Energy Energy correlators in DIS
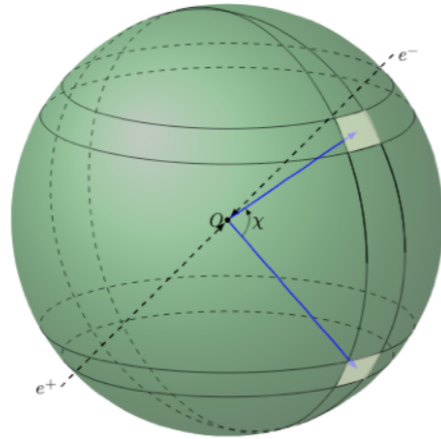
Haitao Li
Shandong University

arXiv:2006.02437, arXiv: 2102.05669, arXiv: 2312.07655
with H. Cao, Y. Makris, Z. Mi, I. Vitev, Y.J. Zhu

ICHEP 2024 | PRAGUE
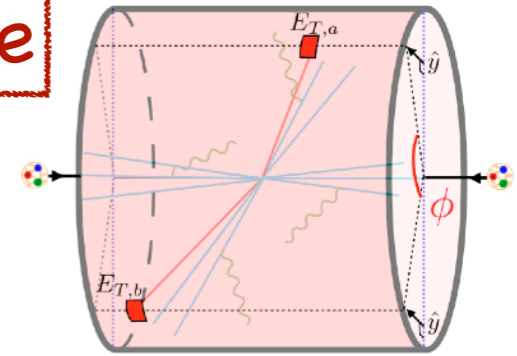
18-July-2024

EEC and TEEC

e^+e^- Collisions

Hadronic initial state

observable

$$\text{EEC} = \sum_{a,b} \int d\sigma_{V\to a+b+X} \frac{2E_a E_b}{Q^2 \sigma_{\text{tot}}} \delta(\cos(\theta_{ab}) - \cos(\chi))$$

$$\text{TEEC} = \sum_{a,b} \int d\sigma_{pp\to a+b+X} \frac{2E_{T,a} E_{T,b}}{|\sum_i E_{T,i}|^2} \delta(\cos\phi_{ab} - \cos\phi)$$

- sum over all the jets for each event
- sum over all the particles for each event
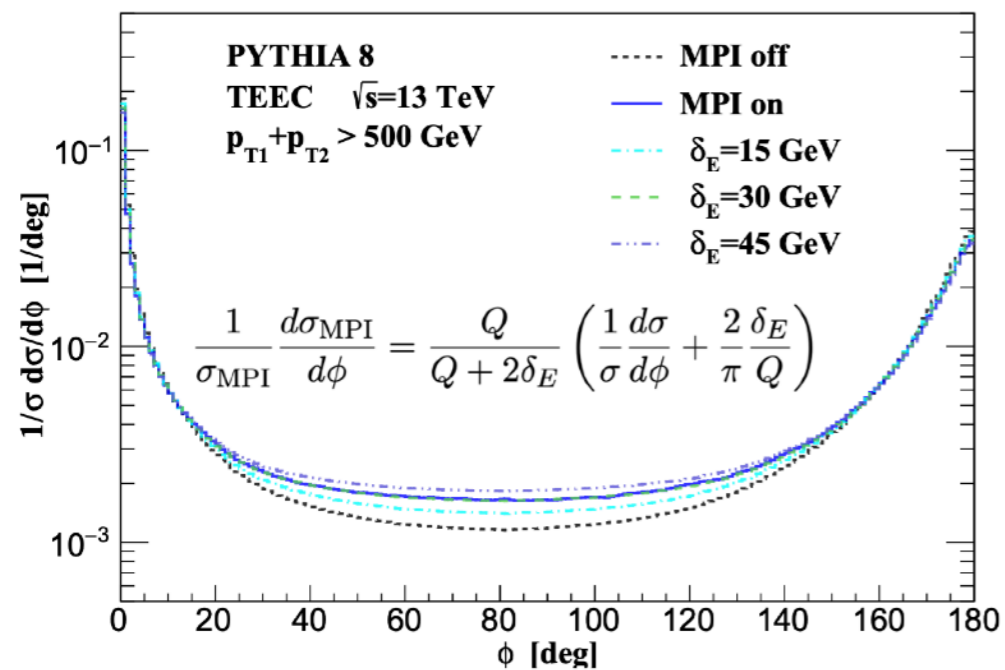
Basham et al 1978
Moult, Zhu, 2018

- weighted cross section
- the soft radiation does not contribute directly to the observable at leading power
- soft gluon contributes only via recoil

Ali et al 1984
Gao, HTL, Moult, Zhu, 2019, 2023

EEC and TEEC

EEC/TEEC is a class of event shape variables

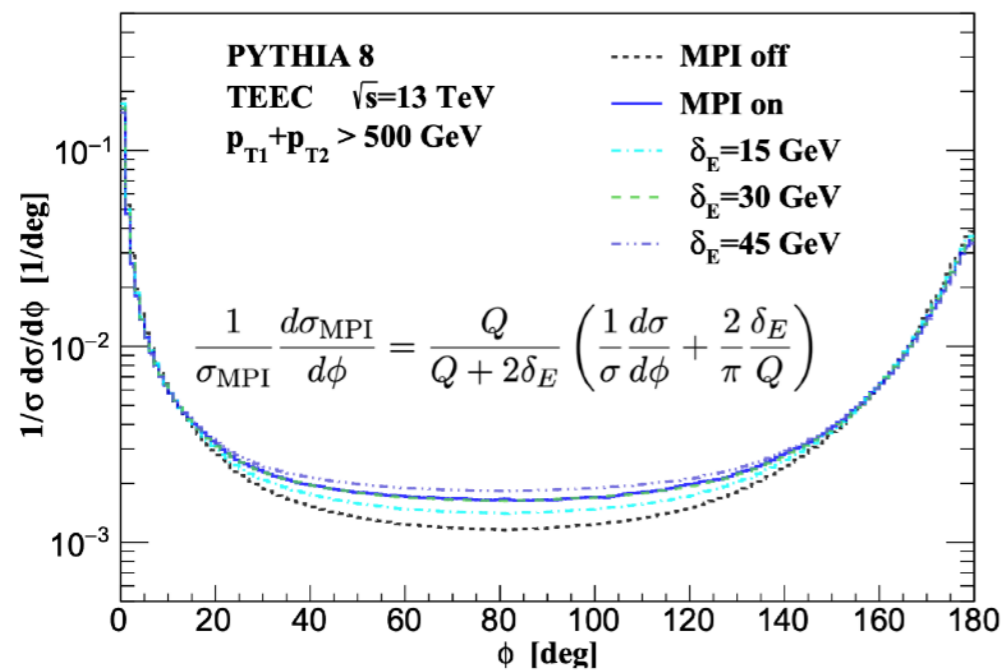where nonperturbative effect is supposed to be small
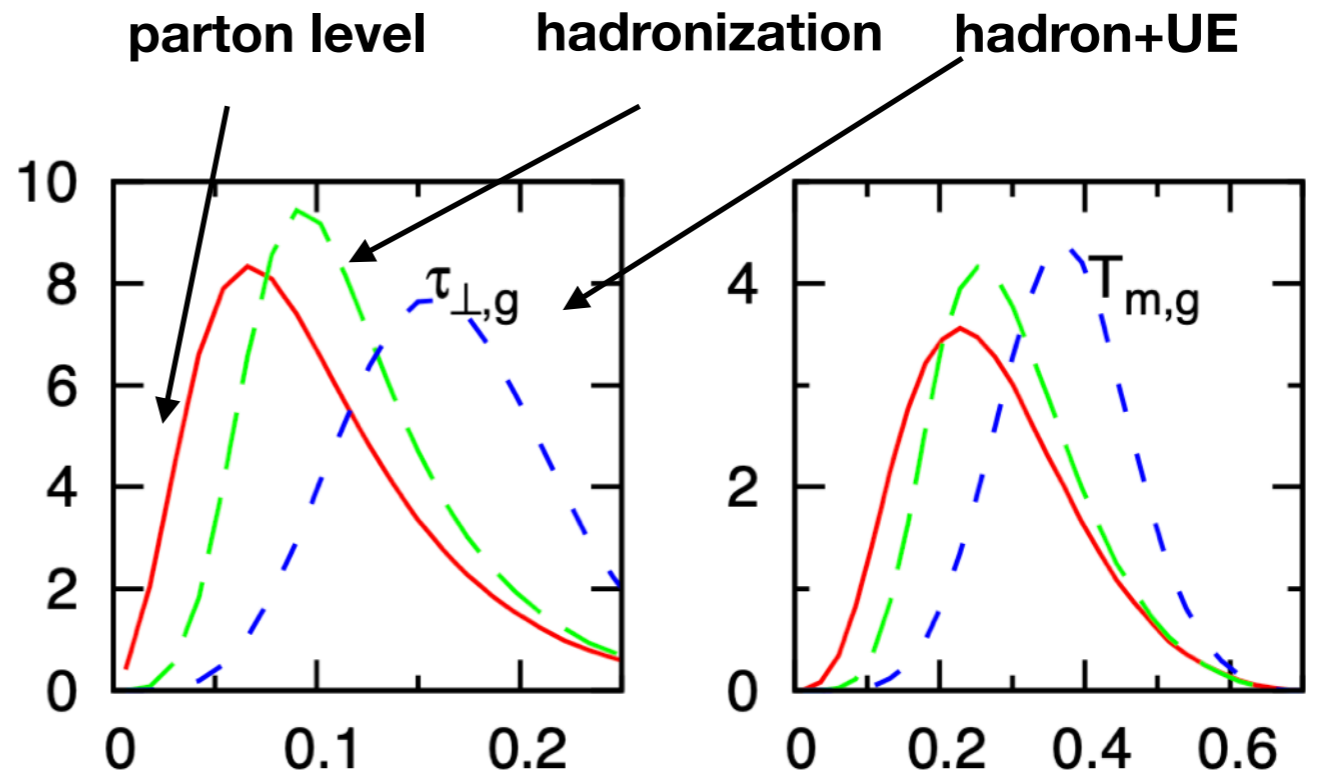

