Chiral Symmetry and Large Magnetic Fields

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Motivation:

Chiral Symmetry in Large Magnetic Fields

- Record-breaking surface magnetic field of magnetar
  Fields of neutron star interiors conjectured >> surface

[Kong, *et al.*, Astrophys. J. Lett. 933 (2022)]

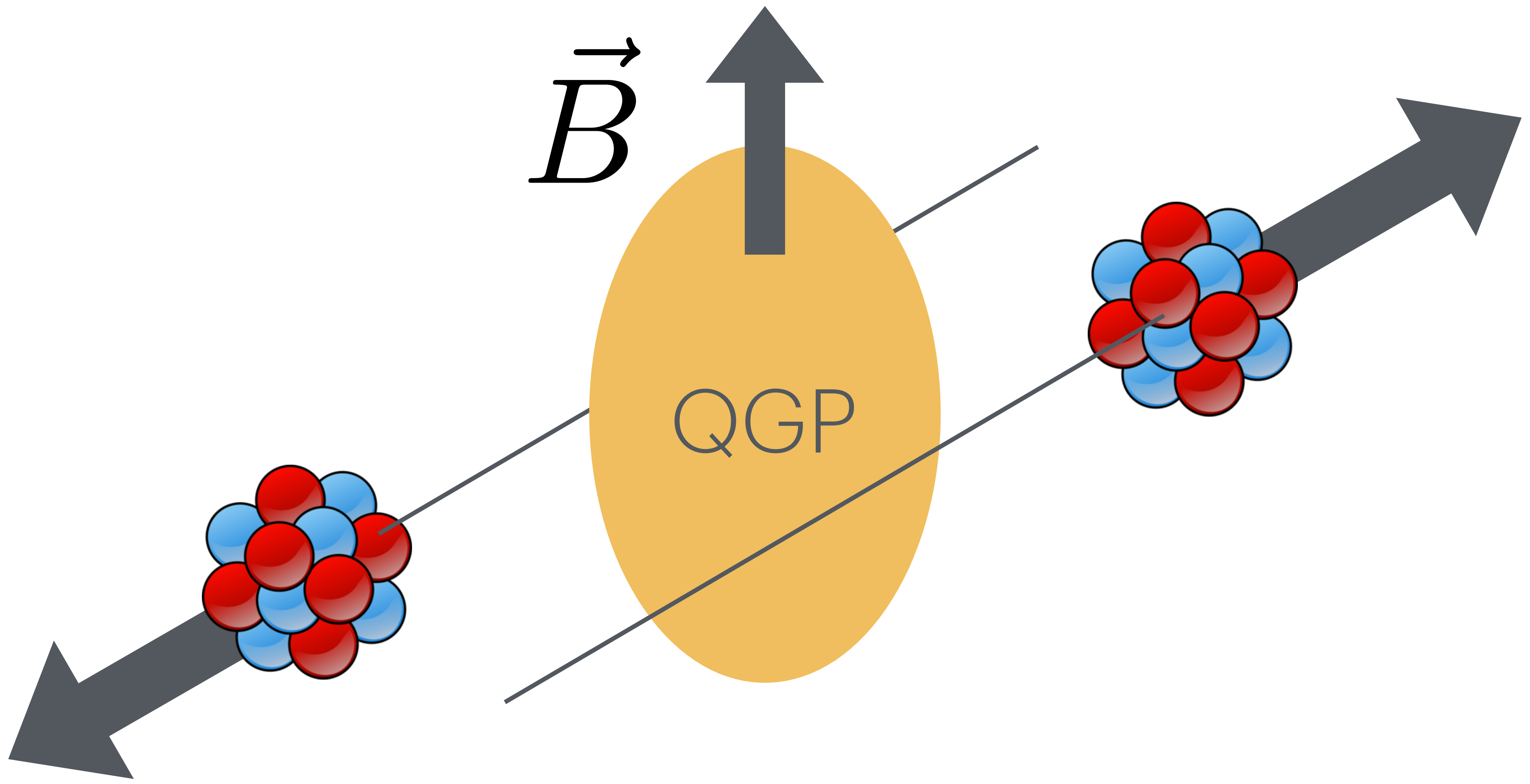
$$1.6 \times 10^{13}\, G \approx 0.3\,\mathrm{MeV}^2$$

- Non-central heavy ion collisions produce magnetic fields $eB \lesssim m_\pi^2$ *RHIC* $eB \lesssim 15 m_\pi^2$ *LHC*

[Skokov, Illarionov, Toneev, Int. J. Mod. Phys. A 24 (2009)]

... Experiments starting to probe imprints of EM-fields on charged particles emitted in HICs

[STAR Collab., Phys. Rev. X 14 (2024)]



