



Beyond the N_1 -dominated leptogenesis with the smallest modular finite group

Minimal seesaw and leptogenesis with the smallest modular finite group
JHEP 05 (2024), 020

S. Marciano

D. Meloni

M. Parriciatu



Prague, 19/07/2024

Outline

- Motivations
- The smallest modular finite group
- Leptogenesis: N_1 -dominated scenario
- Results
- Conclusions and comments



Neutrino masses: who ordered that?

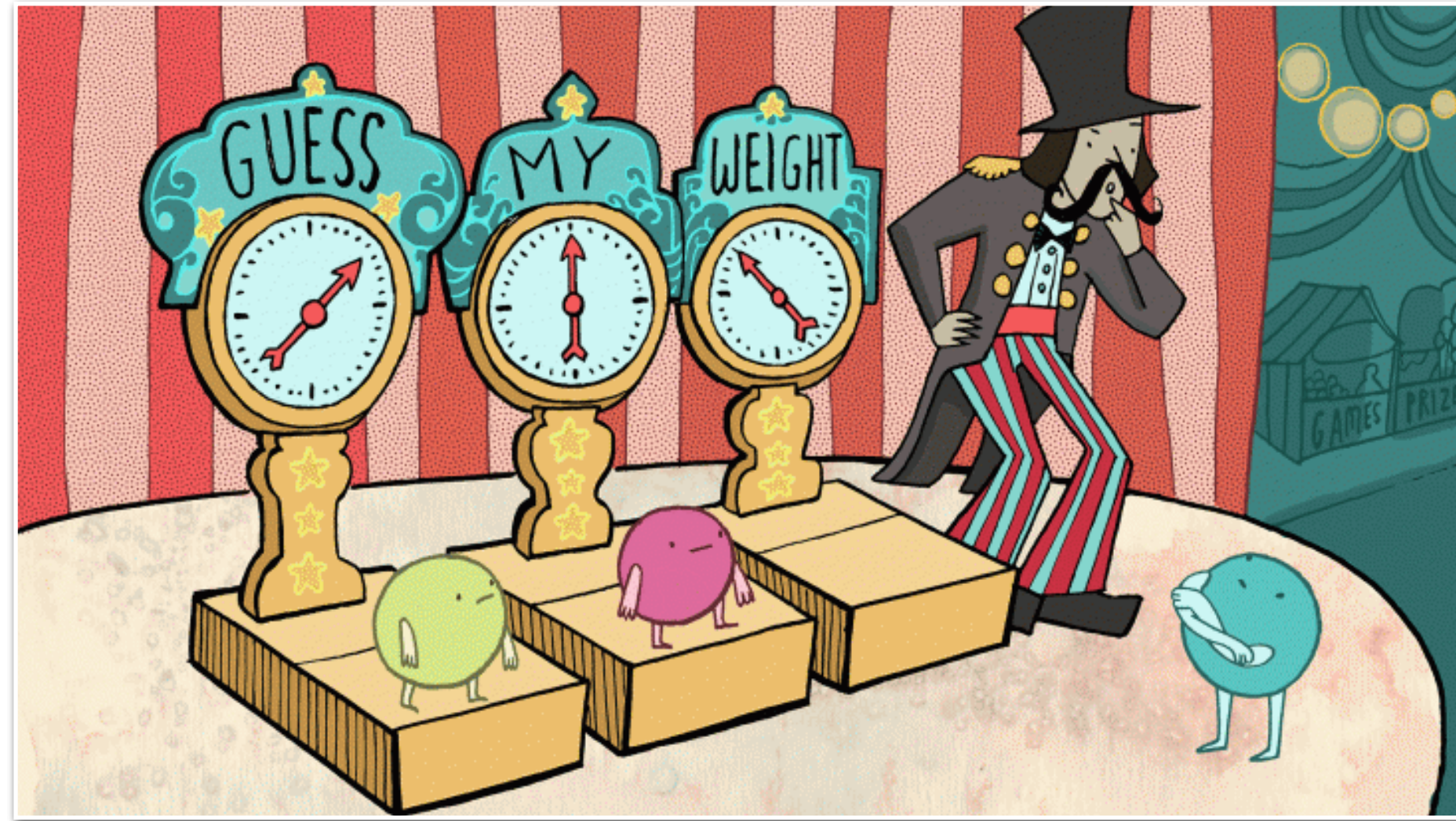
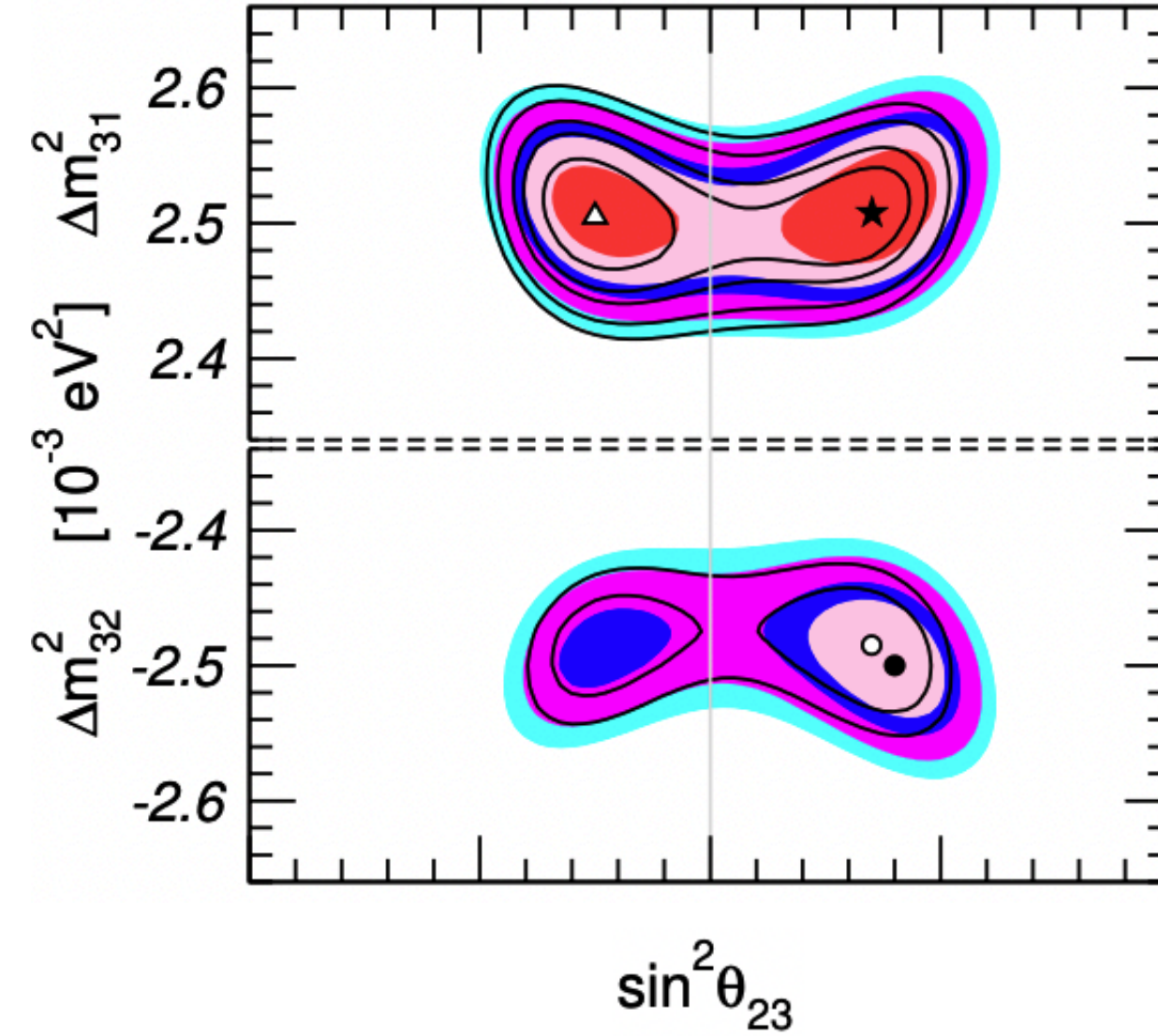
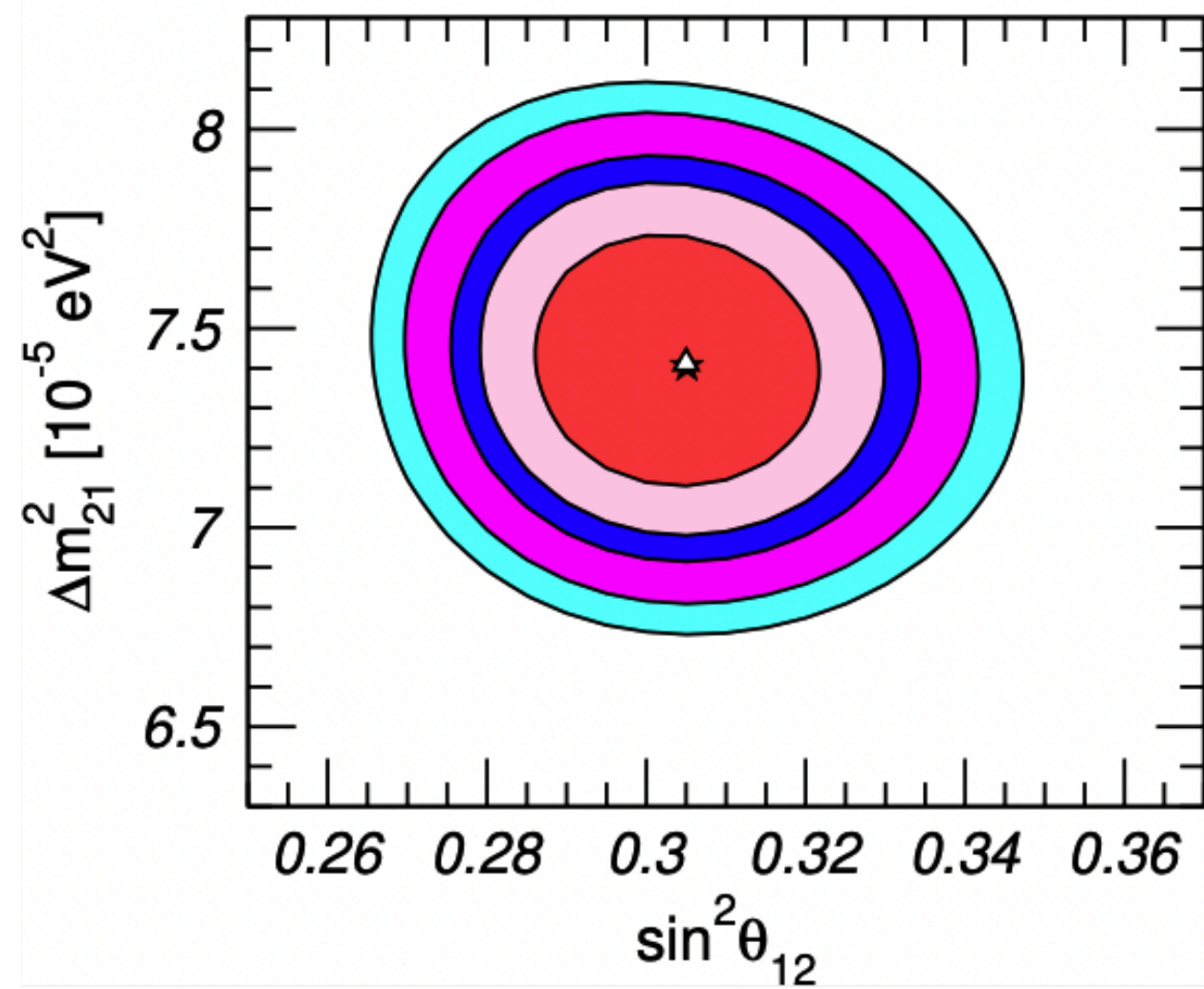


Illustration by Sandbox Studio, Chicago

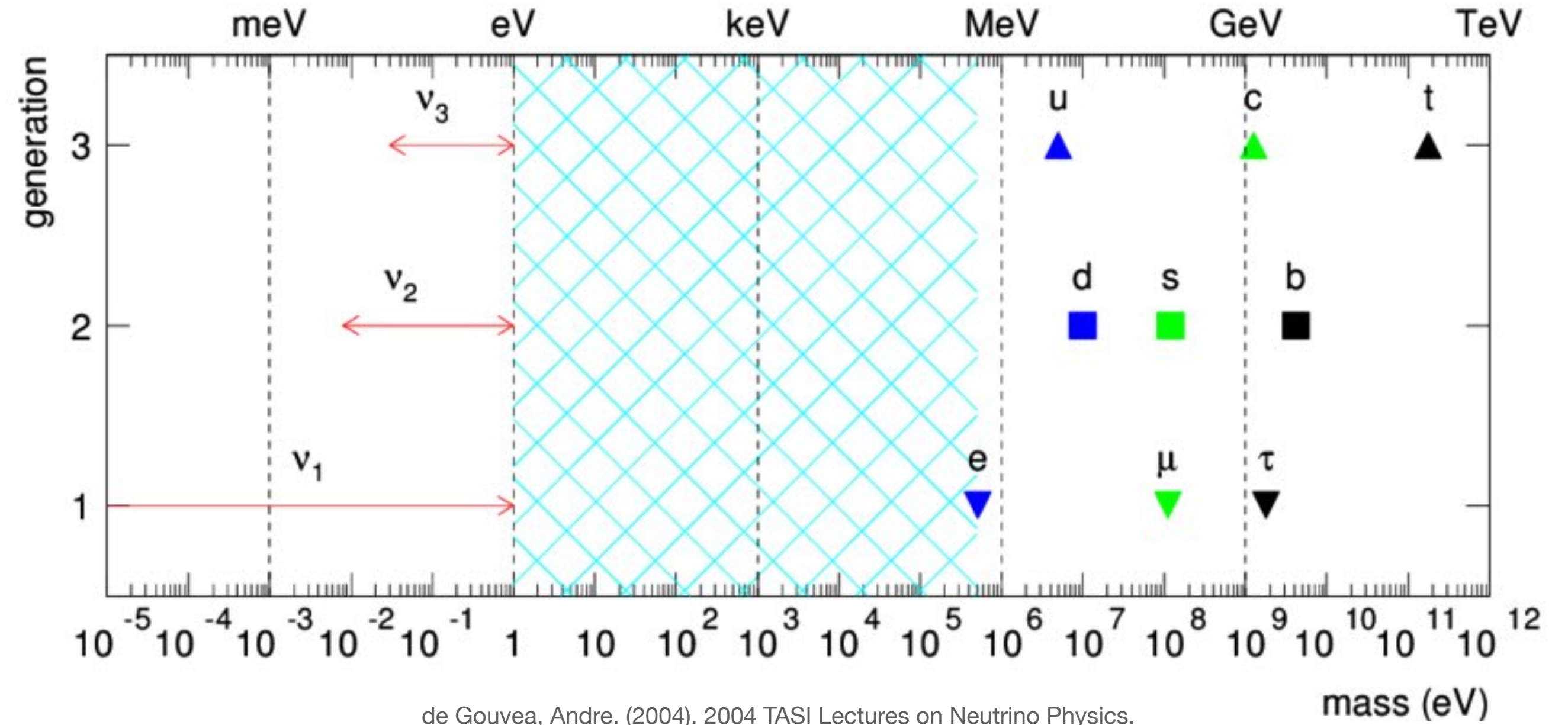
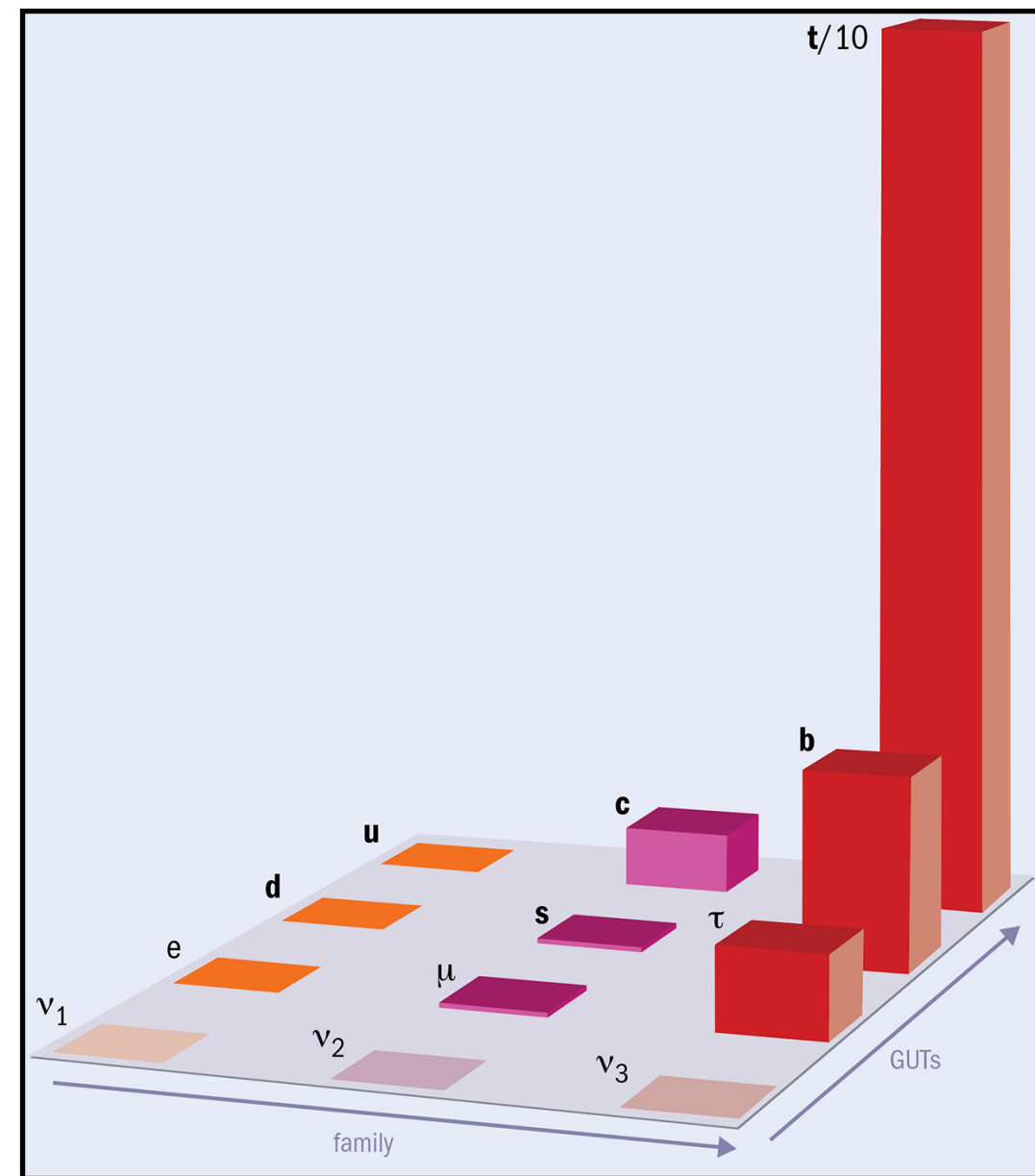


		NuFIT 5.2 (2022)			
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2 \theta_{12}$	$0.303^{+0.012}_{-0.011}$	$0.270 \rightarrow 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \rightarrow 0.341$
	$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \rightarrow 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \rightarrow 35.74$
	$\sin^2 \theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.406 \rightarrow 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \rightarrow 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \rightarrow 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \rightarrow 52.1$
	$\sin^2 \theta_{13}$	$0.02203^{+0.00056}_{-0.00059}$	$0.02029 \rightarrow 0.02391$	$0.02219^{+0.00060}_{-0.00057}$	$0.02047 \rightarrow 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	$8.19 \rightarrow 8.89$	$8.57^{+0.12}_{-0.11}$	$8.23 \rightarrow 8.90$
	$\delta_{CP}/^\circ$	197^{+42}_{-25}	$108 \rightarrow 404$	286^{+27}_{-32}	$192 \rightarrow 360$
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.03$
	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.511^{+0.028}_{-0.027}$	$+2.428 \rightarrow +2.597$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \rightarrow -2.408$
	with SK atmospheric data	$\sin^2 \theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \rightarrow 0.341$	$0.303^{+0.012}_{-0.011}$
$\theta_{12}/^\circ$		$33.41^{+0.75}_{-0.72}$	$31.31 \rightarrow 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \rightarrow 35.74$
$\sin^2 \theta_{23}$		$0.451^{+0.019}_{-0.016}$	$0.408 \rightarrow 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \rightarrow 0.613$
$\theta_{23}/^\circ$		$42.2^{+1.1}_{-0.9}$	$39.7 \rightarrow 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \rightarrow 51.5$
$\sin^2 \theta_{13}$		$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \rightarrow 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \rightarrow 0.02416$
$\theta_{13}/^\circ$		$8.58^{+0.11}_{-0.11}$	$8.23 \rightarrow 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \rightarrow 8.94$
$\delta_{CP}/^\circ$		232^{+36}_{-26}	$144 \rightarrow 350$	276^{+22}_{-29}	$194 \rightarrow 344$
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$		$7.41^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.03$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$		$+2.507^{+0.026}_{-0.027}$	$+2.427 \rightarrow +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \rightarrow -2.406$

JHEP 09 (2020) 178 [arXiv:2007.14792]

Neutrino masses and mixing

Mass hierarchies



de Gouvea, Andre. (2004). 2004 TASI Lectures on Neutrino Physics.