Gao, HTL, Moult, Zhu, 2023

EEC and TEEC

EEC/TEEC is a class of event shape variables

where nonperturbative effect is supposed to be small

parton level   **hadronization**   **hadron+UE**
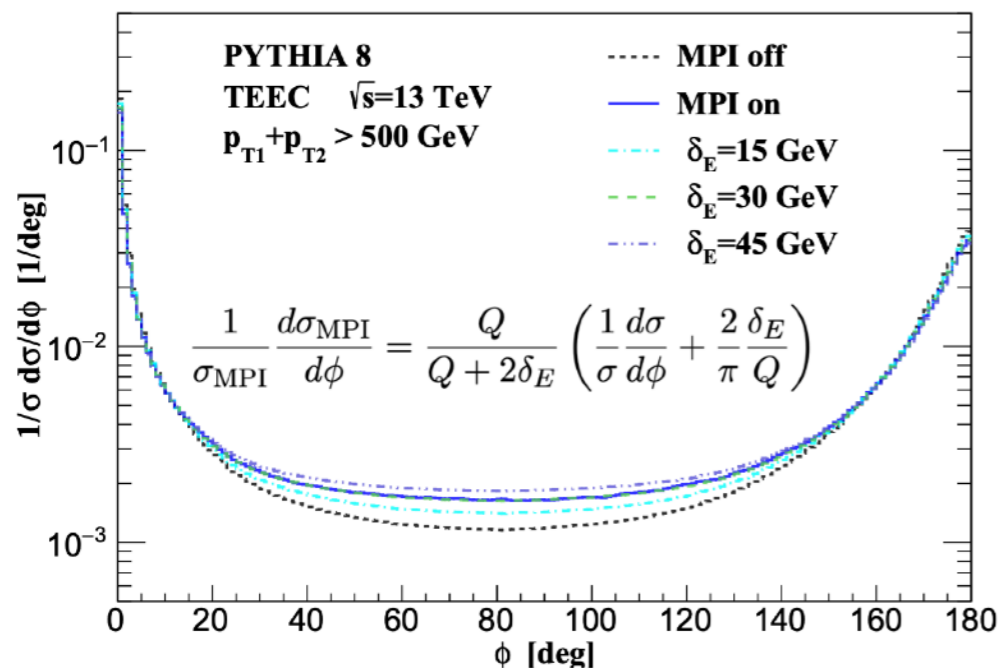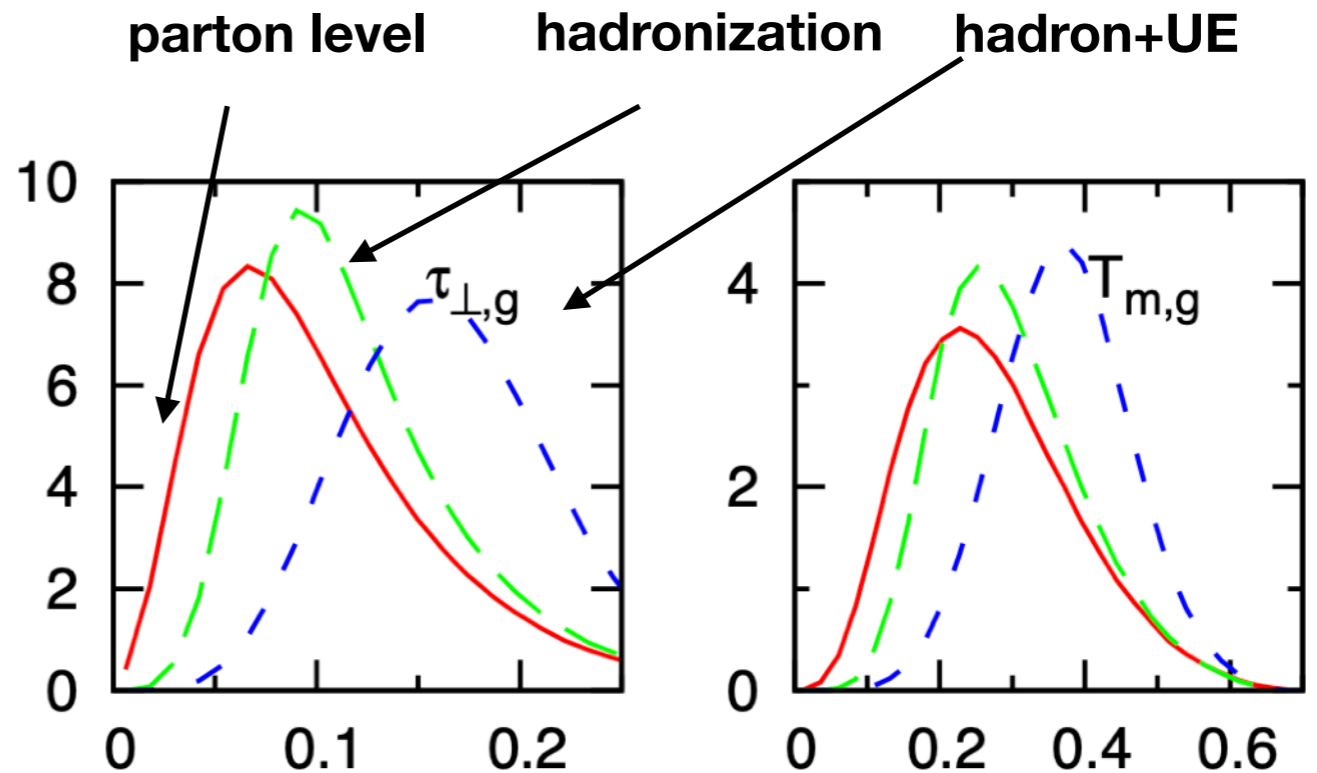

Gao, HTL, Moult, Zhu, 2023

Banfi, Salam and Zanderighi 2010

EEC and TEEC

EEC/TEEC is a class of event shape variables

where nonperturbative effect is supposed to be small

parton level   **hadronization**   **hadron+UE**

$$\frac{1}{\sigma_{MPI}}\frac{d\sigma_{MPI}}{d\phi} = \frac{Q}{Q+2\delta_E}\left(\frac{1}{\sigma}\frac{d\sigma}{d\phi} + \frac{2}{\pi}\frac{\delta_E}{Q}\right)$$

Gao, HTL, Moult, Zhu, 2023

Banfi, Salam and Zanderighi 2010

Recent developments (EEC/TEEC in DIS)

Nucleon Energy Correlators for Color Glass Condensate, Liu, Zhu et al, 2022,2023
Collins-type EEC jet in DIS, Kang et al, 2023
Imaging Cold Nuclear Matter with Energy Correlators, Devereaux et al 2023
TEEC in the Color-Glass Condensate at the Electron-Ion Collider, Kang et al 2023
TMDs from Semi-inclusive Energy Correlators, Liu, Zhu, 2024
NEEC and frature function, Chen, Ma, Tong, 2024   *and many other works*

TEEC in DIS

HTL, Vitev, Zhu, 2020

Definition

$$\text{TEEC} = \sum_a \int d\sigma_{lp \to l+a+X} \frac{E_{T,l} E_{T,a}}{E_{T,l} \sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$$

sum over

all hadrons

energy weighted

measure azimuthal

angle correlations

TEEC in DIS

HTL, Vitev, Zhu, 2020

Definition

$$\text{TEEC} = \sum_a \int d\sigma_{lp \to l+a+X} \frac{E_{T,l} E_{T,a}}{E_{T,l} \sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$$
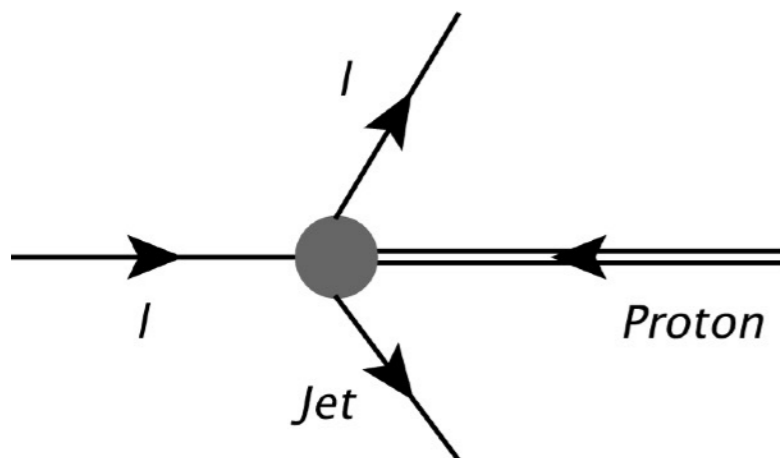
sum over all hadrons