$$eB \sim \Lambda_{\mathrm{QCD}}^2$$

Strong interaction modified

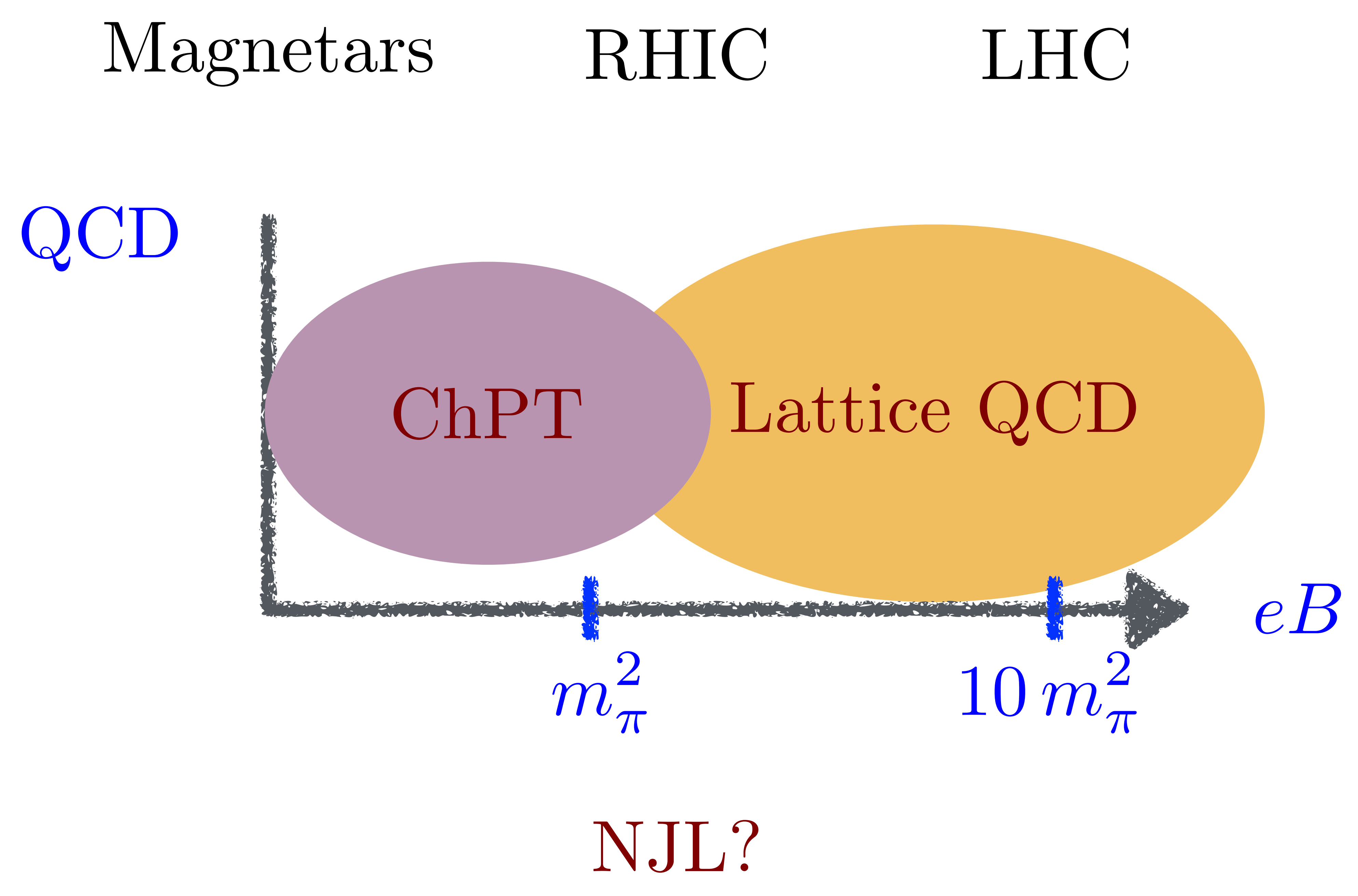
Overview

Chiral Symmetry and Large Magnetic Fields

QCD + B: Models (*NJL*, ...) useful for intuition, but lattice QCD can address: spectrum, phases

Chiral Perturbation Theory
provides constraints (low-energy theorems)

- Systematic inclusion of external magnetic fields
- Magnetic masses (confront NJL model)
- Pion axial and vector transitions (confront models & lattice)
- Finite-volume effects (confront lattice artifacts)

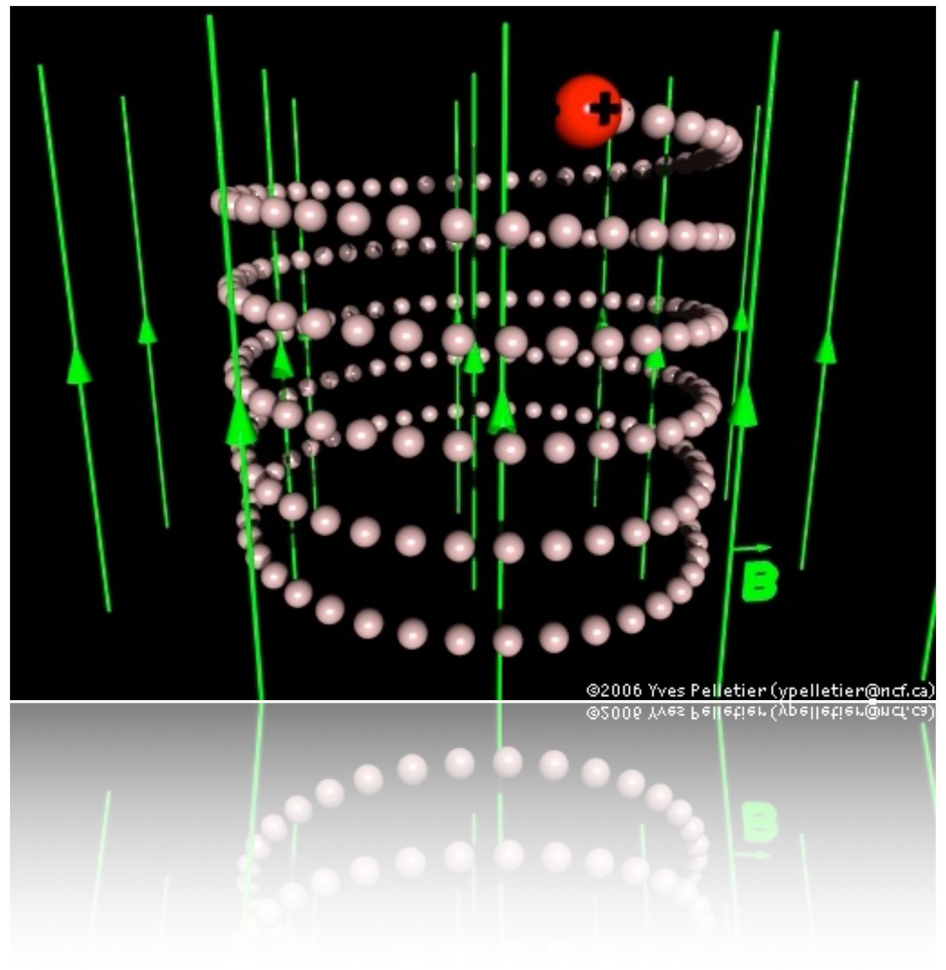


Chiral Perturbation Theory and Large Magnetic Fields

- Pattern of spontaneous and explicit chiral symmetry breaking modified:

