How is CPV in leptogenesis connected to CPV in ν-oscillations?

—— the first general + explicit link in the type-I seesaw ——

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Theorists:
Type-I seesaw mechanism:

- 3 heavy neutrino masses
- 9 active-sterile mixing angles
- 6 CP-violating phases

Hundreds of model-dependent connections on the market

Mr. Seesaw

Experimentalists:
Low-energy measurements:

- 3 light neutrino masses
- 3 active mixing angles
- 3 CP-violating phases

The 1st model-independent link (ZZX, 2406.01142)

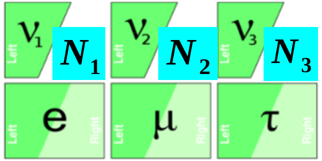
The 42nd International Conference on High Energy Physics (ICHEP), Prague, 18—24.7.2024

Seesaw: high gains + low costs

- **Fundamentals** of the electroweak SM structure → **reasons** for zero ν-mass:
  - The Lorentz invariance
  - Local $SU(2)_L \times U(1)_Y$ gauge symmetries
  - The Higgs mechanism
  - Renormalizability (no d ≥ 5 operators)

Plus *economical* particle content:

- No right-handed neutrino fields
- Only one Higgs doublet



$$\underbrace{\boxed{m_1 = m_2 = m_3 = 0}}_{}$$

Go beyond the d=4 operators

SMEFT

The unique d=5 operator for ν-mass and Majorana nature

$$\boxed{\mathcal{O}_{\mathrm{W}} = \frac{\overline{\ell_{\mathrm{L}}}\widetilde{H}\widetilde{H}^T \ell_{\mathrm{L}}^c}{\Lambda}}$$

Steven Weinberg 1979

R-handed neutrinos + Majorana character

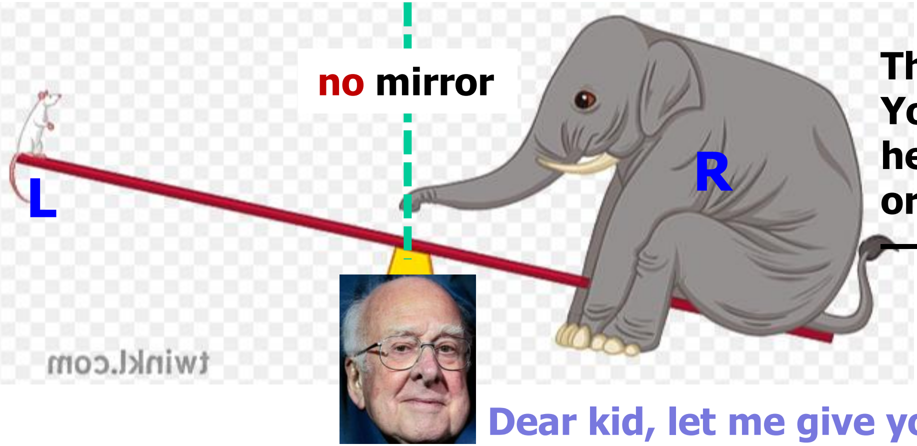
- The most convincing *UV* mechanism = canonical seesaw

fully consistent with the SMEFT spirit, most natural/economical extension of the SM. *Bonus: Leptogenesis, SO(10) GUT-friendly…*

Integrate out the heavy dof

Three key issues of the seesaw

- **Right-handed** neutrino fields are **not** the **mirror** counterparts of the **left-handed** ones



It is said that I was **weightless** at birth, and it was you who fed me up a bit.

