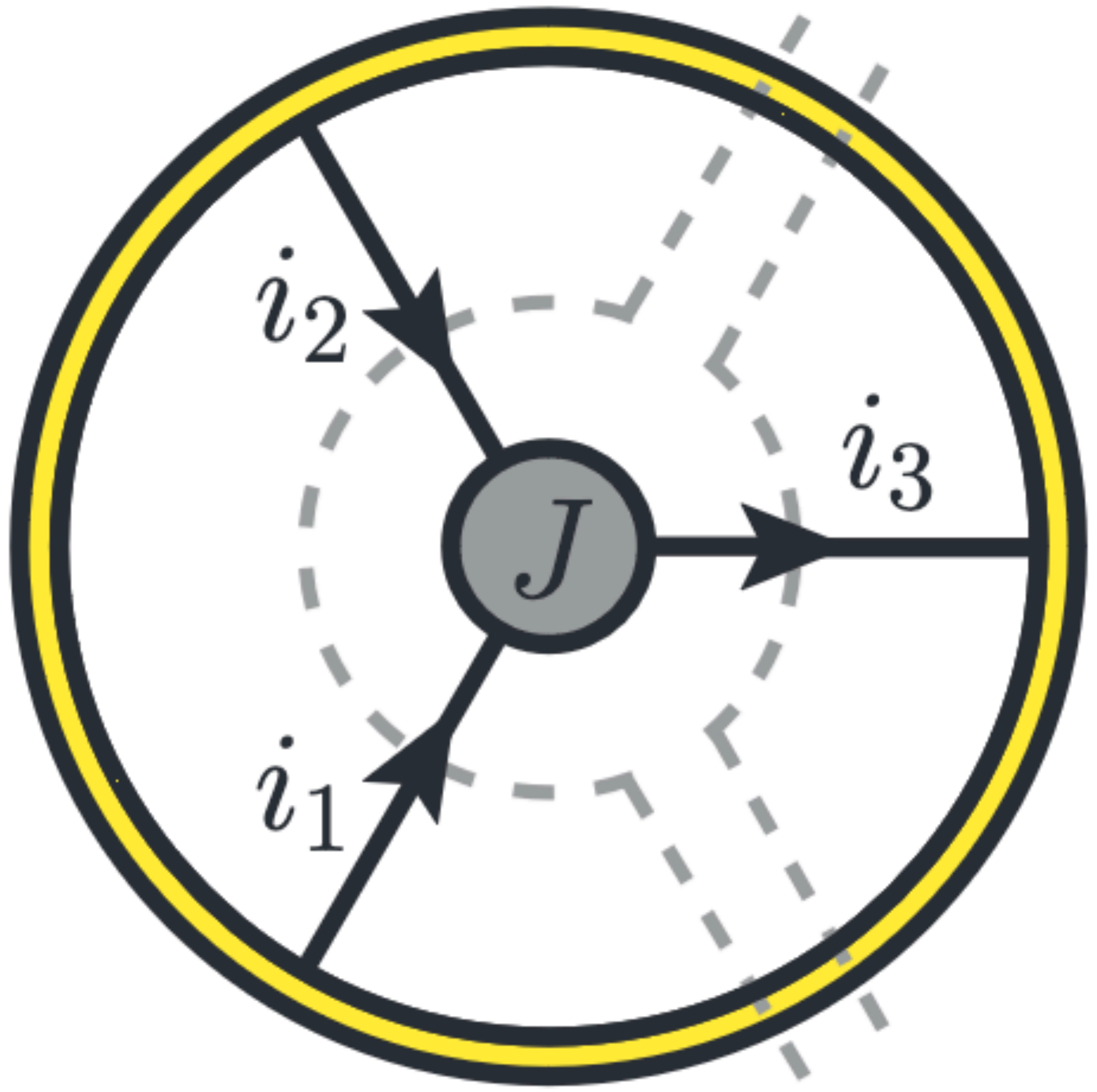


- A vacuum amplitude with the insertion of a trivalent interaction (it could be a multiloop subdiagram or an effective operator). The LTD vacuum amplitude is proportional to

$$\mathcal{A}_{\rm D}^{(\Lambda)} \sim \frac{1}{\lambda_{i_1 i_2 \cdots ab}\, \lambda_{i_3 \cdots ab}} \,,$$

- Each phase-space residue is singular for $\lambda_{i_1 i_2 \bar{i}_3} = q_{i_1,0}^{(+)} + q_{i_2,0}^{(+)} - q_{i_3,0}^{(+)} \to 0$, due to the following identities

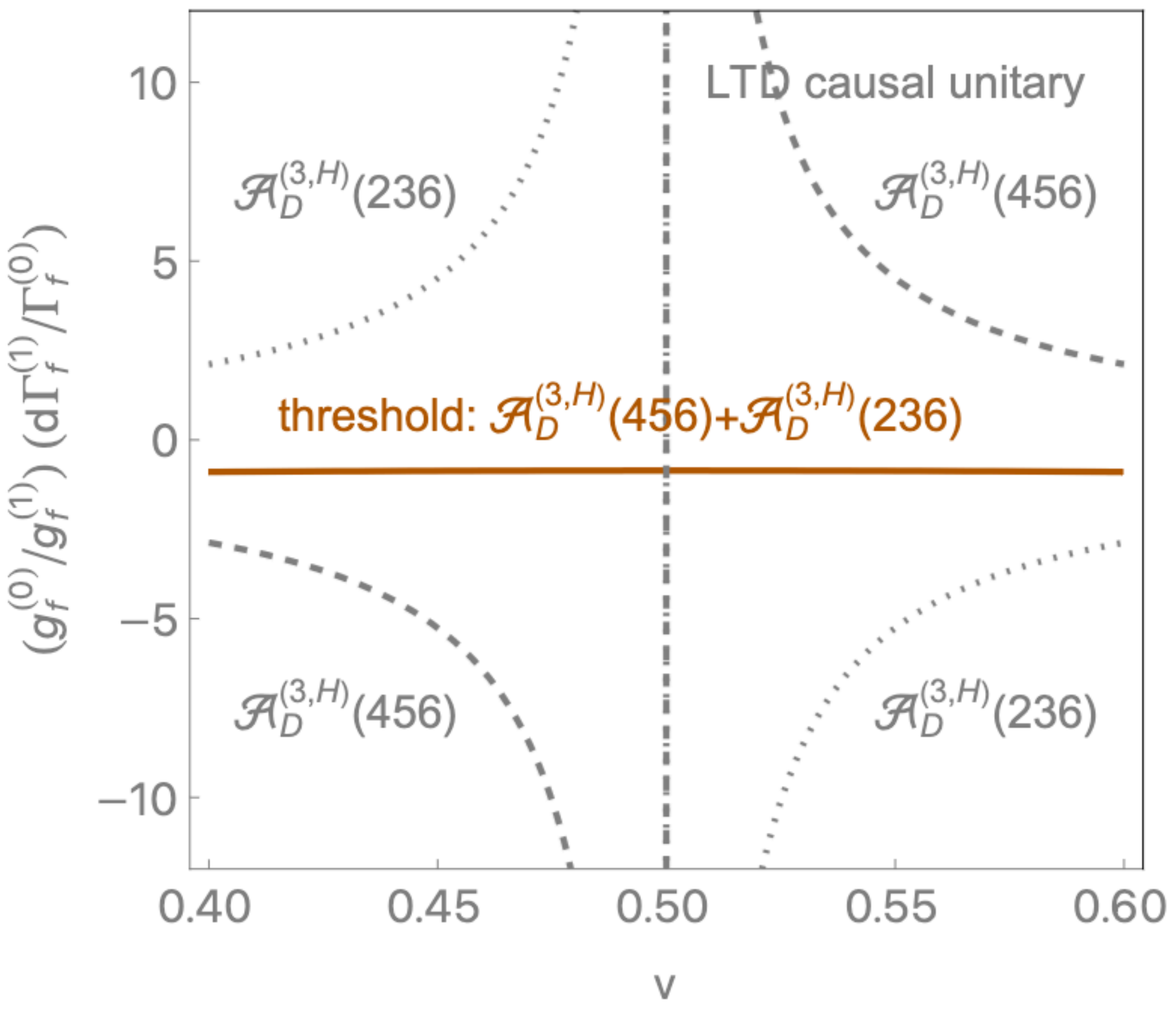
$$\frac{1}{\lambda_{i_1 i_2 \cdots ab}}\bigg|_{\lambda_{i_3 \cdots ab}=0} = \frac{1}{\lambda_{i_1 i_2 \bar{i}_3}} \,, \qquad \frac{1}{\lambda_{i_3 \cdots ab}}\bigg|_{\lambda_{i_1 i_2 \cdots ab}=0} = -\frac{1}{\lambda_{i_1 i_2 \bar{i}_3}} \,,$$

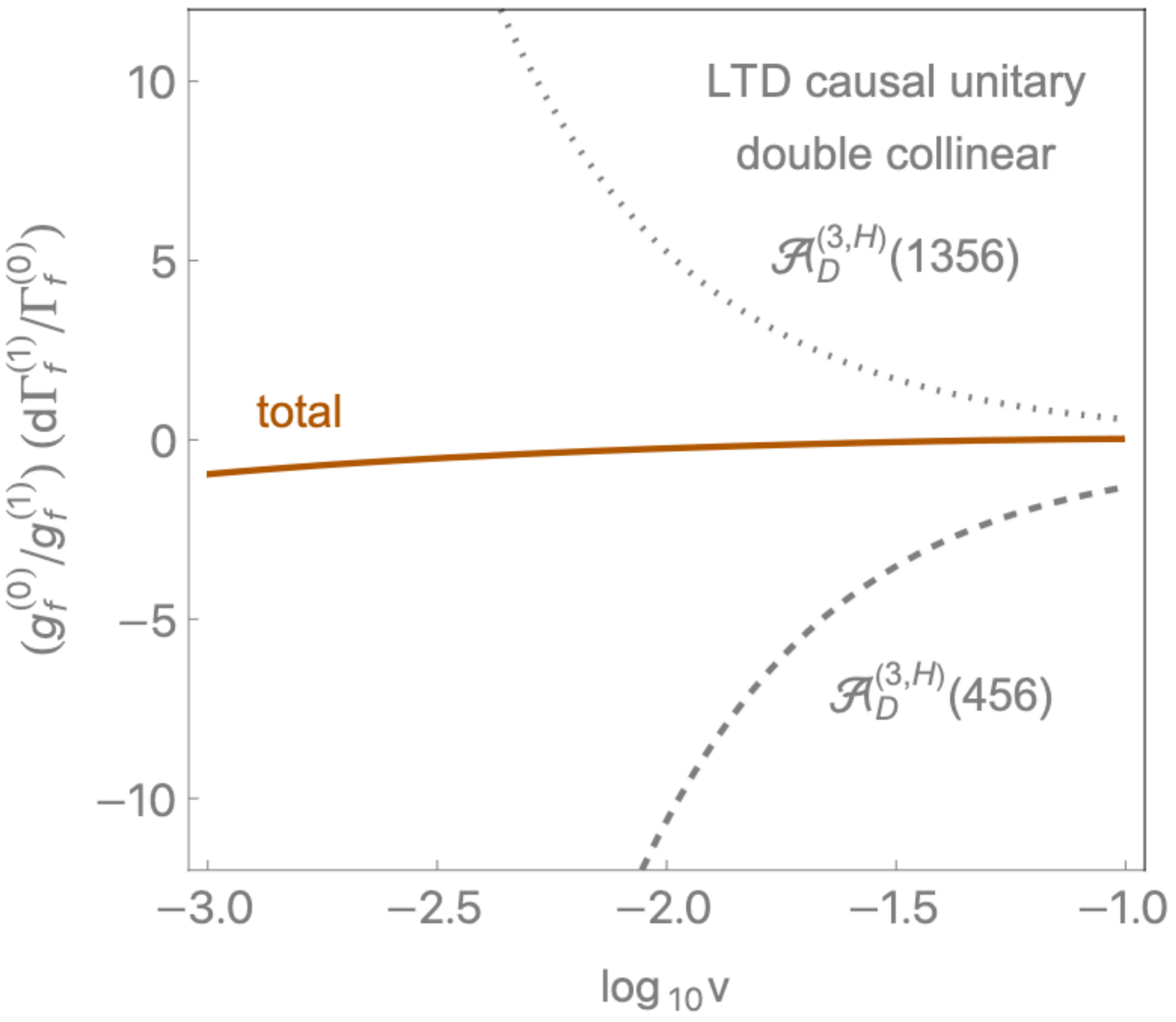
- The sum of phase-space residues is finite in that limit:

$$\lim_{\lambda_{i_1 i_2 \bar{i}_3} \to 0} \left(\mathcal{A}_{\rm D}^{(\Lambda)}(i_1 i_2 \cdots ab)\, \tilde{\Delta}_{i_1 i_2 \cdots \bar{a}\bar{b}} + \mathcal{A}_{\rm D}^{(\Lambda)}(i_3 \cdots ab)\, \tilde{\Delta}_{i_3 \cdots \bar{a}\bar{b}} \right) = \mathcal{O}(\lambda_{i_1 i_2 \bar{i}_3}^{0}) \,.$$