$$
\begin{array}{lccc}
B = 0 & SU(2)_L \times SU(2)_R & \longrightarrow & SU(2)_V \\
 & \downarrow & & \downarrow \\
B \neq 0 & U(1)_R \times U(1)_L & \longrightarrow & U(1)_V
\end{array}
$$

$$
SO(3,1) \xrightarrow{\vec{B}} SO(2)_{\perp} \times SO(1,1)_{\parallel}
$$



- Effective theory of low-energy QCD

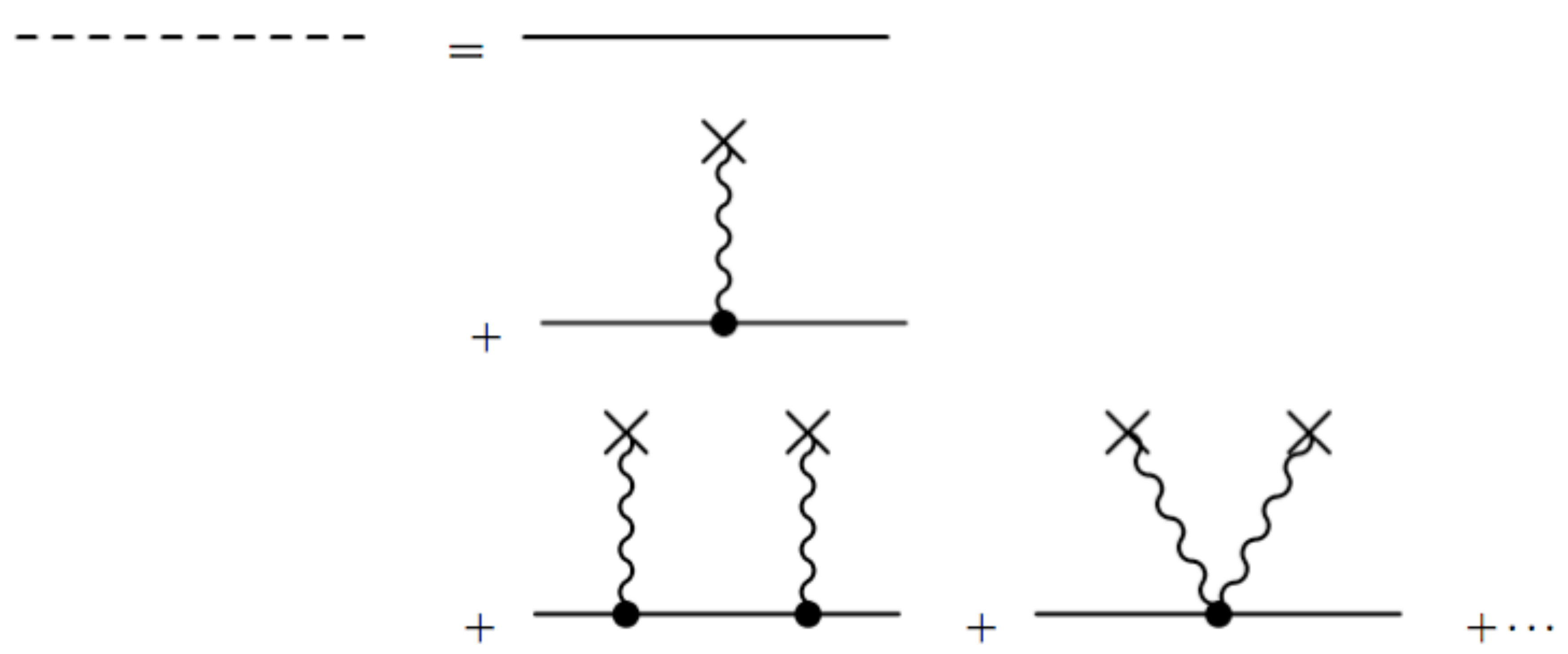
$$
\mathcal{L}_2 = \frac{F^2}{4} \left\langle \mathcal{D}^\mu U^\dagger D_\mu U + U^\dagger \chi + \chi^\dagger U \right\rangle
$$

- Power counting needed for renormalization

$$
p^2 \sim m_\pi^2 \sim eB \ll (4\pi F)^2 \approx 1.1\,\mathrm{GeV}^2
$$

- Allows for magnetic fields $eB/m_\pi^2 \sim 1$

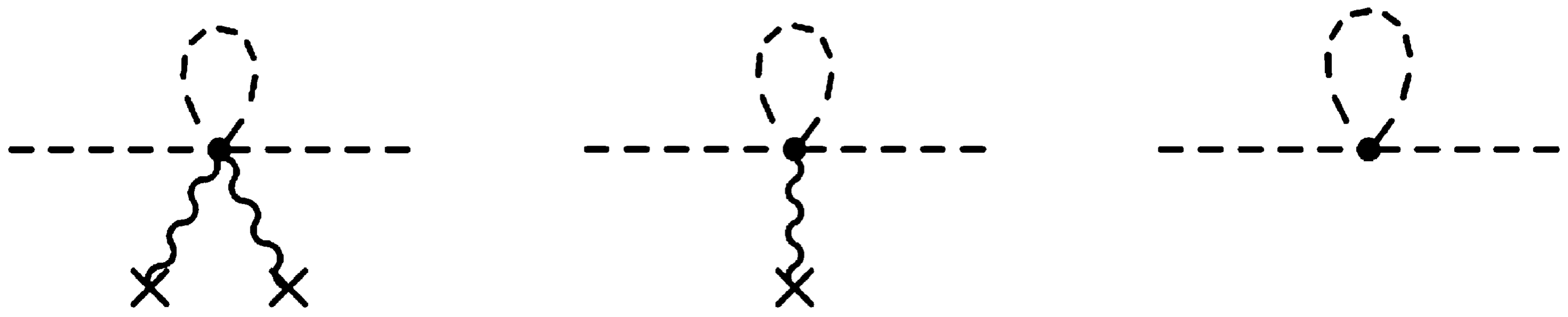
- Leading-order charged pion propagator…



… can be solved in closed form for uniform B, e.g.