energy weighted

measure azimuthal angle correlations
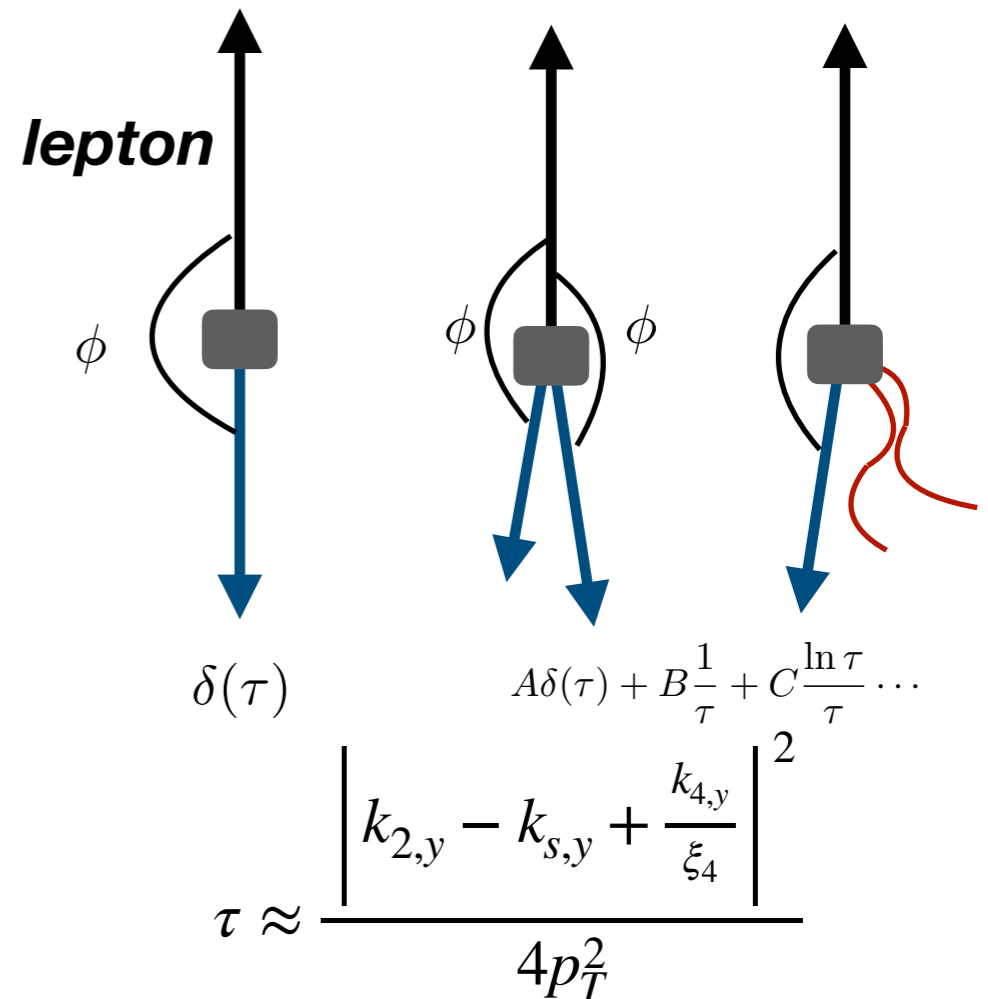
azimuthal angle correction between particles and lepton $\tau = \dfrac{1 + \cos\phi}{2}$

For $\tau \to 1$, large angle radiation
For $\tau \to 0$, small angle radiation

lepton

ϕ ϕ ϕ ϕ

l

l *Proton*

Jet

In transverse plane

$\delta(\tau)$ $A\delta(\tau) + B\frac{1}{\tau} + C\frac{\ln\tau}{\tau}\cdots$

$$\tau \approx \frac{\left| k_{2,y} - k_{s,y} + \dfrac{k_{4,y}}{\xi_4} \right|^2}{4p_T^2}$$

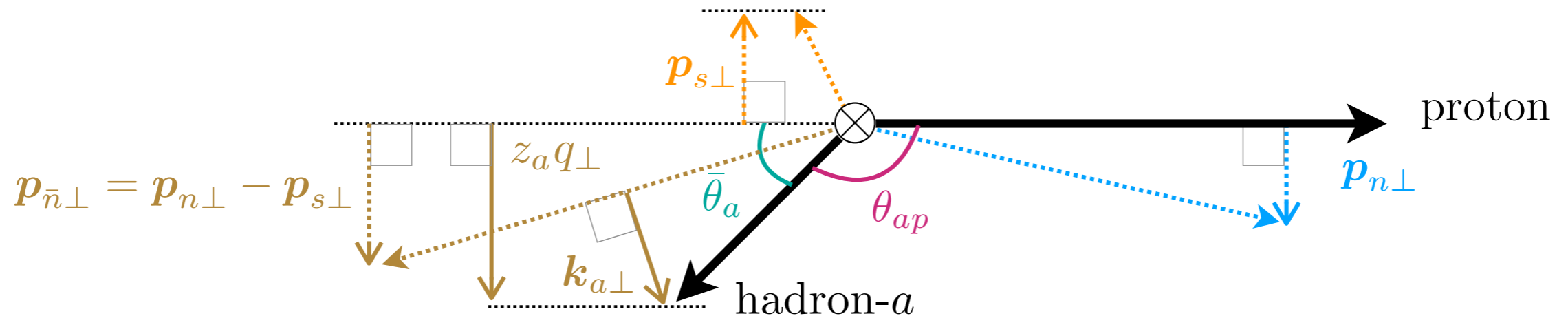# EEC in DIS

HTL, Makris, Vitev 2021

From Lab frame to Breit Frame

- boost the system to proton rest frame
- rotate the system: virtual photon has zero $\vec{q}_T$
- boost along z direction: virtual photon has zero energy

$\gamma^* + \text{proton} \rightarrow \text{jet/hadron} + X$
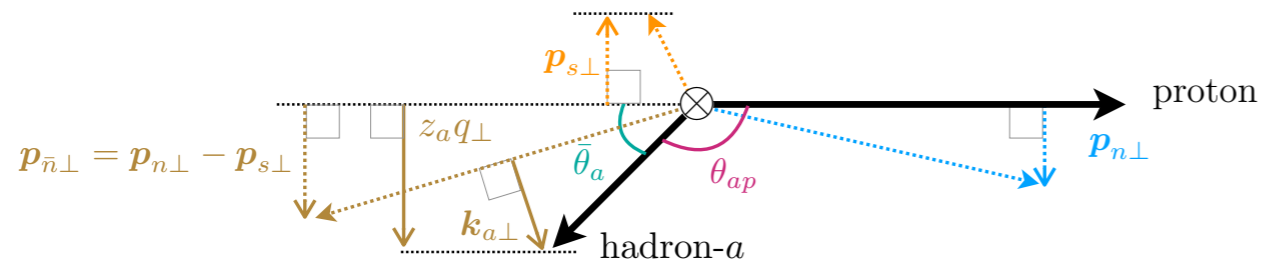

We proposed a new definition of EEC in DIS:

correlation between initial proton and final state hadron

$$\text{EEC} = \sum_a \int d\sigma_{lp \rightarrow l+a+X} \left(\frac{p \cdot p_a}{\sum_i p \cdot p_i} \right) \delta(\cos\chi - \cos\theta_{ap})$$
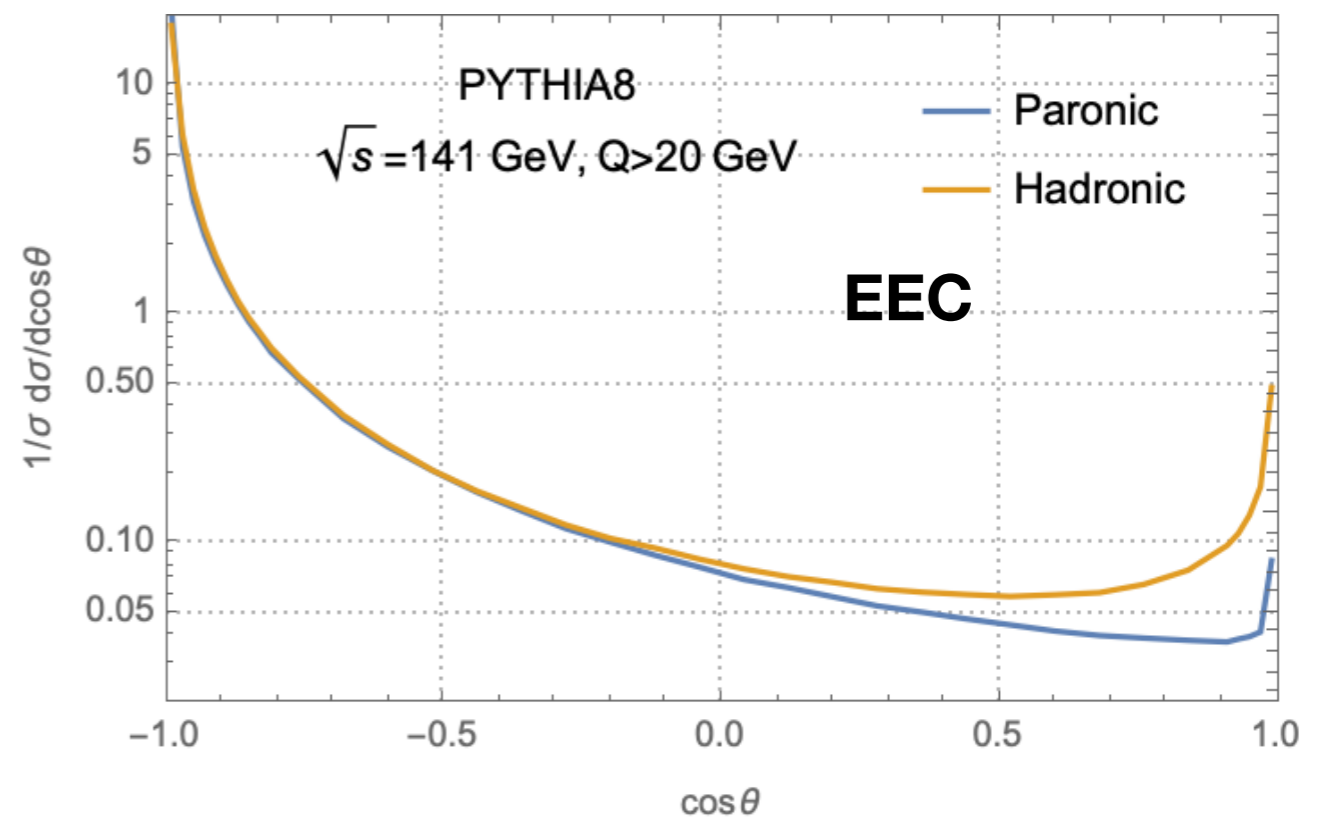
TEEC/EEC in DIS

$$\text{EEC} = \sum_a \int d\sigma_{lp \to l+a+X} \left(\frac{p \cdot p_a}{\sum_i p \cdot p_i} \right) \delta(\cos\chi - \cos\theta_{ap})$$