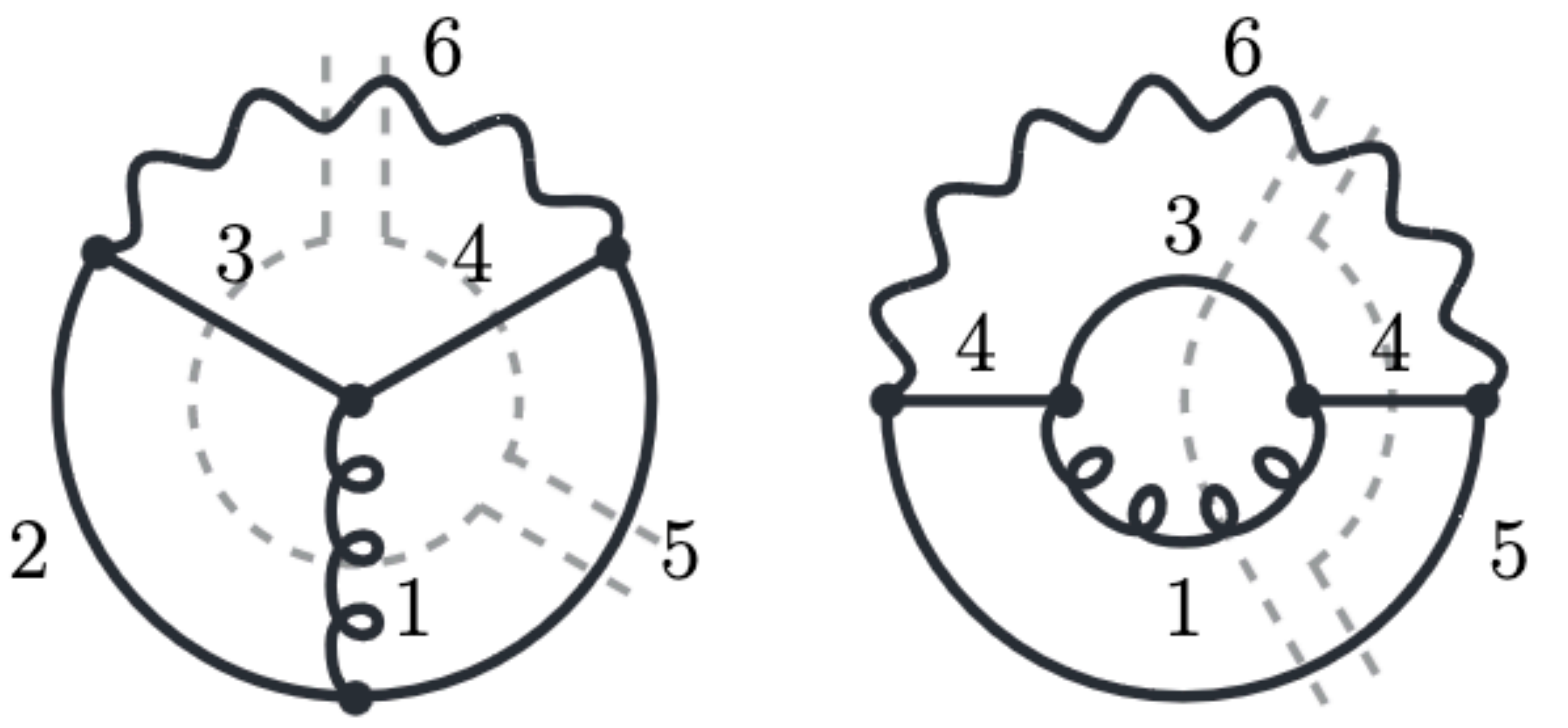
Proof of concept NLO: flat integrands

$$d\Gamma^{(1)}_{f\to q\bar{q}} = \frac{d\Phi_{\ell_1\ell_2}}{2\sqrt{s}} \left[\left(\mathcal{A}_{\rm D}^{(3,f,{\rm R})}(456)\, \tilde{\Delta}_{45\bar{6}} + \mathcal{A}_{\rm D}^{(3,f)}(1356)\, \tilde{\Delta}_{135\bar{6}} \right) + (5 \leftrightarrow 2,\, 4 \leftrightarrow 3) \right] .$$

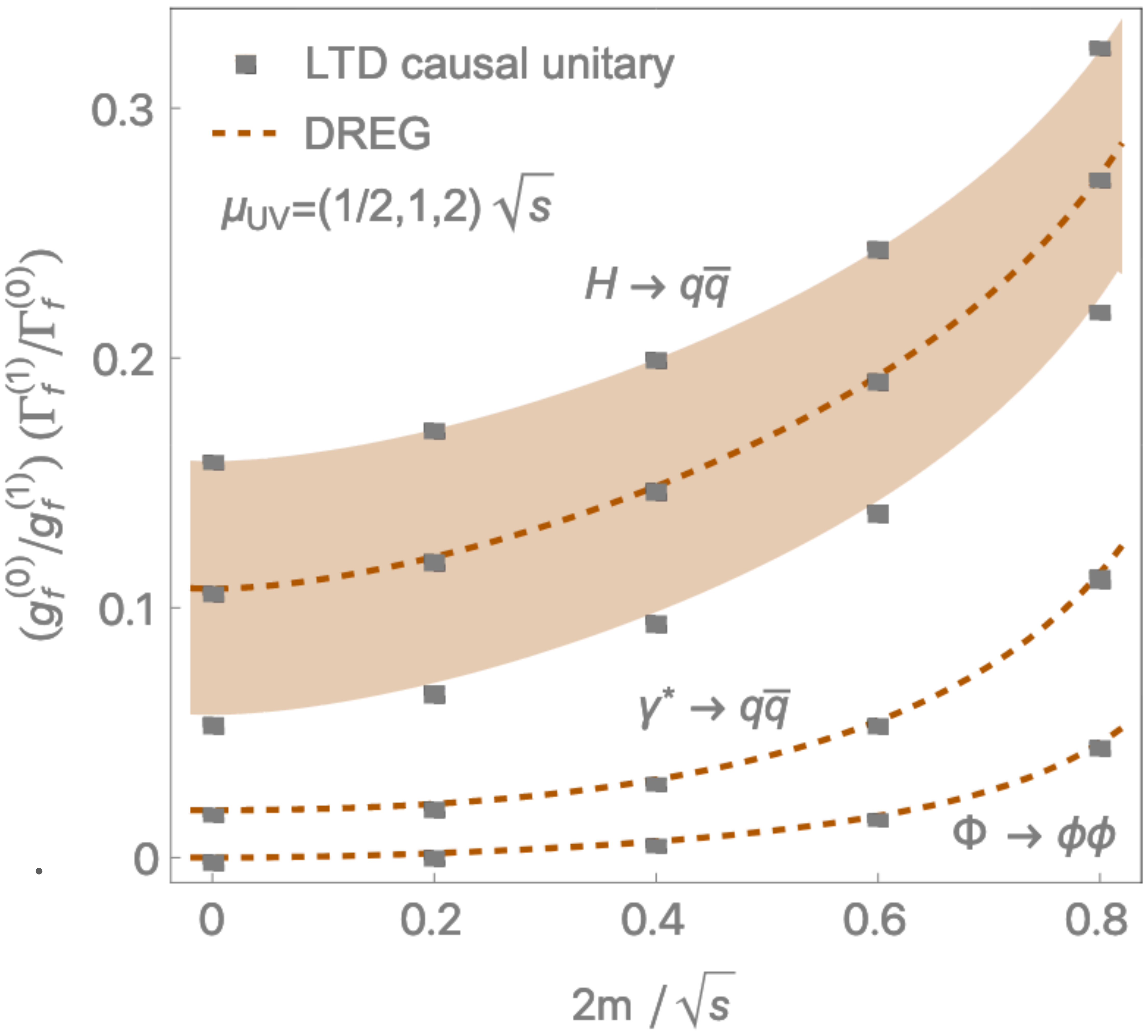




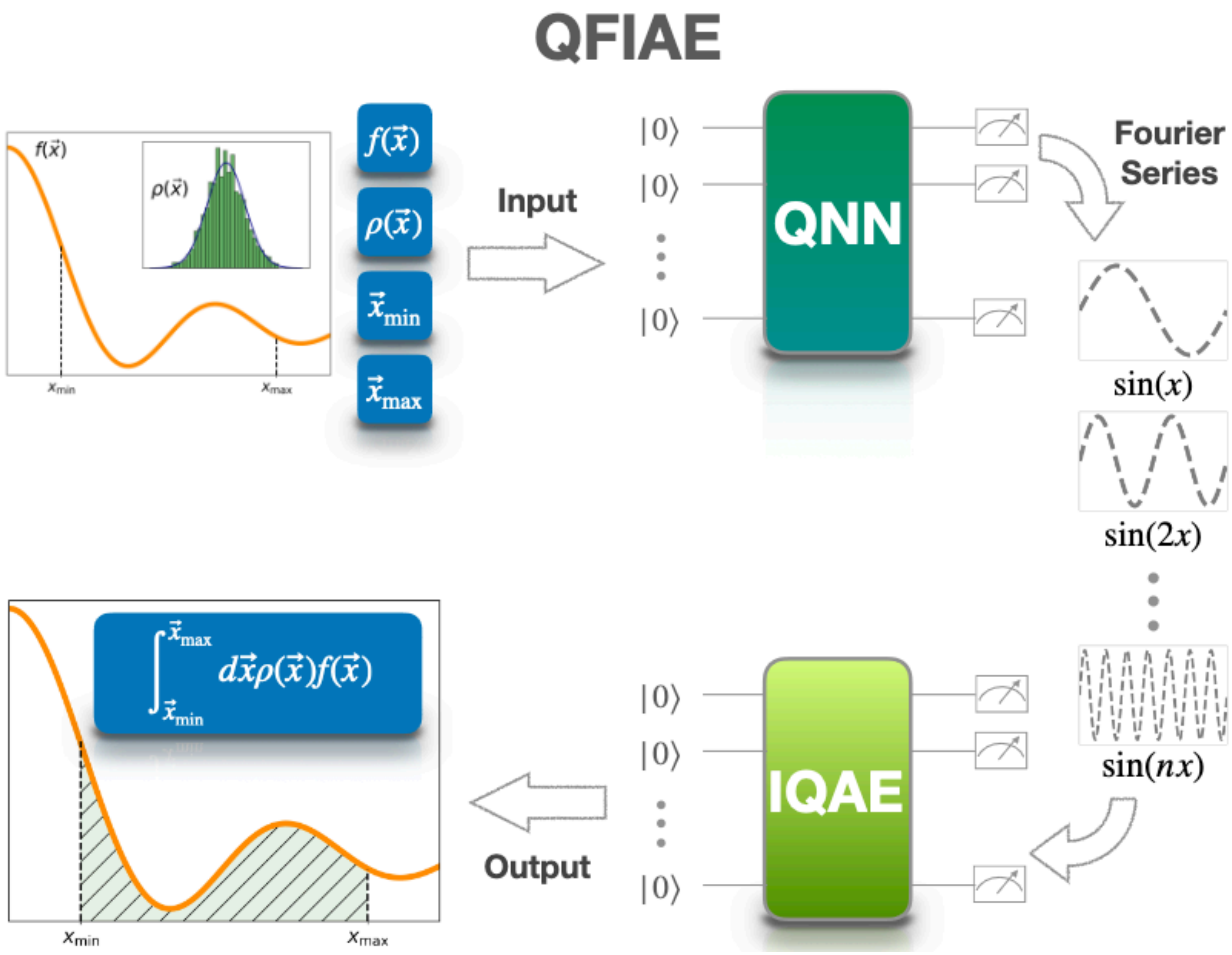
Proof of concept NLO



$$d\Gamma^{(1)}_{f\to q\bar{q}} = \frac{d\Phi_{\ell_1\ell_2}}{2\sqrt{s}}\left[\left(\mathcal{A}_{\rm D}^{(3,f,{\rm R})}(456)\,\tilde{\Delta}_{45\bar{6}} + \mathcal{A}_{\rm D}^{(3,f)}(1356)\,\tilde{\Delta}_{135\bar{6}}\right) + (5\leftrightarrow 2,\, 4\leftrightarrow 3)\right].$$