Magnetic Masses of Pions in Chiral Perturbation Theory

- Neutral pion magnetic mass [Tiburzi, Nucl. Phys. A 814 (2008)]



$$m_{\pi^0}^2(B) = m_\pi^2\left[1 + \frac{2eB}{(4\pi F)^2}\log\Gamma\left(\frac{1}{2} + \frac{m_\pi^2}{2eB}\right) + \cdots\right]$$

Chiral correction

eB/m_π^2 treated to all orders

- Small-field limit $m_{\pi^0}(B) = m_\pi - \frac{1}{2}\beta_M B^2 + \cdots$

$$\beta_M = \frac{e^2}{6m_\pi(4\pi F)^2}$$

magnetic polarizability

- NJL Model *implies* [Coppola, Gómez Dumm, Scoccola, Phys, Lett. B 782 (2018)]

$$(\beta_M)_{\mathrm{NJL}}/\beta_M = (F/M)^2\frac{20/9}{-I_2(\Lambda, M)} \approx \frac{1}{3}$$

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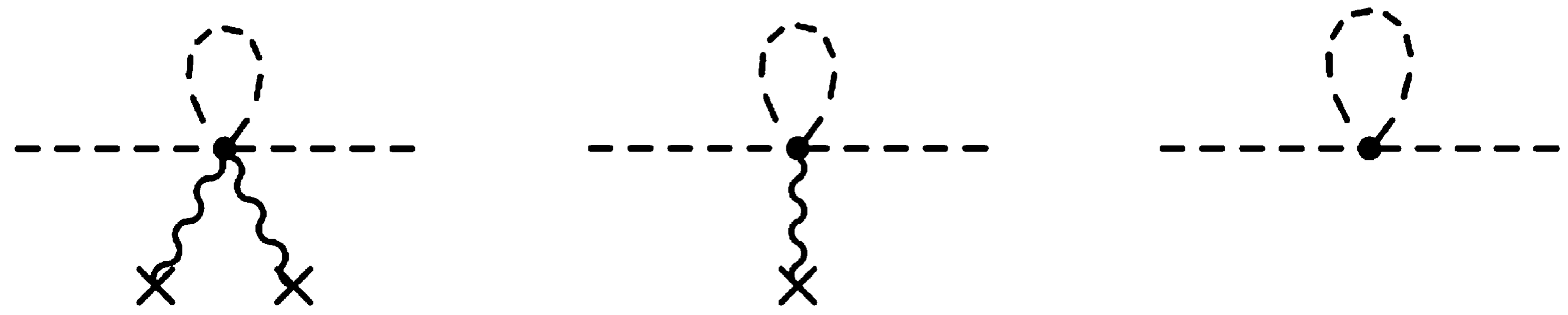

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magnetic polarizability

- Charged pion magnetic mass @NLO

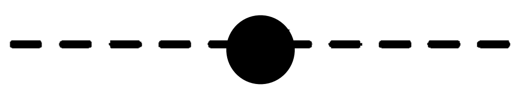
$$E_{\pi^-}^2 = m_{\pi^-}^2(B) + (2n+1)eB + p_\parallel^2$$

In net, only neutral pion loops contribute

$$m_{\pi^-}^2 = m_\pi^2 + \bar{\ell}\left(\frac{eB}{4\pi F}\right)^2$$

$$\bar{\ell} = 1.0 \pm 0.1$$

$\exists$ local operators @NLO, e.g.



$$\mathcal{L}_4 \supset \langle U^\dagger F_{\mu\nu} U F^{\mu\nu} \rangle$$

$$\beta_M = -\frac{e^2\,\bar{\ell}}{m_\pi(4\pi F)^2}$$

- NJL Model *implies* [Coppola, Gómez Dumm, Scoccola, Phys, Lett. B 782 (2018)]

$$(\beta_M)_{\mathrm{NJL}}/\beta_M = (F/M)^2 \frac{20/9}{-I_2(\Lambda, M)} \approx \frac{1}{3}$$

$$(\beta_{\mathrm{NJL}})/\beta_M = (F/M)^2 \frac{17/27}{-I_2(\Lambda, M)} \approx -\frac{1}{10}$$

Axial-Vector and Vector Pion Transition Matrix Elements

[Adhikari & Tiburzi, *arXiv*:2406.00818]

- Motivated by pioneering lattice QCD study of weak pion decay in B

[Bali, *et al.*, Phys. Rev. Lett. 121 (2018)]

Qualitative agreement

$\Gamma(\pi^- \to \mu \overline{\nu}_\mu)$ = larger that B=0 width, due to available energy

$\Gamma(\pi^- \to e \overline{\nu}_e)$ = now same order of magnitude, due to overcoming helicity suppression

- General parameterization of transition matrix elements

[Coppola, *et al.*, Phys. Rev. D 99 (2019)]

In chiral perturbation theory

$$J_A^{\mu -} = - \left[F_{\pi^-}^{(A1)} D^\mu + ie F_{\pi^-}^{(A2)} F^{\mu\nu} D_\nu + e^2 F_{\pi^-}^{(A3)} F^{\mu\nu} F_{\nu\alpha} D^\alpha \right] \pi^-$$

survives B=0

local operators @NLO

same as mass due to chiral symmetry

requires NNLO

$$\overline{\ell} = 1.0 \pm 0.1$$

$$F_{\pi^-}^{(A2)} = \frac{\overline{\ell}}{(4\pi)^2 F_\pi}$$

Compare with B=0

$$J_A^{\mu -} = -F_\pi \, \partial^\mu \pi^-$$

NJL Model Study

[Coppola, *et al.*, Phys. Rev. D 100 (2019)]

$$\left(F_{\pi^-}^{(A2)} \right)_{\text{NJL}} \Big/ F_{\pi^-}^{(A2)} \approx 2$$

Inconsistent with pion mass!