- ☐ weight function is Lorentz Invariant
- ☐ radiation close to the beam direction is suppressed
- ☐ soft radiation/hadronization effect is suppressed
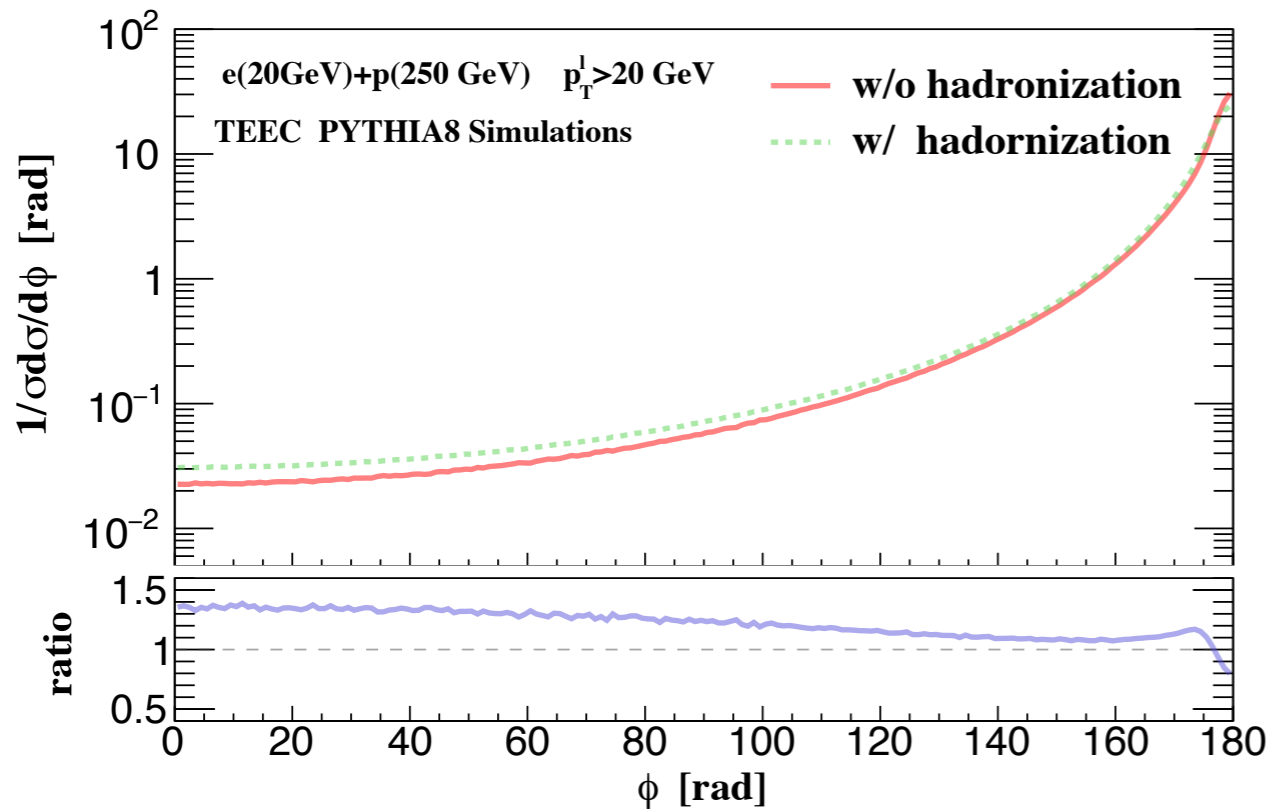
TEEC/EEC in DIS

$$\text{EEC} = \sum_a \int d\sigma_{lp \to l+a+X} \left(\frac{p \cdot p_a}{\sum_i p \cdot p_i} \right) \delta(\cos\chi - \cos\theta_{ap})$$

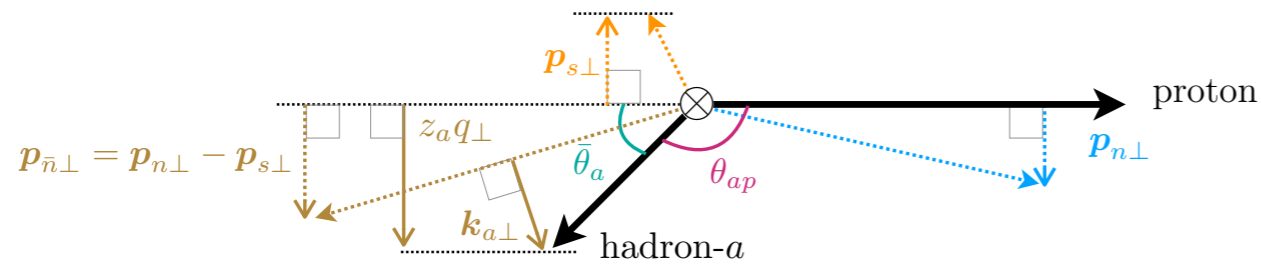☐ weight function is Lorentz Invariant

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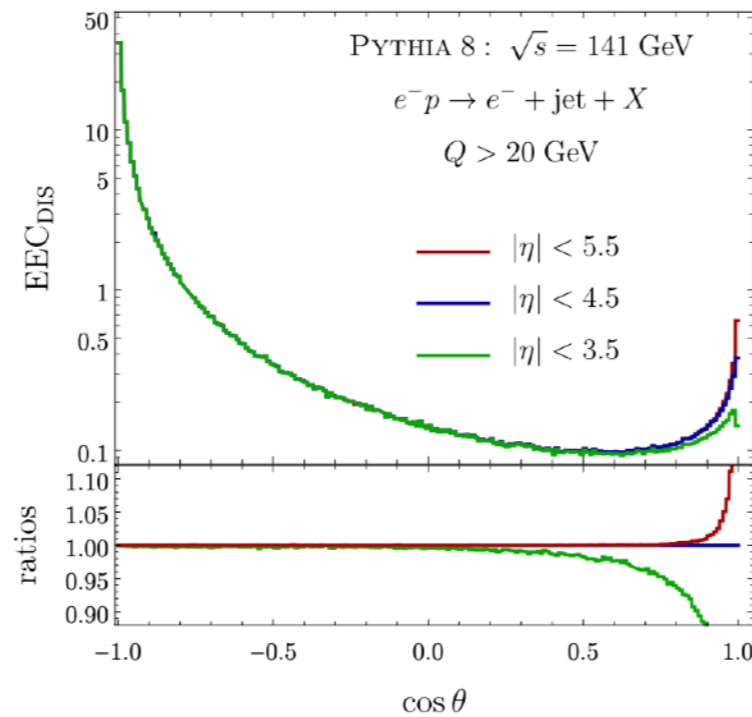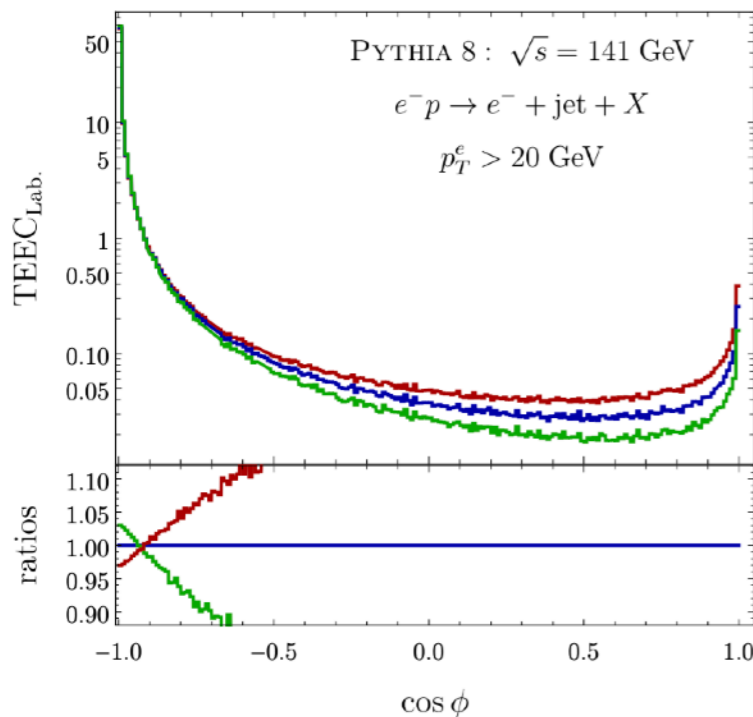☐ soft radiation/hadronization effect is suppressed

normalized to the cross section with cut $|\eta| < 5.5$

In Breit frame, rapidity cut only changes the cross section tail region

TEEC/EEC in DIS

| Definition | $\text{TEEC} = \sum_a \int d\sigma_{lp \to l+a+X} \dfrac{E_{T,l} E_{T,a}}{E_{T,l} \sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$ |

