Radiative corrections for $K^+ \to \pi^+ \ell^+ \ell^-$ decays revisited

Tomáš Husek
Charles University, Prague and *University of Birmingham*

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FACULTY OF MATHEMATICS AND PHYSICS
Charles University
UNIVERSITY OF BIRMINGHAM
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Radiative decays $K^+ \to \pi^+ \ell^+ \ell^-$
Motivation

Flavor-changing (strangeness-changing) neutral-current weak transitions
$\hookrightarrow$ absent at tree level in Standard Model
$\hookrightarrow$ manifest in radiative non-leptonic kaon decays like $K^+ \to \pi^+ \ell^+ \ell^- (\gamma)$, $\ell = e, \mu$
  $\hookrightarrow$ interesting probe of SM quantum corrections and beyond

Underlying long-distance-dominated radiative modes (transitions) $K^+ \to \pi^+ \gamma^* (\gamma)$ studied before
$\hookrightarrow$ calculated in Chiral Perturbation Theory (ChPT) enriched with electroweak perturbations
$\hookrightarrow$ *Ecker, Pich, de Rafael*, NPB 291 (1987), 303 (1988), at leading order (LO) (at one-loop level)

beyond LO: including the dominant unitarity corrections from $K \to 3\pi$
$\hookrightarrow$ *D'Ambrosio, Ecker, Isidori, Portolés*, JHEP 08 (1998)
$\hookrightarrow$ *Gabbiani*, PRD 59 (1999)

$K^+ \to \pi^+ \gamma^*$ transition

$$\mathcal{M}_\rho\big(K^+(P) \to \pi^+(r)\gamma^*_\rho(k)\big) \equiv i \int \mathrm{d}^4x\, e^{ikx} \langle \pi(r) \big| T\big[J_\rho^{\text{EM}}(x) \mathcal{L}^{\Delta S=1}(0)\big] \big| K(P) \rangle$$
$$= \frac{e}{2} F(k^2) \big[(P-r)^2 (P+r)_\rho - (P^2 - r^2)(P-r)_\rho\big]$$

Radiative decays $K^+ \to \pi^+\ell^+\ell^-$

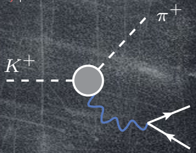
Form-factor parametrization

LO appears at $\mathcal{O}(p^4)$ + unitarity loop correction from $\pi\pi$ rescattering

$\hookrightarrow$ universal parametrization: $F(s) \propto W_+(s/M_K^2)$, $W_+(z) = G_\mathsf{F} M_K^2 (a_+ + b_+ z) + W_+^{\pi\pi}(z)$

$\hookrightarrow$ *Ecker et al.*, NPB 291 (1987), *D'Ambrosio et al.*, JHEP 08 (1998)

$$\frac{\mathrm{d}\Gamma_+}{\mathrm{d}z} = \frac{G_\mathsf{F}^2 \alpha^2 M_K^5}{3(4\pi)^5} \lambda^{3/2}(z) \sqrt{1 - \frac{4r_\ell^2}{z}} \left(1 + \frac{2r_\ell^2}{z}\right) |W_+(z)|^2$$



LFU

$\hookrightarrow$ a_+ and b_+ should be the same for both (e and μ) channels

$\hookrightarrow$ discrepancy due to new physics via short-distance effects

Moreover, the ratio deviates significantly from the VMD ansatz

$$\text{VMD:} \quad \frac{b_+}{a_+} = \frac{M_K^2}{M_\rho^2} \approx 0.4\,, \qquad \text{exp.:} \quad \frac{b_+}{a_+} \approx 1.25$$

Measurement of quadratic term $c_+ z^2$ may further test the VMD hypothesis

ℓ	a_+	b_+	exp.
e	$-0.587(10)$	$-0.655(44)$	E865
e	$-0.578(16)$	$-0.779(66)$	NA48/2
μ	$-0.575(39)$	$-0.813(145)$	NA48/2
μ	$-0.575(13)$	$-0.722(43)$	NA62(2022)

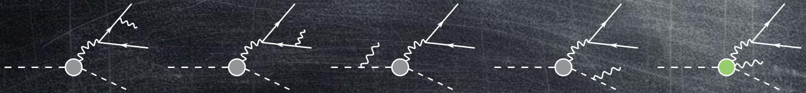
$\leftarrow$ JHEP 11 (2022) 011

improve precision $\longrightarrow$ radiative corrections, studied earlier: *Kubis and Schmidt*, EPJC 70 (2010)

Radiative decays $K^+ \to \pi^+ \ell^+ \ell^-$

Bremsstrahlung

$\hookrightarrow$ LO (scalar) QED contributions where the real photon is radiated from lepton (meson) legs



$\hookrightarrow$ radiation from effective $K^+ \to \pi^+ \gamma^*$ vertex $\longrightarrow$ gauge invariant

$\hookrightarrow$ represented in terms of $F(s)$?