Quantum Computing approx



Quantum results

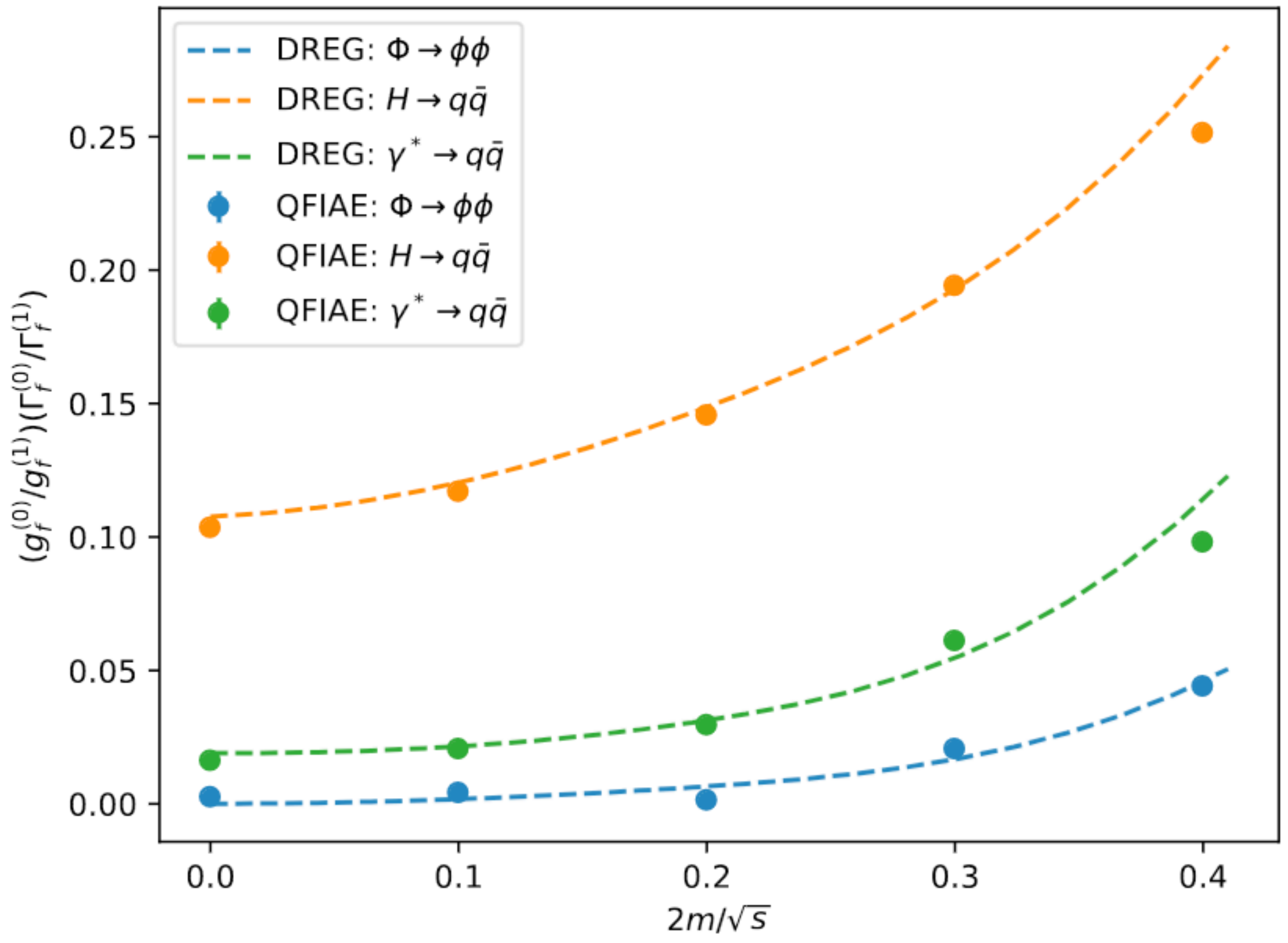


FIG. 3. Quantum-integrated decay rates in a quantum simulator for the three decay processes $H \to q\bar{q}(g)$, $\gamma^* \to q\bar{q}(g)$ and $\Phi \to \phi\phi(\phi)$ at NLO as a function of the final state mass, using QFIAE and LTD causal unitary. The dashed lines are the theoretical predictions in dimensional regularization. The parameters used in the quantum implementation are: $max\_steps = 15000$, $step\_size = 0.001$, $layers = n_{Fourier} = 20$, $n_{qubits} = 6$ for the QNN and $n_{qubits} = 5$, $n_{shots} = 10^3$, $\epsilon = 0.01$, $\alpha = 0.05$ for the IQAE module.

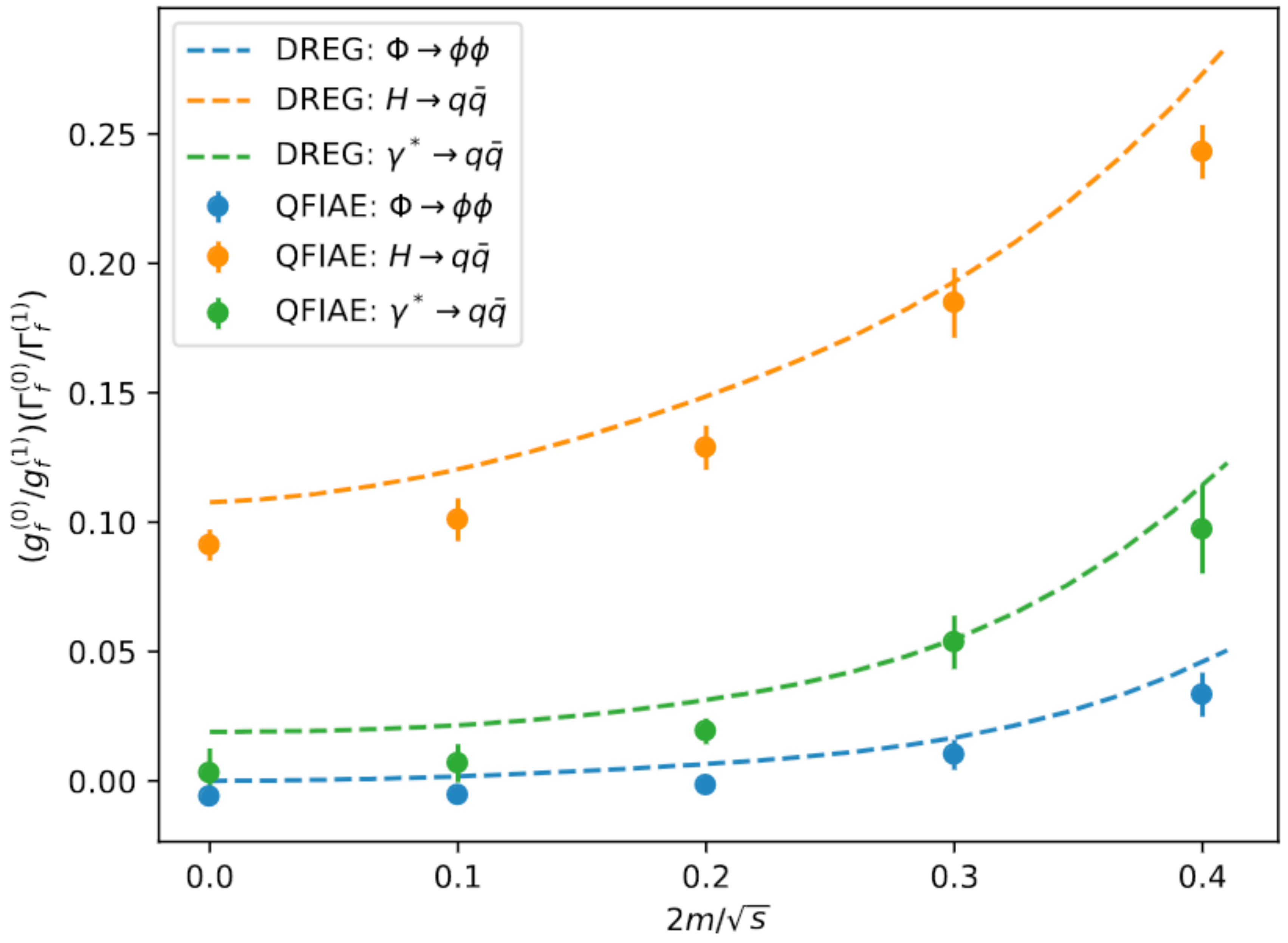


FIG. 4. Quantum-integrated decay rates in quantum simulator (hardware) for the QNN (IQAE) module of the QFIAE, for the three decay processes $H \to q\bar{q}(g)$, $\gamma^* \to q\bar{q}(g)$ and $\Phi \to \phi\phi(\phi)$ at NLO as a function of the final state mass, using QFIAE and LTD causal unitary. The dashed lines are the theoretical predictions in dimensional regularization. The parameters used in the quantum implementation are: $max\_steps = 15000$, $step\_size = 0.001$, $layers = n_{Fourier} = 20$, $n_{qubits} = 6$ for the QNN and $n_{qubits} = 5$, $n_{shots} = 10^3$, $\epsilon = 0.01$, $\alpha = 0.05$ for the IQAE module.

Conclusions

- **Vacuum amplitudes** in the loop-tree duality are the optimal building blocks for assembling theoretical predictions at high-energy colliders
- This hypothesis is strongly supported by the **manifestly causal properties** of LTD
- The vacuum amplitude act as a kernel that generates all final states contributing to a scattering or decay process through **residues in the on-shell energies** of internal particles, after analytic continuation to negative values of those in the initial state
- The **unitary sum** over all phase-space residues ensures the preservation of the competitive advantage of the vacuum amplitude: local cancellation of soft and collinear singularities, absence of threshold discontinuities …
- A novel representation of differential observables, which is well defined in the **four physical dimensions** of the space-time
- First proof-of-concept results presented, more to come