In back-to-back limit, it is similar to 1-dimensional TMD factorization

hadron with small p_T

TMD PDF

$$\frac{d\sigma_h}{d^2 p_\perp} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int \frac{db}{2\pi} e^{ib_\perp \cdot p_\perp} f_{f/N}(b,\xi,\mu,\nu)$$

$$S\left(b, \frac{n_2 \cdot n_4}{2}, \mu, \nu\right) \int dz F_{h/f}(z, b/z, E_4, \mu, \nu)$$

TMD soft   TMDFF

TEEC/EEC in DIS

Definition	$\text{TEEC} = \sum_a \int d\sigma_{lp \to l+a+X} \dfrac{E_{T,l} E_{T,a}}{E_{T,l} \sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$

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TMD PDF

sum over all hadrons in the final state

$$\frac{d\sigma_h}{d^2 p_\perp} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int \frac{db}{2\pi} e^{ib_\perp \cdot p_\perp} f_{f/N}(b,\xi,\mu,\nu)$$

$$S\left(b, \frac{n_2 \cdot n_4}{2}, \mu, \nu\right) \quad \int dz F_{h/f}(z, b/z, E_4, \mu, \nu)$$

TMD soft   TMDFF

$$\frac{d\sigma_h}{d\tau} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int dk_y \int \frac{db}{2\pi} e^{-ib_y \cdot k_y} f_{f/N}(b,\xi,\mu,\nu)$$

$$S\left(b, \frac{n_2 \cdot n_4}{2}, \mu, \nu\right) \sum_h \int dz\ z F_{h/f}(z, b/z, E_4, \mu, \nu)\ \delta(\tau - \tau(k_y))$$

TEEC/EEC in DIS

Definition	$\text{TEEC} = \sum_a \int d\sigma_{lp \to l+a+X} \dfrac{E_{T,l} E_{T,a}}{E_{T,l} \sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$

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TMD soft   TMDFF

sum over all hadrons in the final state

$$\frac{d\sigma_h}{d\tau} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int dk_y \int \frac{db}{2\pi} e^{-ib_y \cdot k_y} f_{f/N}(b, \xi, \mu, \nu)$$

$$S\left(b, \frac{n_2 \cdot n_4}{2}, \mu, \nu\right) \sum_h \int dz\ z F_{h/f}(z, b/z, E_4, \mu, \nu)\, \delta(\tau - \tau(k_y))$$
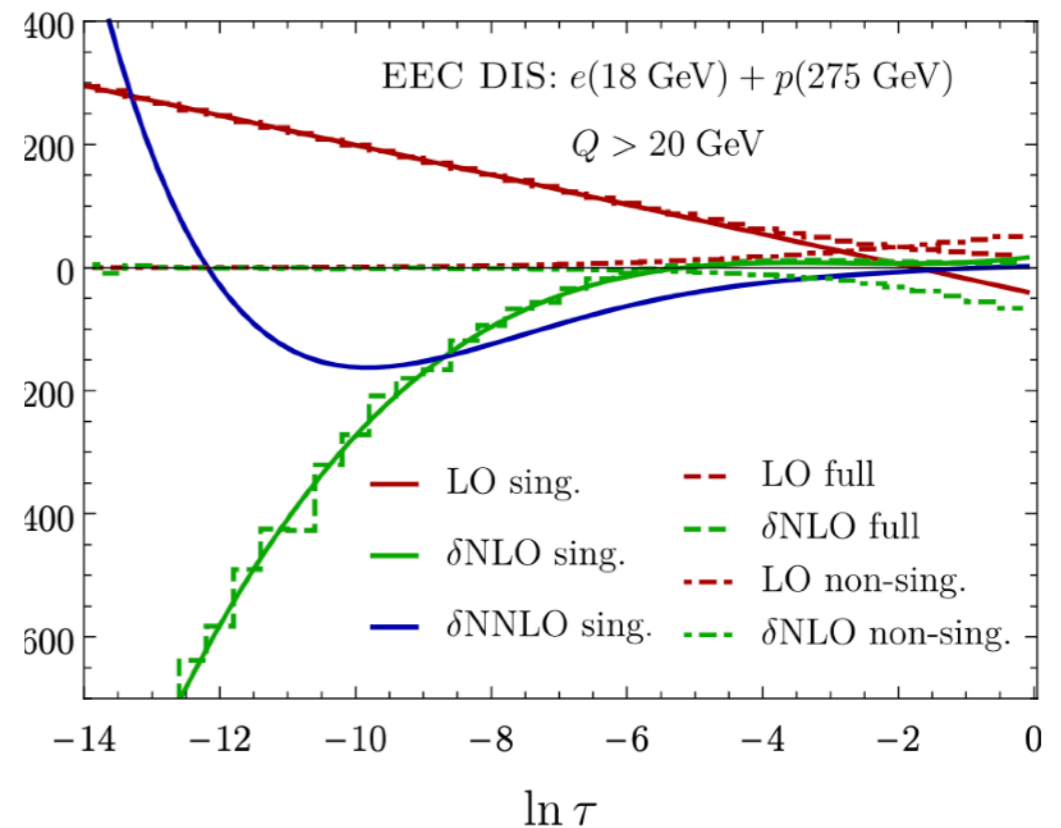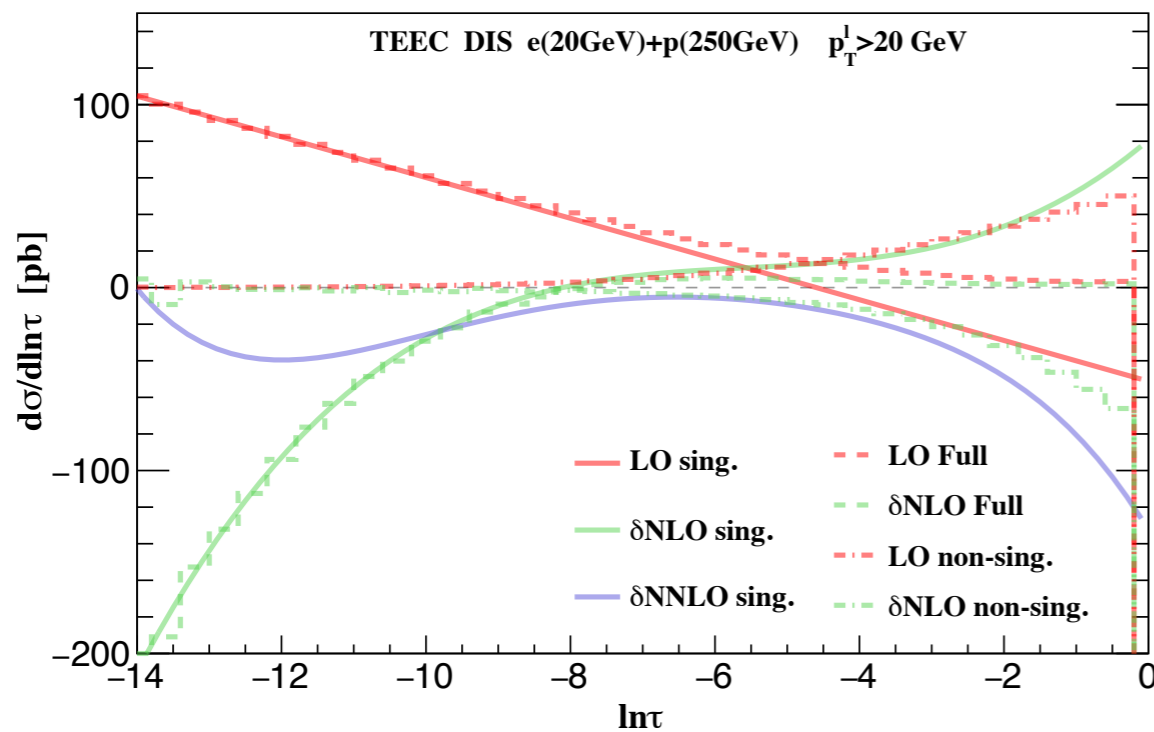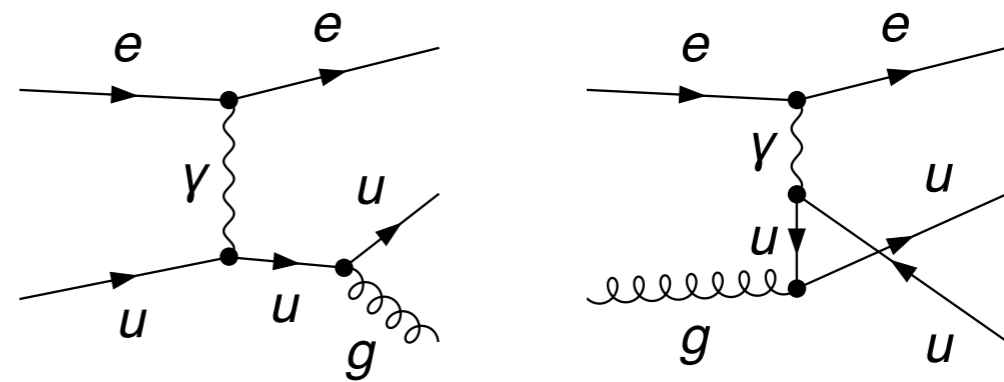
Jet function   the second Mellin-Moment of the TMDFFs

$$\sum_N \int_0^1 dz z F_{N/q}(z, b_\perp/z, \nu) = \sum_{i,N} \int_0^1 dz z \int_z^1 \frac{d\xi}{\xi}\ d_{N/i}(z/\xi)\ \mathscr{C}_{iq}(\xi, b_\perp/\xi, \nu) + \mathcal{O}\left(b_T^2 \Lambda_{\text{QCD}}^2\right)$$

$$= \sum_{i,N} \int_0^1 dx x \mathscr{C}_{iq}(x, b_\perp/\xi, \nu) \int_0^1 d\xi \xi d_{N/i}(\xi) + \mathcal{O}\left(b_T^2 \Lambda_{\text{QCD}}^2\right)$$