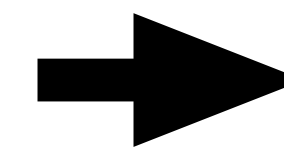
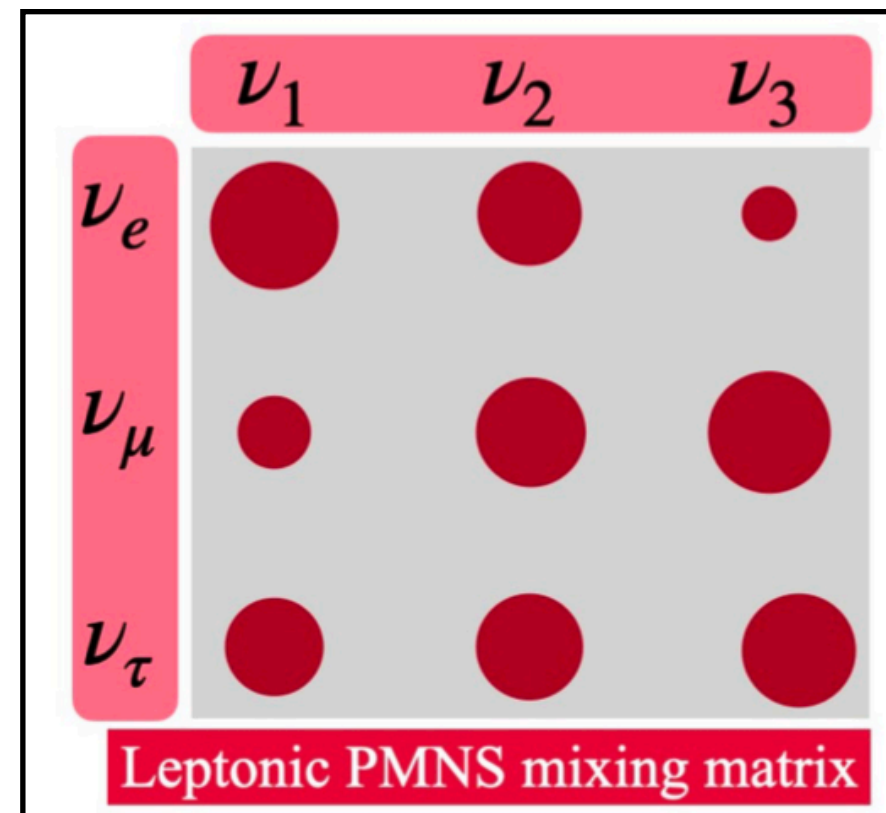
$$m_d \ll m_s \ll m_b, \quad \frac{m_d}{m_s} = 5.02 \times 10^{-2},$$

$$m_u \ll m_c \ll m_t, \quad \frac{m_u}{m_c} = 1.7 \times 10^{-3},$$

$$\frac{m_s}{m_b} = 2.22 \times 10^{-2}, \quad m_b = 4.18 \text{ GeV};$$

$$\frac{m_c}{m_t} = 7.3 \times 10^{-3}, \quad m_t = 172.9 \text{ GeV};$$

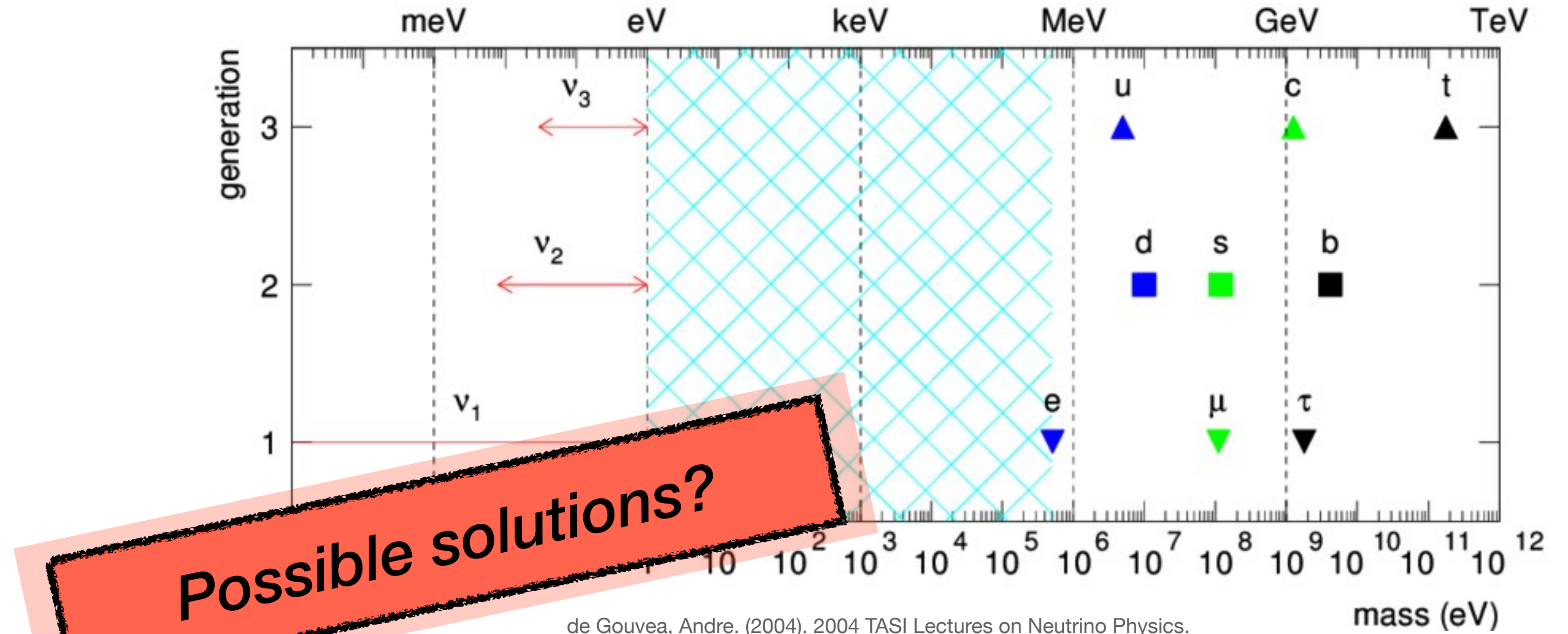
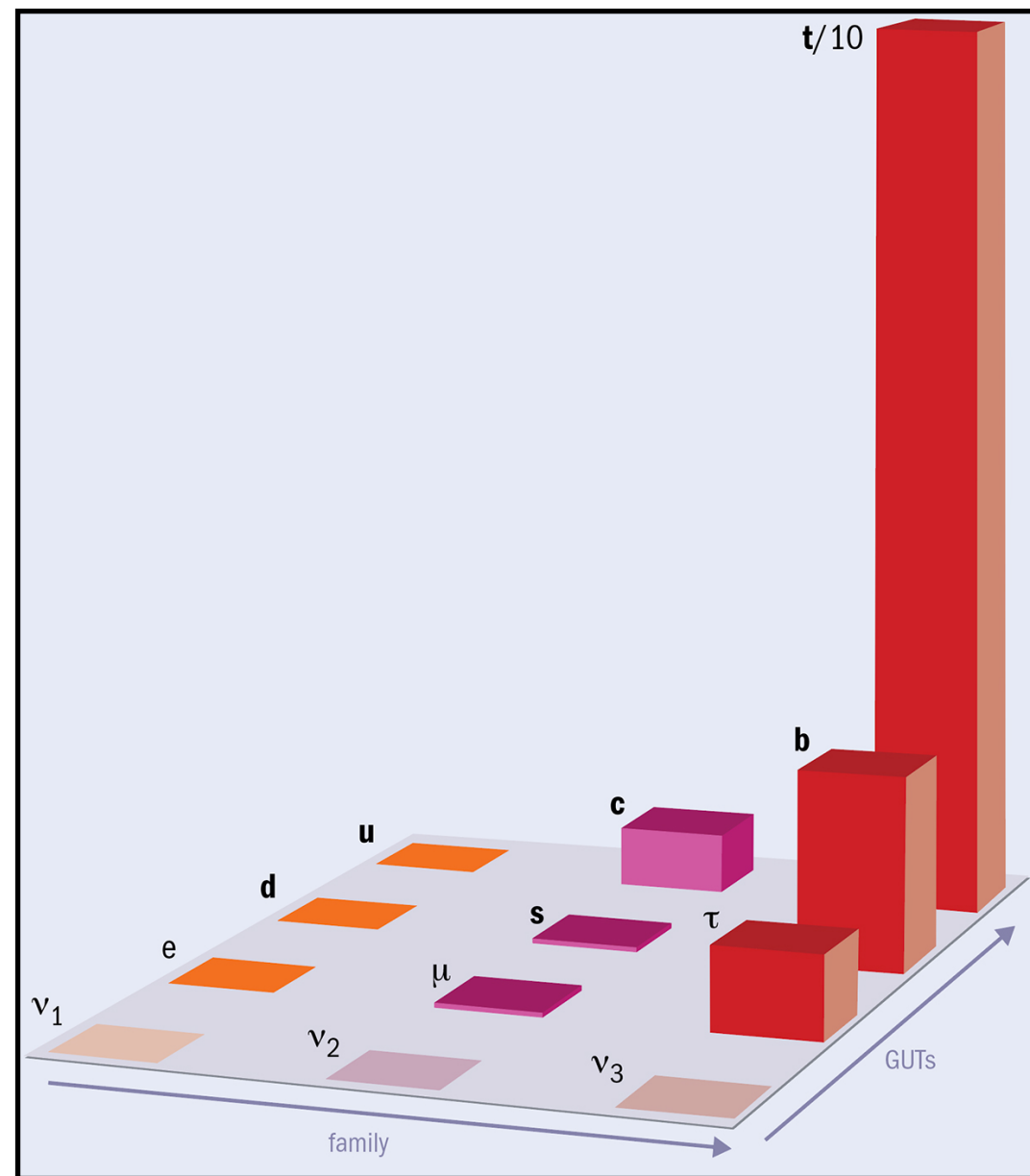
Fermion mixing



All mixing are large
but the 13 element

Neutrino masses and mixing

Mass hierarchies



de Gouvea, Andre. (2004). 2004 TASI Lectures on Neutrino Physics.

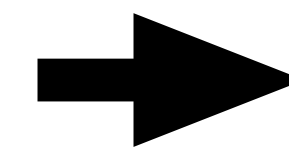
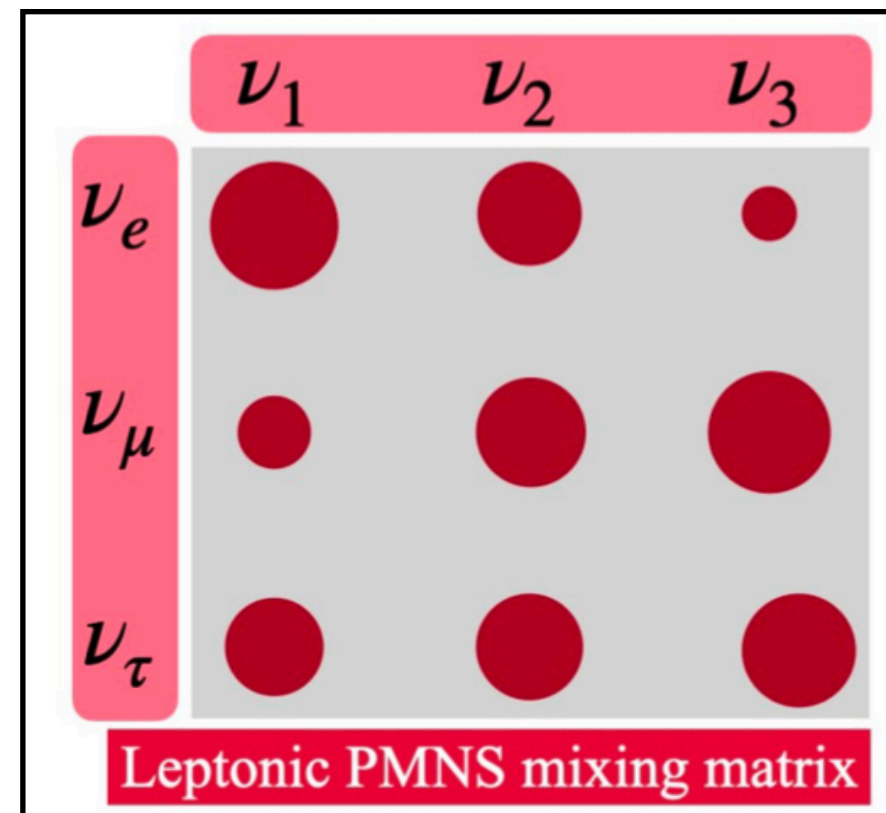
$$m_d \ll m_s \ll m_b, \quad \frac{m_d}{m_s} = 5.02 \times 10^{-2},$$