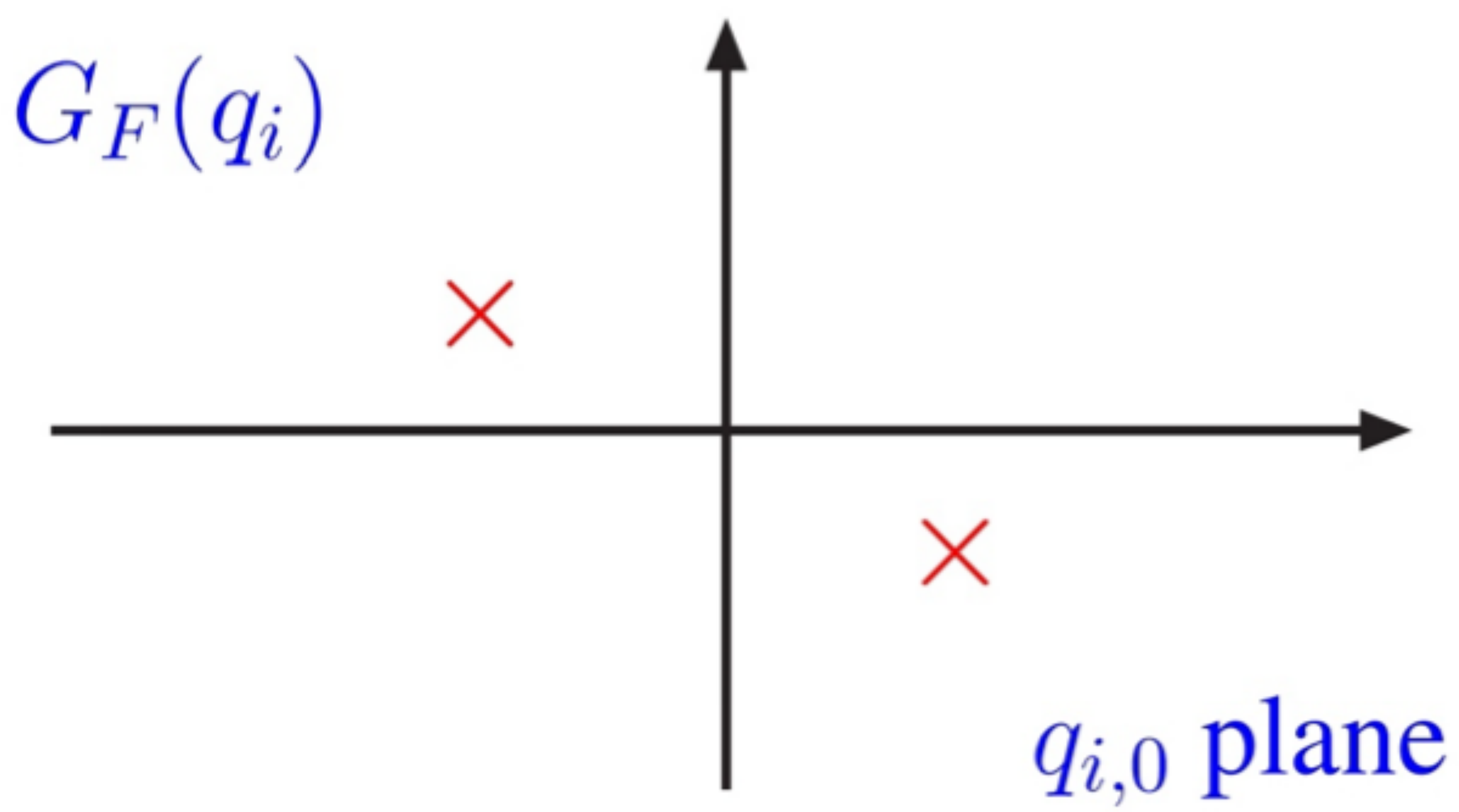
Thanks

Fundamental concepts

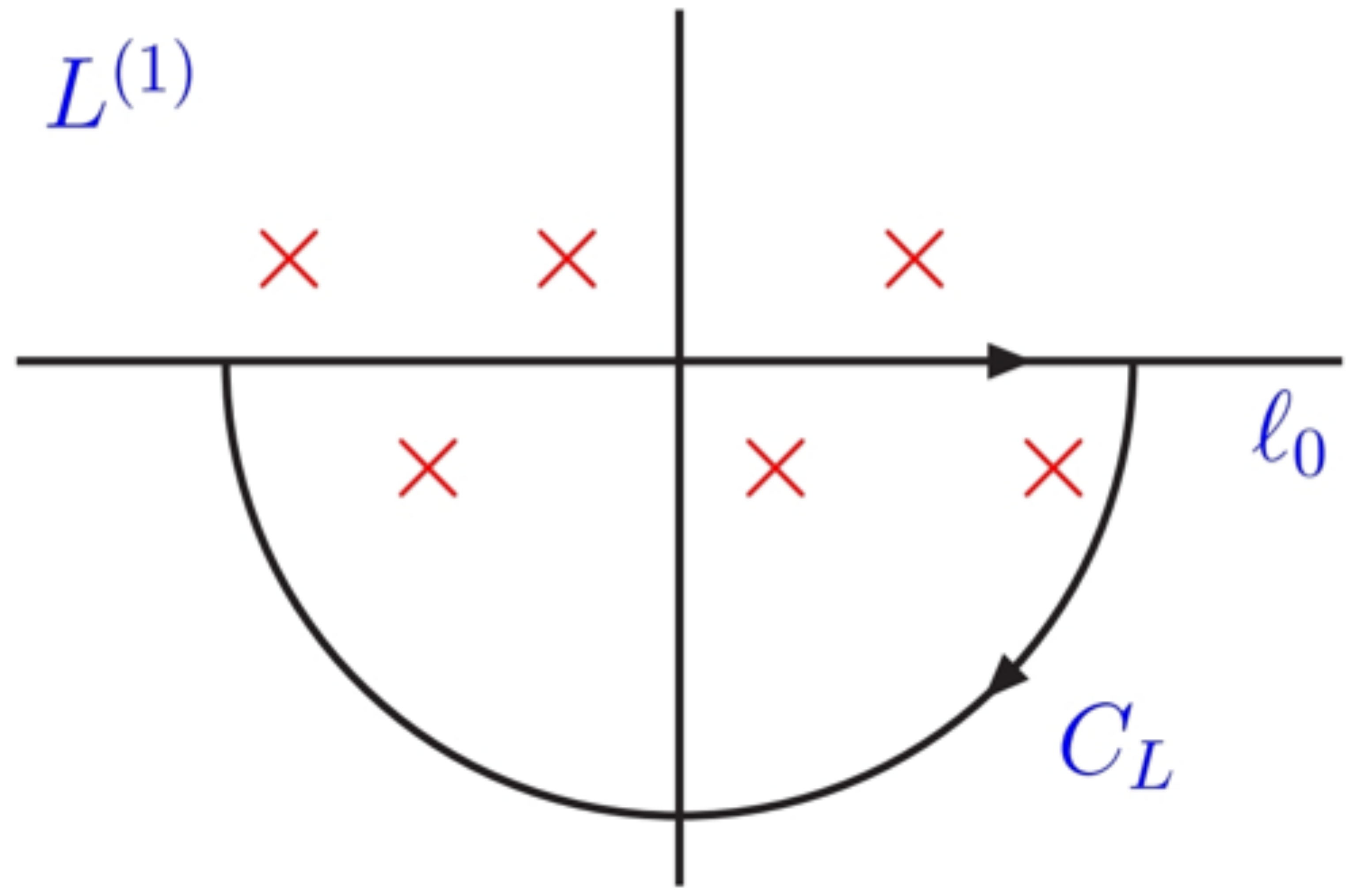
Loop-Tree Duality relation (LTD)

○ Catani, Gleisberg, Krauss, G. Rodrigo and Winter, JHEP 09 (2008) 065

$$L^{(1)} = \int_\ell \mathcal{N}(\ell) \prod_{i=1}^{n} G_F(q_i)$$



Cauchy's residue theorem



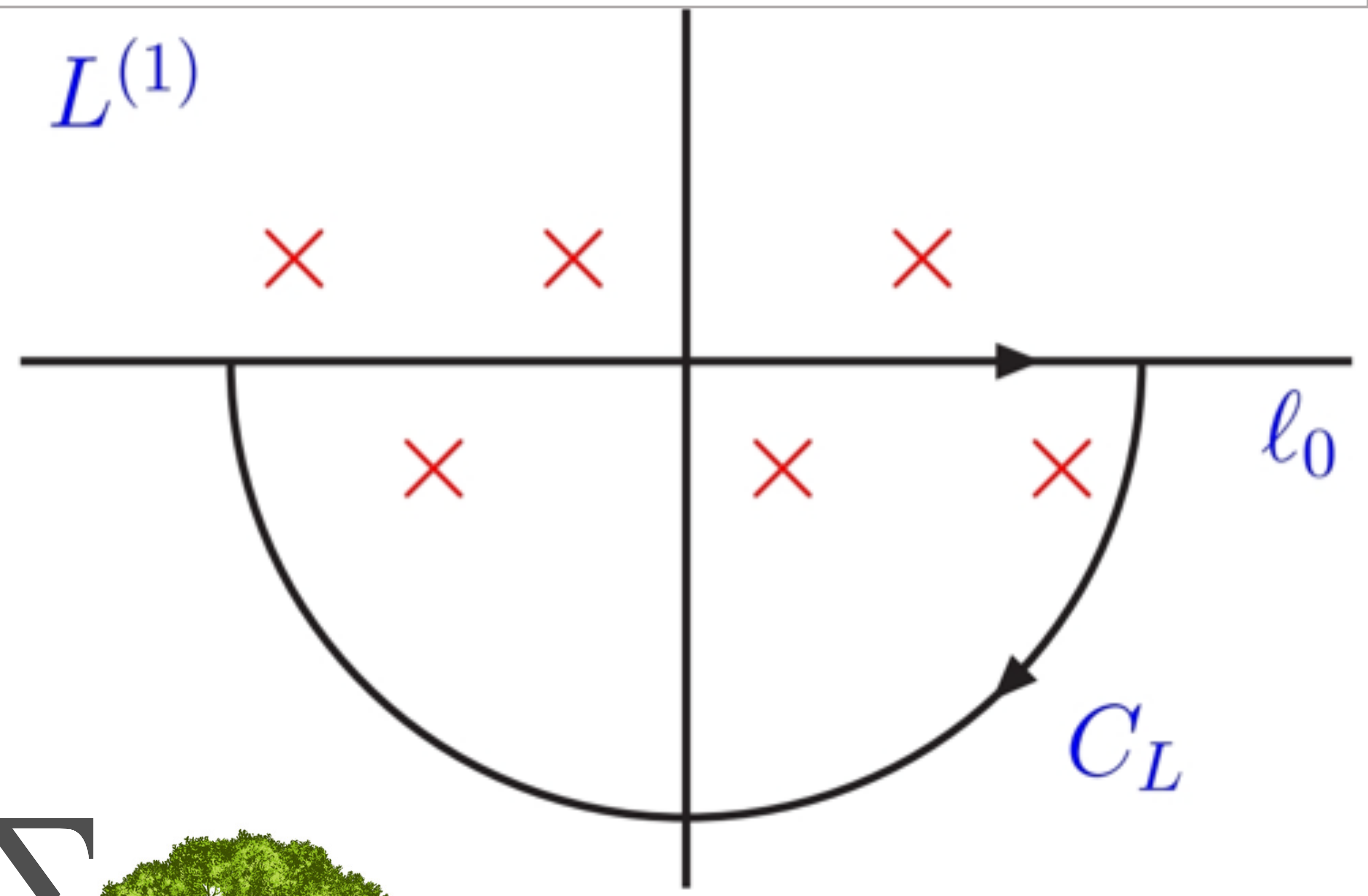
$$G_F(q_i) = \frac{1}{\left(q_{i,0} - q_{i,0}^{(+)}\right)\left(q_{i,0} + q_{i,0}^{(+)}\right)}$$

◎ $q_i = \ell + k_i$

◎ $q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + m_i^2 - \imath 0}$

On-shell energy

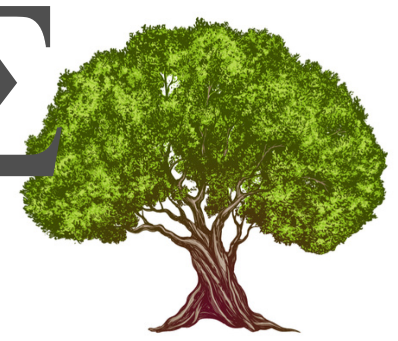
Fundamental concepts



Cauchy's residue theorem

in the loop energy complex plane

$$\int_{C_L} \prod_{i=1}^{n} G_F(q_i) = -2\pi i \sum \text{Res}\left(\prod_{i=1}^{n} G_F(q_i),\ \text{Im}\left(\eta \bullet q_i\right) < 0 \right) = \quad \sum$$



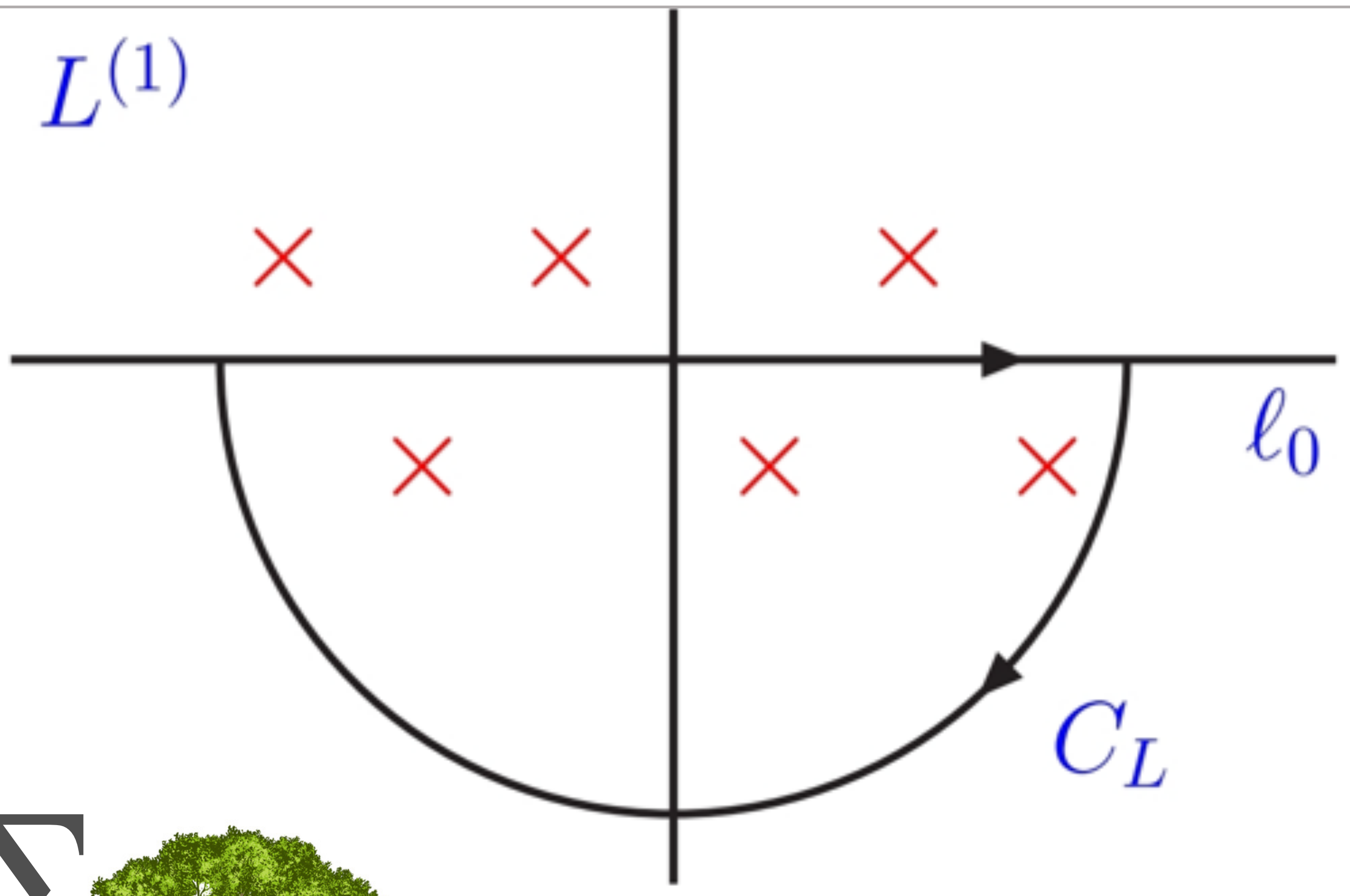
$\text{Im}\left(\eta \bullet q_i\right) < 0$

Selects the poles with negative imaginary components

$\eta^\mu = \left(1, \mathbf{0}\right)$

Euclidean space instead of Minkowski space

Fundamental concepts



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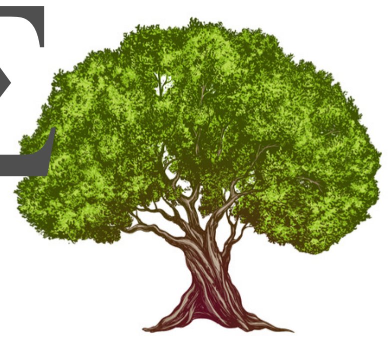