Effective Lagrangian for $K^+ \to \pi^+ \gamma^* (\gamma)$ transition

$$\mathcal{L} = \underbrace{ieF(0) D_\mu K^+ (w\, \partial_\nu F^{\mu\nu}) \pi^-}_{\text{minimal term leading to gauge-invariant amplitude}} - \frac{e^2}{2} \kappa F(0) K^+ F_{\mu\nu} F^{\mu\nu} \pi^-$$

$$\mathcal{M}_{\rho\sigma}(K^+(P) \to \pi^+(r) \gamma^*_\rho(k_1) \gamma_\sigma(k_2)) = e^2 F(k_1^2) \left\{ k_1^2 \left(r_\rho \frac{P_\sigma}{P \cdot k_2} - P_\rho \frac{r_\sigma}{r \cdot k_2} + g_{\rho\sigma} \right) \right\} + e^2 \tilde{\kappa} F(k_1^2) \left[(k_1 \cdot k_2) g_{\rho\sigma} - k_{1\sigma} k_{2\rho} \right]$$

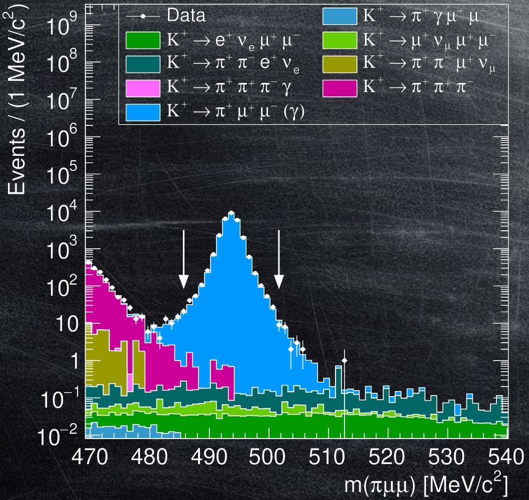
$\hookrightarrow$ term proportional to $\tilde{\kappa}$

$\hookrightarrow$ mimic the behavior of additional form factors adequately at given order

$\hookrightarrow$ ideally negligible effects in given set-up $\longrightarrow$ estimate of associated uncertainty

Radiative decays $K^+ \to \pi^+ \ell^+ \ell^-$

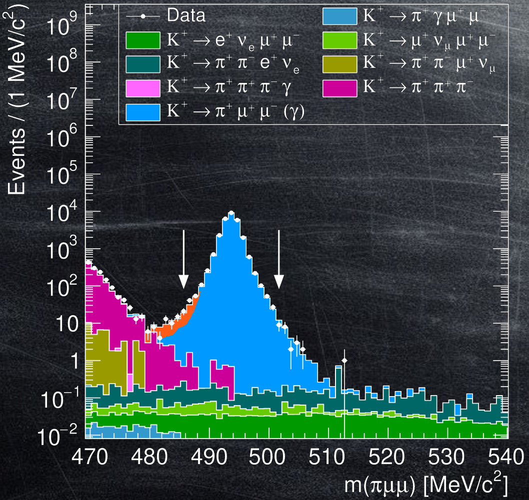
Spectra



(Thanks to *Ľ. Bičian (NA62)*)