$$m_u \ll m_c \ll m_t, \quad \frac{m_u}{m_c} = 1.7 \times 10^{-3},$$

$$\frac{m_s}{m_b} = 2.22 \times 10^{-2}, \quad m_b = 4.18 \text{ GeV};$$

$$\frac{m_c}{m_t} = 7.3 \times 10^{-3}, \quad m_t = 172.9 \text{ GeV};$$

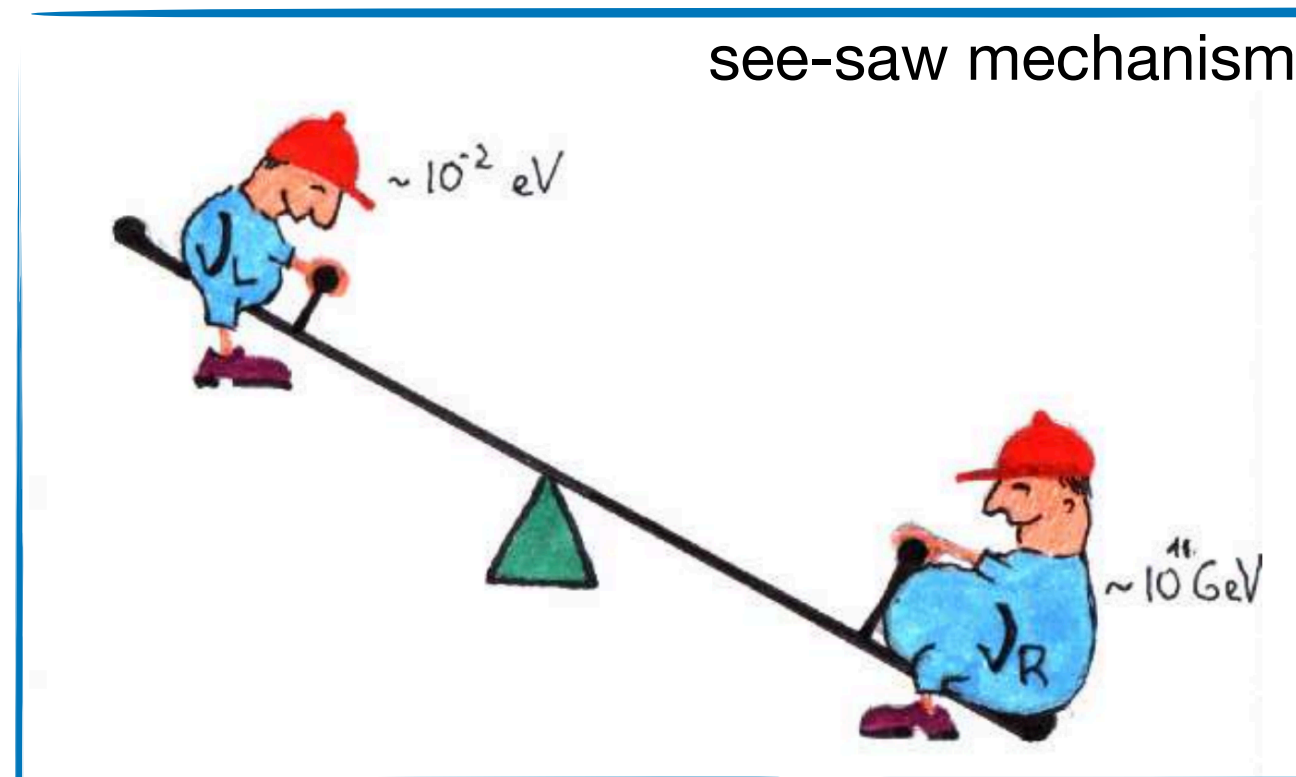
Fermion mixing



All mixing are large
but the 13 element

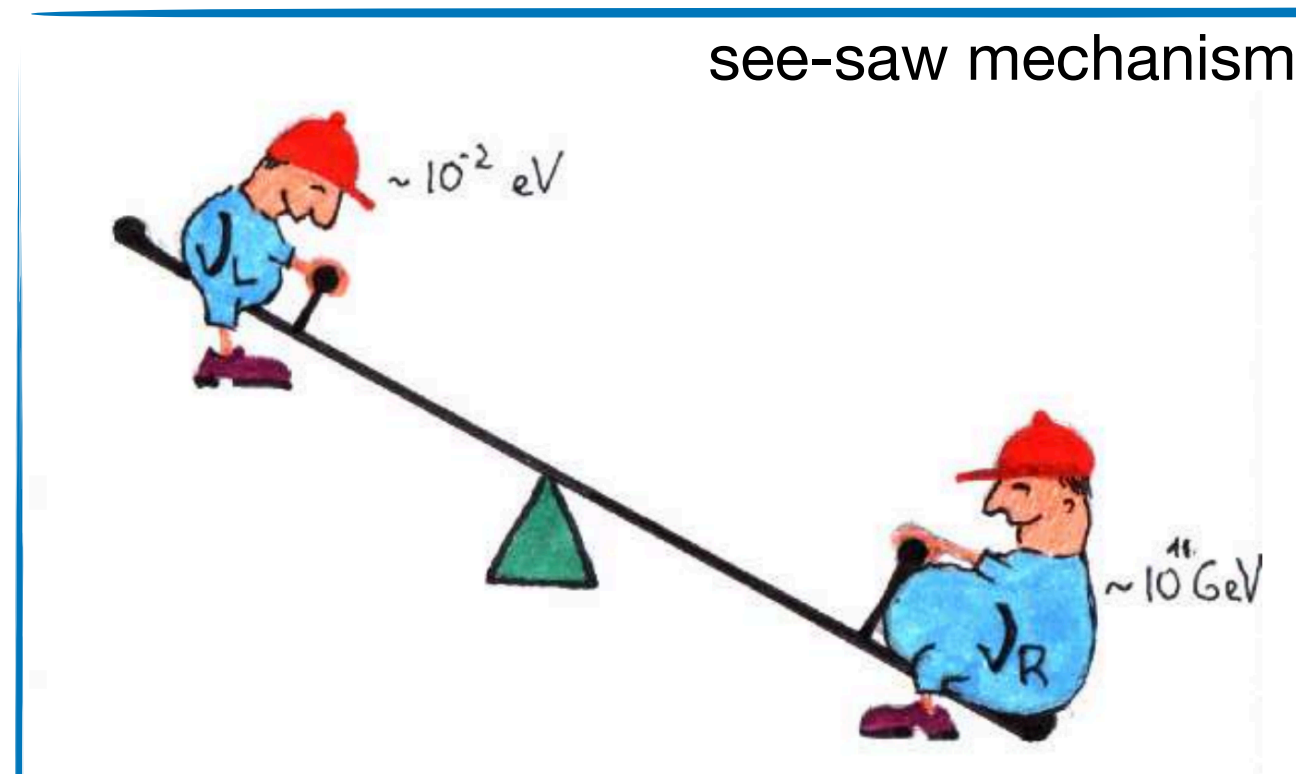
Suggested solutions

Smallness of neutrino masses



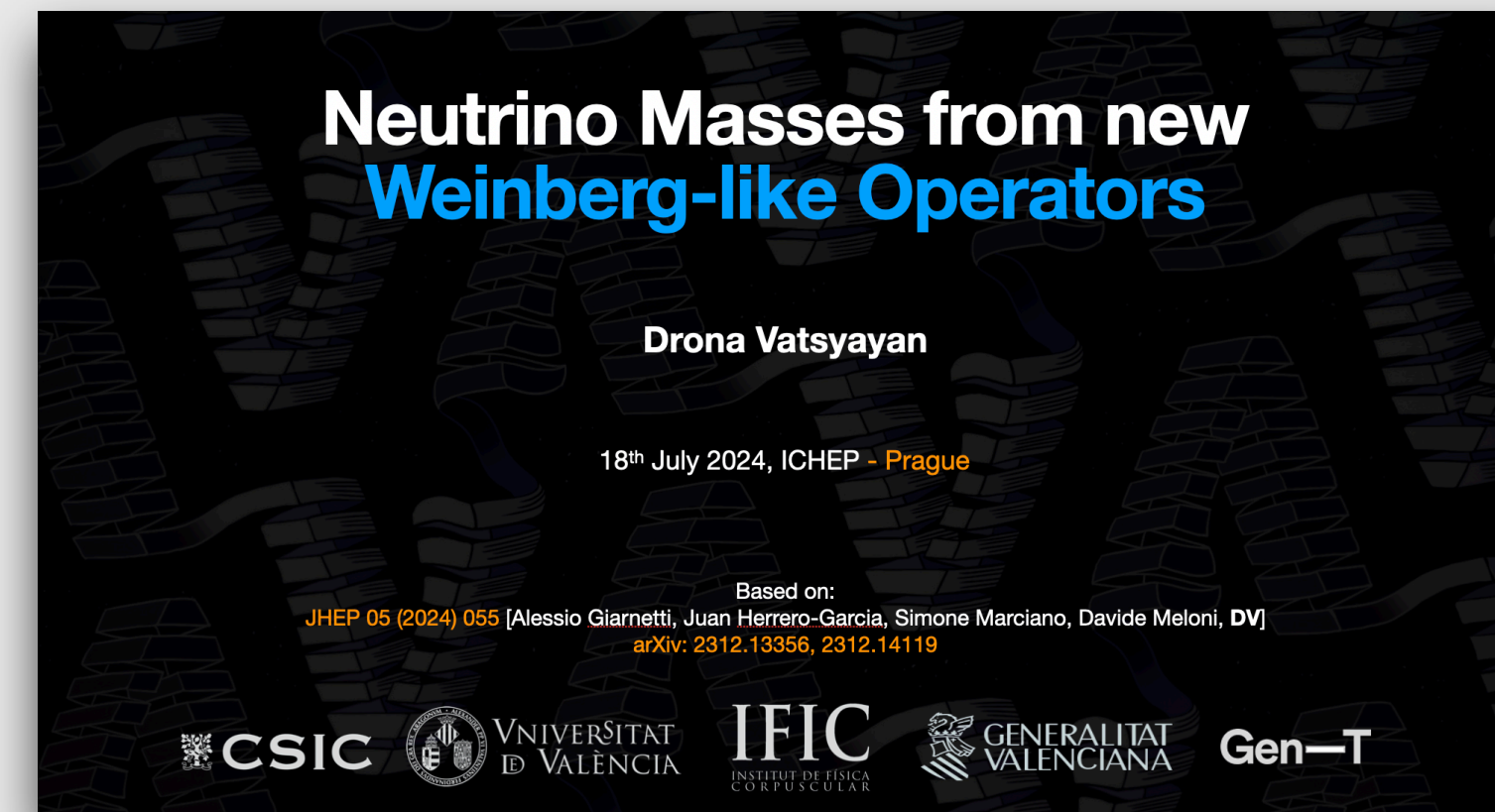
$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}$$
$$m_{\text{light}} \approx \frac{m_D^2}{M_R}$$

Smallness of neutrino masses



$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}$$
$$m_{\text{light}} \approx \frac{m_D^2}{M_R}$$

see also the talk by D. Vatsyayan on novel Weinberg-like operators

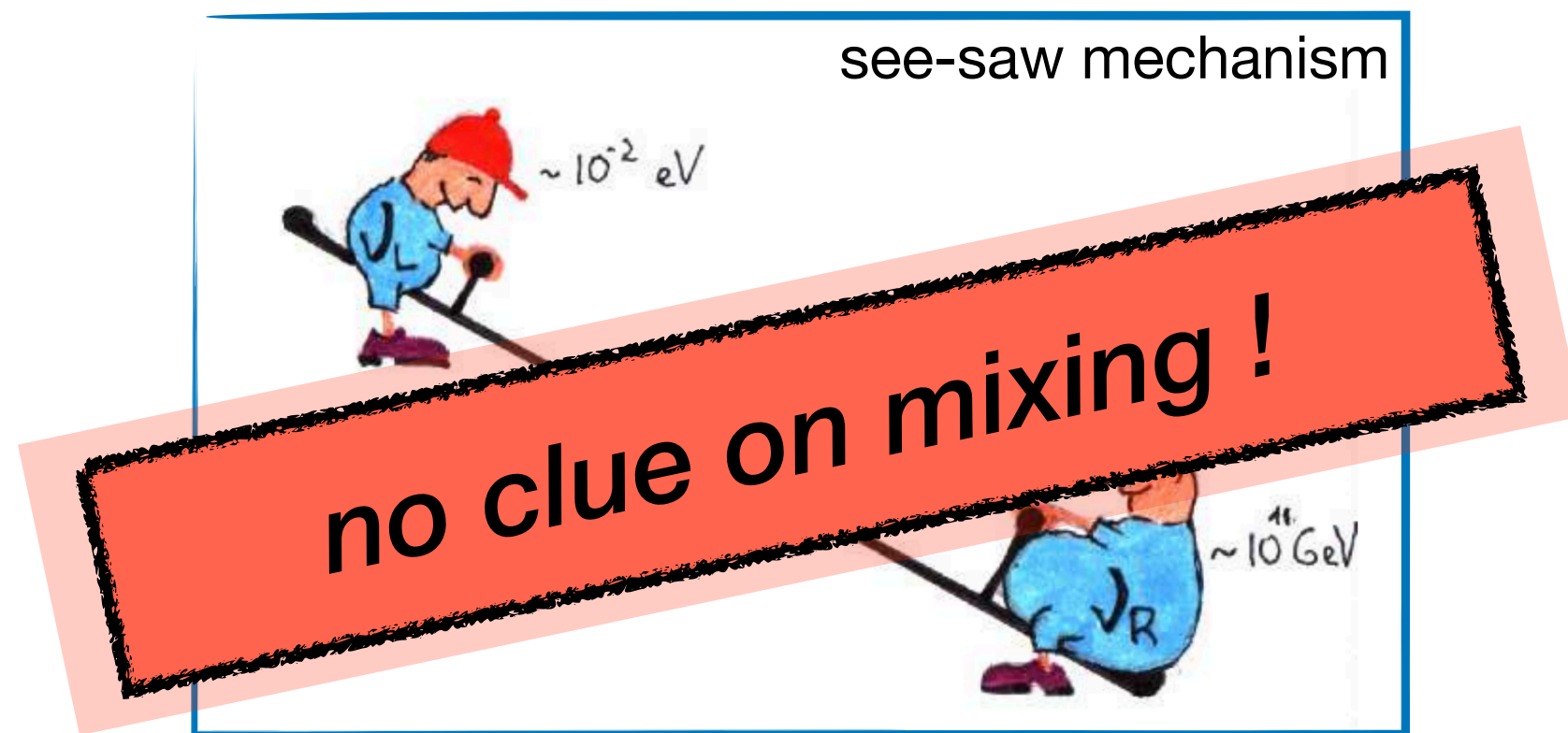


Neutrino masses from new seesaw models: Low-scale variants and phenomenological implications, [arXiv:2312.14119 [hep-ph]]

Neutrino masses from new Weinberg-like operators: phenomenology of TeV scalar multiplets JHEP 05 (2024), 055

A. Giarnetti, J. Herrero-Garcia, S. Marciano, D. Meloni and D. Vatsyayan

Smallness of neutrino masses



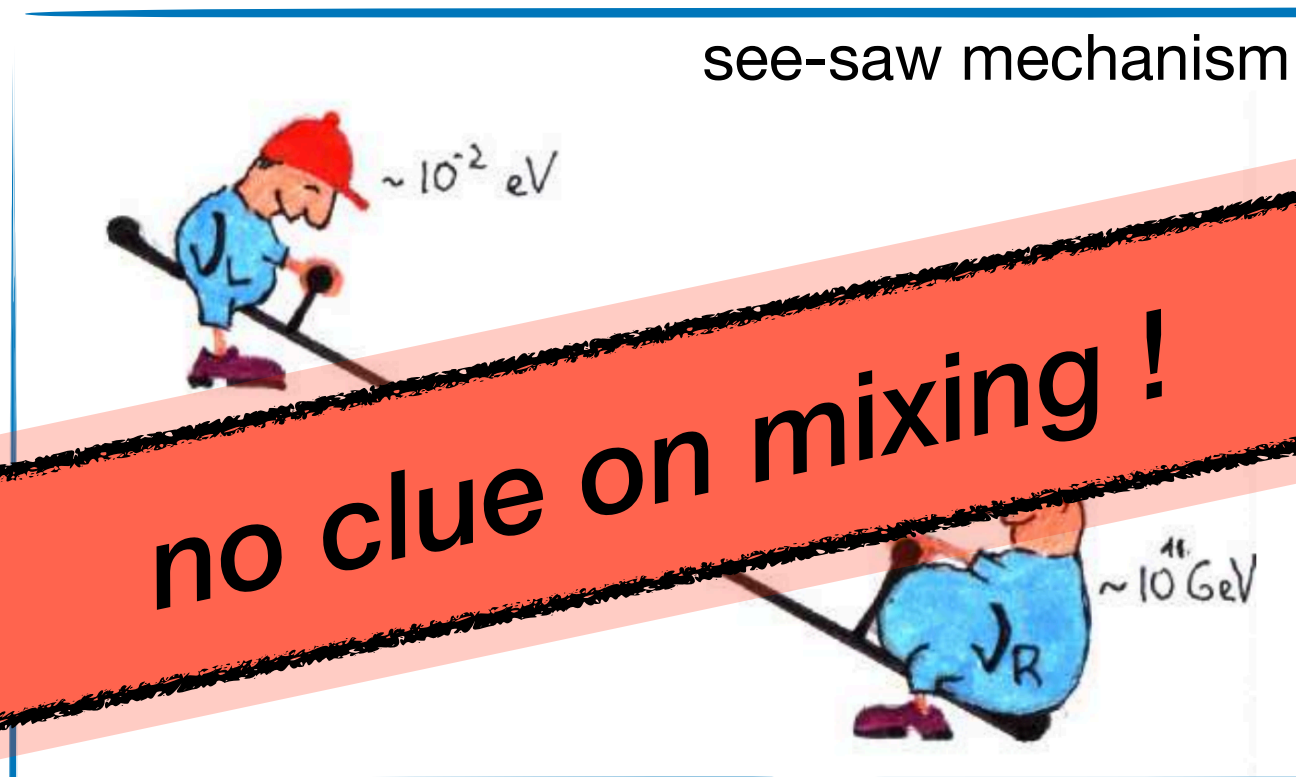
$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}$$

$$m_{\text{light}} \approx \frac{m_D^2}{M_R}$$

Suggested solutions

Hierarchical pattern and mixing angles

Smallness of neutrino masses



$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}$$

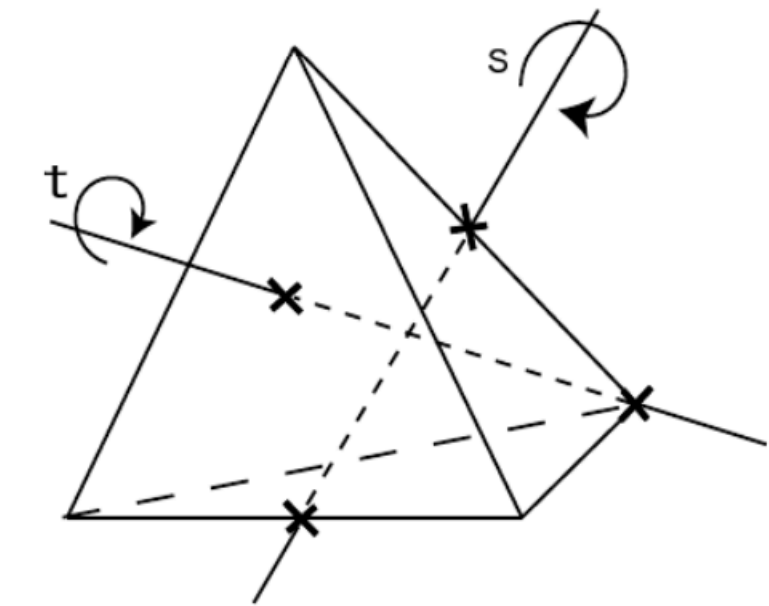
$$m_{\text{light}} \approx \frac{m_D^2}{M_R}$$

Froggatt-Nielsen mechanism

$$L \sim \bar{\Psi}_L H \Psi_R \left(\frac{\Theta}{\Lambda} \right)^n$$

too many $\mathcal{O}(1)$ coefficients

non-Abelian discrete flavor symmetries



complicated scalar sector

Modular symmetry ?

Feruglio, 1706.08749

Modular finite group

The modular group $\bar{\Gamma} = \text{SL}(2, \mathbb{Z}) / \{\pm \mathbb{1}\}$ acts on the modulus τ restricted to the upper-half complex plan through the transformation $\gamma : \tau \rightarrow \gamma(\tau)$

$$\gamma(\tau) = \frac{a\tau + b}{c\tau + d}, \quad a, b, c, d \in \mathbb{Z}, \quad ad - bc = 1,$$

$$S : \tau \rightarrow -\frac{1}{\tau} \quad T : \tau \rightarrow \tau + 1 \quad S^2 = (ST)^3 = \mathbb{1}$$

A modular form is a holomorphic function of τ such that:

$$f(\gamma(\tau)) = (c\tau + d)^k f(\tau),$$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}(2, \mathbb{Z}) \mid \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\},$$

Feruglio ([1]) has shown that it is always possible to find a basis where modular forms of a given level N transform in unitary representations of the finite groups which, for $N < 6$, are isomorphic to the non-Abelian discrete groups

$$\Gamma_2 \simeq S_3 \quad \Gamma_3 \simeq A_4 \quad \Gamma_4 \simeq S_4 \quad \Gamma_5 \simeq A_5$$

for further details, see the talk by M. Parriciatu

[1] F. Feruglio. "Are neutrino masses modular forms?", 1706.08749 [hep-ph]

Smallest modular finite group

The Model in a nutshell

The modular group $\bar{\Gamma} = \text{SL}(2, \mathbb{Z}) / \{\pm \mathbb{1}\}$ acts on the modulus τ restricted to the upper-half complex plan through the transformation $\gamma : \tau \rightarrow \gamma(\tau)$

$$\gamma(\tau) = \frac{a\tau + b}{c\tau + d}, \quad a, b, c, d \in \mathbb{Z}, \quad ad - bc = 1,$$

$$S : \tau \rightarrow -\frac{1}{\tau} \quad T : \tau \rightarrow \tau + 1 \quad S^2 = (ST)^3 = \mathbb{1}$$

A modular form is a holomorphic function of τ such that:

$$f(\gamma(\tau)) = (c\tau + d)^k f(\tau),$$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}(2, \mathbb{Z}) \mid \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\},$$

Feruglio ([1]) has shown that it is always possible to find a basis where modular forms of a given level N transform in unitary representations of the finite groups which, for $N < 6$, are isomorphic to the non-Abelian discrete groups

$$\Gamma_2 \simeq S_3 \quad \Gamma_3 \simeq A_4 \quad \Gamma_4 \simeq S_4 \quad \Gamma_5 \simeq A_5$$

$$\Gamma_2 \simeq S_3 \quad \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_2 = \begin{pmatrix} \frac{7}{100} + \frac{42}{25}q + \frac{42}{25}q^2 + \frac{168}{25}q^3 + \dots \\ \frac{14\sqrt{3}}{25}q^{1/2}(1 + 4q + 6q^2 + \dots) \end{pmatrix},$$