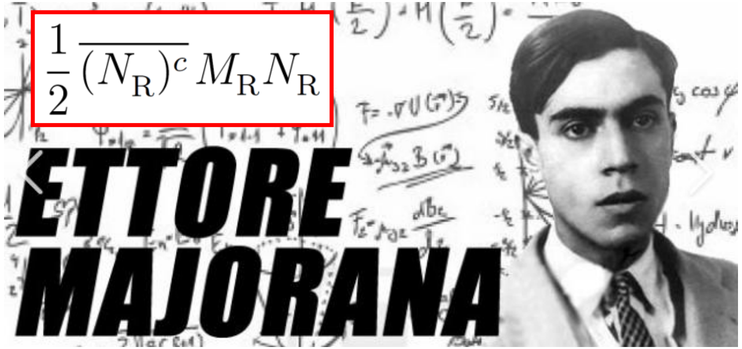
That is true, my love!
You may call me
heavy neutrinos
or heavy neutral leptons
—— both sound strange!

Dear kid, let me give you mass on behalf of God

- **Yukawa interactions** —— the **Higgs** fields play a crucial role, as they do in generating masses for the charged fermions in the SM.

- The **Majorana** nature of massive neutrinos:
  N and N^c may have *self-interactions*, respecting all the fundamental symmetries of the SM.

> **Gell-Mann**'s **totalitarian principle** (**1956**)
> *Everything not forbidden is compulsory!*



$$\frac{1}{2}\overline{(N_{\rm R})^c}\,M_{\rm R}N_{\rm R}$$

It works both before and after SSB

The seesaw mechanism formally works far above the Fermi scale, before SSB (ZZX, 2301.10461):

$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}}\, Y_l H\, l_{\rm R} + \overline{\ell_{\rm L}}\, Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c}\, M_{\rm R} N_{\rm R} + {\rm h.c.}$$

$$= \overline{l_{\rm L}}\, Y_l\, l_{\rm R} \underset{\uparrow}{\phi^0} + \frac{1}{2}\overline{\begin{bmatrix}\nu_{\rm L} & (N_{\rm R})^c\end{bmatrix}} \underline{\begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix}} \begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + \overline{\nu_{\rm L}}\, Y_l\, l_{\rm R} \underset{\uparrow}{\phi^+} - \overline{l_{\rm L}}\, Y_\nu N_{\rm R} \underset{\uparrow}{\phi^-} + {\rm h.c.}$$

The basis transformation related to the origin of active Majorana neutrino masses even before SSB:

$$\mathbb{U}^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix} \mathbb{U}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix}$$

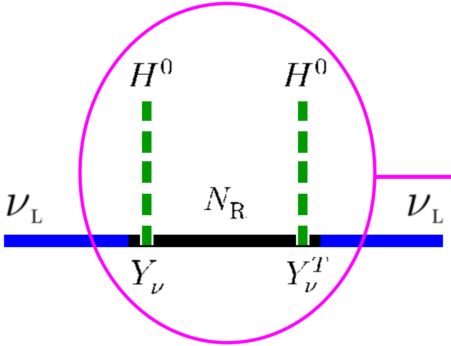
working masses:

$$\begin{cases} D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\} & \textbf{light} \\ D_N \equiv {\rm Diag}\{M_4, M_5, M_6\} & \textbf{heavy} \end{cases}$$

$$\Big\downarrow \ \textbf{SSB}$$

$$\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix}$$

6 × 6 mass matrix



Integrating out the heavy degrees of freedom:

$$\boxed{-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}} M_\nu \nu_{\rm L}^c + {\rm h.c.}} \qquad M_\nu \simeq -Y_\nu \frac{\langle H\rangle^2}{M_{\rm R}} Y_\nu^T$$

Consistent with the dim-5 Weinberg operator!

If you can untie Weinberg's knot, you will find new heavy Majorana neutrinos at a superhigh scale.