Radiative decays $K^+ \to \pi^+\ell^+\ell^-$

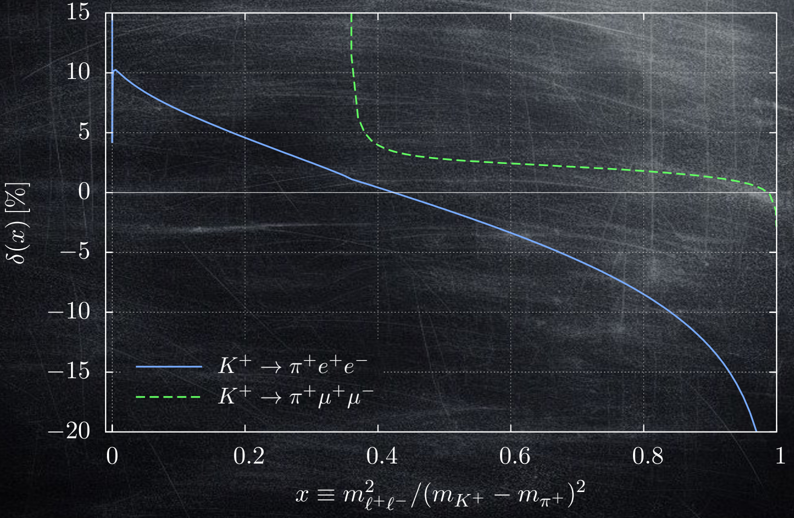
Spectra



(Thanks to *Ľ. Bičian (NA62)*)

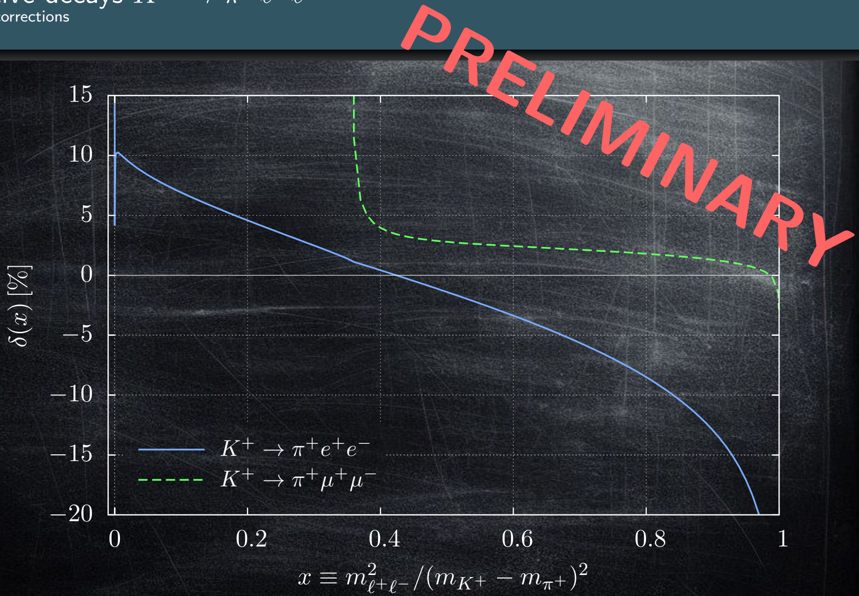
Radiative decays $K^+ \to \pi^+\ell^+\ell^-$

Radiative corrections

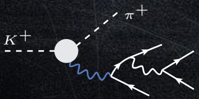


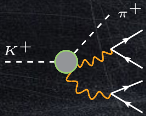
Radiative decays $K^+ \to \pi^+ \ell^+ \ell^-$

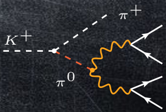
Radiative corrections



$K^+ \to \pi^+ 4e$ decay







$K^+ \to \pi^+ e^+ e^- e^+ e^-$
Introduction

The long-distance-dominated $K^+ \to \pi^+ \gamma^*$ transition essential also for $K^+ \to \pi^+ 4e$
$\hookrightarrow$ one also needs to consider $K^+ \to \pi^+ \gamma^* \gamma^*$ transition

Challenging to observe $K^+ \to \pi^+ 4e$ away from $m_{4e} \simeq M_{\pi^0}$
$\hookrightarrow$ suppressed decay rate $\longrightarrow$ attractive to study possible effects of BSM physics
$\hookrightarrow$ to identify new-physics-scenario contribution $\longrightarrow$ need for (rough) estimate of SM rate
$\hookrightarrow$ new-physics effects spotted as deviations from such SM predictions

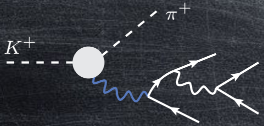
$$B(K^+ \to \pi^+ 4e, \text{non-res.}) = 7.2(7) \times 10^{-11}$$

$\longrightarrow$ possible BSM scenarios are being explored

Hostert, Pospelov, PRD 105 (2022)
$\hookrightarrow$ $K \to \pi 4e$ decays proceed via $K \to \pi(X' \to XX)$ intermediate states
$\hookrightarrow$ cascade of dark-sector particles $X^{(\prime)}$
$\hookrightarrow$ underlying dynamics potentially significantly enhanced compared to the SM case