SUSY framework

$$\begin{array}{ll} \Phi(\tau, \varphi) & \varphi^{(I)} \rightarrow (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \\ \text{chiral superfields} & \text{matter supermultiplets} \end{array}$$

$$\text{Superpotential} \quad \mathcal{W}(\Phi) = \sum (Y_{I_1 \dots I_n}(\tau) \varphi^{(I_1)} \dots \varphi^{(I_n)})_{\mathbf{1}},$$

$$\text{Yukawa Couplings as modular forms} \quad Y_{I_1 \dots I_n}(\gamma(\tau)) = (c\tau + d)^{k_Y} \rho(\gamma) Y_{I_1 \dots I_n}(\tau),$$

The novelty compared to classic non-Abelian discrete groups model building is that the invariance is achieved by satisfying not only the existence of a singlet contraction between all the irreps involved, but also that every operator must be *weightless*

[1] F. Feruglio. "Are neutrino masses modular forms?", 1706.08749 [hep-ph]

Smallest modular finite group

The Model in a nutshell

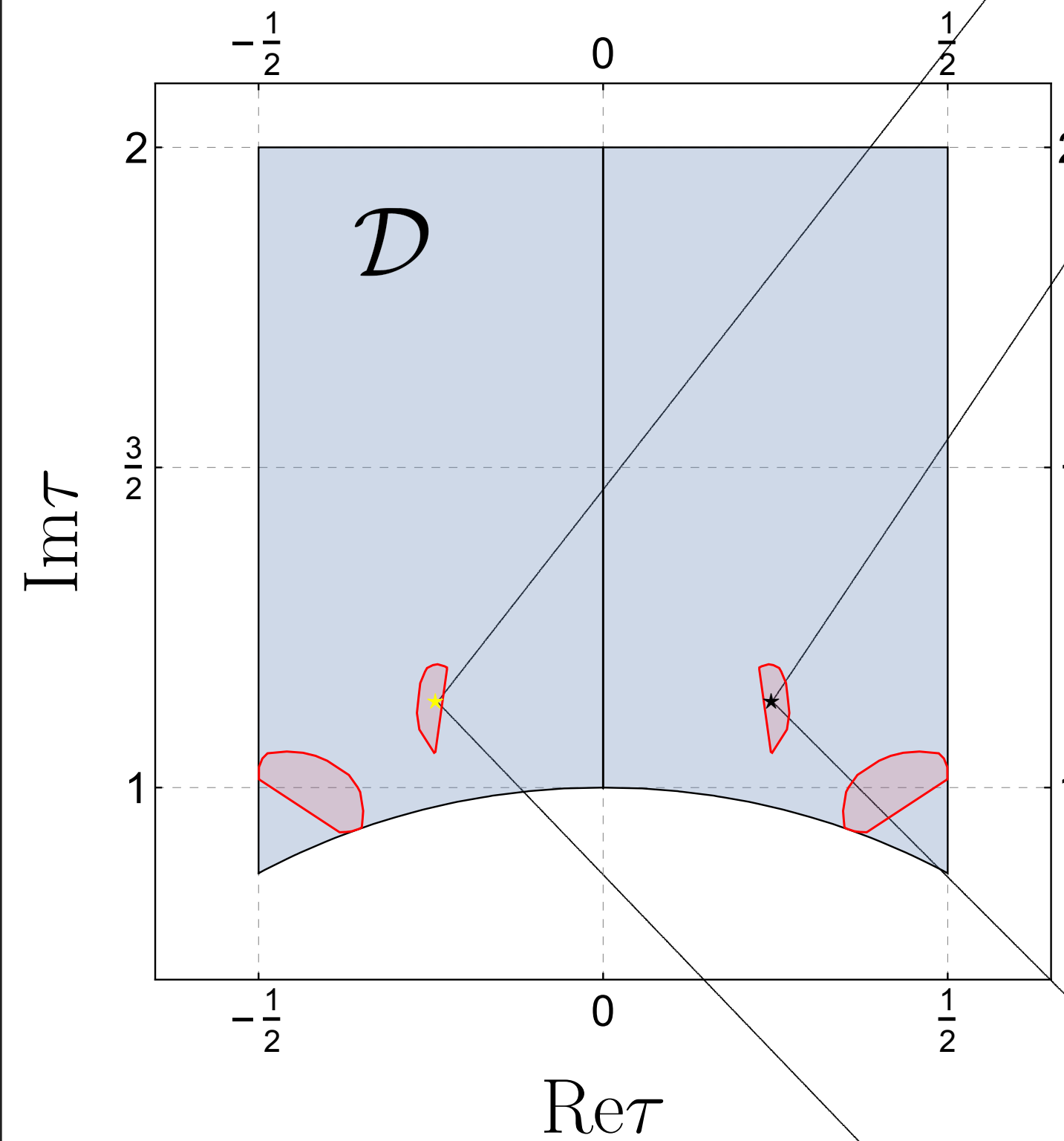
	E_1^c	E_2^c	E_3^c	D_ℓ	ℓ_3	$H_{d,u}$	N^c
$SU(2)_L \times U(1)_Y$	(1, +1)	(1, +1)	(1, +1)	(2, -1/2)	(2, -1/2)	(2, $\mp 1/2$)	(1, 0)
$\Gamma_2 \cong S_3$	1	1'	1'	2	1'	1	2
k_I	4	0	-2	2	2	0	2

$$\mathcal{W}_e^H = \alpha E_1^c H_d (D_\ell Y_2^{(3)})_1 + \beta E_2^c H_d (D_\ell Y_2)_1 + \gamma E_3^c H_d \ell_3 + \alpha_D E_1^c H_d \ell_3 Y_1^{(3)},$$

$$\mathcal{W}_\nu = g H_u N^c D_\ell Y_2^{(2)} + g' H_u (N^c Y_2^{(2)})_1 \ell_3 + g'' H_u (N^c D_\ell)_1 Y_1^{(2)} + \Lambda [(N^c N^c)_2 Y_2^{(2)} + \lambda (N^c N^c)_1 Y_1^{(2)}]$$

Parameter	Best-fit value and 1 σ range	
$\Delta m_{\text{sol}}^2 / (10^{-5} \text{ eV}^2)$	7.41 $^{+0.21}_{-0.20}$	
	NO	IO
$ \Delta m_{\text{atm}}^2 / (10^{-3} \text{ eV}^2)$	2.507 $^{+0.026}_{-0.027}$	2.486 $^{+0.025}_{-0.028}$
$r \equiv \Delta m_{\text{sol}}^2 / \Delta m_{\text{atm}}^2 $	0.0295 \pm 0.0008	0.0298 \pm 0.0008
$\sin^2 \theta_{12}$	0.303 $^{+0.012}_{-0.012}$	0.303 $^{+0.012}_{-0.011}$
$\sin^2 \theta_{13}$	0.02225 $^{+0.00056}_{-0.00059}$	0.0223 $^{+0.00058}_{-0.00058}$
$\sin^2 \theta_{23}$	0.451 $^{+0.019}_{-0.016}$	0.569 $^{+0.016}_{-0.021}$
m_e/m_μ	0.0048 \pm 0.0002	
m_μ/m_τ	0.0565 \pm 0.0045	

I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, "The fate of hints: updated global analysis of three-flavor neutrino oscillations," KHEP 09 (2020) 178, 2007.14792 [hep-ph]



$\star(-)$ $\star(+)$	Best-fit and 1 σ range
	$\text{Re } \tau$ $\pm 0.244^{+0.012}_{-0.067}$
	$\text{Im } \tau$ $1.132^{+0.027}_{-0.297}$
	β/α $0.92^{+0.85}_{-0.03}$
	γ/α $-1.20^{+0.06}_{-2.14}$
	$\log_{10}(\alpha_D/\alpha)$ $-13.4^{+13.2}_{-76.3}$
	g'/g $2.76^{+0.21}_{-0.23}$
	g''/g $-2.53^{+0.13}_{-0.03}$
	$\log_{10}(\lambda)$ $-12.2^{+10.9}_{-59.2}$
	$v_d \alpha$, [GeV] $1.08^{+0.06}_{-0.69}$
	$v_u^2 g^2/\Lambda$ [eV] $3.46^{+0.55}_{-1.65}$
	$\sin^2 \theta_{12}$ $0.305^{+0.011}_{-0.011}$
	$\sin^2 \theta_{13}$ $0.0221^{+0.0006}_{-0.0005}$
	$\sin^2 \theta_{23}$ $0.448^{+0.014}_{-0.016}$
	r $0.0296^{+0.0006}_{-0.0008}$
	m_e/m_μ $0.0048^{+0.0001}_{-0.0002}$
	m_μ/m_τ $0.0574^{+0.0032}_{-0.0050}$
	Ordering NO
	J_{CP} $-0.018^{+0.002}_{-0.002}$
	α_1/π 0
	α_2/π $\pm 0.112^{+0.792}_{-0.014}$
	m_1 [meV] 0
	m_2 [meV] $8.620^{+0.095}_{-0.123}$
	m_3 [meV] $50.806^{+0.016}_{-0.021}$
	$\sum_i m_i$ [eV] $0.0594^{+0.0001}_{-0.0001}$
	$ m_{\beta\beta} $ [meV] $3.61^{+0.09}_{-0.09}$
	m_β^{eff} [meV] $8.90^{+0.10}_{-0.09}$
	d_{FT} 3.03
	χ_{min}^2 0.98

Thermal Leptogenesis

“Baryogenesis Without Grand Unification”, Phys.Lett.B174:45,1986, by Fukugita and Yanagida.

Volume 174, number 1

PHYSICS LETTERS B

26 June 1986

BARYOGENESIS WITHOUT GRAND UNIFICATION

M. FUKUGITA

Research Institute for Fundamental Physics, Kyoto University, Kyoto 606, Japan

and

T. YANAGIDA

Institute of Physics, College of General Education, Tohoku University, Sendai 980, Japan and Deutscher Elektronen-Synchrotron DESY, D-2000 Hamburg, Fed. Rep. Germany

Received 8 March 1986

A mechanism is pointed out to generate cosmological baryon number excess without resorting to grand unified theories. The lepton number excess originating from Majorana mass terms may transform into the baryon number excess through the unsuppressed baryon number violation of electroweak processes at high temperatures.

The current view ascribes the origin of cosmological baryon excess to the microscopic baryon number violation process in the early stage of the Universe [1,2]. The grand unified theory (GUT) of particle interactions is regarded as the standard candidate to account for this baryon number violation. The theory can give the correct order of magnitude for baryon to entropy ratio. If the Universe undergoes the inflation epoch after the baryogenesis, however, generated baryon numbers are diluted by a huge factor. The reheating after the inflation is unlikely to ease the temperature above the GUT energy scale. A more interesting problem is that no evidences are given so far experimentally for the baryon number violation, which might cast some doubt on the GUT idea.

Some time ago 't Hooft suggested that the instanton-like effect violates baryon number in the Weinberg-Salam theory through the anomaly term, although the effect is suppressed by a large factor [3]. It has been pointed out, however, that this effect is not suppressed and can be efficient at high temperatures above the Weinberg-Salam energy scale [4]. This baryon number violating process conserves $B-L$, but it erases rapidly the baryon asymmetry which would have been generated at the early Universe with $B-L$

conserving baryon number violation processes as in the standard SU(5) GUT. (Baryon numbers would remain, if the baryon production takes place at low temperatures $T \lesssim O(100 \text{ GeV})$, e.g., after reheating [5,6].) The process itself can not produce the baryon asymmetry, since it is unlikely to suppose a particular mechanism leading to departures from equilibrium [4].

In this letter, we point out that the electroweak baryon number violation process, if it is supplemented by a lepton number generation at an earlier epoch, can generate the cosmological baryon asymmetry without resorting to the GUT scenario. The lepton number excess in the earlier stage can efficiently be transformed into the baryon number excess. It is rather easy to find an agent leading to the lepton number generation. A candidate is the decay process involving Majorana mass terms.

Let us present a specific model which gives lepton number generation. We assume the presence of a right-handed Majorana neutrino N_R^c ($c = 1, \dots, n$) in addition to the conventional leptons. We take the Lagrangian to be

Volume 174, number 1

PHYSICS LETTERS B

26 June 1986

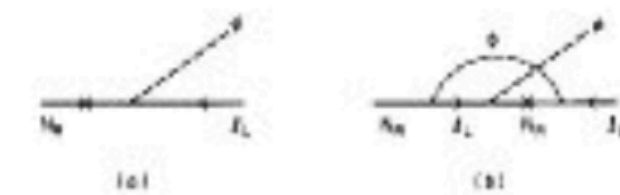


Fig. 1. The simplest diagram giving rise to a net lepton number production. The cross denotes the Majorana mass insertion.

$$\mathcal{L} = \mathcal{L}_{\text{WS}} + N_R^c \bar{N}_R^c + M_i N_R^c N_R^c + \text{h.c.} + k_{ij} N_R^c \bar{L}_i^c \phi^j + \text{h.c.}, \quad (1)$$

where \mathcal{L}_{WS} is the standard Weinberg-Salam Lagrangian, and ϕ the standard Higgs doublet. For simplicity we assume three generations of flavors and the mass hierarchy $M_1 < M_2 < M_3$. In the decay of N_R^c ,

$$N_R^c \rightarrow \bar{\nu}_L^c + \phi^c, \quad (2a)$$

$$N_R^c \rightarrow \bar{\nu}_L^c + \phi^c, \quad (2b)$$

there appears a difference between the branching ratios for (2a) and (2b), if CP is violated, through the one-loop radiative correction by a Higgs particle. The net lepton number production due to the decay of a lightest right-handed neutrino N_R^1 arises from the interference of the two diagrams in fig. 1, and its magnitude is calculated as [7]

$$\epsilon = (9/4\pi) \text{Im}(h_{33} h_{11}^* h_{21}^* h_{12}) / (M_1^2 M_2^2) (\delta h^2)_{11}, \quad (3)$$

with

$$f(x) = x^{3/2} [1 + (1+x) \ln|x/(1+x)|].$$

If we assume h_{33} to be the largest entry of the Yukawa coupling matrix and $M_3 \gg M_1$, (3) reduces to

$$\epsilon \approx (9/8\pi) |h_{33}|^2 (M_3/M_1)^6. \quad (4)$$

with δ the phase causing CP violation.

We apply the delayed decay mechanism [8] to generate the baryon asymmetry in the Universe. The out-of-equilibrium condition is satisfied, if the temperature T is smaller than the mass M_1 so that the inverse decay is blocked at the time when the decay rate $\Gamma = (\delta h^2)_{11} / 16\pi$ is equal to the expansion rate of the Universe $\dot{n}/n \sim 1.7\sqrt{g}T^2/m_{\text{pl}}$ ($g =$ number of degrees of freedom), i.e.,

$$(\Gamma/m_{\text{pl}})^{-1/2} < M_1. \quad (5)$$

To obtain numerical factors for this condition, one has to solve the Boltzmann equation. Let us borrow the results of ref. [9] to obtain a rough number. The lepton number to entropy ratio is given as

$$\hat{k}(\Delta L)_i/n \sim 10^{-3} \epsilon K^{-1.2}, \quad (6)$$

with $K = \frac{1}{2} \Gamma/\dot{n}(x)$ for $K \gg 1$. The parameters in (4) and in the expression of Γ are not directly constrained by low-energy experiments. One may have an idea, however, on the mass scale M_1 as follows: With the parameter in a reasonable range, one may obtain $\epsilon \leq 10^{-6}$. Then to obtain our required number for $\hat{k}(\Delta L)_i/n \sim 10^{-10.5}$ (see below), $K \leq 30$ is necessary, which gives $M_1 \gtrsim 2.4 \times 10^{14} \text{ GeV} (\delta h^2)_{11}^{-1/2}$. If we assume $|\delta h_{12}|^2, |\delta h_{13}|^2 \lesssim |\delta h_{11}|^2$ and take $(\delta h^2)_{11} \approx |\delta h_{11}|^2 \sim (10^{-2})^2$, then we are led to $M_1 \gtrsim 2 \times 10^4 \text{ GeV}$. This constant can also be expressed in terms of the left-handed Majorana neutrino mass m_{ν}^{ν} as $m_{\nu}^{\nu} \approx \delta h_{11}^2 (\phi^2)/M_1 \leq 0.1 \text{ eV}$. If the lightest left-handed neutrino has a Majorana mass smaller than this value, the required asymmetry can be generated.

Now let us discuss the generation of the baryon asymmetry. In the presence of an instanton-like electroweak effect the baryon asymmetry changes as [4]

$$\Delta B(t) = \frac{1}{2} \Delta(B-L)_i + \frac{1}{2} \Delta(B+L)_i \exp(-\gamma t), \quad (7)$$

with $\gamma \sim T$. At the time of the Weinberg-Salam epoch the exponent is $m_{\text{pl}}/T\sqrt{g} \sim 10^{16}$ and the second term practically vanishes. Therefore we obtain

$$\Delta B = -(\Delta L)_i/2, \quad (8)$$

which survives up to the present epoch, and should give $\hat{k}(\Delta B)_i \sim 10^{-10.8}$.

⁴¹ Here we assumed the dominance of the diagonal matrix elements. More precisely speaking, the matrix element constraint by our condition differs from that which appears in the observable neutrino mass: The left-handed neutrino mass matrix is given by $(m_{\nu}^{\nu})_{ij} = \sum_k (\delta h_{1k}^2)_{ij} A_k \phi^k / M_k$ [10]. The double beta decay experiment restricts the matrix element $(m_{\nu}^{\nu})_{11} = (\delta h_{11}^2)_{11} / M_1 + \sum_{i=2,3} (\delta h_{1i}^2)_{11} / M_i \leq m_{\nu}^{\nu}$, while eq. (5) refers to $(\delta h_{11}^2)_{11} + |\delta h_{12}|^2 + |\delta h_{13}|^2 \leq m_{\nu}^{\nu} M_1$ and $\delta h_{ij} = \delta h_{ji}$ in general. (Here we took the basis where the charged-lepton mass matrix is diagonal.) Therefore, the double beta decay experiment does not constrain directly the parameters in eq. (5). The tritium beta decay experiment restricts the eigenvalue of the mass matrix $(m_{\nu}^{\nu})_{11}$ (see ref. [11]).

Volume 174, number 1

PHYSICS LETTERS B

26 June 1986

A primordial lepton number excess existed before the epoch of the right-handed neutrino mass scale should have been washed out by the equilibrium of process (2) and its inverse process, if the Yukawa coupling $(\delta h^2)_{22}$ or $(\delta h^2)_{33}$ is large enough. The equilibrium condition $\Gamma \exp(-M_i/T) \gtrsim 1.7\sqrt{g}T^2/m_{\text{pl}}$ ($i = 2$ or 3) leads to a constraint similar to (5) but with the inequality reversed. The net baryon number destruction factor behaves as $\sim \exp(-\alpha K)$ ($\alpha \sim O(1)$) [9]. For $K \geq 20-30$, the equilibrium practically erases the whole pre-existing lepton number excess. This condition is expressed as $(m_{\nu}^{\nu})_i > 0.1 \text{ eV}$ for the largest entry of the Majorana mass matrix.

In the presence of unsuppressed instanton-like electroweak effects, the lepton number equilibrium implies that the baryon excess which existed at this epoch should also be washed out, even if it was produced in the process with $B-L \neq 0$. Namely, if there are neutrinos with the Majorana mass heavier than $\sim 0.1 \text{ eV}$ both baryon and lepton numbers which existed before this epoch are washed out irrespective of their $B-L$ properties.