A full parameterization of seesaw

A ***block parametrization*** **of active-sterile flavor mixing in the seesaw framework:**

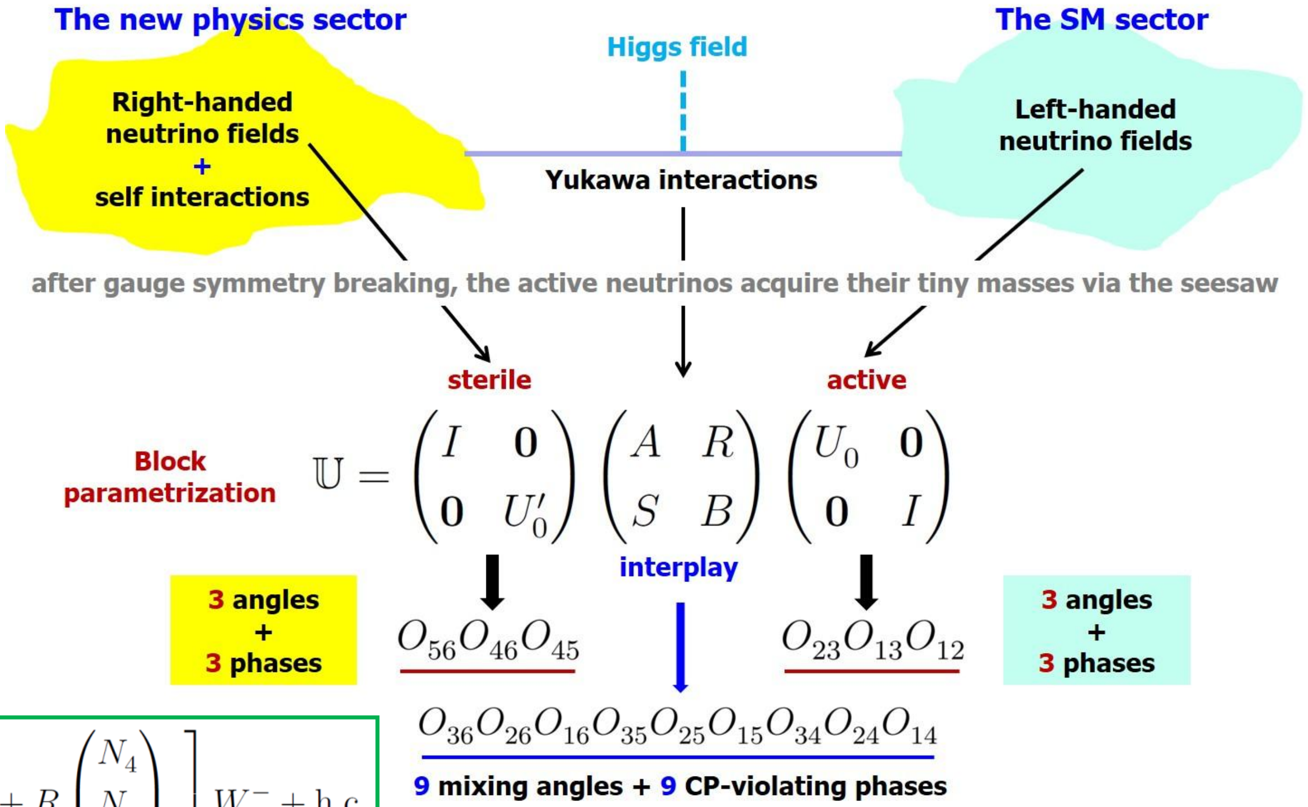
- ♦ **reflects salient features of the seesaw dynamics**
- ♦ **offers *generic* + *explicit* expressions of observables using the Euler-like angles and phases (ZZX, 1110.0083)**

The weak charged-current interactions of leptons:

$U = AU_0$: the PMNS matrix;
R : an analogue for heavy.