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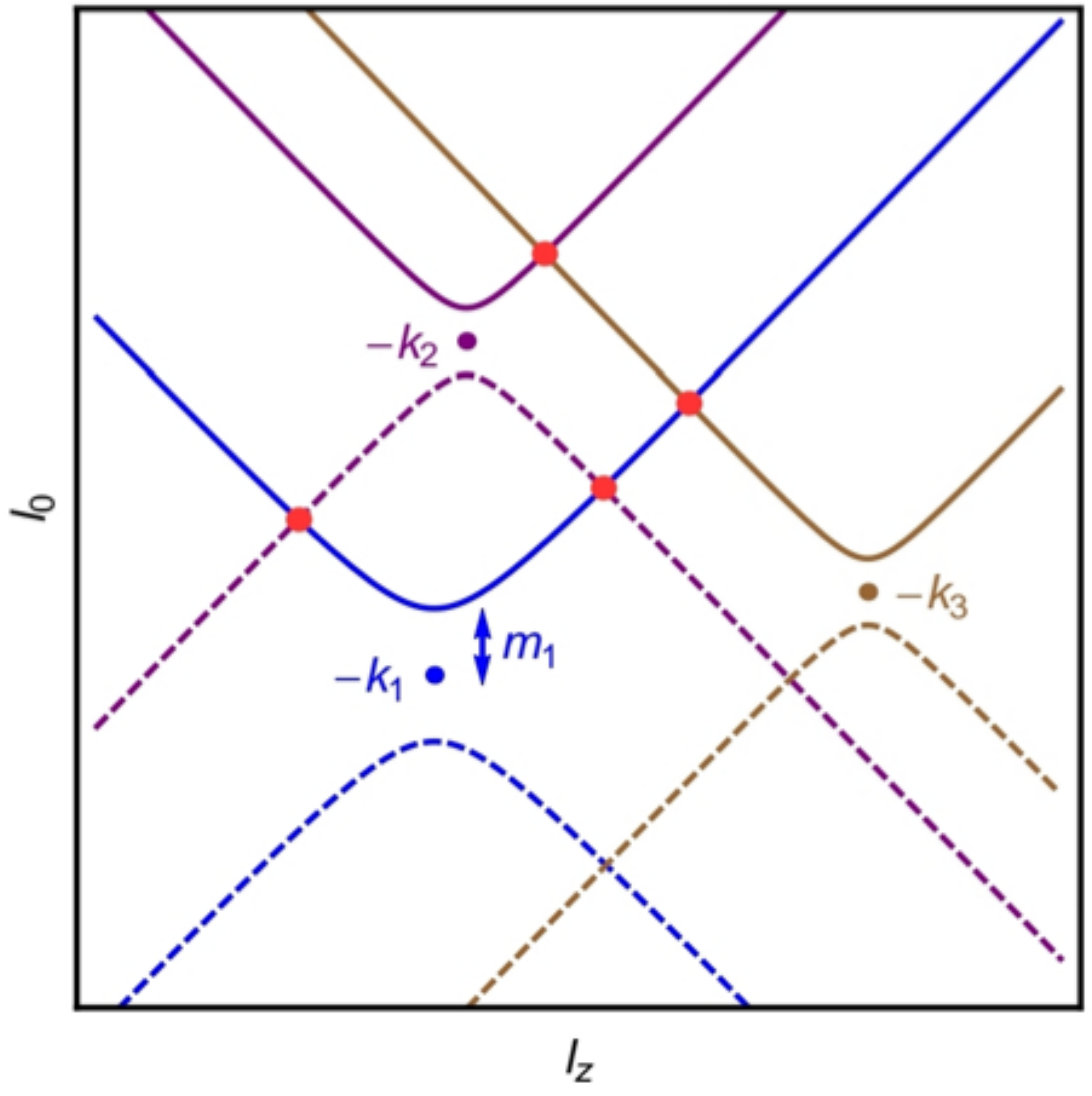
Multiple poles — ○ Bierenbaum, Buchta, Draggiotis, Malamos and G. Rodrigo, JHEP 03 (2013) 025

Singular behaviour in the loop momentum space

○ Buchta, Chachamis, Draggiotis, Malamos and Rodrigo, JHEP 11 (2014) 014



The loop integrand becomes singular when subsets of internal propagators go on-shell

$$G_F^{-1}(q_i) = \left(q_{i,0} - q_{i,0}^{(+)}\right)\left(q_{i,0} + q_{i,0}^{(+)}\right) = 0$$

The two on-shell modes are graphically represented by an hyperboloid in the loop momentum space

◎ Origin at $-k_i \qquad (q_i = \ell + k_i)$

◎ Forward on-shell hyperboloid (solid) $\quad q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + m_i^2 - \imath 0}$

$q_{i,0}^{(-)} = -\, q_{i,0}^{(+)}$

◎ Backward on-shell hyperboloid (dashed)

◎ Hyperboloids degenerate to light-cones for massless propagators

Notation

A generic L-loop scattering amplitude with … ◎ P external legs $\{p_j\}_P$

◎ $L+k$ sets of internal momenta

$$\mathcal{A}^{(L)}(1,\ldots,L+k) = \int_{\ell_1,\ldots,\ell_L} \mathcal{A}_F^{(L)}(1,\ldots,L+k)$$

$$\mathcal{N}\left(\{\ell_i\}_L, \{p_j\}_P\right) G_F(1,\ldots,L+k)$$

$$\int_{\ell_i} = -i\mu^{4-d}\int \frac{d^d\ell_i}{(2\pi)^d}$$

$$\prod_{i\in 1\cup\ldots\cup(L+k)} \left(G_F(q_i)\right)^{a_i}$$

◎ The internal structure of $\mathcal{A}_F^{(L)}$ is implicitly specified via the overall tagging of the different sets of internal momenta

Cauchy's residue theorem

The direct **iterative application** of Cauchy's residue theorem

The LTD representation is written in terms of nested residues

One propagator on shell in each set

All propagators off shell

$$\mathscr{A}_D^{(L)}(1,\ldots,r;r+1,\ldots,L+k) = -2\pi\imath \sum_{i_r\in r} \mathrm{Res}\left(\mathscr{A}_D^{(L)}(1,\ldots,r-1;r,\ldots,L+k),\ \mathrm{Im}\left(\eta\bullet q_{i_r}\right)<0\right)$$

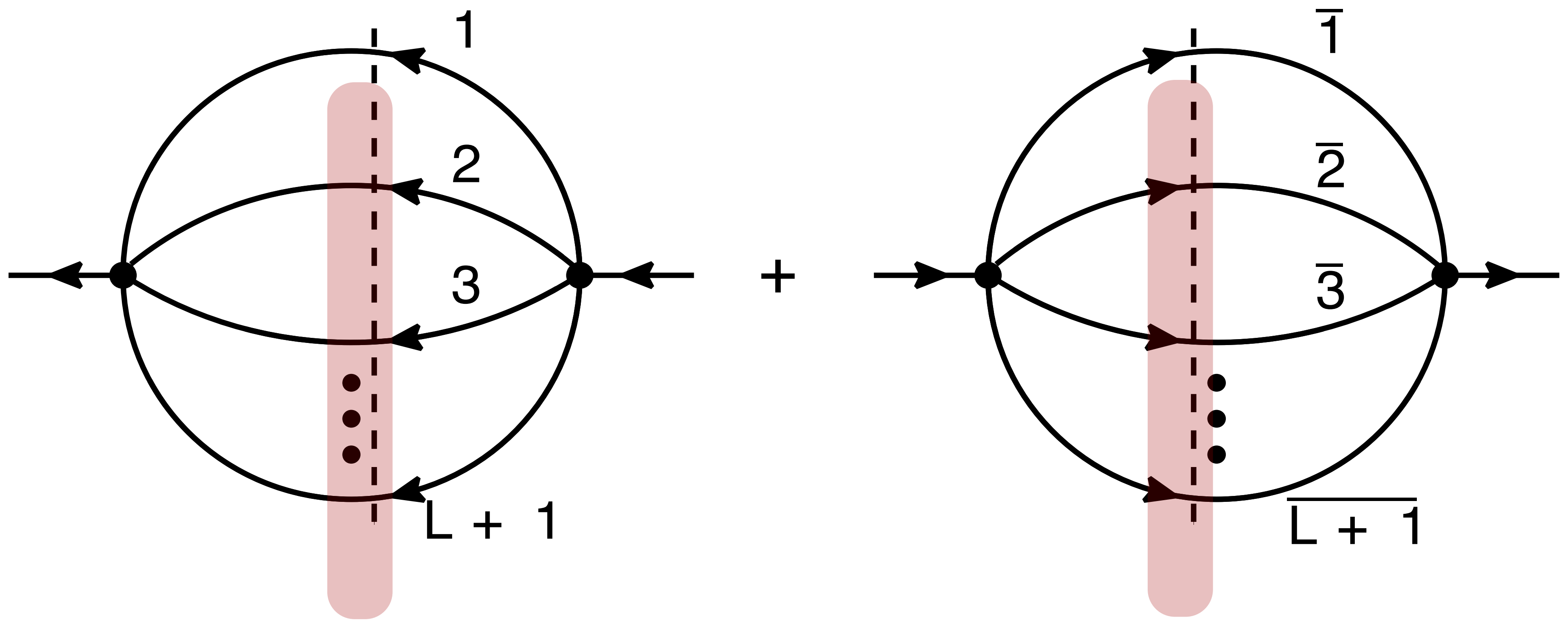
starting from

$$\mathscr{A}_D^{(L)}(1;2,\ldots,L+k) = -2\pi i \sum_{i_1\in 1} \mathrm{Res}\left(\mathscr{A}_F^{(L)}(1,\ldots,L+k),\ \mathrm{Im}\left(\eta\bullet q_{i_1}\right)<0\right)$$

◎ The sum over all possible on-shell configurations in $\mathscr{A}_D^{(L)}$ is understood through the sum of residues.

Causal Maximal Loop Topology

$$\mathscr{A}^{(L)}_{MLT}(1,\ldots,L+1) = \int_{\vec{\ell}_1,\ldots,\vec{\ell}_L} \frac{1}{x_{L+1}}\left(\frac{1}{\lambda^+_{L+1}} + \frac{1}{\lambda^-_{L+1}}\right) =$$



$$x_{L+1} = \prod_{i=1}^{L+1} 2\, q^{(+)}_{i,0}, \quad \lambda^{\pm}_{L+1} = \sum_{i=1}^{L+1} q^{(+)}_{i,0} \pm k_{L+1,0}$$

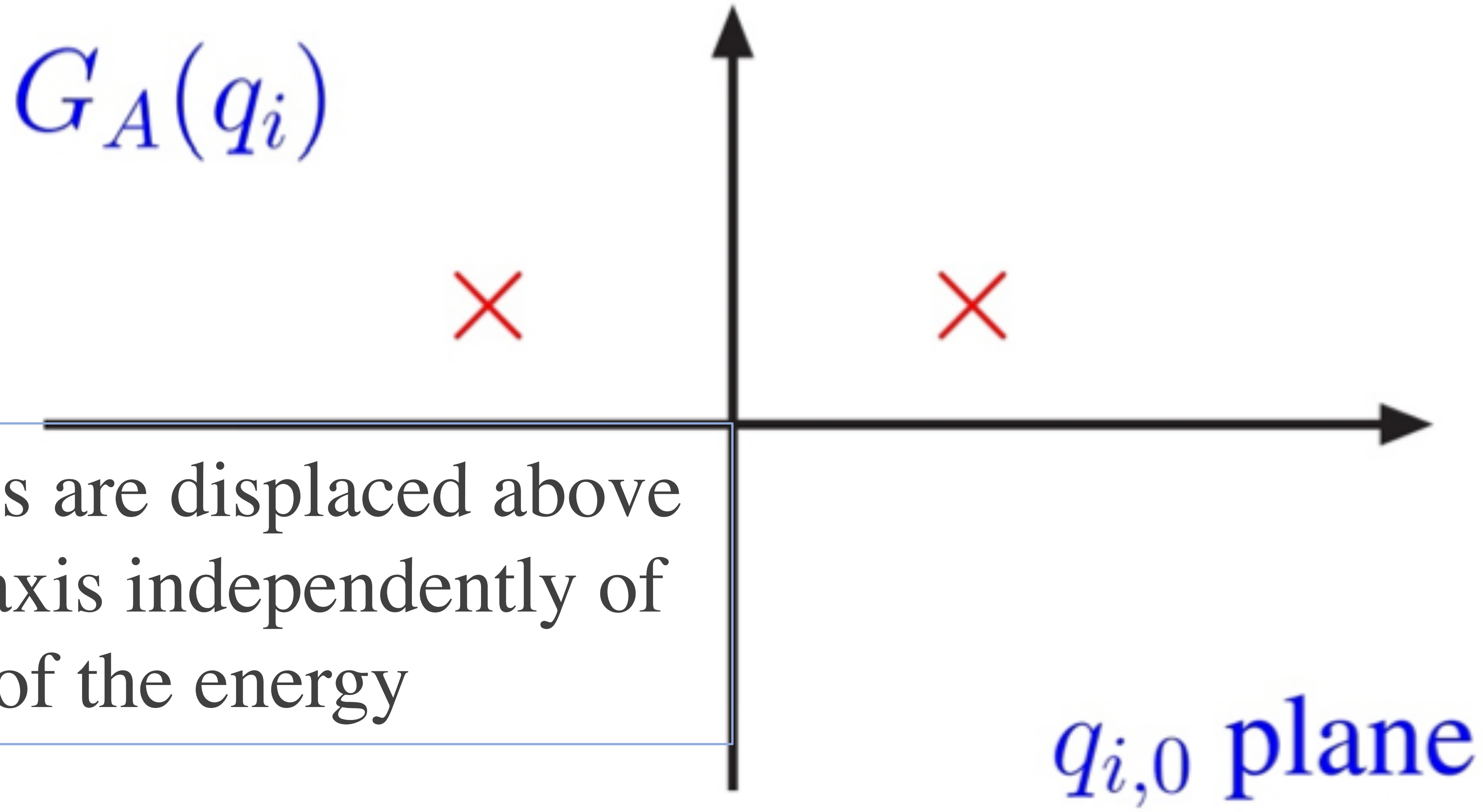
with k_{L+1} external momentum

and $$\int_{\vec{\ell}_s} \equiv -\mu^{4-d}\int \frac{d^{d-1}\ell_s}{(2\pi)^{d-1}}$$