In summary, we have the following possible scenarios for the cosmological baryon number excess:

- (1) At a temperature above the mass scale M (= scale of right-handed Majorana neutrino), we started with $\Delta B = \Delta L = 0$. (The inflationary universe would give this initial condition.) Then the lepton number is generated through the Majorana mass term, and is transformed into the baryon number due to the unsuppressed instanton-like electroweak effect.
- (2) At the scale $> M$, baryon and lepton numbers are generated by the grand unification, or alternatively we start with a $\Delta B \neq 0, \Delta L \neq 0$ Universe. The equilibrium of $N_R^c \rightleftharpoons \bar{\nu}_L^c + \phi^c, \phi + \bar{\nu}_L^c$, together with the electroweak process washes out both baryon and lepton numbers. Then the lepton number is newly generated by the out-of-equilibrium scenario, and it turns into the baryon number.

(3) The baryon number with $B-L \neq 0$ is generated by the grand unification (e.g., the SU(10) model [12]). If the scale M is too large to establish the equilibrium of N_R^c and $\phi + \bar{\nu}_L^c$, then the initial $\Delta(B-L)$ will not be erased. The electroweak process does not affect $B-L$, and hence the initial baryon

number remains. This case is the original GUT baryon number generation scenario. To achieve this, however, all neutrino mass matrix elements (Majorana mass) should be smaller than $\sim 0.1 \text{ eV}$. If the double beta experiment would observe a Majorana mass greater than this value, this scenario fails.

In conclusion we have suggested a mechanism of cosmological baryon number generation without resorting to grand unification. In our scenario the cosmological baryon number can be generated, even if proton decay does not happen at all.

One of us (M.F.) would like to thank V.A. Rubakov for discussions on baryon number nonconservation in electroweak processes.

References

- [1] A.D. Sakharov, *Pisma Zh. Eksp. Teor. Fiz.* 5 (1967) 32, *V.A. Kouzin, Pisma Zh. Eksp. Teor. Fiz.* 12 (1970) 355.
- [2] M. Yoshimura, *Phys. Rev. Lett.* 41 (1978) 281, A.Yu. Izrael et al., *Phys. Lett.* 83B (1978) 436.
- [3] G. 't Hooft, *Phys. Rev. Lett.* 37 (1976) 8.
- [4] V.A. Kuz'min, V.A. Rubakov and M.E. Shaposhnikov, *Phys. Lett.* B155 (1985) 36.
- [5] I. Affleck and M. Dine, *Nucl. Phys.* B249 (1985) 361, A.D. Linde, *Phys. Lett.* B160 (1985) 243.
- [6] M. Fukugita and V.A. Rubakov, *Phys. Rev. Lett.* 56 (1986) 988.
- [7] T. Yanagida and M. Yoshimura, *Phys. Rev.* D23 (1981) 2048, A. Masiero and T. Yanagida, *Phys. Lett.* B112 (1982) 336.
- [8] D. Toussaint, S.B. Treiman, P. Wilczek and A. Zee, *Phys. Rev.* D19 (1979) 1056, S. Weinberg, *Phys. Rev. Lett.* 42 (1979) 859, M. Yoshimura, *Phys. Lett.* B 88 (1979) 284.
- [9] J.N. Fry, K.A. Olive and M.S. Turner, *Phys. Rev. Lett.* 45 (1980) 2074.
- [10] T. Yanagida, in: *Proc. Workshop on the Unified theory and the baryon number in the universe* (Tsukuba, 1979), eds. O. Sawada and S. Sugawara, Report KEK-79-18 (1979), M. Gell-Mann, P. Ramond and R. Slansky, in: *Supergravity*, eds. D.Z. Freedman and F. van Nieuwenhuizen (North-Holland, Amsterdam, 1979).
- [11] L. Wolfenstein, *Caracas-Melken University report CMU-HEG 82-9* (1982), M. Fukugita and T. Yanagida, *Phys. Lett.* B 144 (1984) 286.
- [12] E.g., M. Fukugita, T. Yanagida and M. Yoshimura, *Phys. Lett.* B106 (1981) 183.

Thermal Leptogenesis

“Baryogenesis Without Grand Unification”, Phys.Lett.B174:45,1986, by Fukugita and Yanagida.

Sakharov Conditions

Baryon Number violation

C and CP violation

Departure from thermal eq.

C and CP violation

if C or CP are conserved,

$$\Gamma(X \rightarrow Y + B) = \Gamma(\bar{X} \rightarrow \bar{Y} + \bar{B})$$

No net effect

Departure from thermal eq.

In thermal equilibrium, the production rate of baryons is equal to the destruction rate:

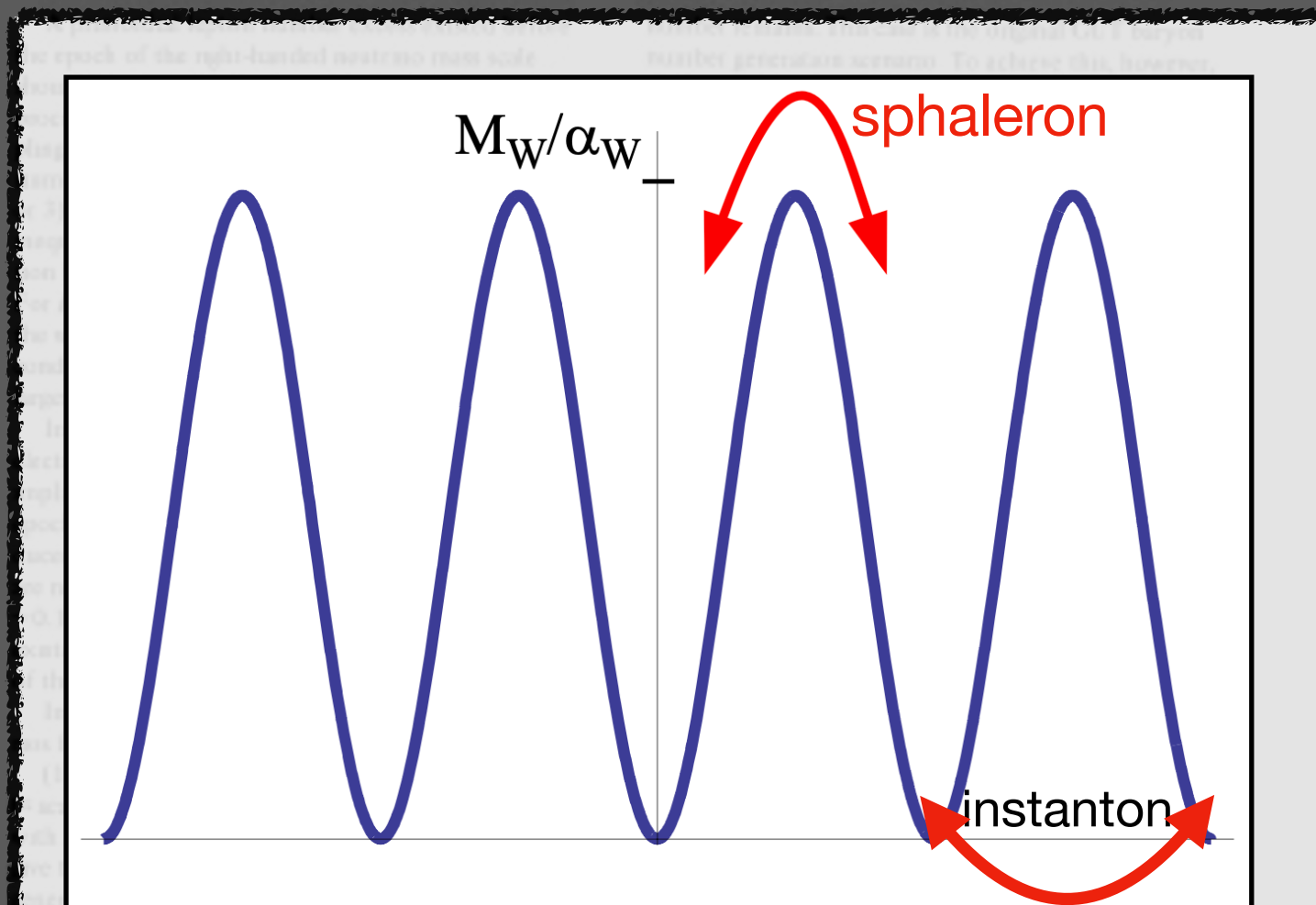
$$\Gamma(X \rightarrow Y + B) = \Gamma(Y + B \rightarrow X)$$

No net effect

Baryon Number violation

if baryon asymmetry is conserved, then no baryon number can be dynamically generated.

$$X^{B=0} \rightarrow Y^{B=0} + B^{\neq 0}$$



$$\Delta B = \Delta L = 3$$

At high temperatures, transitions violating B+L (and preserving B-L) occur very often

$$T < T_{EW} \quad \frac{\Gamma}{V} \sim \text{Exp}[-M_W/(\alpha kT)]$$

$$T > T_{EW} \quad \frac{\Gamma}{V} \sim \alpha^5 (\ln \alpha^{-1}) T^4$$

Leptogenesis from modular symmetry

$$N_{B-L}^f = \sum_i \epsilon_i k_i^f \quad , \quad i = 1,2$$

$$k_i^f = k_i(z = \infty) = - \int_{z_{in} \rightarrow 0}^{z_{fin} \rightarrow \infty} \frac{dN_i}{dz'} \text{Exp} \left[\sum_i \int_{z'}^z W_i(z'') dz'' \right] dz'$$

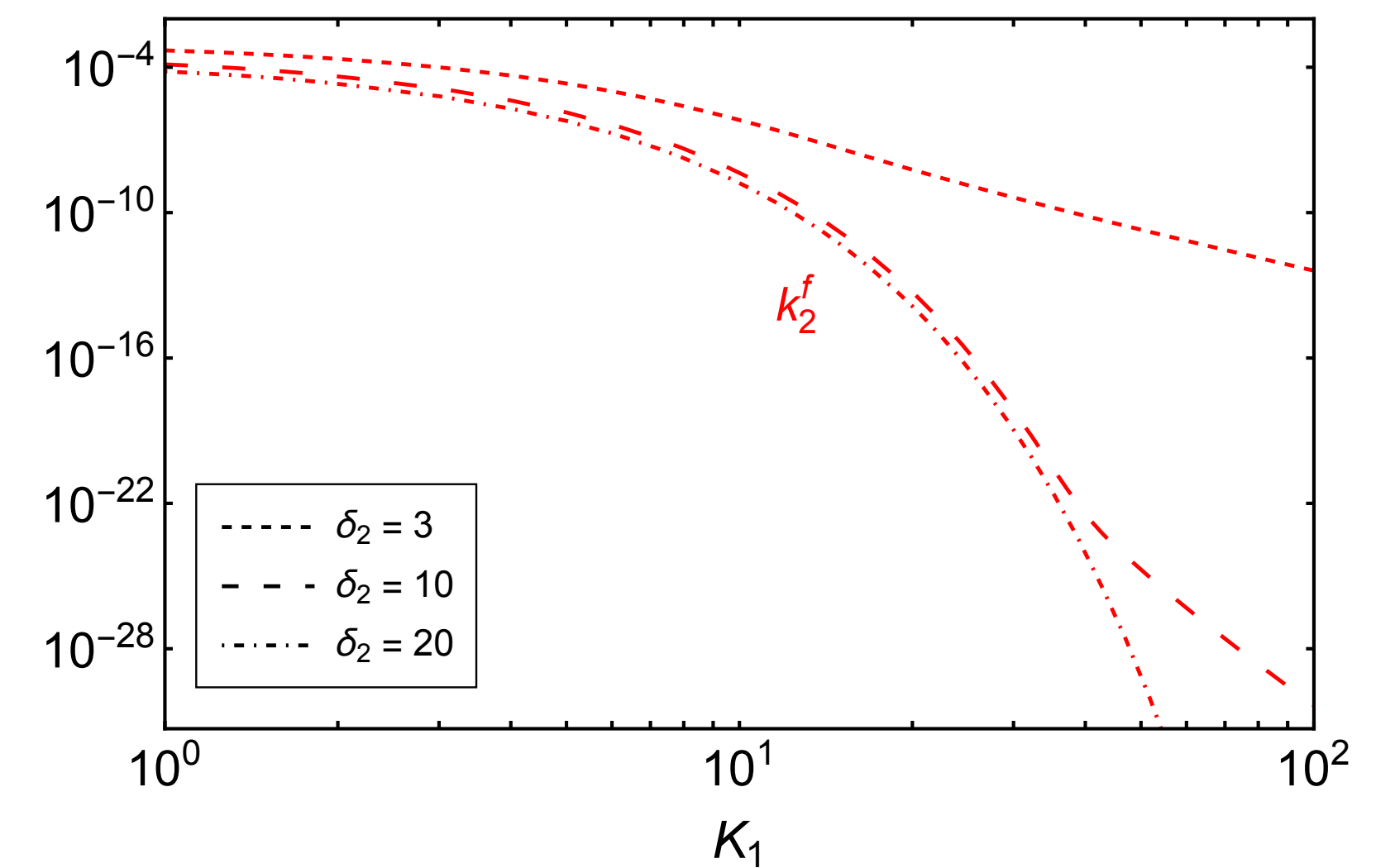
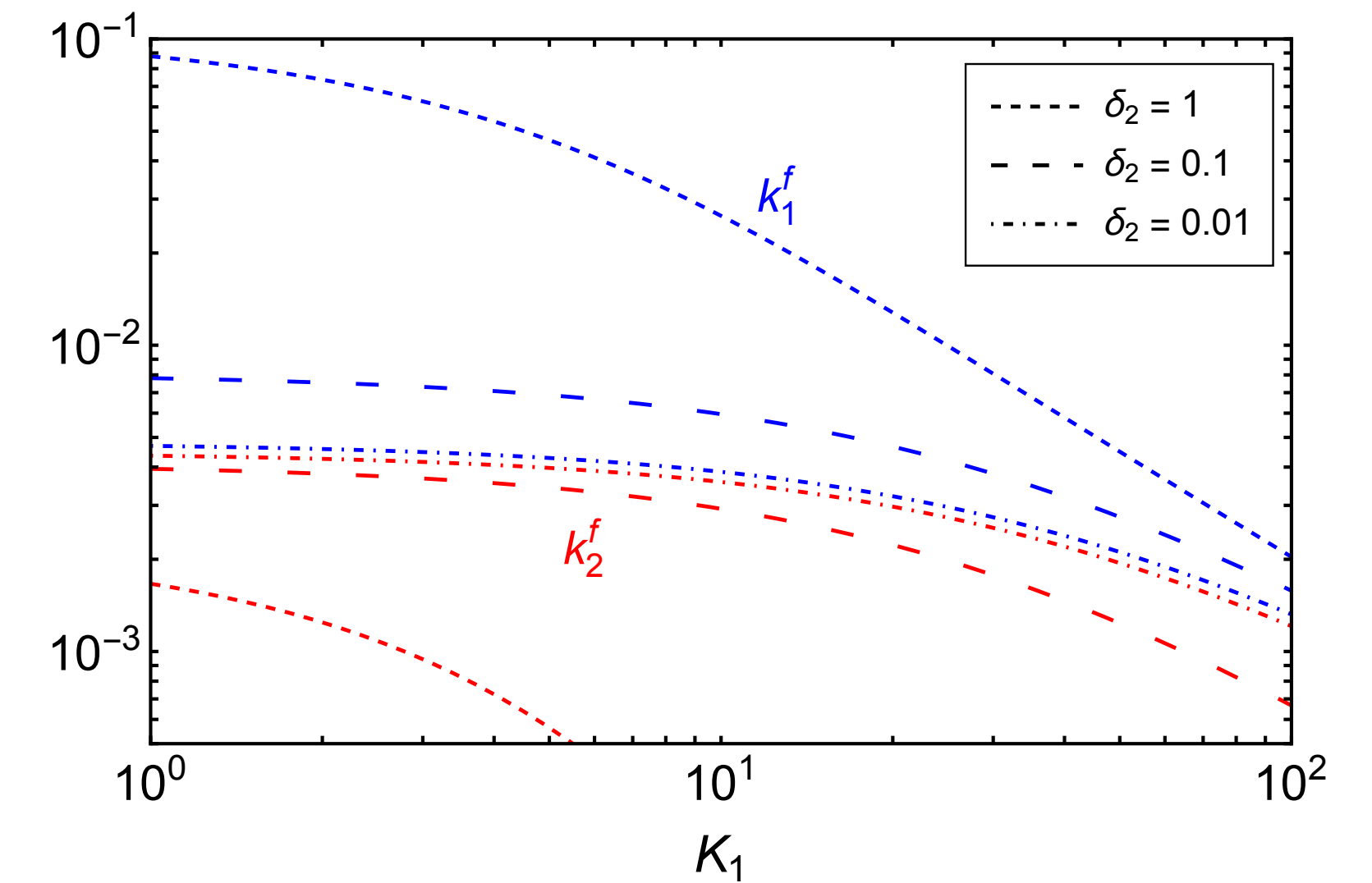
Leptogenesis from modular symmetry

$$N_{B-L}^f = \sum_i \epsilon_i k_i^f, \quad i = 1, 2$$

$$k_i^f = k_i(z = \infty) = - \int_{z_{in} \rightarrow 0}^{z_{fin} \rightarrow \infty} \frac{dN_i}{dz'} \text{Exp} \left[\sum_i \int_{z'}^z W_i(z'') dz'' \right] dz'$$