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\,\overline{\left(e \quad \mu \quad \tau\right)_{\rm L}}\,\gamma^{\mu}\left[U\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}_{\rm L} + R\begin{pmatrix}N_4\\ N_5\\ N_6\end{pmatrix}_{\rm L}\right]W^{-}_{\mu} + {\rm h.c.}$$

oscillations ⟵ light **heavy ⟶ leptogenesis**



The new physics sector

Right-handed neutrino fields + self interactions

Higgs field

Yukawa interactions

The SM sector

Left-handed neutrino fields

after gauge symmetry breaking, the active neutrinos acquire their tiny masses via the seesaw

sterile **active**

Block parametrization

$$\mathbb{U} = \begin{pmatrix} I & 0\\ 0 & U_0'\end{pmatrix}\begin{pmatrix} A & R\\ S & B\end{pmatrix}\begin{pmatrix} U_0 & 0\\ 0 & I\end{pmatrix}$$

interplay

3 angles + 3 phases

$$\underline{O_{56}O_{46}O_{45}} \qquad \underline{O_{23}O_{13}O_{12}}$$

3 angles + 3 phases

$$\underline{O_{36}O_{26}O_{16}O_{35}O_{25}O_{15}O_{34}O_{24}O_{14}}$$

9 mixing angles + 9 CP-violating phases

seesaw + unitarity:
$$\begin{cases} UD_{\nu}U^{T} + RD_{N}R^{T} = \mathbf{0}\\ UU^{\dagger} + RR^{\dagger} = I\end{cases}$$

Original vs derivational seesaw parameters

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

derivational parameters!

$$c_{ij} \equiv \cos\theta_{ij}$$

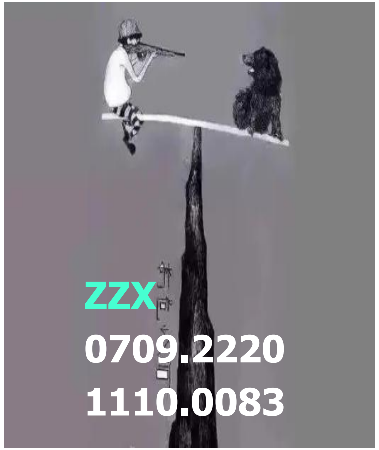
$$\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin\theta_{ij} \ (\text{for } 1 \leq i < j \leq 6)$$

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ \begin{array}{l} -c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ -\hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & c_{24}c_{25}c_{26} & 0 \\ \begin{array}{l} -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + \hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} \\ -\hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & c_{34}c_{35}c_{36} \end{pmatrix}$$

$$R = \begin{pmatrix} \hat{s}^*_{14}c_{15}c_{16} & \hat{s}^*_{15}c_{16} & \hat{s}^*_{16} \\ \begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ +c_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & -\hat{s}^*_{15}\hat{s}_{16}\hat{s}^*_{26} + c_{15}\hat{s}^*_{25}c_{26} & c_{16}\hat{s}^*_{26} \\ \begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} - c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} + c_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -\hat{s}^*_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +c_{15}c_{25}\hat{s}^*_{35}c_{36} \end{array} & c_{16}c_{26}\hat{s}^*_{36} \end{pmatrix}$$

The original seesaw parameters in A+R:

9 angles + 6 phases



ZZX

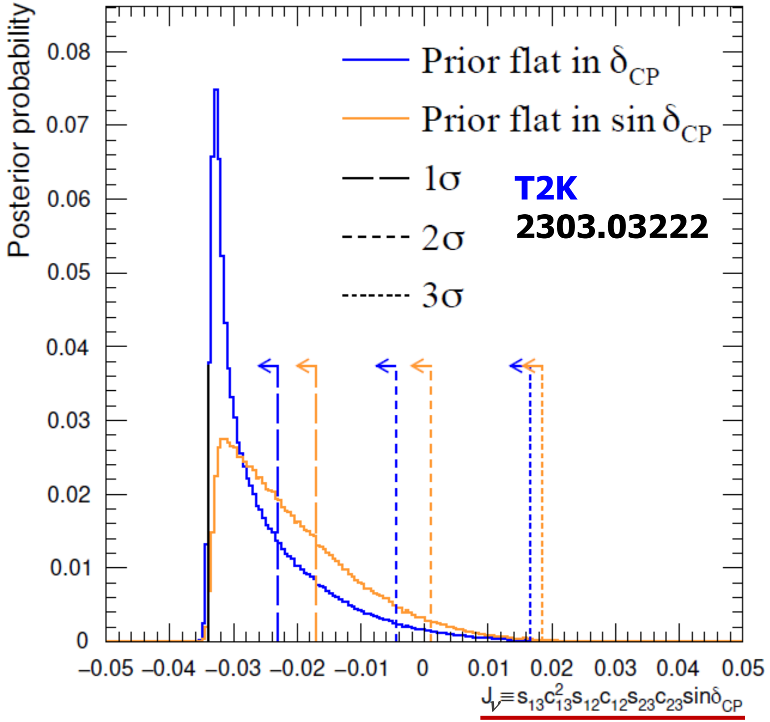
0709.2220

1110.0083

You may calculate everything that can in principle be measured, in terms of 18 seesaw parameters.

Two kinds of fundamental CP violation

- ♦ **3-flavor ν-oscillations are established and a 2σ hint for CPV is achieved.**



T2K 2303.03222

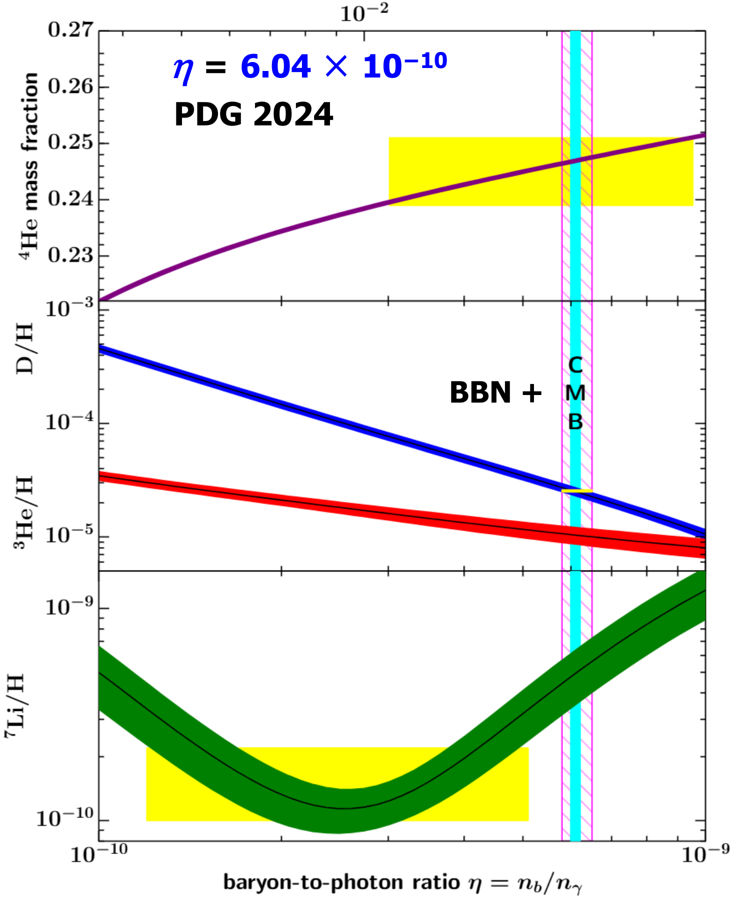
$$P(\nu_\mu \to \nu_e) = -4\sum_{i<j}\left(\mathcal{R}_{ij}\sin^2\frac{\Delta_{ji}L}{4E}\right) - 8\underline{\mathcal{J}_\nu}\prod_{i<j}\sin\frac{\Delta_{ji}L}{4E}$$

direct connection
via
seesaw
+
Leptogenesis
?