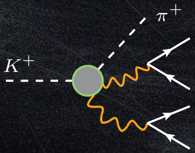
$\longrightarrow$ searches in suitable experiments
$\hookrightarrow$ more precise knowledge of SM background essential
$\hookrightarrow$ ideally at level suited for Monte Carlo (MC) implementation

One-photon-exchange topology

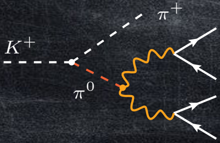


(1)

Two-photon-exchange topology



(2a)



(2b)

TH, PRD 106 (2022)

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Two-photon-exchange topology: Matrix element

The two-photon transition of topology (2a) can be written, approximately, as follows:

$$\mathcal{M}^{(a)}_{\rho\sigma}\big(K(P) \to \pi(r)\gamma^*_\rho(k_1)\gamma^*_\sigma(k_2)\big)$$
$$\simeq e^2 F(k_1^2)\Big\{(k_1^2 r_\rho - r\cdot k_1 k_{1\rho})\,\frac{(2P-k_2)_\sigma}{2P\cdot k_2 - k_2^2} - (k_1^2 P_\rho - P\cdot k_1 k_{1\rho})\,\frac{(2r+k_2)_\sigma}{2r\cdot k_2 + k_2^2}$$
$$+ \big(k_1^2 g_{\rho\sigma} - k_{1\rho}k_{1\sigma}\big)$$
$$+ \widetilde{\kappa}\big[(k_1\cdot k_2)g_{\rho\sigma} - k_{1\sigma}k_{2\rho}\big]\Big\}$$
$$+ \{k_1 \leftrightarrow k_2, \rho \leftrightarrow \sigma\}$$

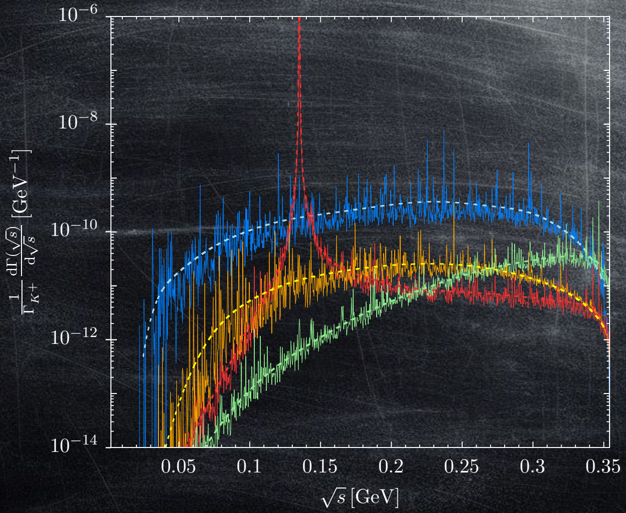
↪ in this model depends on a single form factor (the same $F(s)$)
↪ useful when measuring $F(s)$ ⟶ radiative corrections for the $K^+ \to \pi^+\ell^+\ell^-$ decay
↪ one of the photons on-shell

Soft-photon regime ⟶ approximation justified
Hard photons ⟶ free parameter $|\widetilde{\kappa}| \lesssim 1$ introduced to cover model uncertainty
↪ physical results do not seem to be sensitive to this parameter

For $K^+ \to \pi^+ 4e$, we assume it is good enough (at least) as an order-of-magnitude guess
↪ numerically negligible (one order of magnitude) compared to the topology (1)

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Contributions to the branching ratio



[large MC samples generated by A. Shaikhiev, E. Goudzovski] *TH*, PRD 106 (2022)

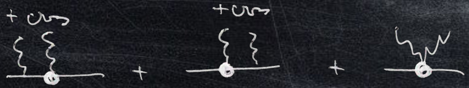
$K^+ \to \pi^+\gamma^*\gamma^*$ transition

$K^+ \to \pi^+\gamma^*\gamma^*$ transition at LO in ChPT

$K^+ \to \pi^+\gamma^{(*)}$ transition

$$= ieF(k^2)\big[k^2 r_\rho - (k\cdot r)k_\rho\big] + e\widetilde{F}(k^2)(M_K^2 + M_\pi^2)(P+r)_\rho$$