Strong dependence on

$$\delta_2 = \frac{M_2 - M_1}{M_1}$$



Leptogenesis from modular symmetry

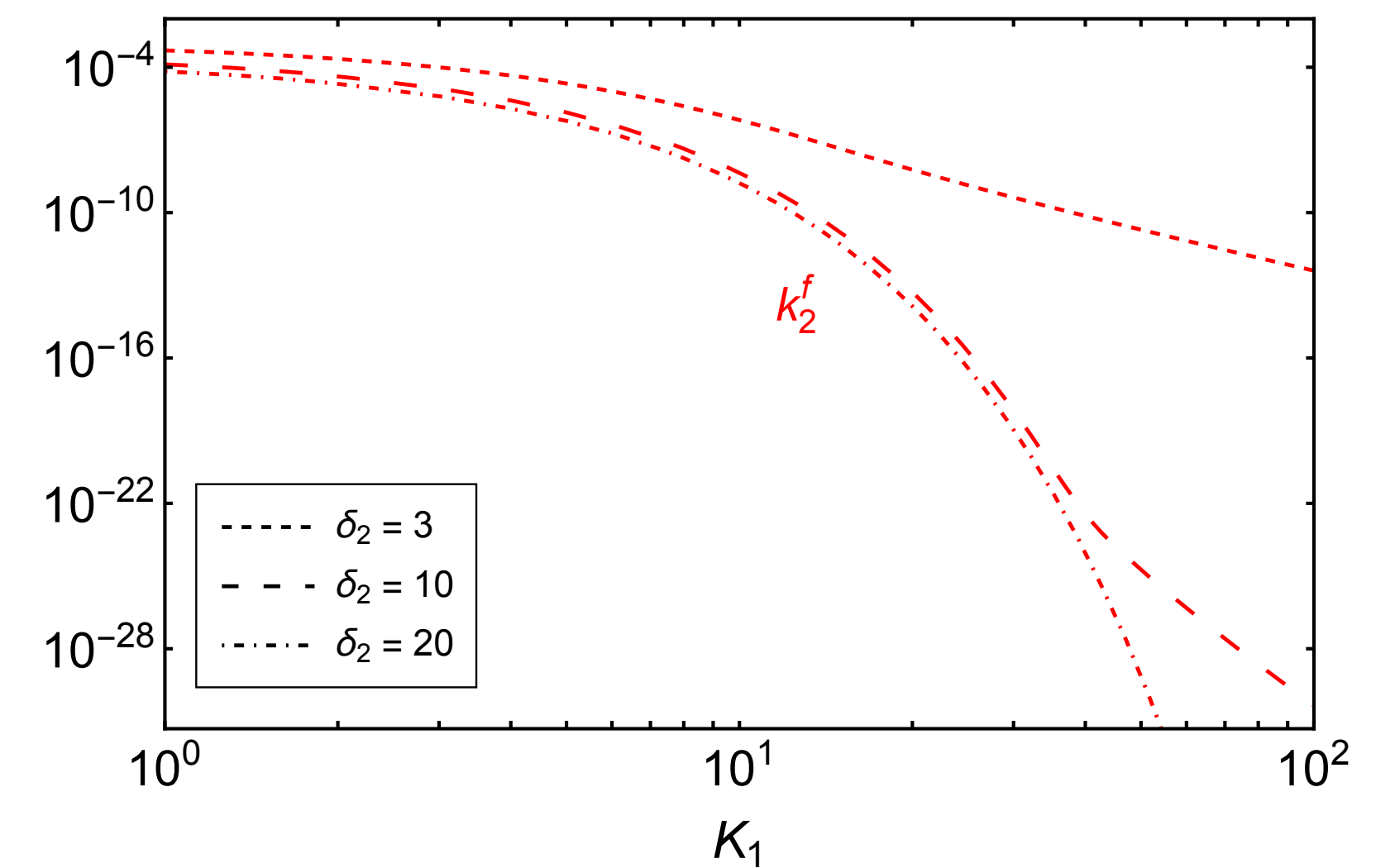
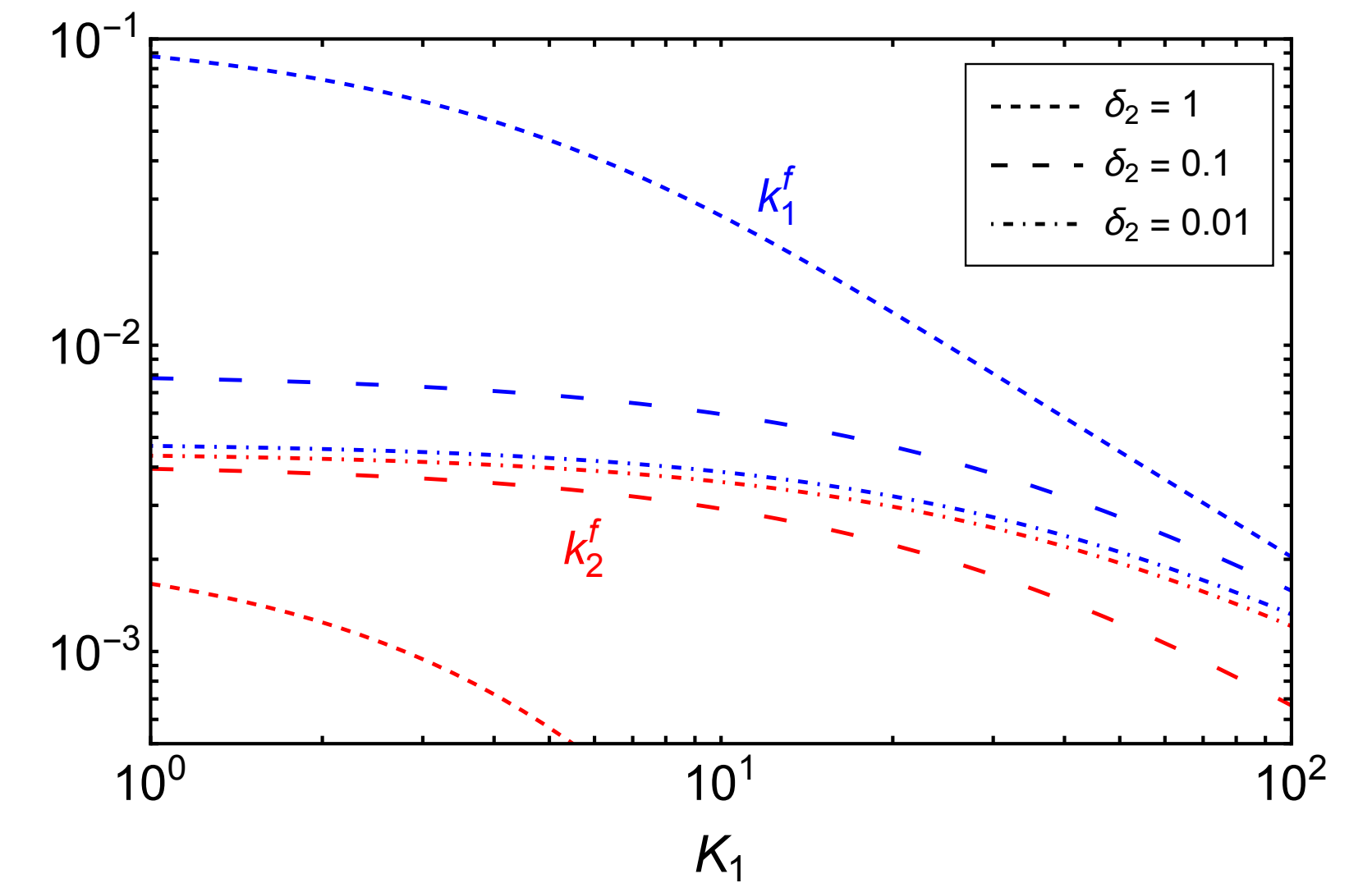
N_1 - dominated scenario?

$$N_{B-L}^f = \sum_i \epsilon_i k_i^f, \quad i = 1, 2$$

$$k_i^f = k_i(z = \infty) = - \int_{z_{in} \rightarrow 0}^{z_{fin} \rightarrow \infty} \frac{dN_i}{dz'} \text{Exp} \left[\sum_i \int_{z'}^z W_i(z'') dz'' \right] dz'$$

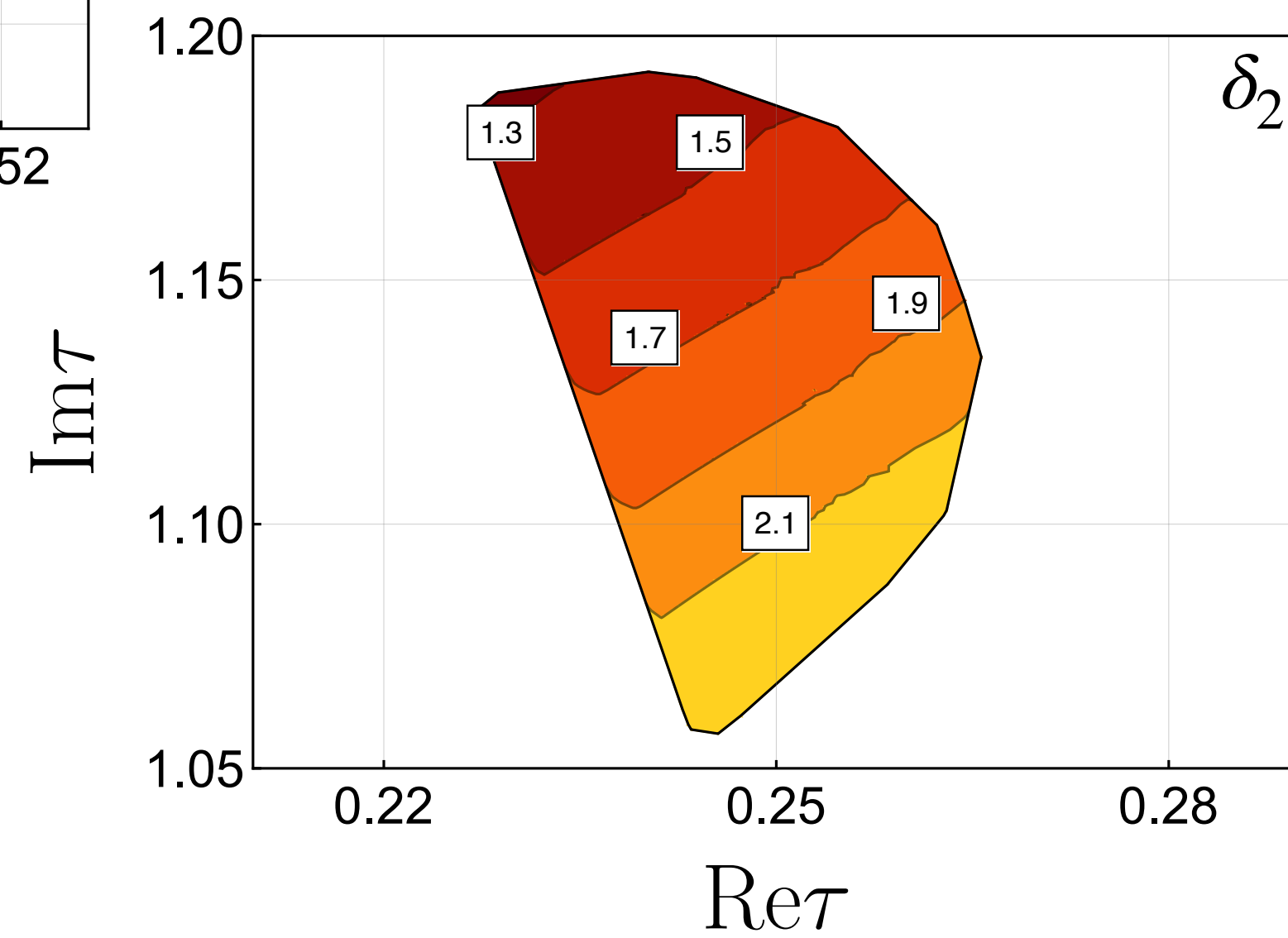
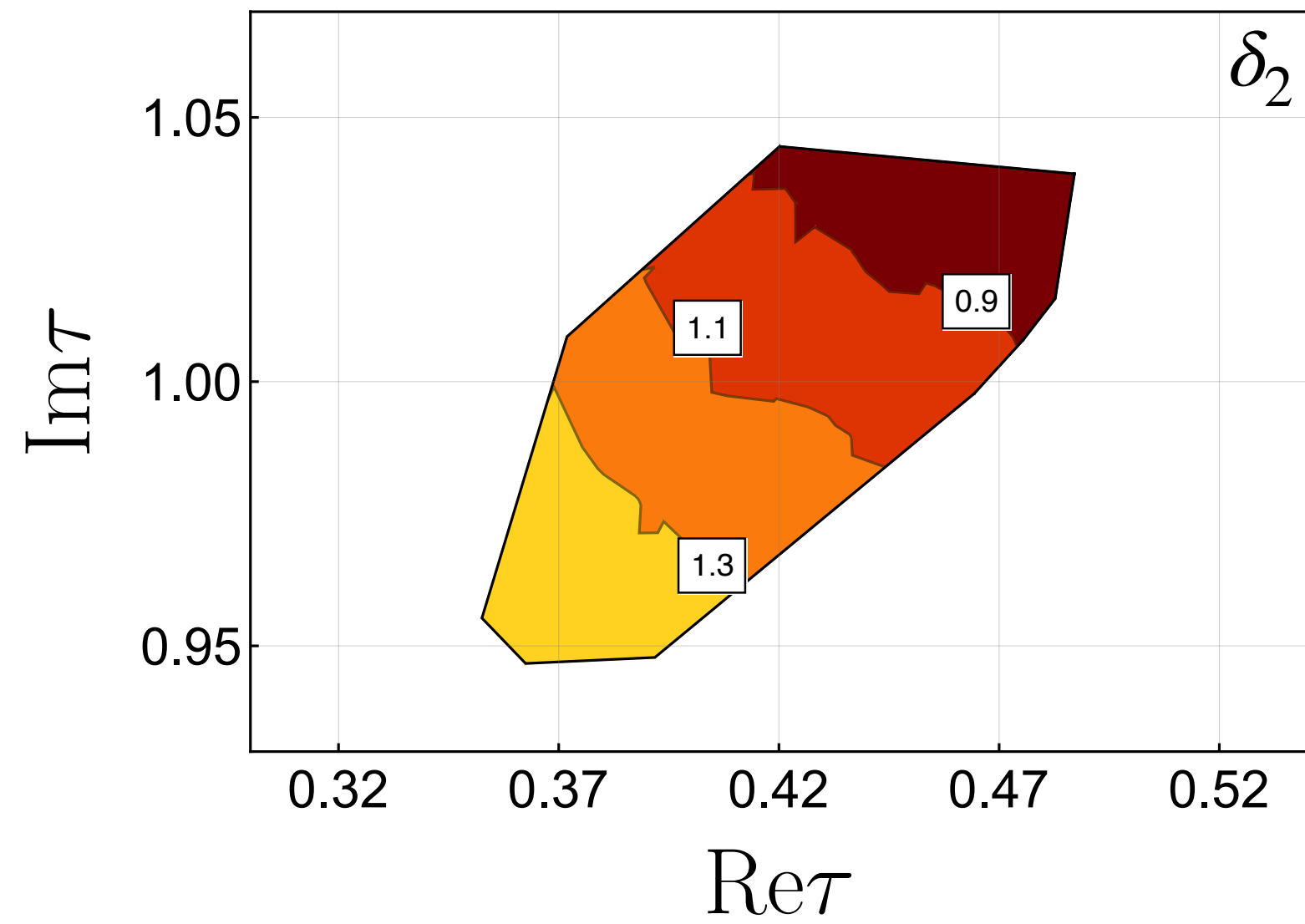
Strong dependence on

$$\delta_2 = \frac{M_2 - M_1}{M_1}$$

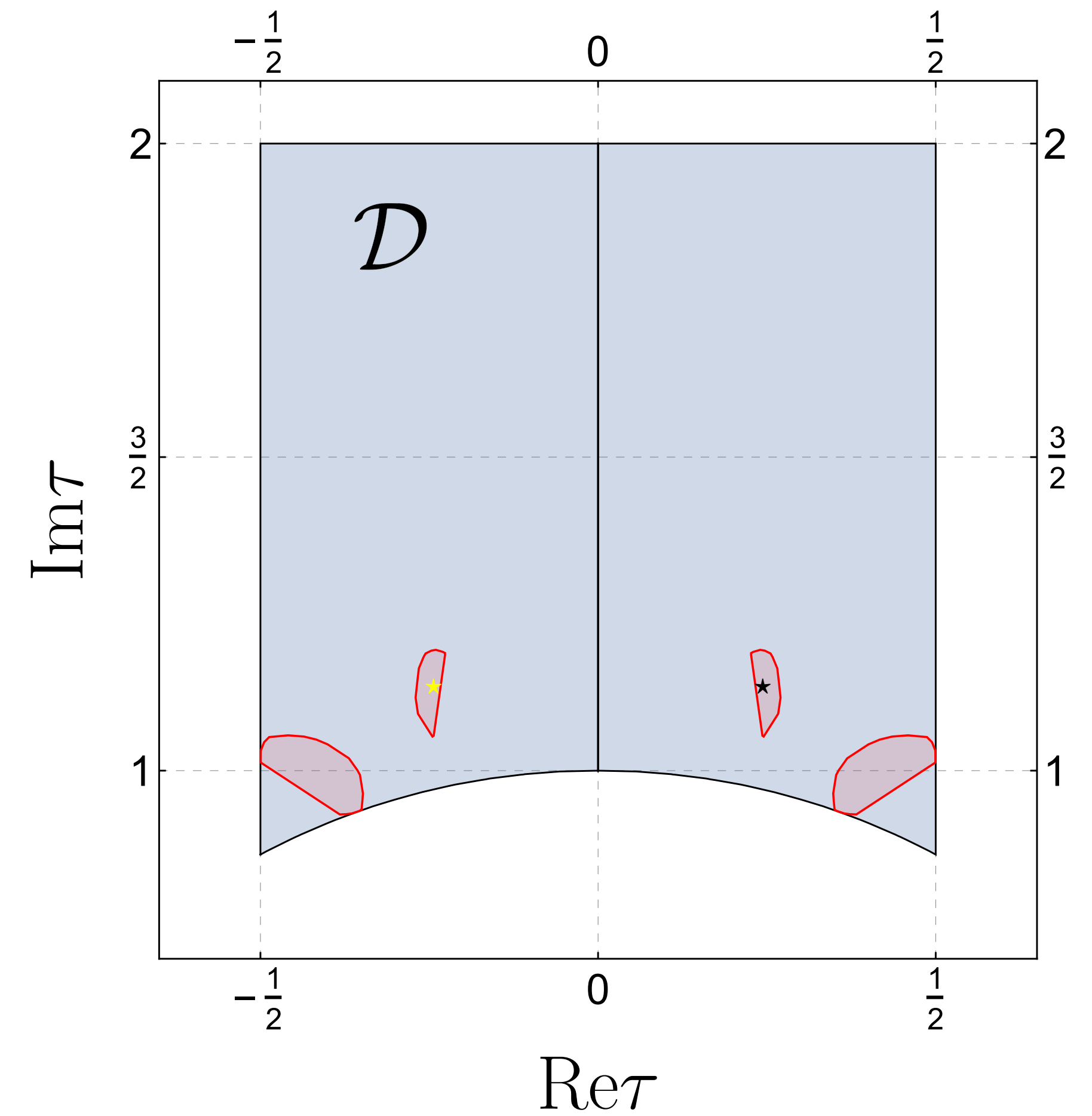


Leptogenesis from modular symmetry

N_1 - dominated scenario?