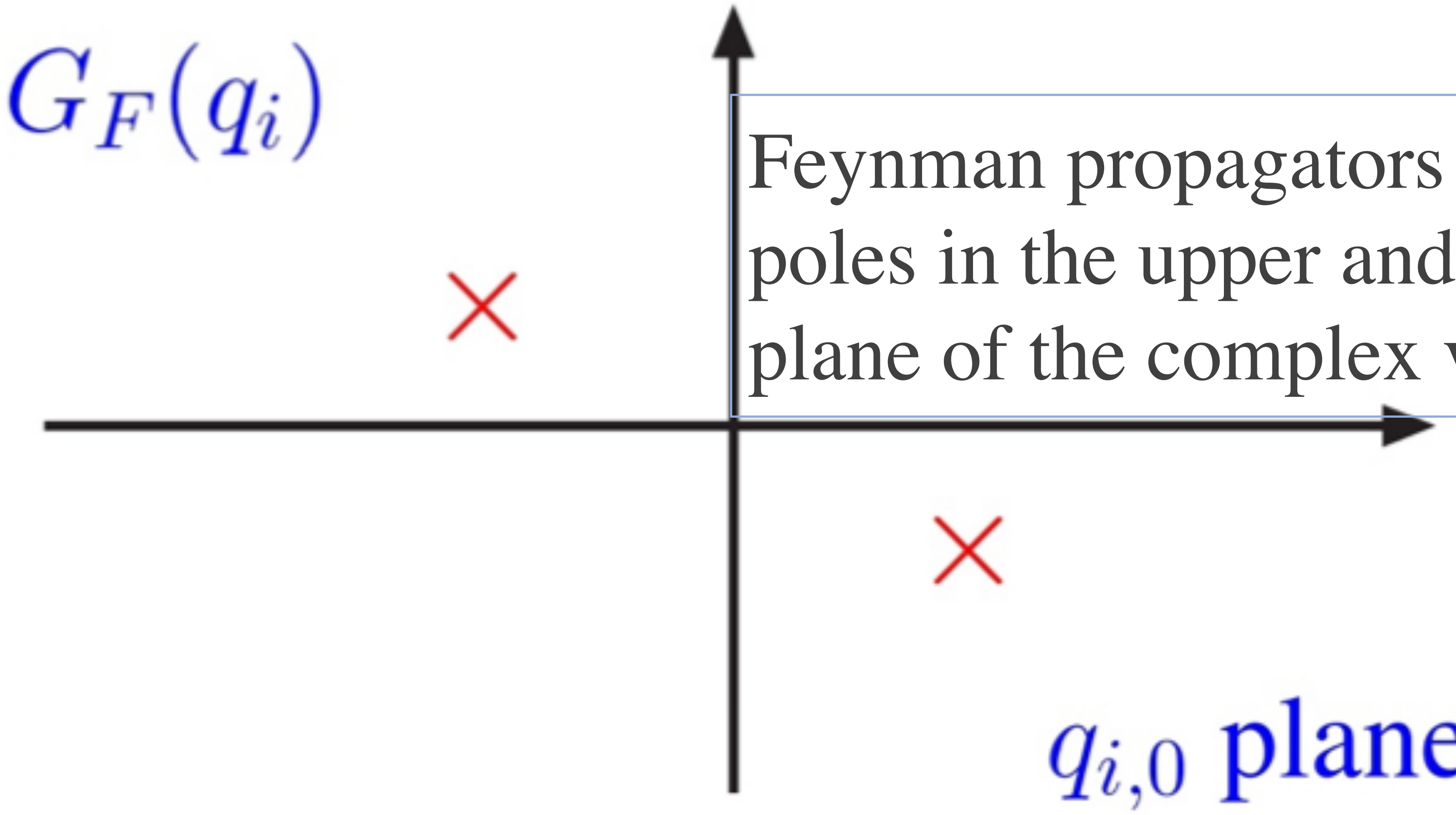
- ◎ Manifestly free of unphysical singularities.
- ◎ Causal singularities occur when either λ^+_{L+1} or λ^-_{L+1} vanishes, depending on the sign of $k_{L+1,0}$.
- ◎ The same expression regardless the number of loops.

Fundamental concepts

○ Catani, Gleisberg, Krauss, G. Rodrigo and Winter, JHEP 09 (2008) 065



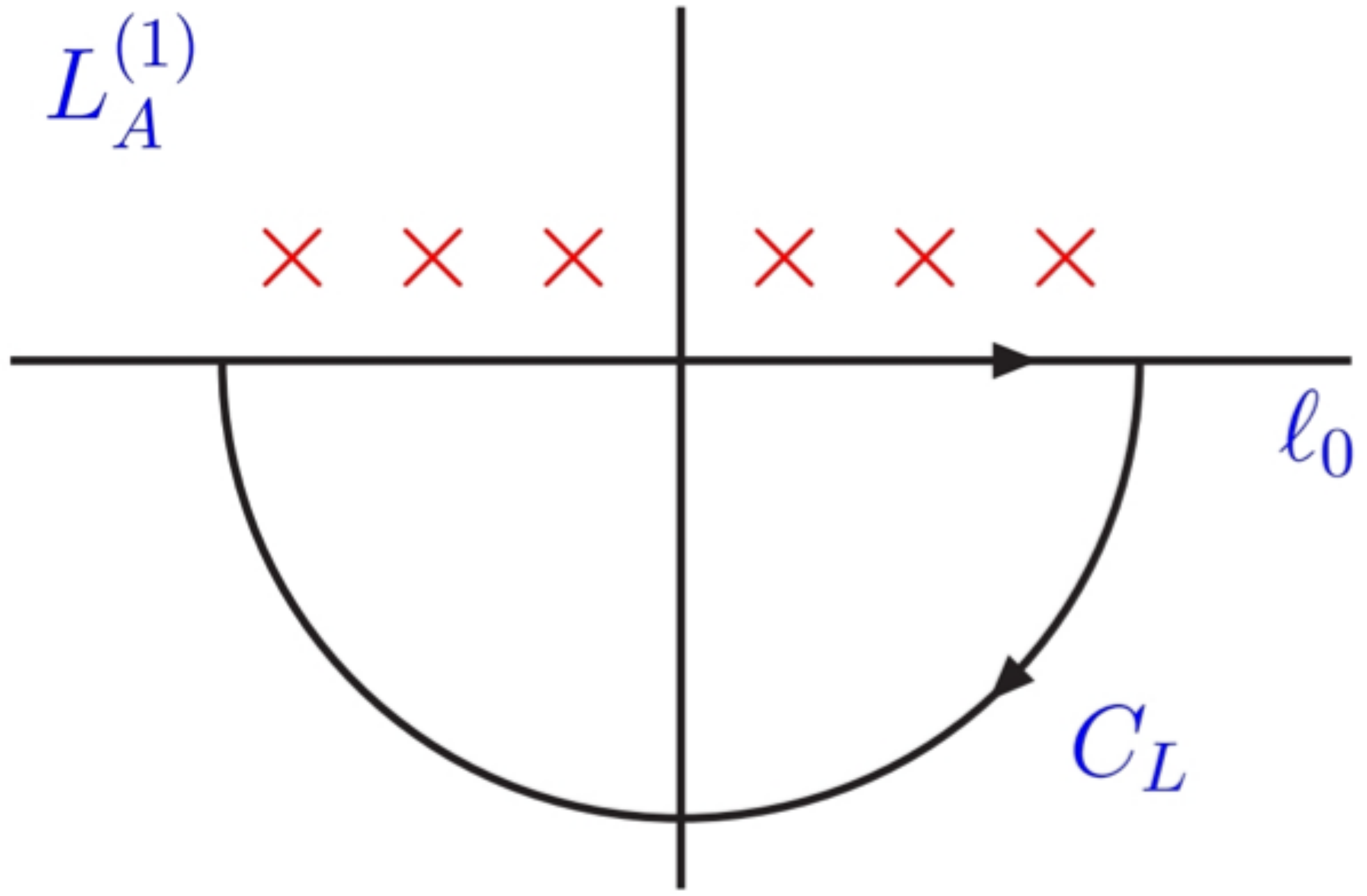
The poles are displaced above the real axis independently of the sign of the energy



Feynman propagators have single poles in the upper and lower half-plane of the complex variable $q_{i,0}$

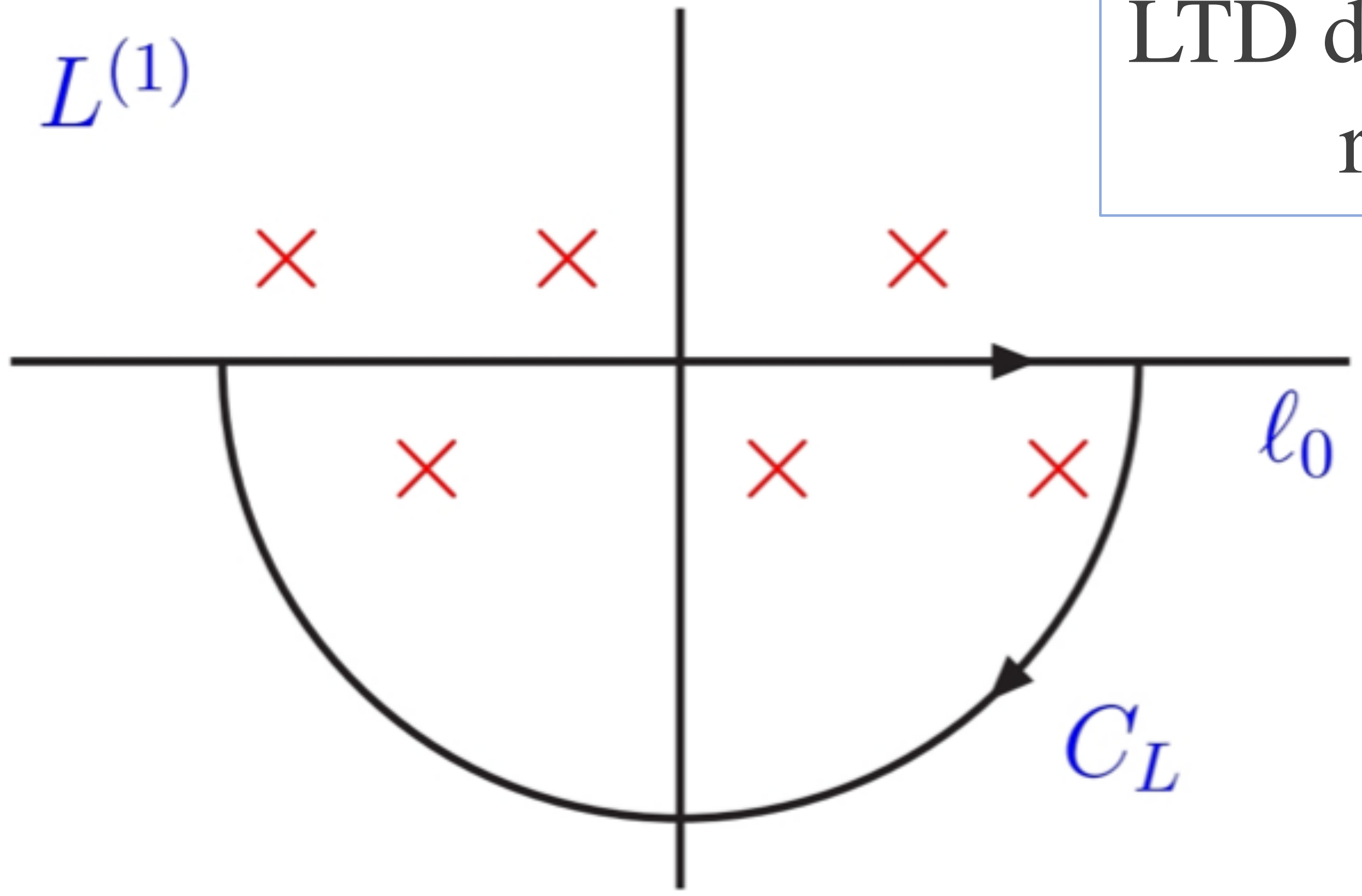
FTT starts from the advanced loop integral

$$L_A^{(1)} = \int_\ell \prod_{i=1}^{n} G_A(q_i)$$



The integral of $L_A^{(1)}$ vanishes along C_L.

LTD directly applies the Cauchy residue theorem to $L^{(1)}$

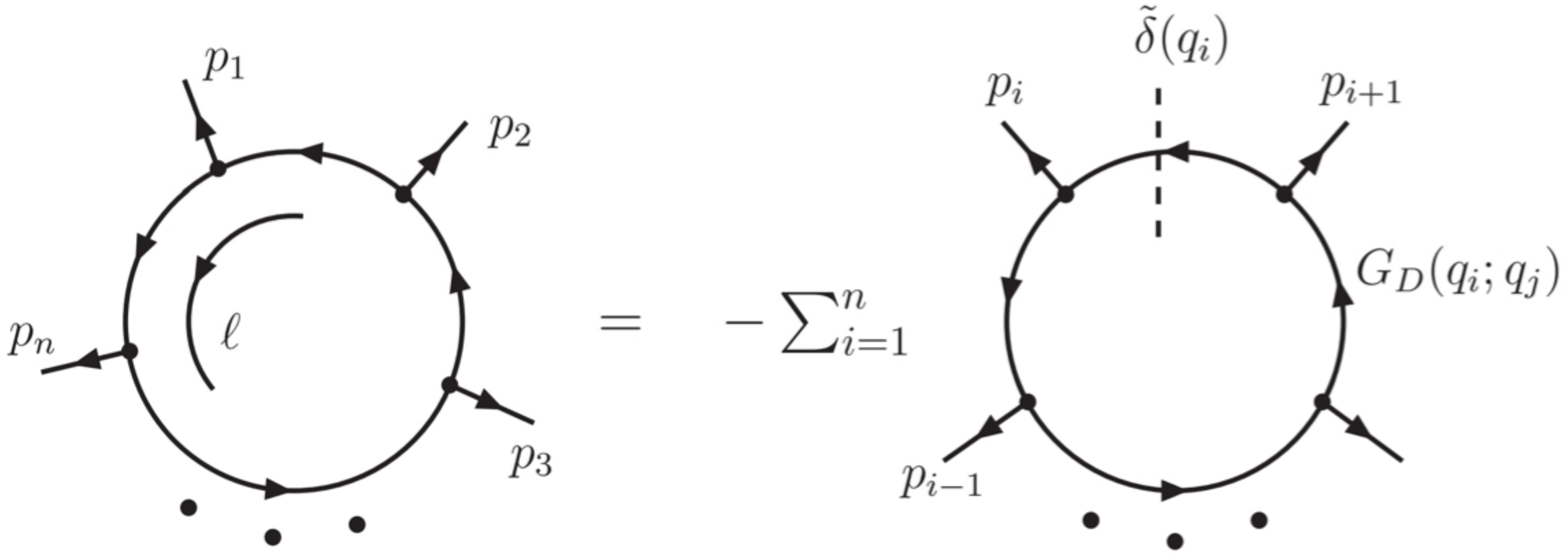


$$L^{(1)} = \int_\ell \prod_{i=1}^{n} G_F(q_i)$$

The poles in the lower half-plane contribute to the computation of $L^{(1)}$, the residues associated to the poles with negative imaginary part of each $G_F(q_i)$.