NO
W. Buchmüller
M. Plümacher:
hep-ph/9608308

YES
ZZX, 2406.01142
↓

- ♦ **Cosmic CPV is already established.**



η = 6.04 × 10^{-10}
PDG 2024

BBN + CMB

A bridge between light and heavy

- **The exact seesaw formula —— a bridge between the original and derivational flavor parameters:**

$$\boxed{U D_\nu U^T + R D_N R^T = \mathbf{0}} \longrightarrow \boxed{M_\nu \equiv U_0 D_\nu U_0^T = \left(\mathrm{i} A^{-1} R\right) D_N \left(\mathrm{i} A^{-1} R\right)^T}$$

Degrees of freedom (mass + mixing angle + CPV phase): 3 + 3 + 3 (derivational) ⟵ 3 + 9 + 6 (original)

- **The Jarlskog invariant of CP violation in ν-oscillations:**

$$\boxed{\mathcal{J}_\nu \equiv \mathrm{Im}\left[\left(U_0\right)_{\alpha i} \left(U_0\right)_{\beta i^\prime} \left(U_0\right)^*_{\alpha i^\prime} \left(U_0\right)^*_{\beta i}\right]}$$

(α, β) run cyclically over (e, μ, τ), $(i, i^\prime)$ run cyclically over $(1, 2, 3)$

$$\left\{\begin{array}{l} D_\nu \equiv \mathrm{Diag}\{m_1, m_2, m_3\} \\ D_N \equiv \mathrm{Diag}\{M_4, M_5, M_6\} \\ \Delta_{ii^\prime} \equiv m_i^2 - m_{i^\prime}^2 \end{array}\right.$$

- **On the one hand, we use the light degrees of freedom to get the relation (ZZX, 2306.02362)**

$$\mathcal{J}_\nu = \frac{\mathrm{Im}\left[\left(M_\nu M_\nu^\dagger\right)_{e\mu} \left(M_\nu M_\nu^\dagger\right)_{\mu\tau} \left(M_\nu M_\nu^\dagger\right)_{\tau e}\right]}{\Delta_{21} \Delta_{31} \Delta_{32}} \longleftarrow \text{already measured}$$

- **On the other hand, we use the original seesaw-related parameters to calculate the same quantity *in the leading order approximation* of *$A^{-1}R$*, because the non-unitarity of *U* characterized by *$R \neq 0$* has been well constrained by precision measurements (M. Blennow et al, 2306.01040)**

$$A^{-1} R \simeq \begin{pmatrix} \hat{s}^*_{14} & \hat{s}^*_{15} & \hat{s}^*_{16} \\ \hat{s}^*_{24} & \hat{s}^*_{25} & \hat{s}^*_{26} \\ \hat{s}^*_{34} & \hat{s}^*_{35} & \hat{s}^*_{36} \end{pmatrix}$$

Is this approximation really safe?

- Of course, one may use the non-unitary **PMNS** matrix ***U*** = ***AU***$_0$ to define the more general ***Jarlskog*** invariants to describe CP violation in neutrino oscillations. But one can show that their leading terms are the same, coming from the unitarity limit (**ZZX**, 1110.0083):

$$\boxed{\mathcal{J}^{ii'}_{\alpha\beta} \equiv \mathrm{Im}\left(U_{\alpha i} U_{\beta i'} U^*_{\alpha i'} U^*_{\beta i}\right)}$$

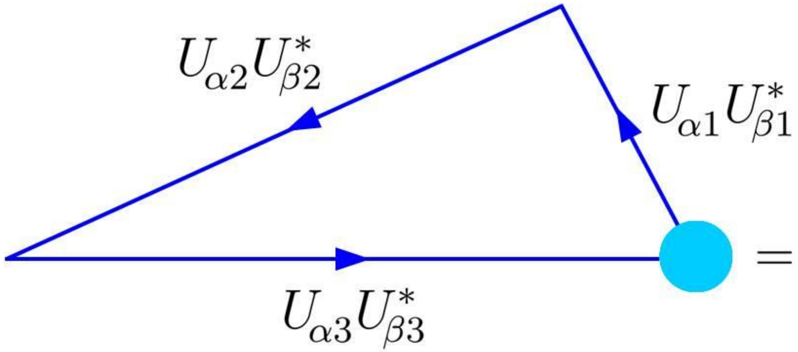
(α, β) run cyclically over (e, μ, τ)