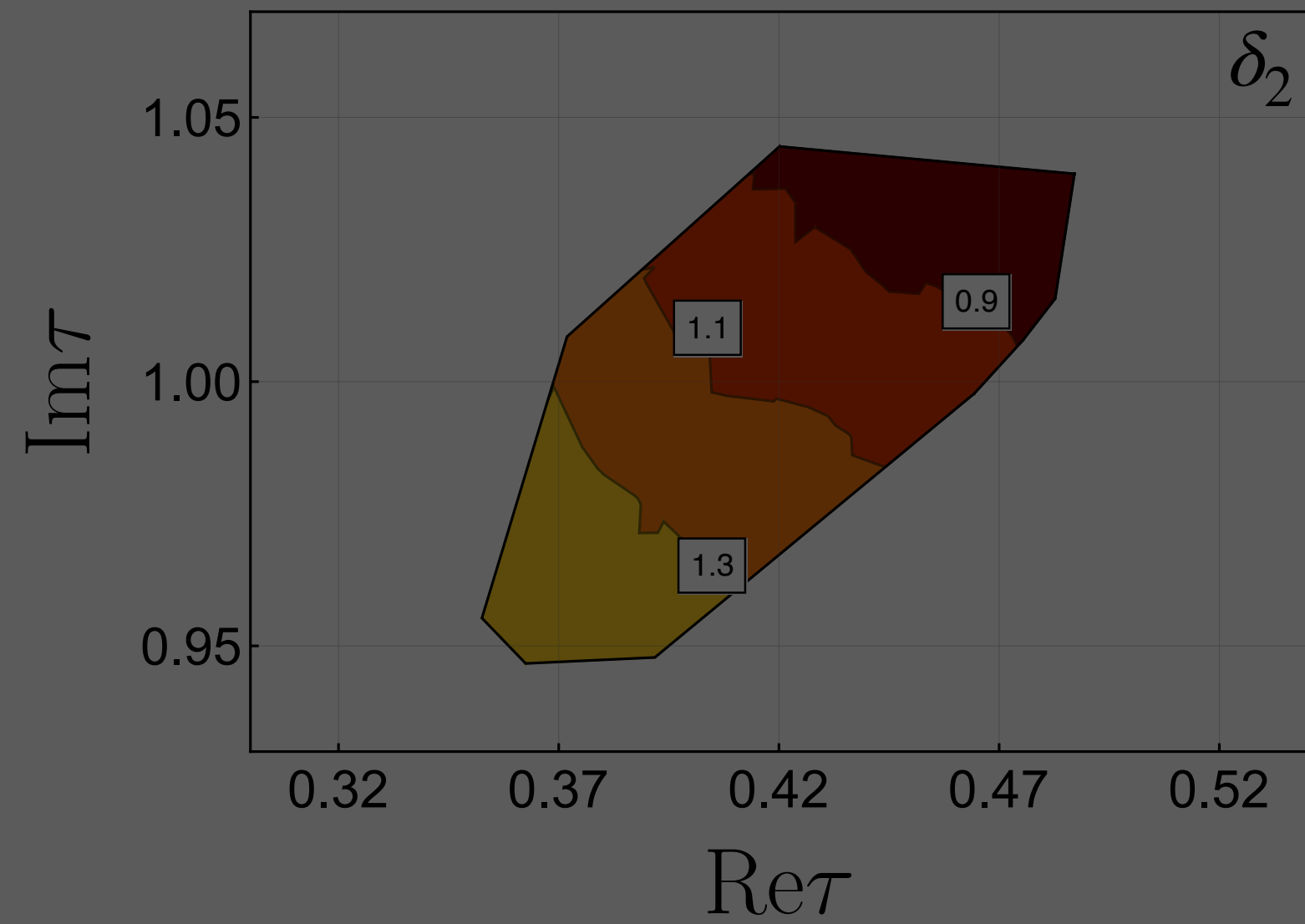


3σ points

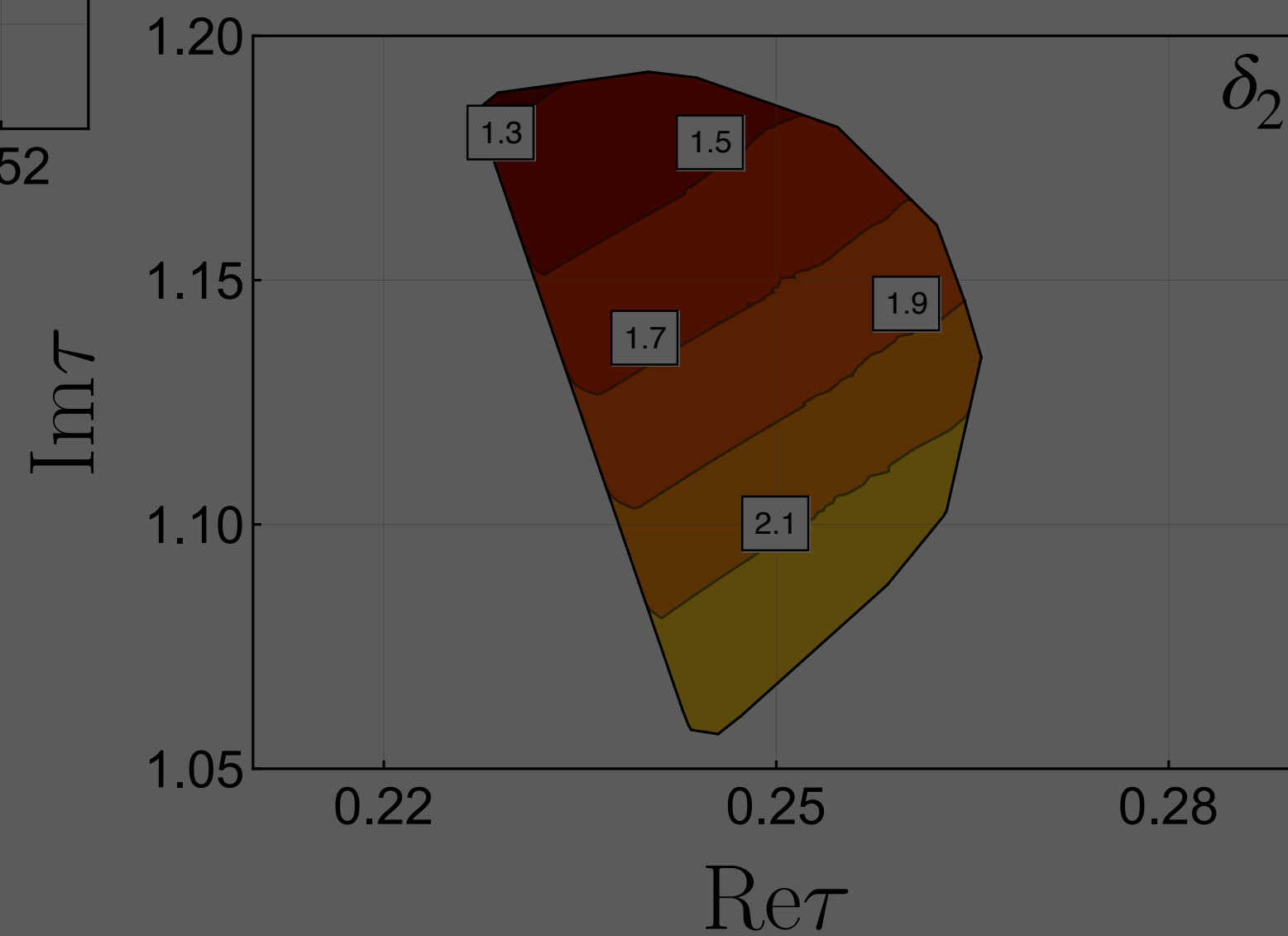


Leptogenesis from modular symmetry

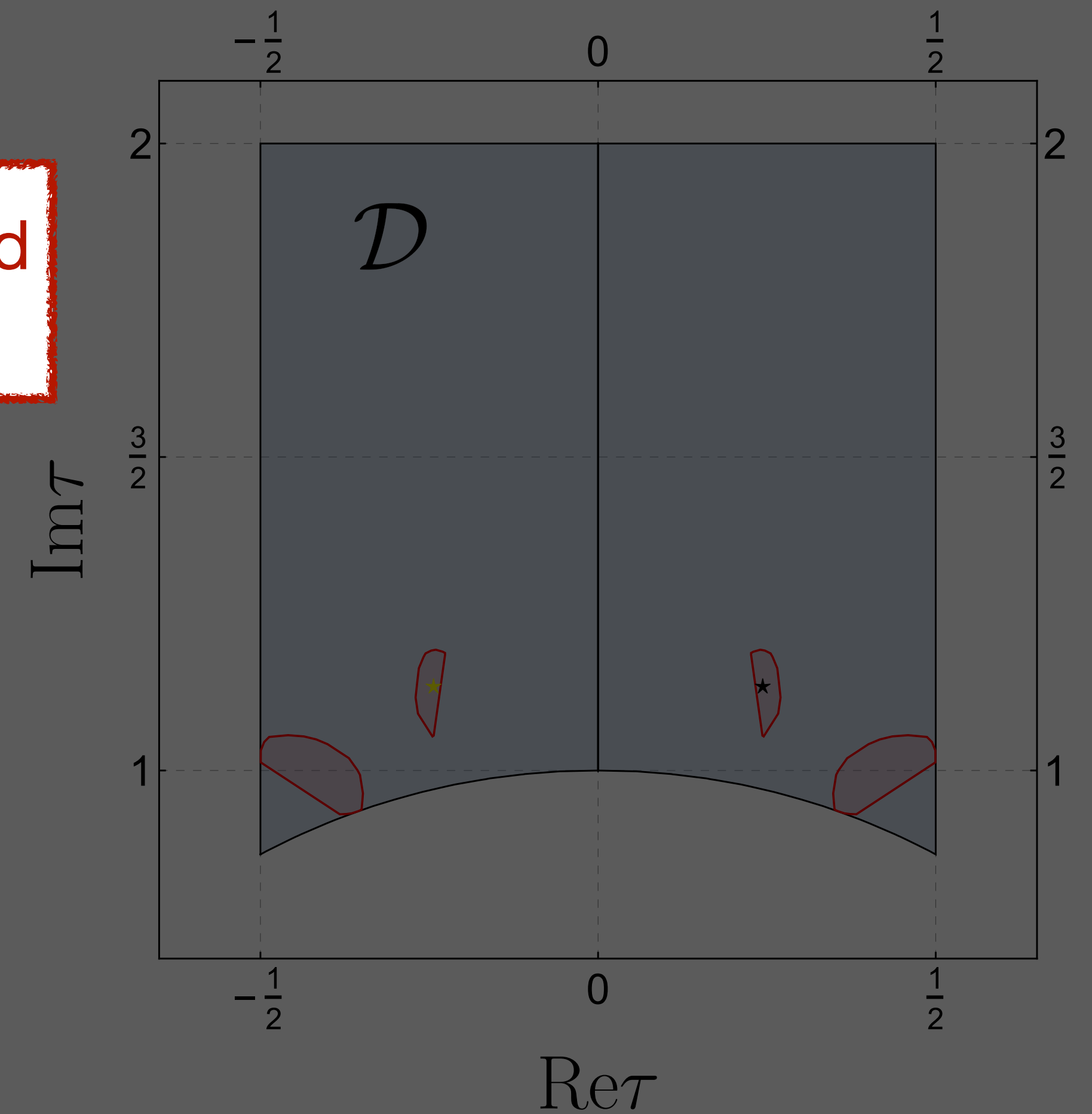
N_1 - dominated scenario?



"Mild" N_1 - dominated scenario



3σ points



Results

$$M_D = gv_u \begin{pmatrix} -(Y_2^2 - Y_1^2) + \frac{g''}{g}(Y_1^2 + Y_2^2) & 2Y_1Y_2 & \frac{g'}{g}(2Y_1Y_2) \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) + \frac{g''}{g}(Y_1^2 + Y_2^2) & -\frac{g'}{g}(Y_2^2 - Y_1^2) \end{pmatrix}_{\text{RL}}$$

$$\mathcal{M}_R = \Lambda \begin{pmatrix} -(Y_2^2 - Y_1^2) + \lambda(Y_1^2 + Y_2^2) & 2Y_1Y_2 \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) + \lambda(Y_1^2 + Y_2^2) \end{pmatrix}_{\text{RR}}$$

$$m_\nu = -M_D^T \mathcal{M}_R^{-1} M_D$$

Low-energy neutrino mixing and mass splittings will be mainly dictated by 5 parameters:

$$g^2 v_u^2 / \Lambda, \text{Re}\tau, \text{Im}\tau, g'/g, g''/g, \lambda$$

Results

$$M_D = g v_u \begin{pmatrix} -(Y_2^2 - Y_1^2) + \frac{g''}{g}(Y_1^2 + Y_2^2) & 2Y_1Y_2 & \frac{g'}{g}(2Y_1Y_2) \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) + \frac{g''}{g}(Y_1^2 + Y_2^2) & -\frac{g'}{g}(Y_2^2 - Y_1^2) \end{pmatrix}_{RL}$$

$$\mathcal{M}_R = \Lambda \begin{pmatrix} -(Y_2^2 - Y_1^2) + \lambda(Y_1^2 + Y_2^2) & 2Y_1Y_2 \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) + \lambda(Y_1^2 + Y_2^2) \end{pmatrix}_{RR}$$

$$m_\nu = -M_D^T \mathcal{M}_R^{-1} M_D$$

Low-energy neutrino mixing and mass splittings will be mainly dictated by 5 parameters:

$$g^2 v_u^2 / \Lambda, \text{Re}\tau, \text{Im}\tau, g'/g, g''/g, \lambda$$

$$g^2 v_u^2 / (r \Lambda)$$

$\Lambda \uparrow, g \uparrow :$

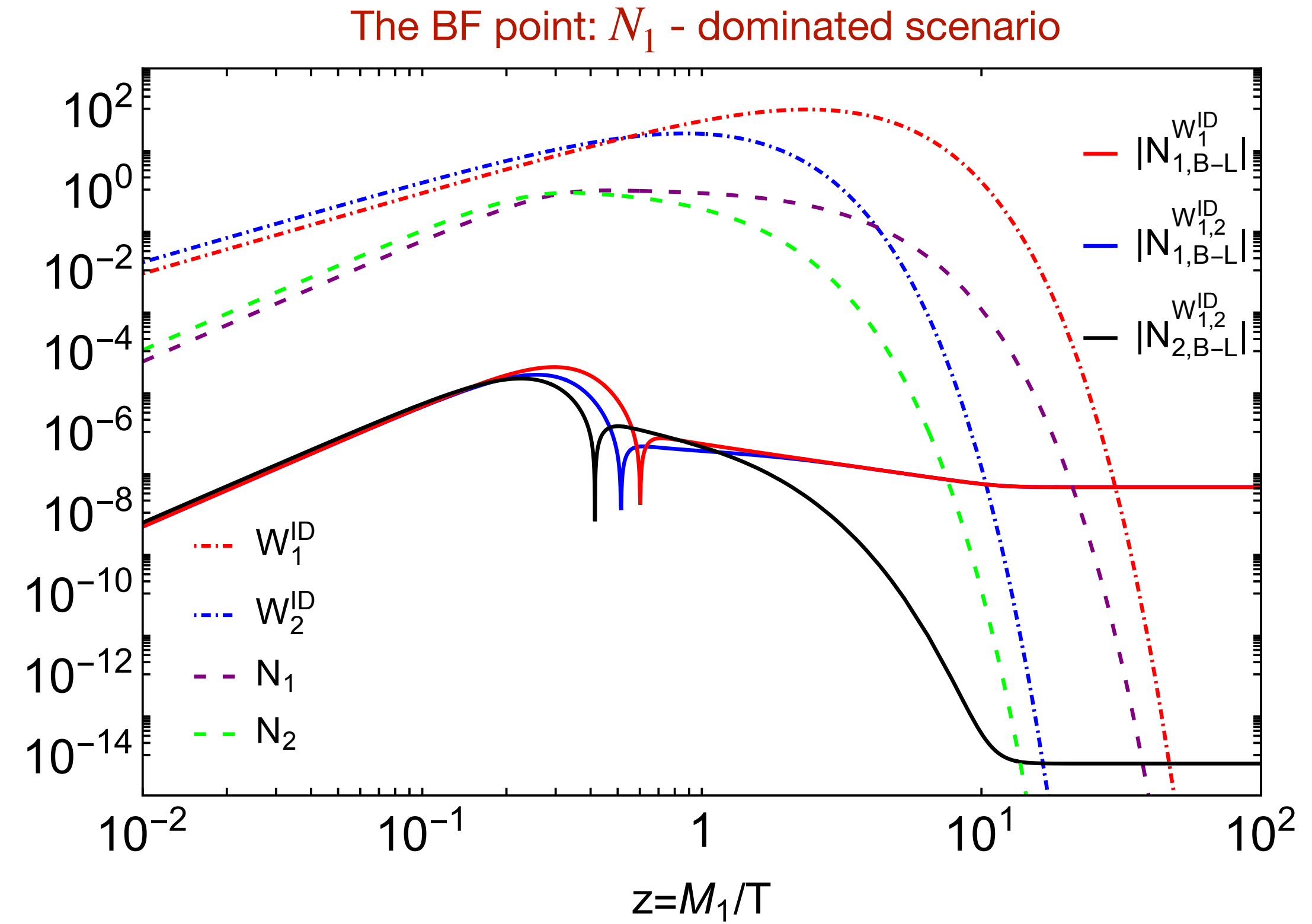
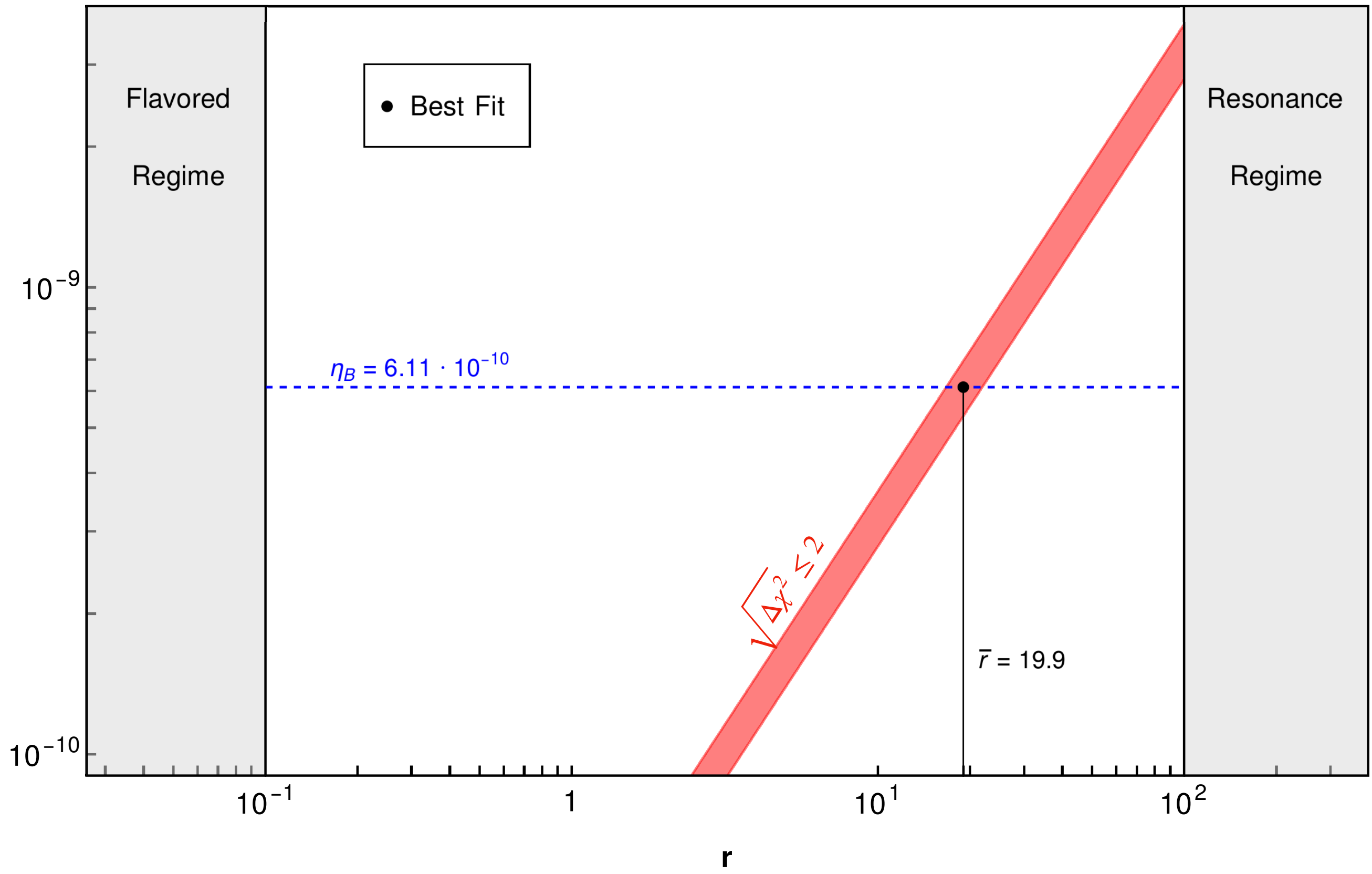
Resonant regime

$\Lambda \downarrow, g \downarrow :$

Flavored regime

Results

$g^2 v_u^2 / (r \Lambda)$ $\Lambda \uparrow, g \uparrow$: Resonant regime $\Lambda \downarrow, g \downarrow$: Flavored regime



Minimalist approach:

despite a small number of modular parameters and fields, the model provides the charged lepton masses, the neutrino masses as well the other low energy observables in great agreement with the measured values, without invoking *ad hoc* assignments

Boltzmann Equations:

a detailed analysis of the leptogenesis scenario is provided. The results are obtained solving numerically the full Boltzmann Equations, finding a realization for which the BAU is predicted inside the 3σ of the observed value. Such a realization, with $\bar{r} = 19.9$, preserve the *naturalness* of the model since it does not imply any further fine-tuning on the other parameters



Thank you!