$K^+ \to \pi^+\gamma^{(*)}\gamma^{(*)}$ transition



$K^+ \to \pi^+\gamma^*\gamma^*$ transition at LO in ChPT

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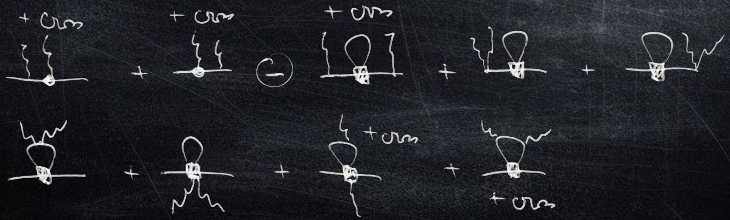
$$= ieF(k^2)\big[k^2 r_\rho - (k\cdot r)k_\rho\big] + e\widetilde{F}(k^2)(M_K^2 + M_\pi^2)(P+r)_\rho$$

$K^+ \to \pi^+\gamma^{(*)}\gamma^{(*)}$ transition



$K^+ \to \pi^+\gamma^*\gamma^*$ transition at LO in ChPT

$$\mathcal{M}_{\rho\sigma}\left(K^+(P) \to \pi^+(r)\gamma^*_\rho(k_1)\gamma^*_\sigma(k_2)\right)$$
$$= e^2 F(k_1^2) T^{(0)}_{\rho\sigma} + \left(k_1 \leftrightarrow k_2, \rho \leftrightarrow \sigma\right)$$
$$+ e^2 C_1 \epsilon_{\rho\sigma(k_1)(k_2)}$$
$$+ e^2\left[T^{(1)}_{\rho\sigma} A^{(+,1)}_{\gamma^*\gamma^*} - 2T^{(2)}_{\rho\sigma} A^{(+,2)}_{\gamma^*\gamma^*}\right] + 4e^2(k_1\cdot k_2) T^{(1*)}_{\rho\sigma}\left[-\hat{c} + \kappa\frac{(k_1+k_2)^2}{2k_1\cdot k_2}\right] - 4e^2 T^{(2)}_{\rho\sigma}\kappa\frac{(k_1+k_2)^2}{2k_1\cdot k_2}$$

$$T^{(0)}_{\rho\sigma} = (k_1^2 r_\rho - r\cdot k_1 k_{1\rho})\frac{(2P-k_2)_\sigma}{2P\cdot k_2 - k_2^2} + (P \leftrightarrow -r) + \left(k_1^2 g_{\rho\sigma} - k_{1\rho}k_{1\sigma}\right)$$

$$T^{(1)}_{\rho\sigma} = g_{\rho\sigma} - \frac{(k_1\cdot k_2)\left(k_{1\sigma}k_{2\rho} + k_{1\rho}k_{2\sigma}\right) - k_1^2 k_{2\rho}k_{2\sigma} - k_2^2 k_{1\rho}k_{1\sigma}}{(k_1\cdot k_2)^2 - k_1^2 k_2^2}, \qquad T^{(1*)}_{\rho\sigma} = g_{\rho\sigma} - \frac{k_{1\sigma}k_{2\rho}}{k_1\cdot k_2}$$

$$T^{(2)}_{\rho\sigma} = \frac{\left[k_1^2 k_{2\rho} - (k_1\cdot k_2)k_{1\rho}\right]\left[k_2^2 k_{1\sigma} - (k_1\cdot k_2)k_{2\sigma}\right]}{(k_1\cdot k_2)^2 - k_1^2 k_2^2}, \qquad T^{(2*)}_{\rho\sigma} = 0$$

$$A^{(+,i)}_{\gamma^*\gamma^*} = \left[M_K^2 - M_\pi^2 + s\right]\hat{A}^{(i)}_{\gamma^*\gamma^*}(M_\pi^2) - \left[M_K^2 - M_\pi^2 - s\right]\hat{A}^{(i)}_{\gamma^*\gamma^*}(M_K^2)$$

$K_\mathsf{L} \to \pi^0\gamma^*\gamma^*$ transition at LO in ChPT

$$\mathcal{M}_{\rho\sigma}\left(K_\mathsf{L}(P) \to \pi^0(r)\gamma^*_\rho(k_1)\gamma^*_\sigma(k_2)\right)$$
$$= -2e^2\left[T^{(1)}_{\rho\sigma}A^{(0,1)}_{\gamma^*\gamma^*} - 2T^{(2)}_{\rho\sigma}A^{(0,2)}_{\gamma^*\gamma^*}\right] - 2e^2\kappa M_K^2 T^{(1)}_{\rho\sigma}$$

$$T^{(0)}_{\rho\sigma} = (k_1^2 r_\rho - r\cdot k_1 k_{1\rho})\,\frac{(2P-k_2)_\sigma}{2P\cdot k_2 - k_2^2} + (P \leftrightarrow -r) + (k_1^2 g_{\rho\sigma} - k_{1\rho}k_{1\sigma})$$