The absence of multiple cuts is due to $\imath 0 \to -\imath 0\, \eta k_{ji}$ of the uncut Feynman propagators

Fundamental concepts



Duality relation between one-loop integrals and phase-spase integrals

$$L^{(1)}(p_1, \dots, p_n) = -\int_{\ell} \sum_{i=1}^{n} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j \neq 1}}^{n} G_D(q_i; q_j)$$

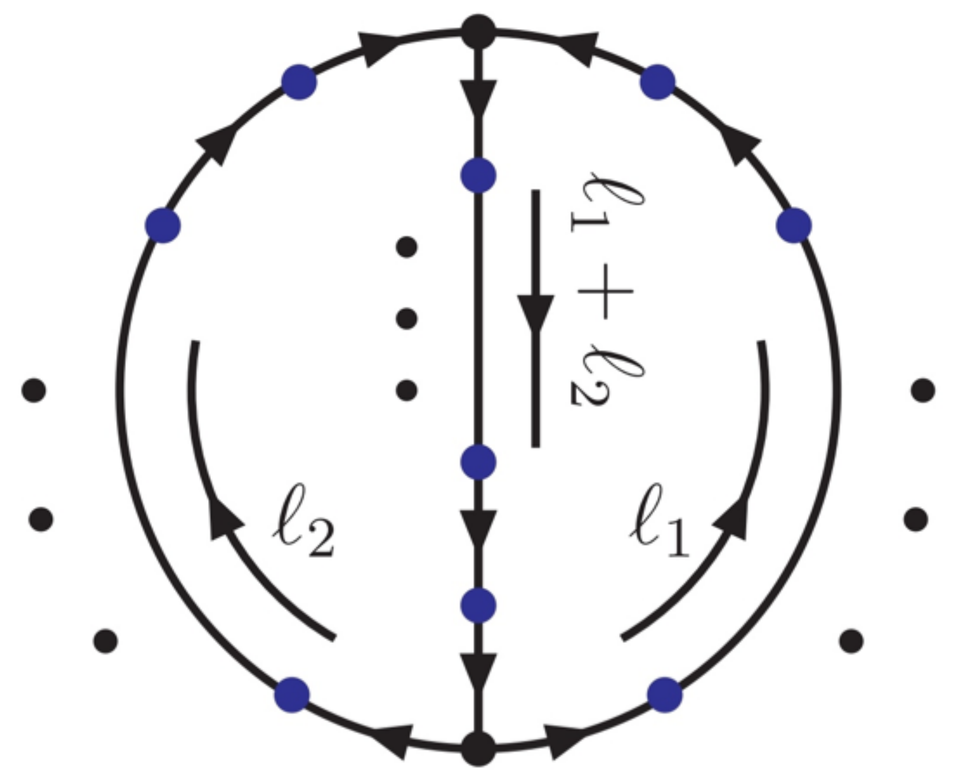
with $G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - \imath 0\, \eta k_{ji}}$ and $k_{ji} = q_j - q_i$

- ➢ The calculation of the residue at the pole of the internal line with momentum q_i changes the propagators of the other lines in the loop integral.
- ➢ The singularity of the uncut Feynman propagator $G_F(q_j) = (q_j^2 - m_j^2 + \imath 0)^{-1}$ is regularized by a modification of the customary Feynman $\imath 0$ prescription, the dual prescription $\to -\imath 0\, \eta k_{ji}$.
- ➢ The dual prescription arises from the fact that $G_F(q_j)$ is evaluated at the complex value of the loop momentum ℓ which is determined by the location of the pole at $q_j^2 - m_j^2 + \imath 0 = 0$.
- ➢ The η_μ dependence of $\imath 0$ is because de residues at each of the poles are not Lorentz-invariance quantities, nevertheless, it is restored after summing over all the residues. Furthermore, ηk_{ji} is independent of ℓ, and depends exclusively on the momenta of the external legs.

Fundamental concepts

○ Bierenbaum, Catani, Draggiotis and G. Rodrigo, JHEP 10 (2010) 073 [84]

Going to higher-orders

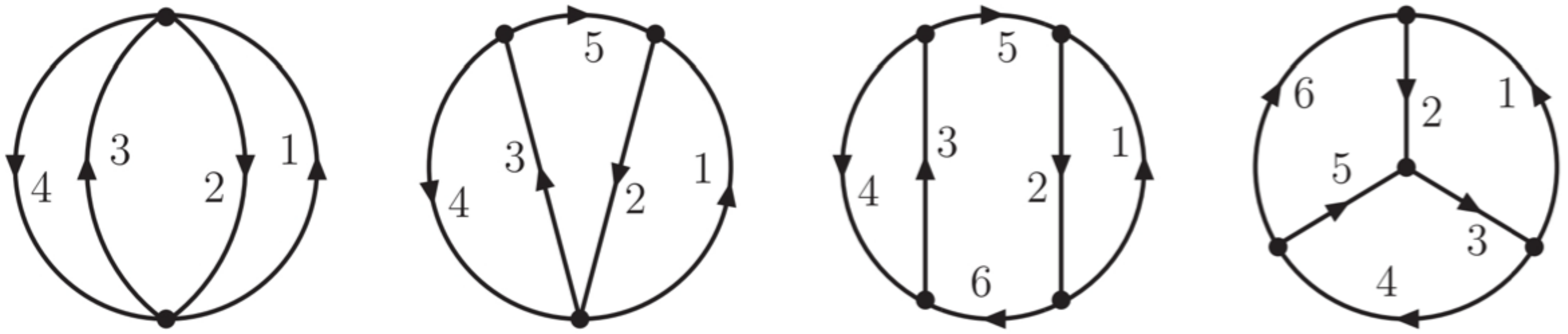


- ➢ Presence of several integration loop momenta.
- ➢ To simplify working with an arbitrary number of propagators we introduce the concept of sets of internal momenta depending on the same integration loop momenta.
- ➢ The Feynman and dual propagator of a set of internal momenta k is given by:

$$G_F(k) = \prod_{i\in k} G_F(q_i), \quad G_D(k) = \sum_{i\in k} \tilde{\delta}(q_i) \prod_{\substack{j\in k \\ j\neq i}} G_D\left(q_i; q_j\right) \quad \text{and} \quad G_D(\bar{k}) = \sum_{i\in k} \tilde{\delta}(-q_i) \prod_{\substack{j\in k \\ j\neq i}} G_D\left(-q_i; -q_j\right)$$

Fundamental concepts

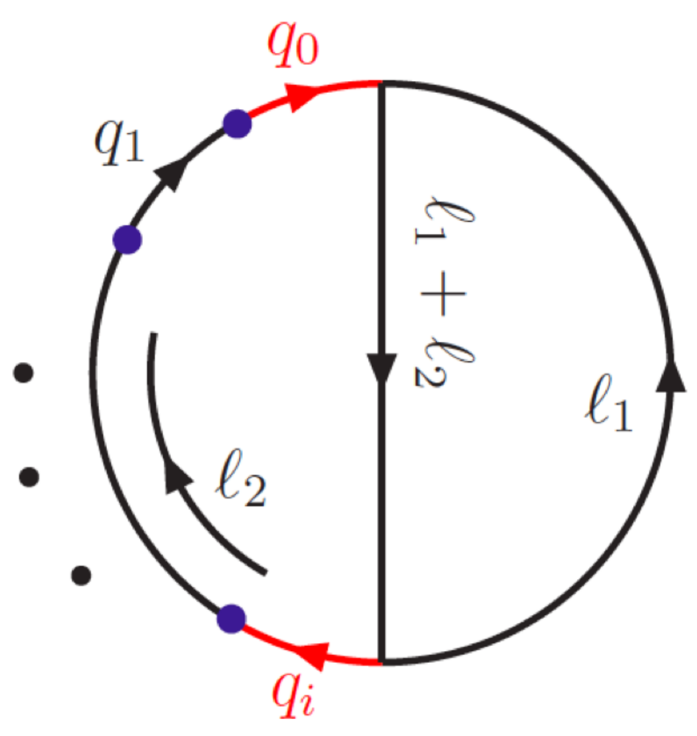
- The strategy followed was the iterative application of
  1. $\int_{\ell_1} [G_F(1) + G_D(1)] = 0$
  2. $\int_{\ell_i} G_F(1 \cup 2 \cup \ldots \cup n) = -\int_{\ell_i} G_D(1 \cup 2 \cup \ldots \cup n)$
  3. $G_D(i \cup j) = G_D(i)G_D(j) + G_D(i)G_F(j) + G_F(i)G_D(j)$.
- We are in a scenario with either, a dual representation expressed in terms of double cuts but possible dual prescription depending on the integration momenta; or a scenario where the dual prescription depends exclusively on the external momenta but considers triple cuts.
- The three-loop level was also explored.

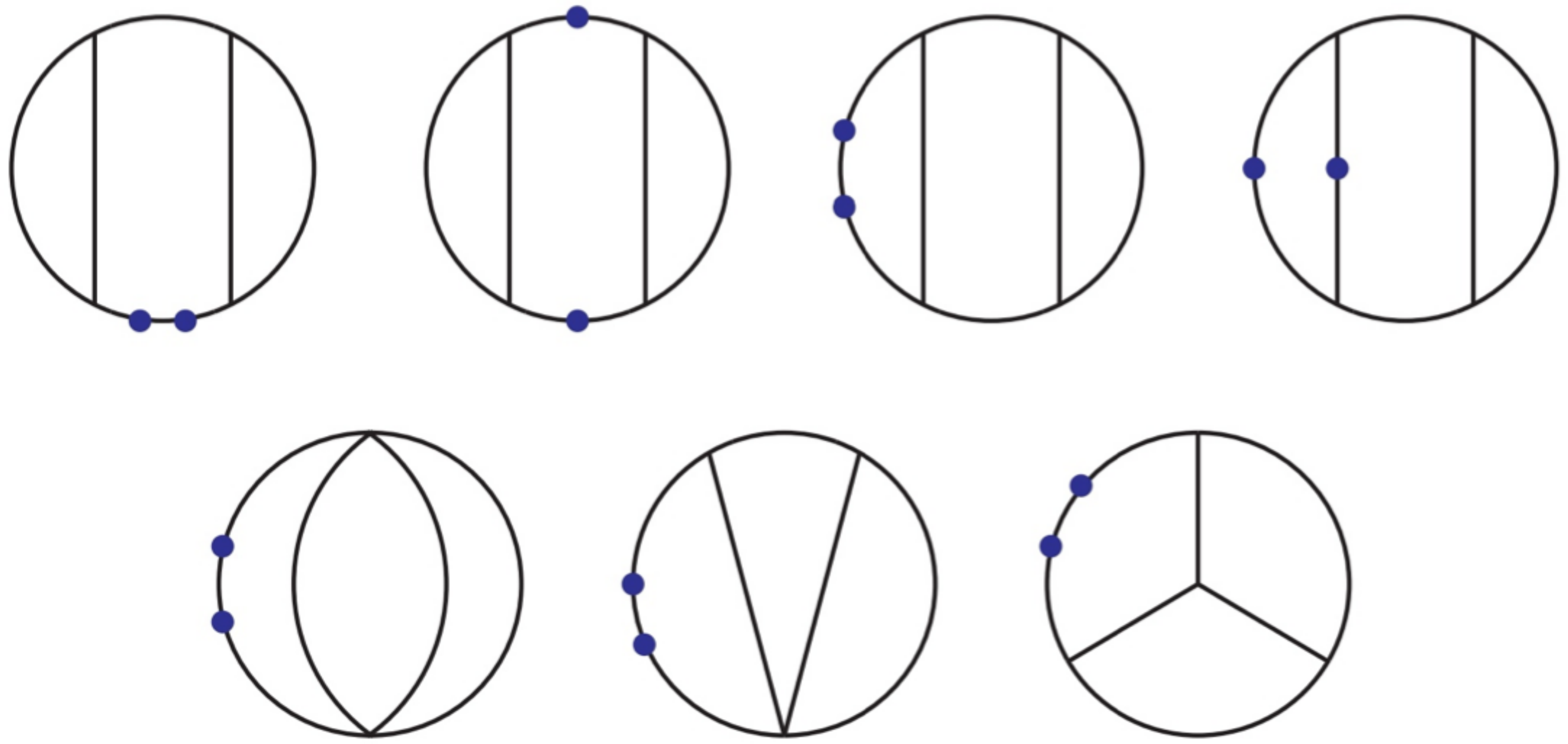


Fundamental concepts

○ Bierenbaum, Buchta, Draggiotis, Malamos and G. Rodrigo, JHEP 03 (2013) 025 [85]