Axial-Vector and Vector Pion Transition Matrix Elements

[Adhikari & Tiburzi, *arXiv*:2406.00818]

$$F_{\pi^-}^{(A1)} = F_\pi \left[1 - \frac{eB}{(4\pi F_\pi)^2} \log\Gamma\left(\frac{1}{2} + \frac{m_\pi^2}{2eB}\right) + \cdots\right] > F_\pi$$

E.g. $eB/m_\pi^2 \ll 1$

$$F_{\pi^-}^{(A1)} = F_\pi \left[1 + \frac{1}{3}\left(\frac{eB}{8\pi m_\pi F_\pi}\right)^2\right]$$

In chiral perturbation theory

$$J_A^{\mu -} = -\left[F_{\pi^-}^{(A1)} D^\mu + ieF_{\pi^-}^{(A2)} F^{\mu\nu} D_\nu + e^2 F_{\pi^-}^{(A3)} F^{\mu\nu} F_{\nu\alpha} D^\alpha\right] \pi^-$$

Compare with B=0

$$J_A^{\mu -} = -F_\pi \, \partial^\mu \pi^-$$

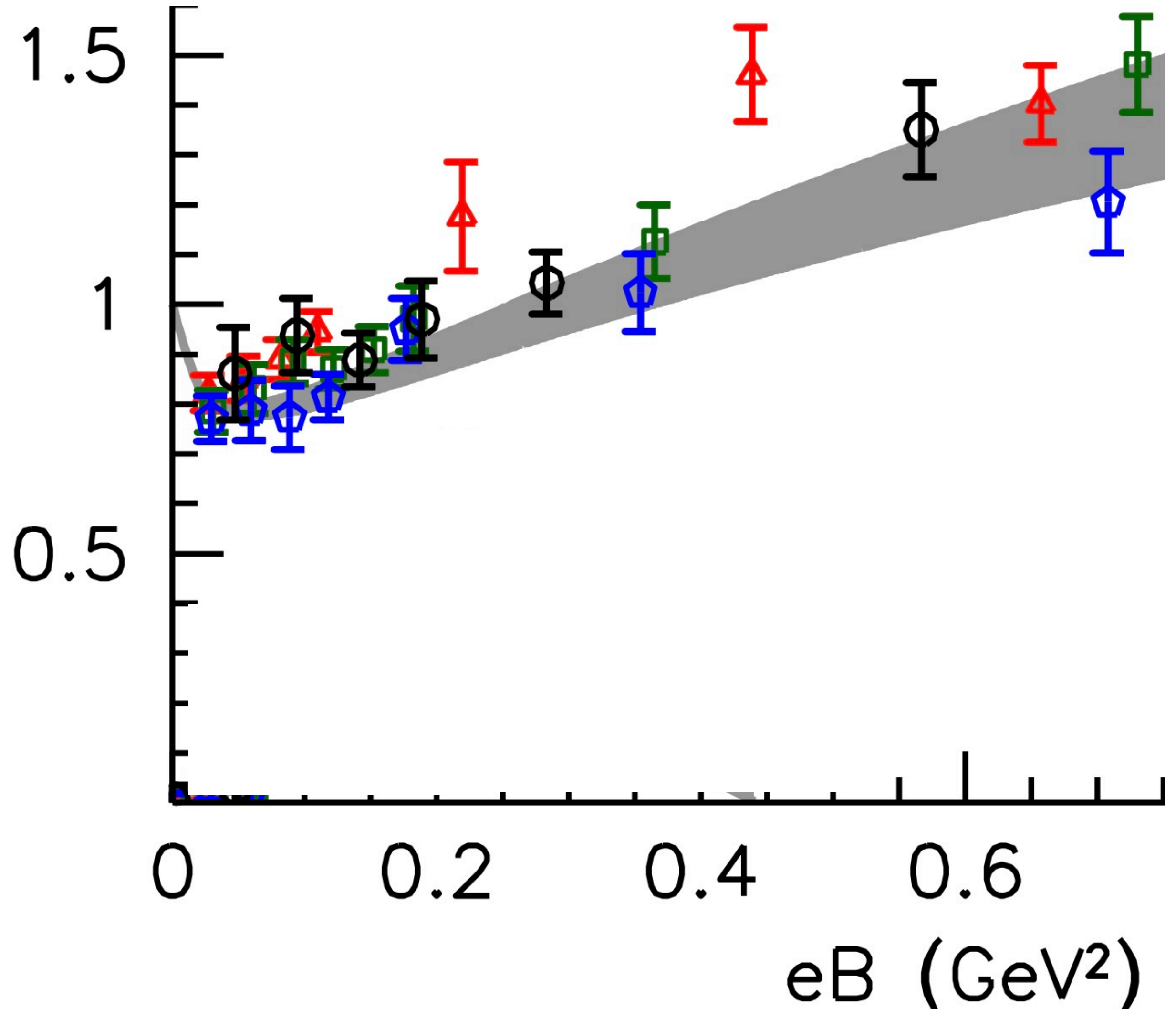
∃ Lattice Data

[Bali, *et al.*, Phys. Rev. Lett. 121 (2018)]

$$\left(F_{\pi^-}^{(A1)}\right)_{\text{lattice}} \Big/ F_\pi < 1$$

$$\sim 1 - eB\,\mathcal{C}$$

for smallest B



Axial-Vector and Vector Pion Transition Matrix Elements

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$\exists$ Lattice Data

Chiral anomaly is the source

Inferred from NJL model

[Coppola, *et al.*, Phys. Rev. D 100 (2019)]

ChPT w/ WZW term

$$F^{(V)}_{\pi^-} = \frac{2}{(4\pi)^2 F_\pi}$$

$$J^{\mu -}_V = -eF^{(V)}_{\pi^-} \widetilde{F}^{\mu\nu} D_\nu \pi^-$$

$$J^{\mu -}_V = 0$$

Single-pion vector and axial-vector currents

$$J^{\mu -}_A = -\left[F^{(A1)}_{\pi^-} D^\mu + ieF^{(A2)}_{\pi^-} F^{\mu\nu} D_\nu + e^2 F^{(A3)}_{\pi^-} F^{\mu\nu} F_{\nu\alpha} D^\alpha \right] \pi^-$$