$$T^{(1)}_{\rho\sigma} = g_{\rho\sigma} - \frac{(k_1\cdot k_2)(k_{1\sigma}k_{2\rho} + k_{1\rho}k_{2\sigma}) - k_1^2 k_{2\rho}k_{2\sigma} - k_2^2 k_{1\rho}k_{1\sigma}}{(k_1\cdot k_2)^2 - k_1^2k_2^2}\,, \qquad T^{(1*)}_{\rho\sigma} = g_{\rho\sigma} - \frac{k_{1\sigma}k_{2\rho}}{k_1\cdot k_2}$$

$$T^{(2)}_{\rho\sigma} = \frac{\left[k_1^2 k_{2\rho} - (k_1\cdot k_2)k_{1\rho}\right]\left[k_2^2 k_{1\sigma} - (k_1\cdot k_2)k_{2\sigma}\right]}{(k_1\cdot k_2)^2 - k_1^2k_2^2}\,, \qquad T^{(2*)}_{\rho\sigma} = 0$$

$$A^{(0,i)}_{\gamma^*\gamma^*} = [s - M_\pi^2]\,\hat{A}^{(i)}_{\gamma^*\gamma^*}(M_\pi^2) + [M_K^2 + M_\pi^2 - s]\,\hat{A}^{(i)}_{\gamma^*\gamma^*}(M_K^2)$$

The above results reduce to special cases present in literature:

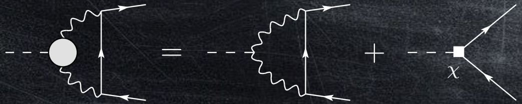
$K^+ \to \pi^+\gamma\gamma$
$\hookrightarrow$ *Ecker, Pich, de Rafael*, Nucl. Phys. B **291** (1987), B **303** (1988)
$\hookrightarrow$ *D'Ambrosio, Portolés*, Phys. Lett. B **386** (1996)

$K^+ \to \pi^+\gamma\gamma^*$
$\hookrightarrow$ *Gabbiani*, Phys. Rev. D **59** (1999)

$K_{\mathsf{L}} \to \pi^0\gamma\gamma$
$\hookrightarrow$ *Ecker, Pich, de Rafael*, Nucl. Phys. B **291** (1987), B **303** (1988)
$\hookrightarrow$ *Gabbiani, Valencia*, Phys. Rev. D **64** (2001)

$K_{\mathsf{L}} \to \pi^0\gamma\gamma^*$
$\hookrightarrow$ *Donoghue, Gabbiani*, Phys. Rev. D **56** (1997), D **58** (1998)

Rare electromagnetic decay $\pi^0 \to e^+e^-$



$\pi^0 \to e^+e^-$ update

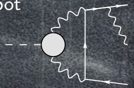
Short review paper on radiative corrections

The neutral-pion decay into electron-positron pair: A review and update

$\hookrightarrow$ arXiv:2405.09650 [hep-ph], accepted for publication in Phys. Rev. D

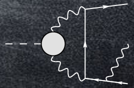
$\hookrightarrow$ accompanies the latest branching-ratio measurement at NA62 (preliminary results)

$\hookrightarrow$ revises, discusses, and summarizes radiative correction for $\pi^0 \to e^+e^-$ in one spot



$\hookrightarrow$ describes relation between the theoretical and experimental observables

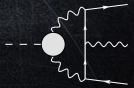
- $\hookrightarrow$ $B(\pi^0 \to e^+e^-(\gamma), x > x_{\text{cut}}) = [1 + \delta(x_{\text{cut}})]\, B(\pi^0 \to e^+e^-)$
- $\hookrightarrow$ updates the critical NLO QED correction $\delta_+(x_{\text{cut}})$ that relates these
  - $\hookrightarrow$ $\delta_+(0.95) = [-6.06(7)_\xi - 0.08\widetilde{\chi}]\% = -6.1(2)\%$



$\hookrightarrow$ calculates the overall correction $\delta = [10.67(7)_\xi - 0.25\widetilde{\chi}]\% = 10.7(1)_\xi(2)_\chi\%$

$\hookrightarrow$ derives other related useful experimental quantities / ratios

$\hookrightarrow$ discusses the latest measurements (KTeV, NA62)



$\hookrightarrow$ properties of the $\pi^0 \to e^+e^-(\gamma)$ amplitude in various limits

- $\hookrightarrow$ discusses its relation with the Dalitz-decay radiative corrections

Thank you for listening!