Fixed order results

The leading order process is

calculated using NLOJET++

- Reproduced the singular behaviors
- Full control of the distributions in the back-back limit at LO and NLO.
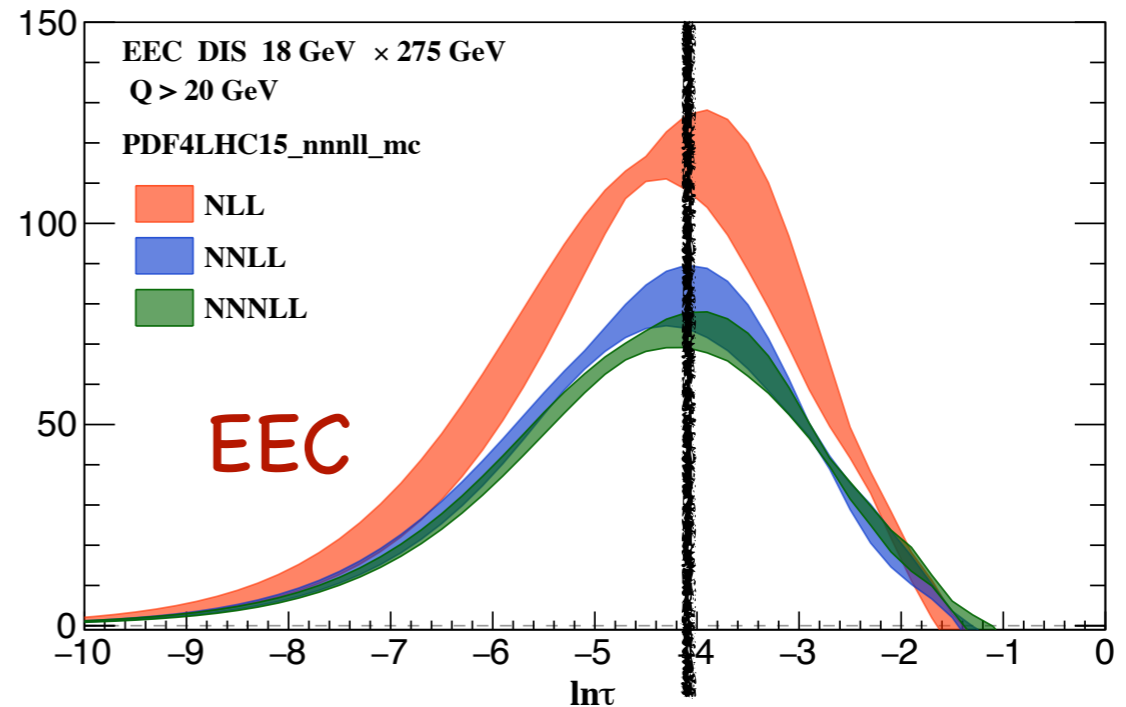- We obtained singular distribution up to NNLO (three loop anomalous dimensions)
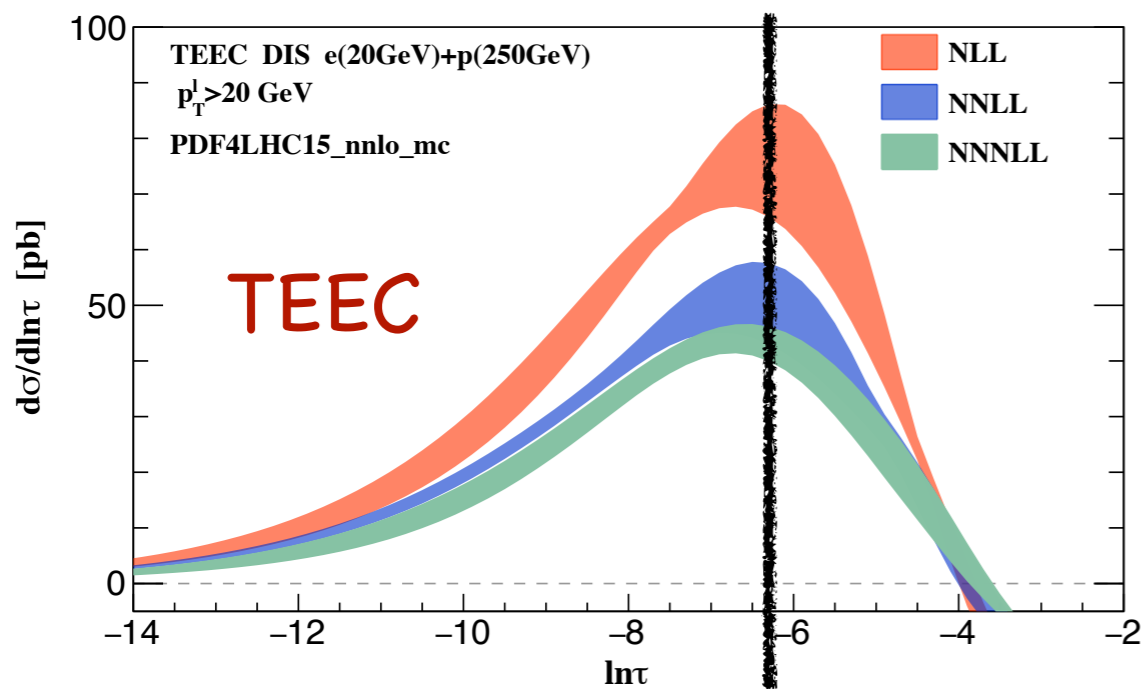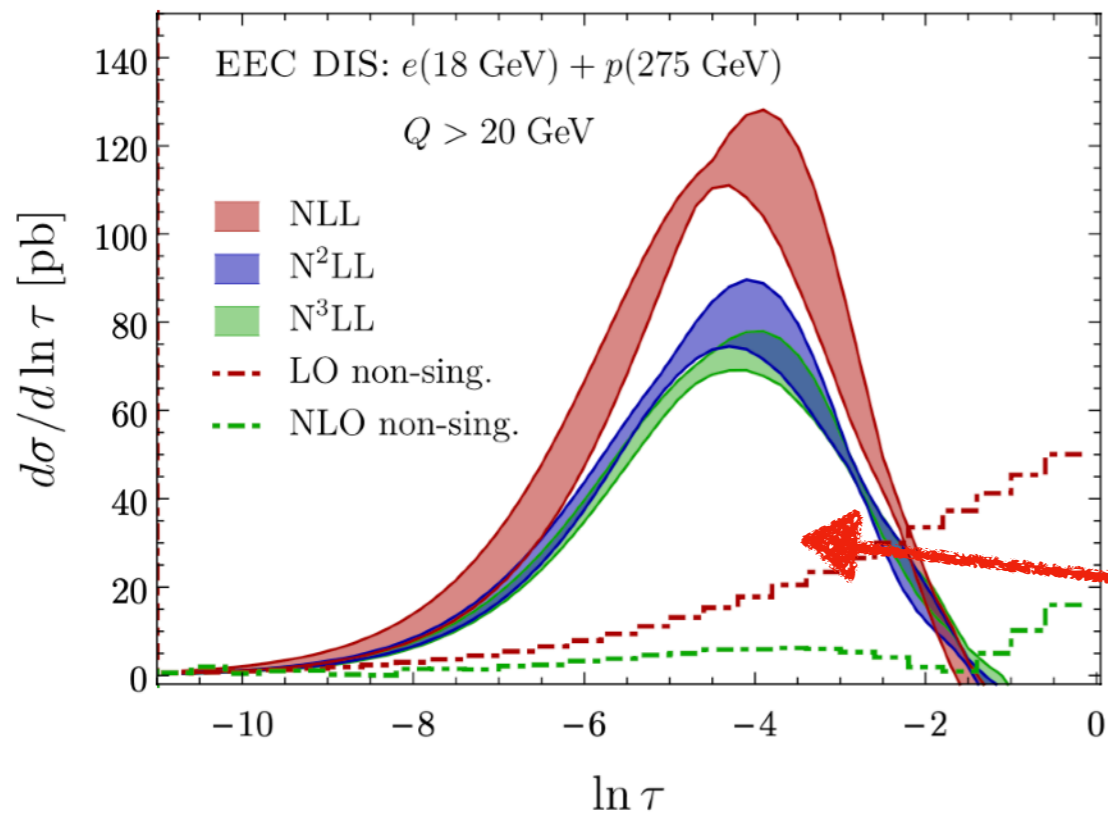
Resummation

resummation accuracy

Accuracy	H, J, S, B	γ_{cusp}	γ	β
LL	Tree	1 loop	-	1 loop
NLL	Tree	2 loop	1 loop	2 loop
NNLL	1 loop	3 loop	2 loop	3 loop
N^3LL	2 loop	4 loop	3 loop	4 loop

Resummation

TEEC DIS e(20GeV)+p(250GeV)
$p_T^l > 20$ GeV
PDF4LHC15_nnlo_mc

- NLL
- NNLL
- NNNLL

TEEC

EEC DIS 18 GeV × 275 GeV
Q > 20 GeV
PDF4LHC15_nnnll_mc

- NLL
- NNLL
- NNNLL

EEC

for EEC peak at larger τ, means small NP effects

EEC DIS: $e(18$ GeV$) + p(275$ GeV$)$
$Q > 20$ GeV

- NLL
- N^2LL
- N^3LL
- LO non-sing.
- NLO non-sing.