(i, i') run cyclically over $(1, 2, 3)$

$$\boxed{\mathcal{J}^{ii'}_{\alpha\beta} = \mathcal{J}_\nu + \text{corrections}}$$

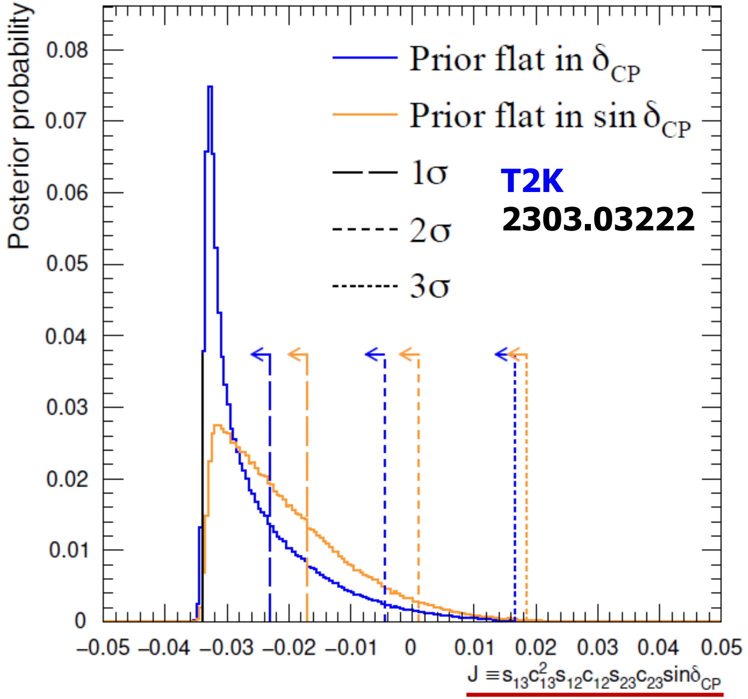
↑ $\leq$ 1%   ↑ $<$ 0.01%

- Yes, absolutely safe, at least by 2044!



$$= \sum_{j=4}^{6} \left(R_{\alpha j} R^*_{\beta j}\right)$$

$$\boxed{\begin{array}{l}\alpha = e \\ \beta = \mu\end{array}}$$



How many terms to be calculated?

- **Let us classify the analytical results in terms of the products of heavy Majorana neutrino masses.**

$$\mathrm{Im}\left[\left(M_\nu M_\nu^\dagger\right)_{e\mu}\left(M_\nu M_\nu^\dagger\right)_{\mu\tau}\left(M_\nu M_\nu^\dagger\right)_{\tau e}\right]$$

$$I_{jk} \equiv \sum_{i=1}^{3} \hat{s}_{ij}^{*}\hat{s}_{ik} = I_{kj}^{*} \; , \; (j,k=4,5,6)$$

$$\left(M_\nu M_\nu^\dagger\right)_{e\mu} = \sum_{j=4}^{6}\sum_{k=4}^{6} M_j M_k I_{jk} \hat{s}_{1j}^{*}\hat{s}_{2k}$$

$$\left(M_\nu M_\nu^\dagger\right)_{\mu\tau} = \sum_{j=4}^{6}\sum_{k=4}^{6} M_j M_k I_{jk} \hat{s}_{2j}^{*}\hat{s}_{3k}$$

$$\left(M_\nu M_\nu^\dagger\right)_{\tau e} = \sum_{j=4}^{6}\sum_{k=4}^{6} M_j M_k I_{jk} \hat{s}_{3j}^{*}\hat{s}_{1k}$$