Compare with B=0

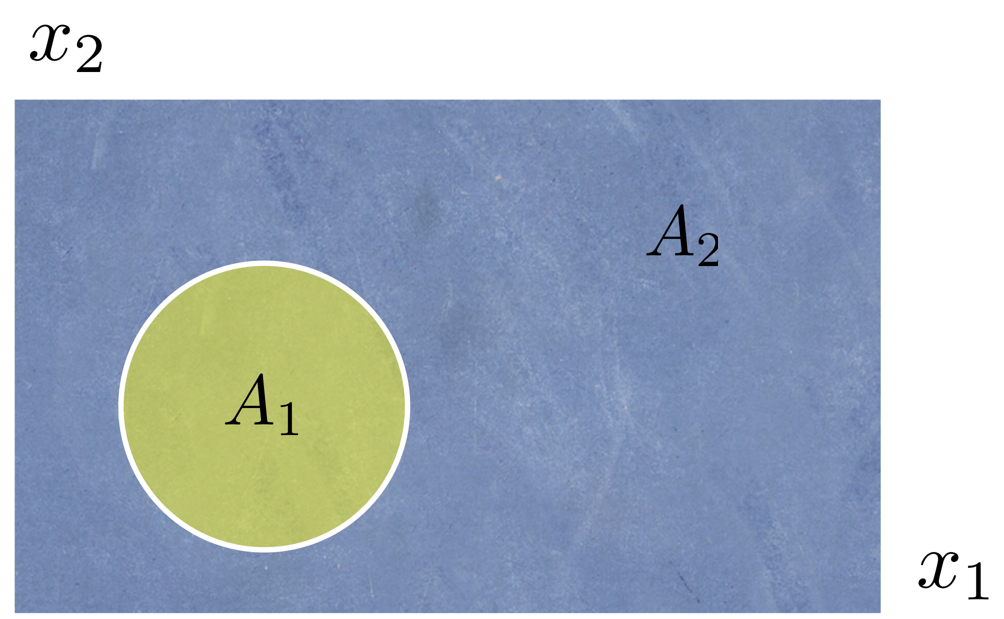
$$J^{\mu -}_A = -F_\pi \, \partial^\mu \pi^-$$

$$\frac{F^{(V)}_{\pi^\pm}(B{=}0)}{F_\pi} = \begin{cases} 1.2(3)\,\texttt{GeV}^{-2} & \text{Wilson quarks: quenched, } m_\pi = 415\,\texttt{MeV} \\ 0.8(2)\,\texttt{GeV}^{-2} & \text{Staggered quarks: fully dynamical, } m_\pi = 135\,\texttt{MeV} \\ 1.49(3)\,\texttt{GeV}^{-2} & \text{Anomaly} \end{cases}$$

Finite-Volume Effects for Lattice QCD from ChPT

[Adhikari & Tiburzi, Phys. Rev. D 107 (2023)]

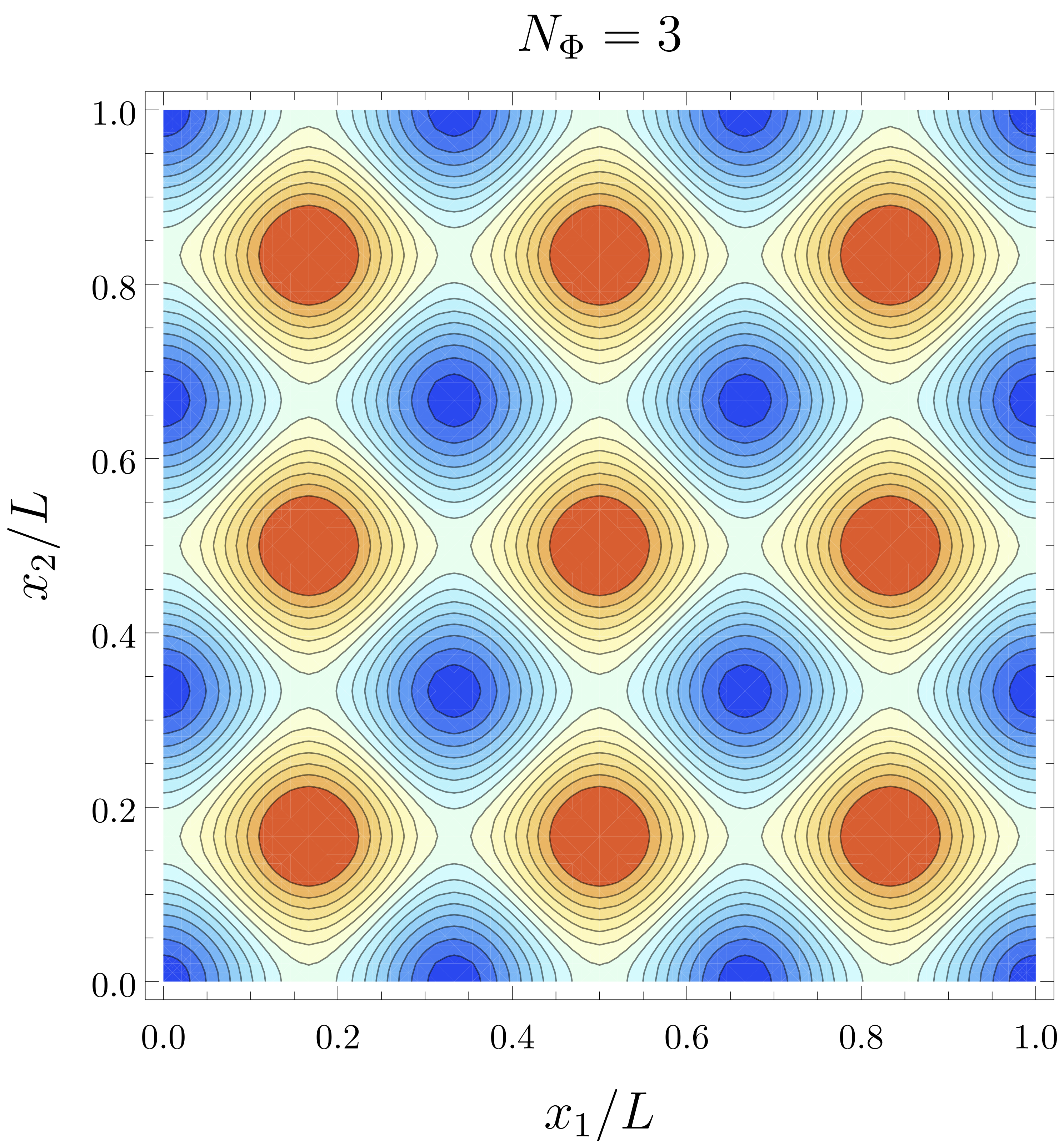
- Uniform B fields on torus require quantization $Q\,eB = \frac{2\pi}{L^2} n_\Phi$
- Charged particles ***not*** periodic, but periodic up to gauge
- Infinite degeneracy of Landau levels broken to $\mathbb{Z}_{n_\Phi}$ remnant translational invariance