- Convergence in back-to-back limit after resummation

- Huge difference from NLL to NNLL and good perturbative convergence from NNLL to NNNLL

- Reduction of scale uncertainties order by order from NLL to NNNLL

Non-singular terms start to contribute which is less important for EEC
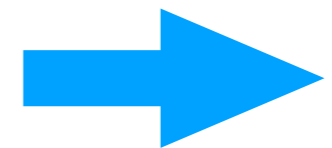
Non-perturbative effects from TMD
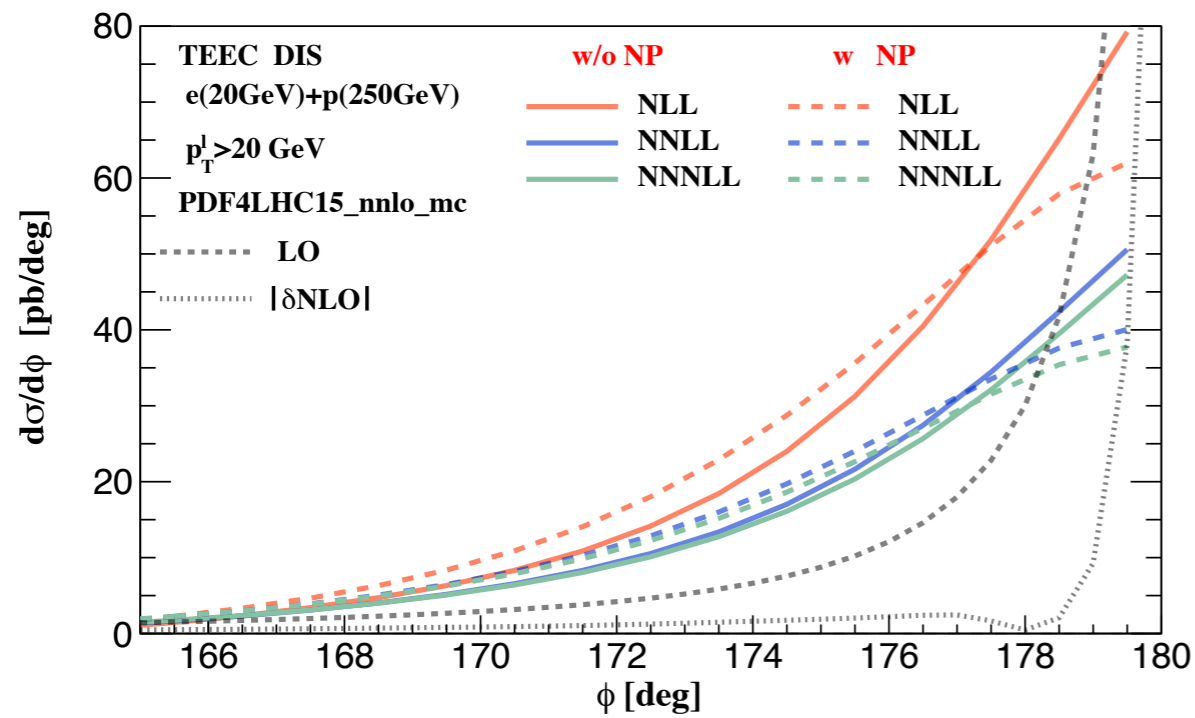
☑ corrections to rapidity evolution
☑ corrections to the TMD matrix element

Non-perturbative form factors, which extracted from the semi-inclusive hadron production in DIS.
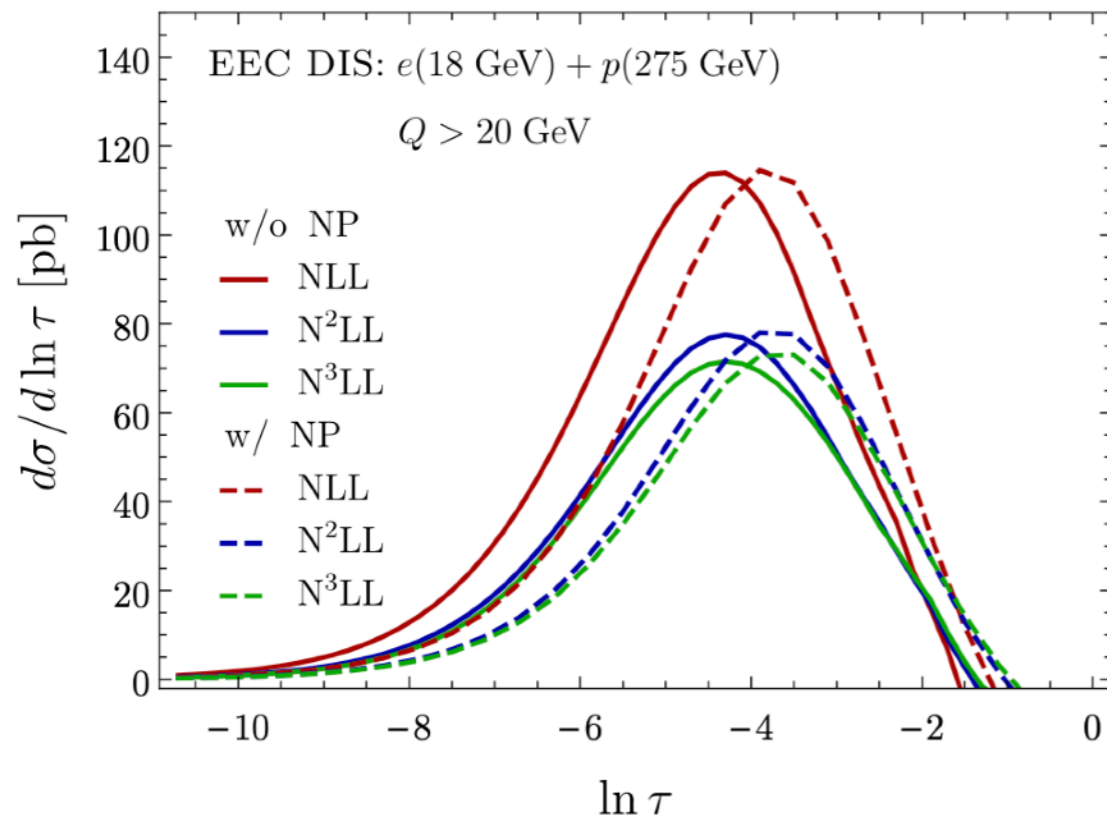
$$S_{\mathrm{NP}} = \exp\left[-0.106\ b^2 - 0.84 \ln Q/Q_0 \ln b/b*\right]$$

from TMD FFs

$$D_{i/a}^{\mathrm{NP}}(y, b) = \exp\left(-0.042\frac{b^2}{y^2}\right)$$
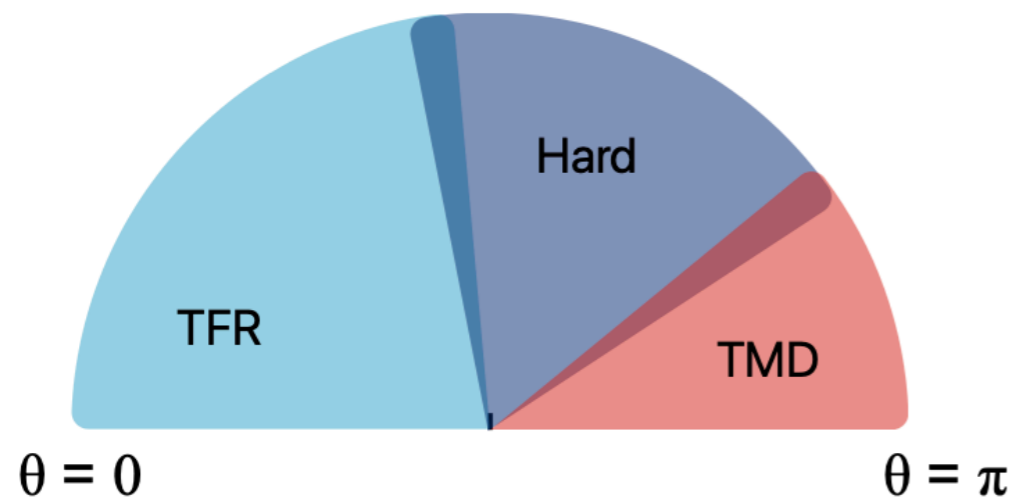
$$j_i(b) = \exp\left(-0.59b - 0.03b^2\right)$$