Multiple poles

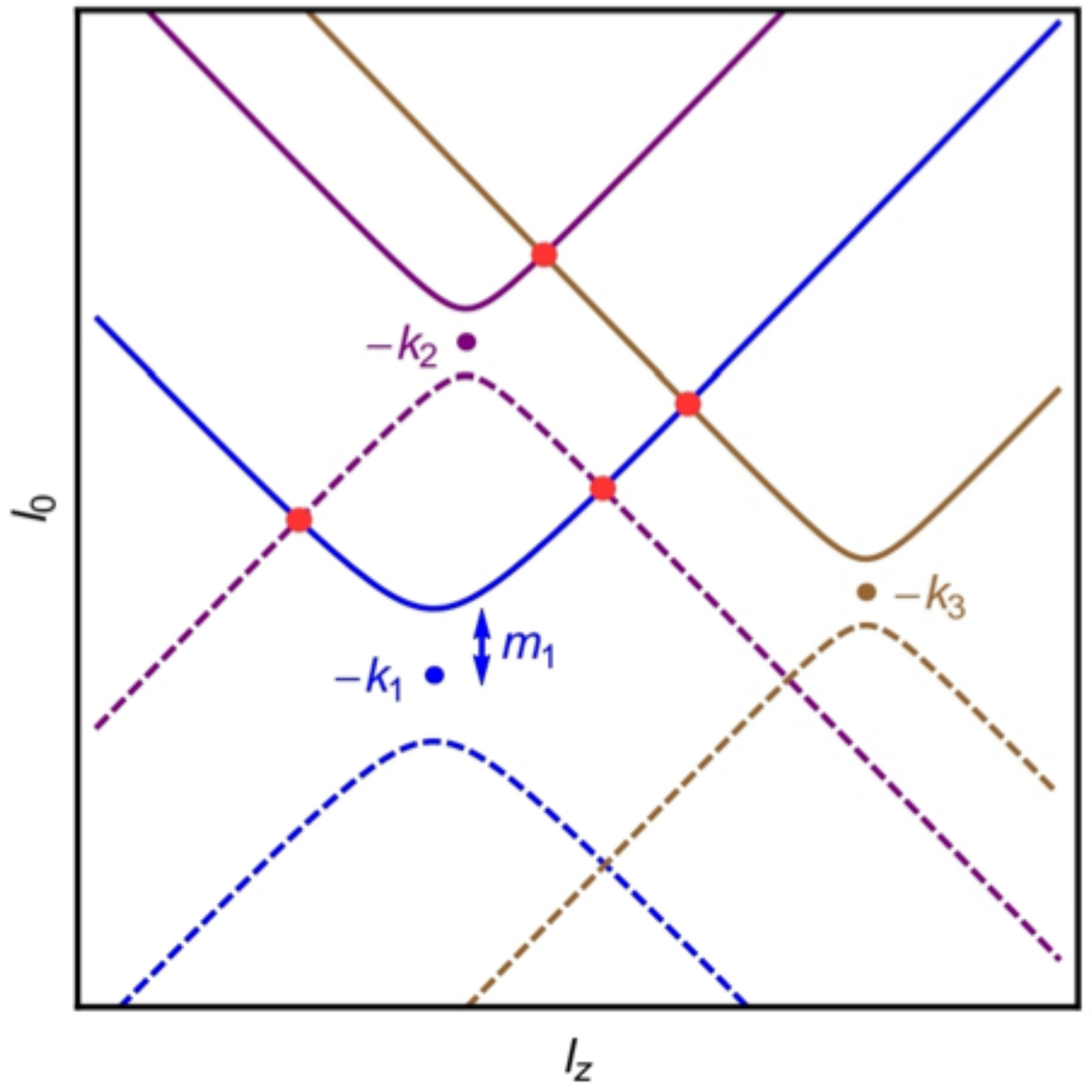




- ➢ Extending the LTD beyond diagrams with single poles requires to evaluate the contribution of the higher order poles which depends on the topology configuration, the nature of the internal propagators and the interaction vertices.
- ➢ For the sole double generic diagram, the procedure was similar to the one-loop case, cutting every propagator line once, included the double propagator, and transforming the rest of the propagators to dual propagators.
- ➢ For triple and higher poles, the calculation of the residue introduces contributions with powers of dual propagators.
- ➢ The tactic followed to deal with higher-order pole integrals was reducing them to single poles integrals by the application of Integration By Parts identities, allowing to use of the LTD theorem in its original form for single pole propagators.

○ Buchta, Chachamis, Draggiotis, Malamos and Rodrigo, JHEP 11 (2014) 014 [88]

Singular behaviour in the loop momentum space

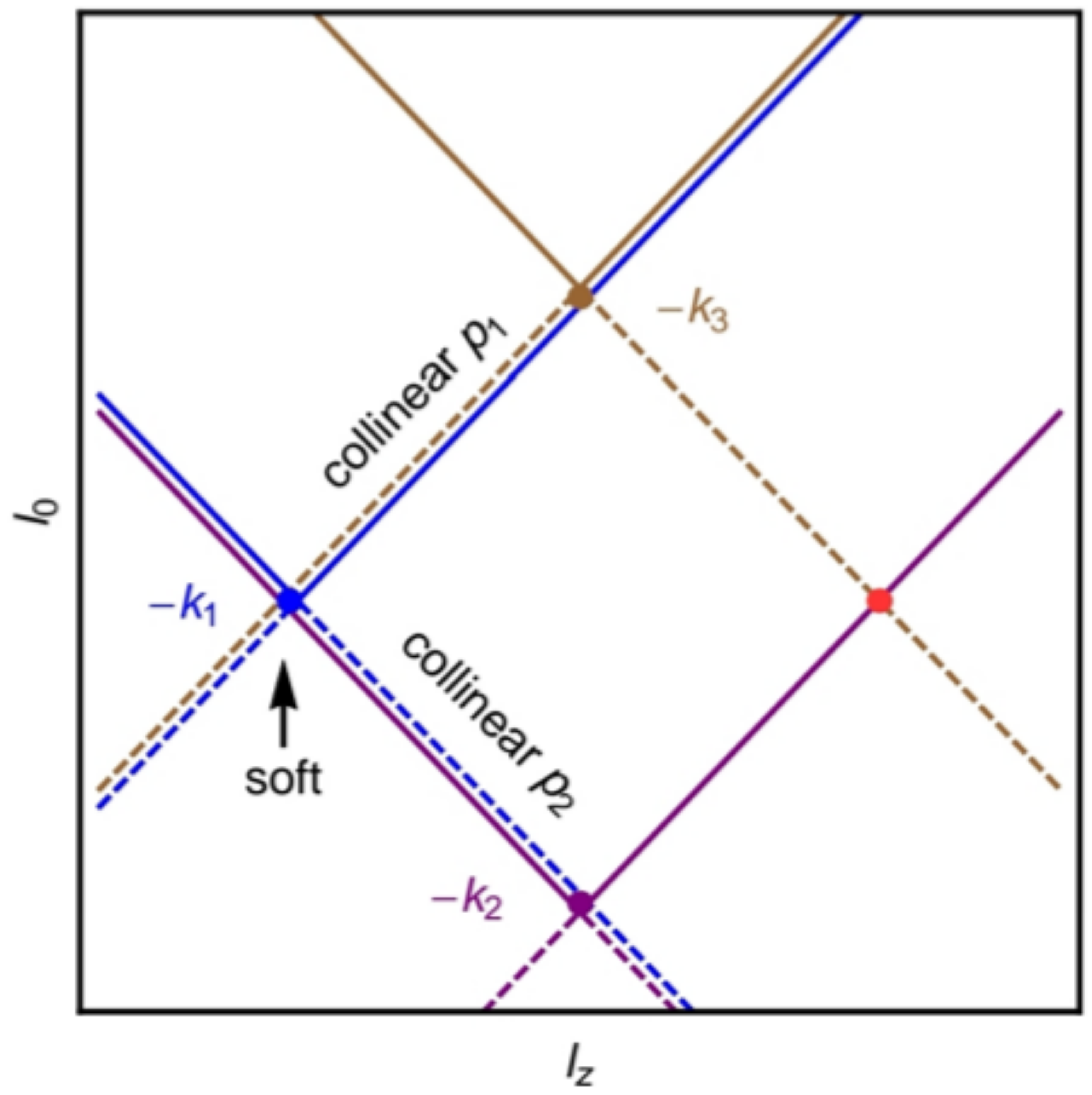


- The integration domain is restricted to the forward on-shell hyperboloids through the Cauchy's residue selection.
- q_1 and q_2 are separated by a time-like distance $(q_2 - q_1)^2 > 0$ and q_3 is space-like separated with respect to the other two $(q_3 - q_1)^2 < 0$ and $(q_3 - q_2)^2 < 0$.
- The advantage of the LTD is that the loop integrand becomes singular only at the intersection of two or more on-shell hyperboloids, where two or more propagators become simultaneously singular.

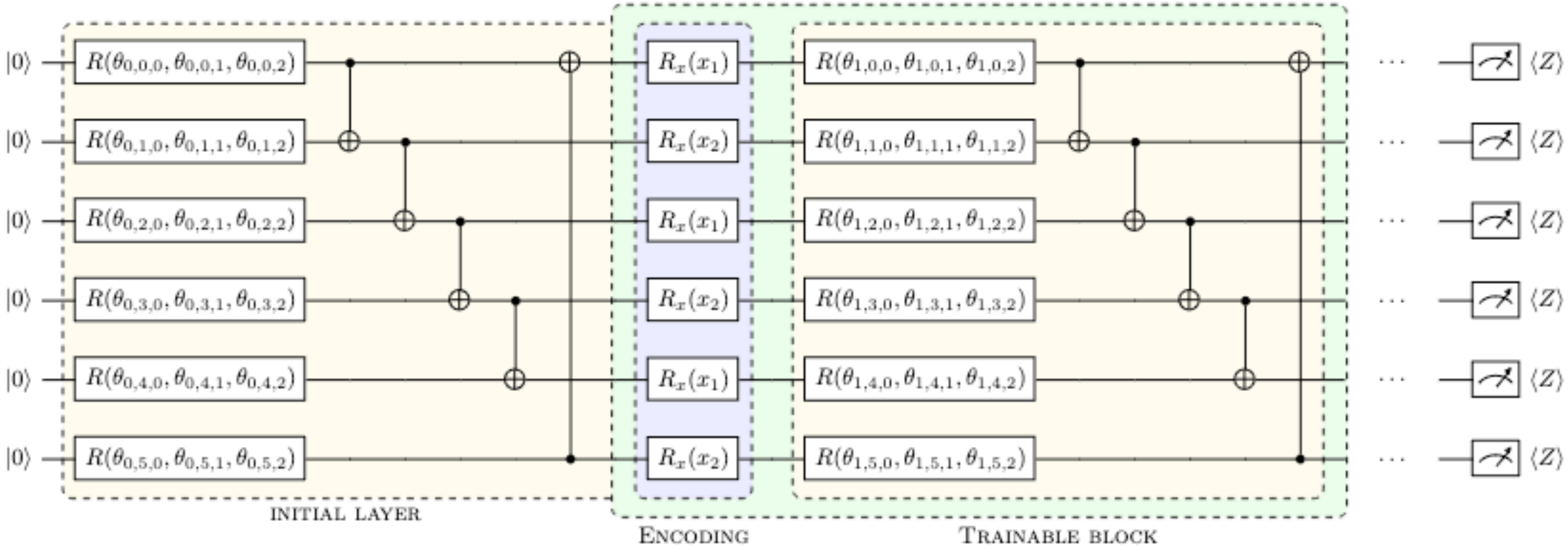
Forward-backward

◎ Sin

Singular behaviour in the loop momentum space



Massless case: ligth-cones


$|0\rangle$
$R(\theta_{0,0,0}, \theta_{0,0,1}, \theta_{0,0,2})$
$R(\theta_{0,1,0}, \theta_{0,1,1}, \theta_{0,1,2})$
$R(\theta_{0,2,0}, \theta_{0,2,1}, \theta_{0,2,2})$
$R(\theta_{0,3,0}, \theta_{0,3,1}, \theta_{0,3,2})$
$R(\theta_{0,4,0}, \theta_{0,4,1}, \theta_{0,4,2})$
$R(\theta_{0,5,0}, \theta_{0,5,1}, \theta_{0,5,2})$
$R_x(x_1)$
$R_x(x_2)$
$R(\theta_{1,0,0}, \theta_{1,0,1}, \theta_{1,0,2})$
$R(\theta_{1,1,0}, \theta_{1,1,1}, \theta_{1,1,2})$
$R(\theta_{1,2,0}, \theta_{1,2,1}, \theta_{1,2,2})$
$R(\theta_{1,3,0}, \theta_{1,3,1}, \theta_{1,3,2})$
$R(\theta_{1,4,0}, \theta_{1,4,1}, \theta_{1,4,2})$
$R(\theta_{1,5,0}, \theta_{1,5,1}, \theta_{1,5,2})$
$\langle Z \rangle$
INITIAL LAYER
ENCODING
TRAINABLE BLOCK