[Minimal seesaw and leptogenesis with the smallest modular finite group](#)
[JHEP 05 \(2024\), 020](#)

S. Marciano

D. Meloni

M. Parriciatu



[Prague, 19/07/2024](#)



BACKUP SLIDES

[Minimal seesaw and leptogenesis with the smallest modular finite group](#)
[JHEP 05 \(2024\), 020](#)

S. Marciano

D. Meloni

M. Parriciatu



[Prague, 19/07/2024](#)

Smallest modular finite group

The Model in a nutshell

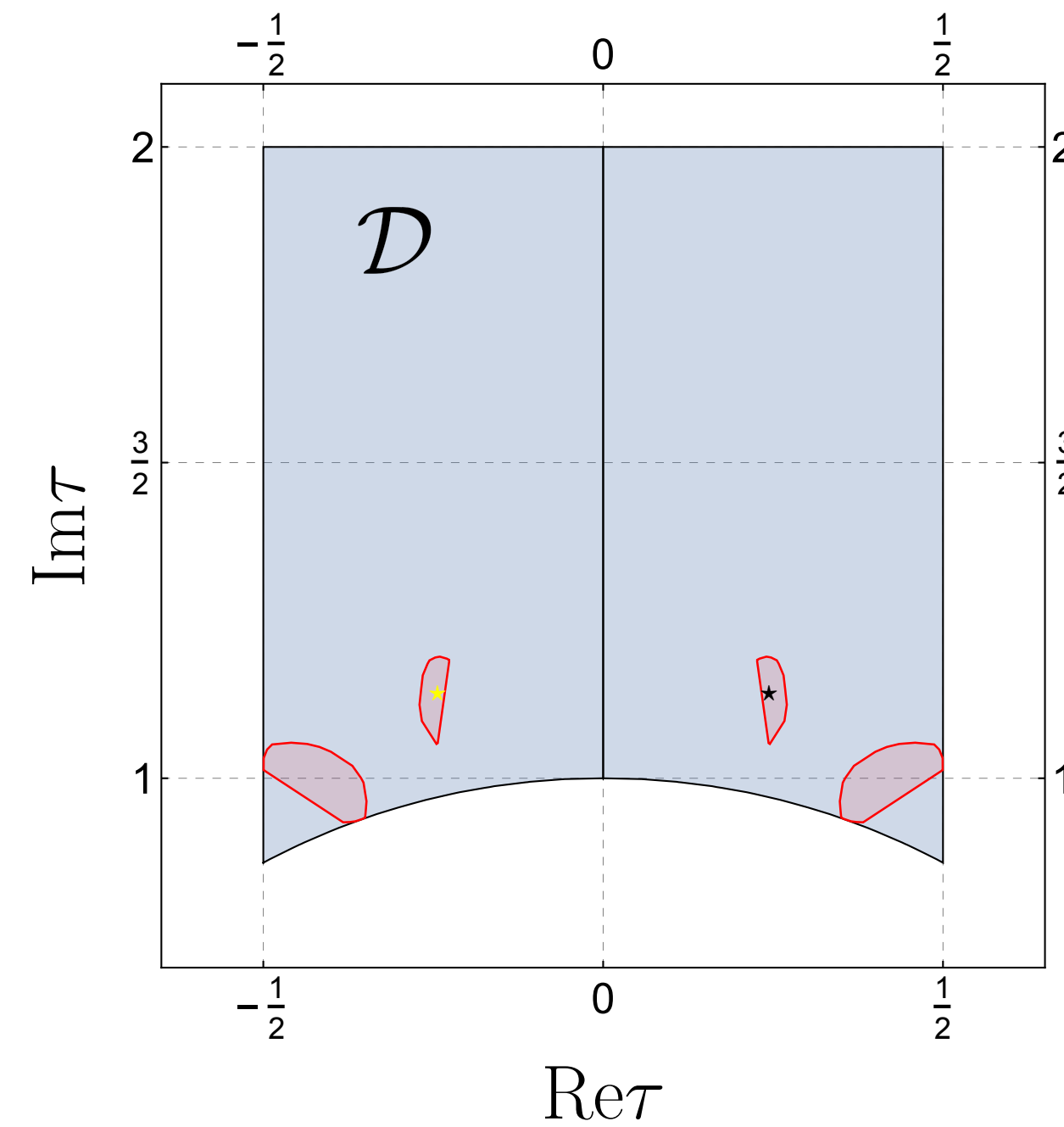
	E_1^c	E_2^c	E_3^c	D_ℓ	ℓ_3	$H_{d,u}$	N^c
$SU(2)_L \times U(1)_Y$	(1, +1)	(1, +1)	(1, +1)	(2, -1/2)	(2, -1/2)	(2, $\mp 1/2$)	(1, 0)
$\Gamma_2 \cong S_3$	1	1'	1'	2	1'	1	2
k_I	4	0	-2	2	2	0	2

$$\mathcal{W}_e^H = \alpha E_1^c H_d (D_\ell Y_2^{(3)})_1 + \beta E_2^c H_d (D_\ell Y_2)_1 + \gamma E_3^c H_d \ell_3 + \alpha_D E_1^c H_d \ell_3 Y_1^{(3)},$$

$$\mathcal{W}_\nu = g H_u N^c D_\ell Y_2^{(2)} + g' H_u (N^c Y_2^{(2)})_1 \ell_3 + g'' H_u (N^c D_\ell)_1 Y_1^{(2)} + \Lambda [(N^c N^c)_2 Y_2^{(2)} + \lambda (N^c N^c)_1 Y_1^{(2)}]$$

Parameter	Best-fit value and 1 σ range	
$\Delta m_{\text{sol}}^2 / (10^{-5} \text{ eV}^2)$	7.41 $^{+0.21}_{-0.20}$	
	NO	IO
$ \Delta m_{\text{atm}}^2 / (10^{-3} \text{ eV}^2)$	2.507 $^{+0.026}_{-0.027}$	2.486 $^{+0.025}_{-0.028}$
$r \equiv \Delta m_{\text{sol}}^2 / \Delta m_{\text{atm}}^2 $	0.0295 \pm 0.0008	0.0298 \pm 0.0008
$\sin^2 \theta_{12}$	0.303 $^{+0.012}_{-0.012}$	0.303 $^{+0.012}_{-0.011}$
$\sin^2 \theta_{13}$	0.02225 $^{+0.00056}_{-0.00059}$	0.0223 $^{+0.00058}_{-0.00058}$
$\sin^2 \theta_{23}$	0.451 $^{+0.019}_{-0.016}$	0.569 $^{+0.016}_{-0.021}$
m_e / m_μ	0.0048 \pm 0.0002	
m_μ / m_τ	0.0565 \pm 0.0045	

I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, "The fate of hints: updated global analysis of three-flavor neutrino oscillations," KHEP 09 (2020) 178, 2007.14792 [hep-ph]



Altarelli-Blankenburg

$$d_{\text{FT}} = \frac{\sum_i \left| \frac{\text{par}_i}{\delta \text{par}_i} \right|}{\sum_i \left| \frac{\text{obs}_i}{\sigma_i} \right|}$$

$\star(-)$ $\star(+)$	Best-fit and 1 σ range
	$\text{Re } \tau$ $\pm 0.244^{+0.012}_{-0.067}$
	$\text{Im } \tau$ $1.132^{+0.027}_{-0.297}$
	β/α $0.92^{+0.85}_{-0.03}$
	γ/α $-1.20^{+0.06}_{-2.14}$
	$\log_{10}(\alpha_D/\alpha)$ $-13.4^{+13.2}_{-76.3}$
	g'/g $2.76^{+0.21}_{-0.23}$
	g''/g $-2.53^{+0.13}_{-0.03}$
	$\log_{10}(\lambda)$ $-12.2^{+10.9}_{-59.2}$
	$v_d \alpha$, [GeV] $1.08^{+0.06}_{-0.69}$
	$v_u^2 g^2 / \Lambda$ [eV] $3.46^{+0.55}_{-1.65}$
	$\sin^2 \theta_{12}$ $0.305^{+0.011}_{-0.011}$
	$\sin^2 \theta_{13}$ $0.0221^{+0.0006}_{-0.0005}$
	$\sin^2 \theta_{23}$ $0.448^{+0.014}_{-0.016}$
	r $0.0296^{+0.0006}_{-0.0008}$
	m_e / m_μ $0.0048^{+0.0001}_{-0.0002}$
	m_μ / m_τ $0.0574^{+0.0032}_{-0.0050}$
	Ordering NO
	J_{CP} $-0.018^{+0.002}_{-0.002}$
	α_1/π 0
	α_2/π $\pm 0.112^{+0.792}_{-0.014}$
	m_1 [meV] 0
	m_2 [meV] $8.620^{+0.095}_{-0.123}$
	m_3 [meV] $50.806^{+0.016}_{-0.021}$
	$\sum_i m_i$ [eV] $0.0594^{+0.0001}_{-0.0001}$
	$ m_{\beta\beta} $ [meV] $3.61^{+0.09}_{-0.09}$
	m_β^{eff} [meV] $8.90^{+0.10}_{-0.09}$
	d_{FT} 3.03
	χ_{min}^2 0.98

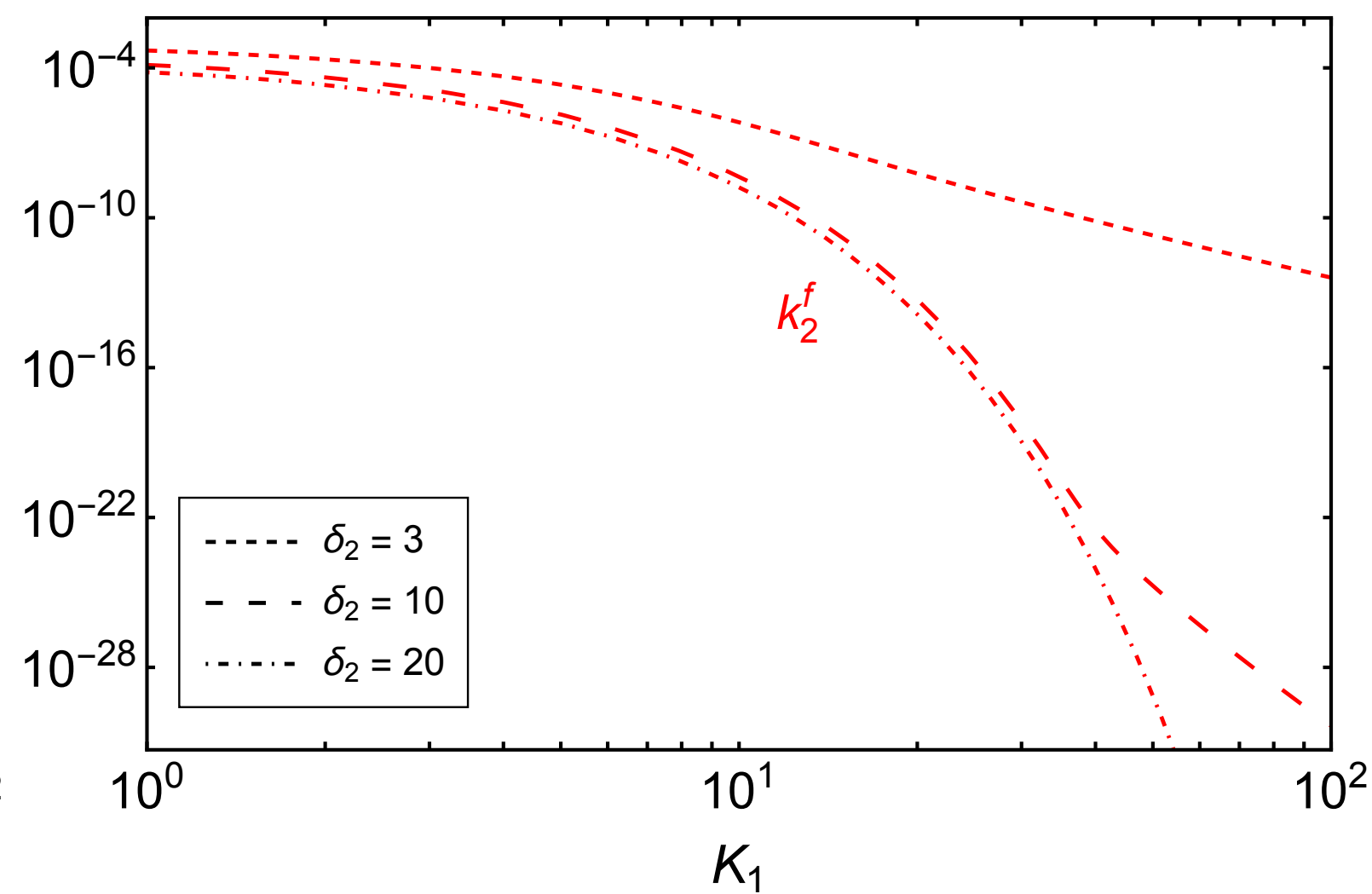
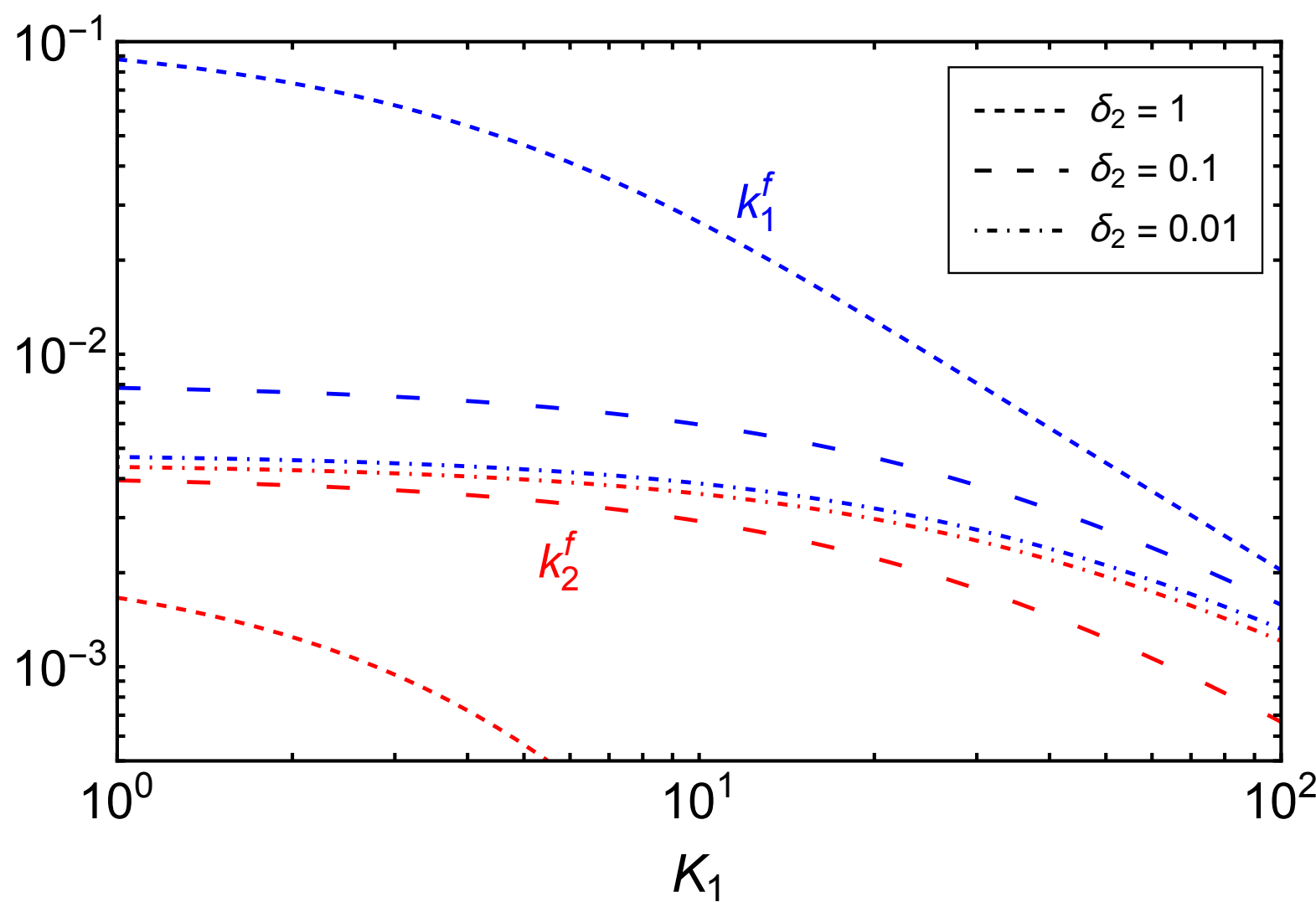
Leptogenesis from modular symmetry

N_1 - dominated scenario?

$$N_{B-L}^f = \sum_i \epsilon_i k_i^f, \quad i = 1, 2$$

$$k_i^f = k_i(z = \infty) = - \int_{z_{in} \rightarrow 0}^{z_{fin} \rightarrow \infty} \frac{dN_i}{dz'} \text{Exp} \left[\sum_i \int_{z'}^z W_i(z'') dz'' \right] dz'$$

Strong dependence on $\delta_2 = \frac{M_2 - M_1}{M_1}$



Leptogenesis from modular symmetry

N_1 - dominated scenario?

$$N_{B-L}^f = \sum_i \epsilon_i k_i^f, \quad i = 1, 2$$

$$k_i^f = k_i(z = \infty) = - \int_{z_{in} \rightarrow 0}^{z_{fin} \rightarrow \infty} \frac{dN_i}{dz'} \text{Exp} \left[\sum_i \int_{z'}^z W_i(z'' dz'') \right] dz'$$

Strong dependence on $\delta_2 = \frac{M_2 - M_1}{M_1}$