Consequences

Finite-Volume Effect on Chiral Condensate

$$\langle \overline{\psi}(x)\psi(x) \rangle$$



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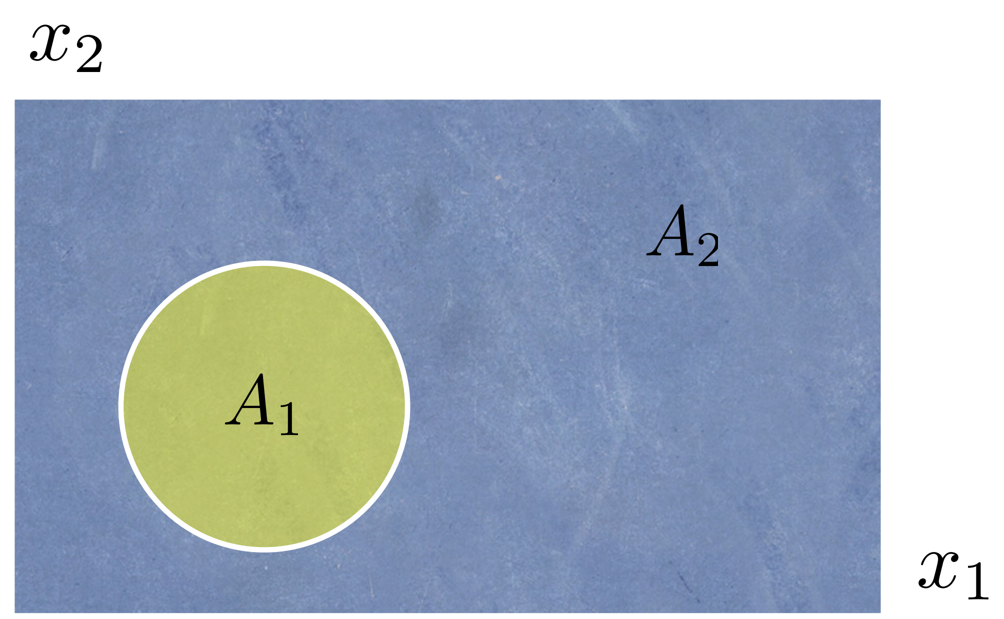


Consequences

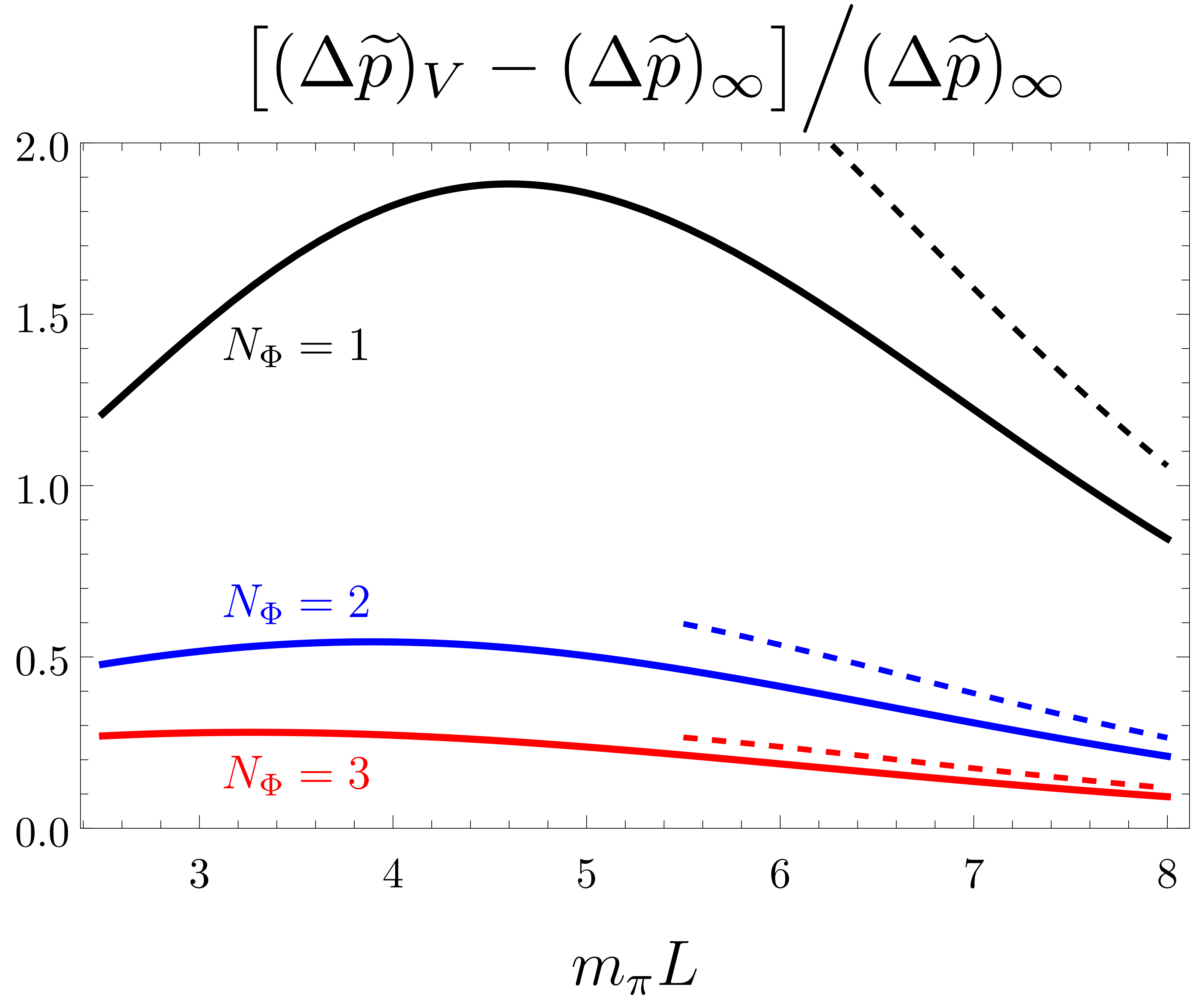
Magnetization strictly inaccessible $\mathcal{M} = -\left(\frac{\partial \mathcal{F}}{\partial B}\right)_V$