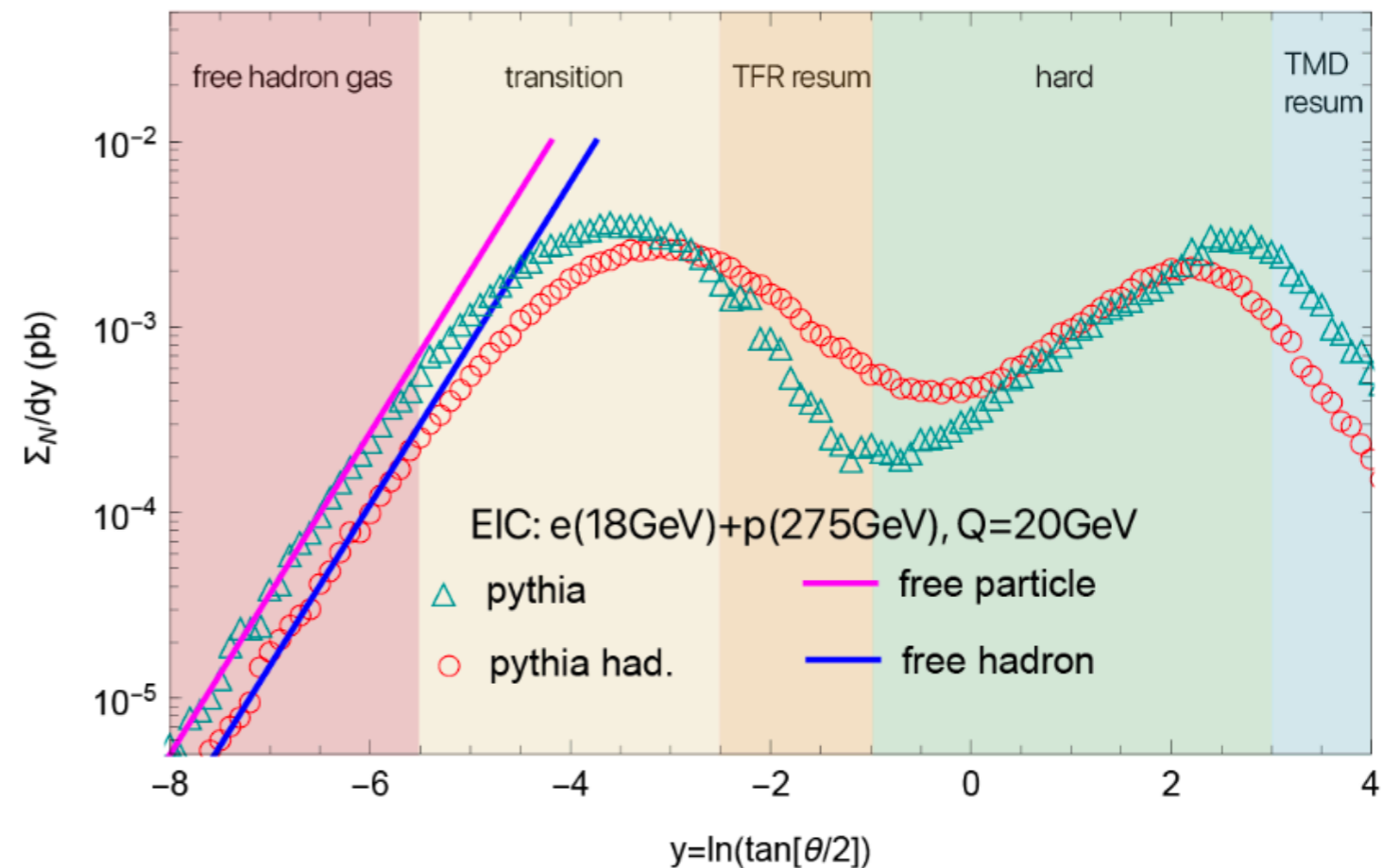NP shifts the cross section

Sizable NP effects in back-to-back limit

Bjorken x weighted EEC

$$\mathrm{EEC} = \frac{1}{\sigma}\sum_a \int d\sigma(\ell + h \to \ell + a + X) x_B^{N-1}\frac{P \cdot p_a}{P \cdot q}\delta\left(\cos\theta_{ap} - \cos\theta\right)$$

NEEC, Liu, Zhu, arXiv:2209.02080; Cao, Liu, Zhu, arXiv:2303.01530.

Cao, HTL, Mi, arXiv:2312.07655

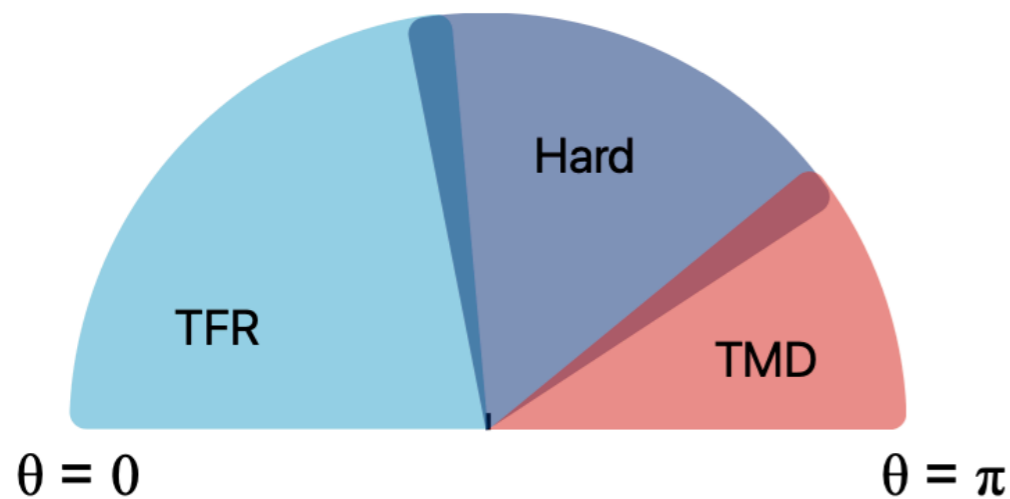TFR: the correlation of the energy flows from the initial nucleon.

Hard: measures the perturbative behavior of QCD

TMD: measures perturbative and nonperturbative TMD physics

Bjorken x weighted EEC

$$\mathrm{EEC} = \frac{1}{\sigma} \sum_a \int d\sigma(\ell + h \to \ell + a + X) x_B^{N-1} \frac{P \cdot p_a}{P \cdot q} \delta\left(\cos\theta_{ap} - \cos\theta\right)$$

NEEC, Liu, Zhu, arXiv:2209.02080; Cao, Liu, Zhu, arXiv:2303.01530.

Cao, HTL, Mi, arXiv:2312.07655

TFR: the correlation of the energy flows from the initial nucleon.

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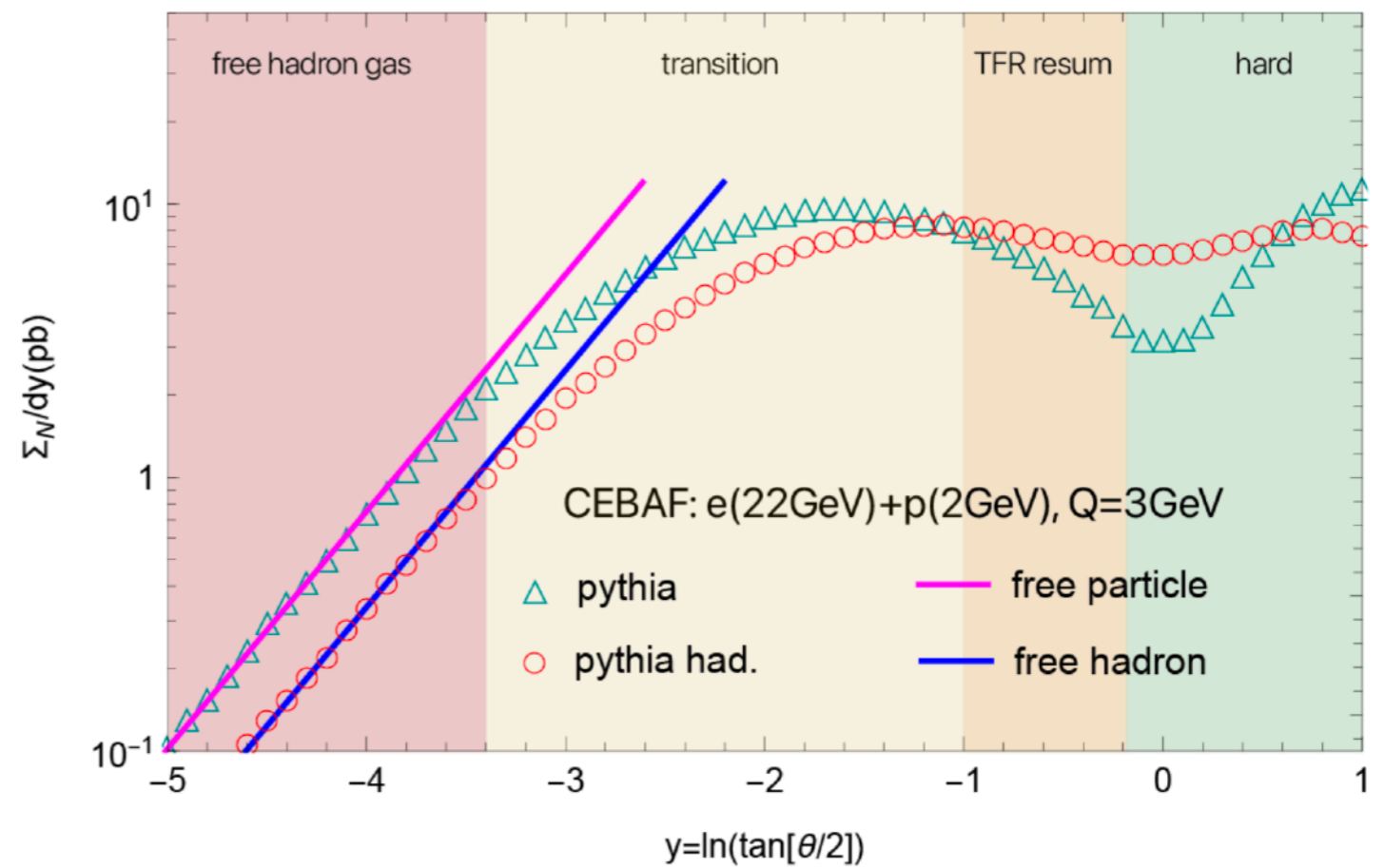TMD: measures perturbative and nonperturbative TMD physics

Conclusion

Motivation

> ☐ Event shapes serve as a QCD laboratory, a tool for QCD study
>
> ☐ EEC/TEEC can been studied for various processes

Observables

> ☐ TEEC and EEC in DIS

Application

> ☐ investigate QCD in low and high energy limits
>
> ☐ test and study TMD factorization
>
> ☐ extract TMD PDFs and TMD FFs

Thank you!