Ingenious workaround: magnetic pressure at fixed flux

[Bali, *et al.*, JHEP 04 (2013)]

$$\widetilde{p}_i = -\frac{L_i}{V}\left(\frac{\partial \mathcal{F}}{\partial L_i}\right)_{L_j, n_\Phi}$$

$$\Delta\widetilde{p} \equiv \widetilde{p}_\perp - \widetilde{p}_\parallel = -B\,\mathcal{M} + \cdots$$



Pressure anisotropy exploits FV effect to obtain magnetization ...

... but can be subject to large FV correction

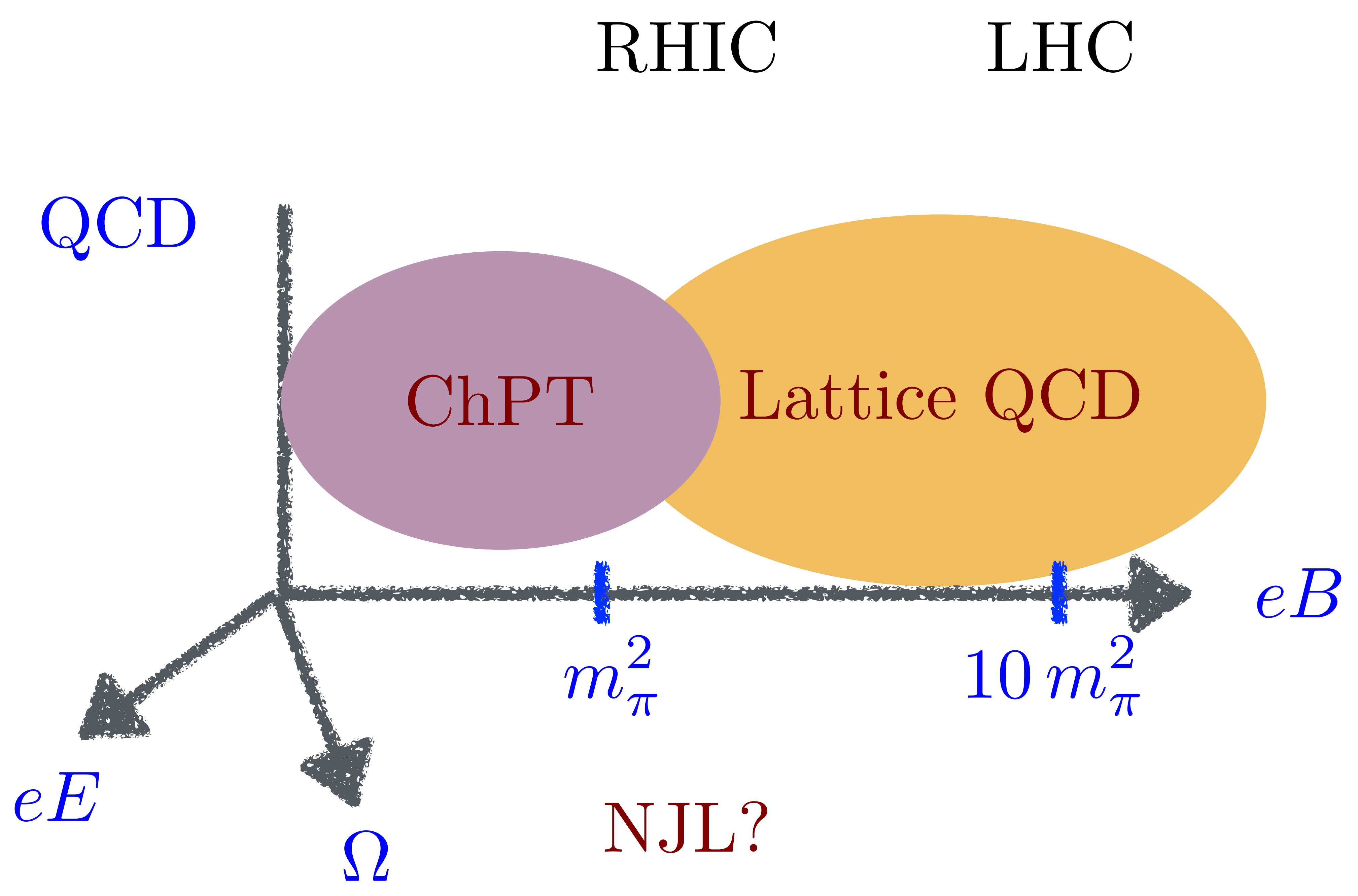
Summary

Chiral Symmetry and Large Magnetic Fields

Chiral Perturbation Theory

Low-energy theorems for QCD + B

- Magnetic polarizabilities of charged and neutral pions predicted from QCD by chiral symmetry not reproduced by NJL model
- Pion vector transition in magnetic field arises due to chiral anomaly: captured correctly in NJL model, tension with staggered lattice QCD result
- Pion axial-vector transition: NJL is discrepant, behavior is even qualitatively different than in lattice QCD



Models adaptable, trustworthy?

Dedicated lattice calculations resolve?

Other axes

Lattice sign problem