CPV from 2-family or (and) 3-family interferences

Term 6 0 0: $M_4^6, \; M_5^6, \; M_6^6$ ×

Term 5 1 0: $M_4^5 M_5, \; M_4^5 M_6, \; M_5^5 M_4, \; M_5^5 M_6, \; M_6^5 M_4, \; M_6^5 M_5$ ×

Term 4 2 0: $M_4^4 M_5^2, \; M_4^4 M_6^2, \; M_5^4 M_4^2, \; M_5^4 M_6^2, \; M_6^4 M_4^2, \; M_6^4 M_5^2$ √

Term 3 3 0: $M_4^3 M_5^3, \; M_4^3 M_6^3, \; M_5^3 M_6^3$ √

Term 4 1 1: $M_4^4 M_5 M_6, \; M_5^4 M_4 M_6, \; M_6^4 M_4 M_5$ √

Term 3 2 1: $M_4^3 M_5^2 M_6, \; M_4^3 M_6^2 M_5, \; M_5^3 M_4^2 M_6, \; M_5^3 M_6^2 M_4, \; M_6^3 M_4^2 M_5, \; M_6^3 M_5^2 M_4$ √

Term 2 2 2: $M_4^2 M_5^2 M_6^2$ √

- **There are totally 6 independent original CP-violating phases in the canonical seesaw mechanism, measuring the *inter-family interference effects* in all processes of heavy and light Majorana neutrinos.**

$$\left\{\begin{array}{l} \alpha_i \equiv \delta_{i4} - \delta_{i5} \\ \beta_i \equiv \delta_{i5} - \delta_{i6} \\ \gamma_i \equiv \delta_{i6} - \delta_{i4} \end{array}\right.$$

$$\alpha_i + \beta_i + \gamma_i = 0 \; (\mathrm{for} \; i = 1, 2, 3)$$

So we arrive at … (1)

- **The general and explicit expression of the Jarlskog invariant in the seesaw mechanism is a linear combination of the above 5 terms:**

$$\mathcal{J}_\nu = \frac{T_{\mathbf{33}} + T_{\mathbf{42}} + T_{\mathbf{411}} + T_{\mathbf{321}} + T_{\mathbf{222}}}{\Delta_{21}\Delta_{31}\Delta_{32}}$$

- **The two *2-family interference* terms are:**

$$\begin{aligned} T_{\mathbf{33}} = {} & \underline{M_4^3 M_5^3} \left(I_{44} I_{55} - |I_{45}|^2\right) \left[\sum_{i=1}^{3} s_{i4} s_{i5} s_{i'4} s_{i'5} \left(s_{i4}^2 s_{i''5}^2 + s_{i''4}^2 s_{i'5}^2 - s_{i'4}^2 s_{i''5}^2 - s_{i''4}^2 s_{i5}^2\right) \underline{\sin\left(\alpha_i + \alpha_{i'}\right)} \right. \\ & \left. - \sum_{i=1}^{3} s_{i4}^2 s_{i5}^2 \left(s_{i'4}^2 s_{i''5}^2 - s_{i''4}^2 s_{i'5}^2\right) \underline{\sin 2\alpha_i} \right] \\ & + \mathrm{term}\{4 \to 5,\ 5 \to 6;\ \alpha_i \to \beta_i\} + \mathrm{term}\{4 \to 6,\ 5 \to 4;\ \alpha_i \to \gamma_i\} \end{aligned}$$

i, i' and i'' run cyclically over 1, 2 and 3

$$\begin{aligned} T_{\mathbf{42}} = {} & \underline{M_4^2 M_5^2} \left(I_{44} I_{55} - |I_{45}|^2\right) \left[\sum_{i=1}^{3} s_{i4} s_{i5} s_{i'4} s_{i'5} \left(\underline{M_4^2} I_{44} s_{i''4}^2 - \underline{M_5^2} I_{55} s_{i''5}^2\right) \underline{\sin\left(\alpha_i - \alpha_{i'}\right)}\right] \\ & + \mathrm{term}\{4 \to 5,\ 5 \to 6;\ \alpha_i \to \beta_i\} + \mathrm{term}\{4 \to 6,\ 5 \to 4;\ \alpha_i \to \gamma_i\} \end{aligned}$$

Switching off the 3rd heavy neutrino species "6", we can immediately arrive at the results in the *minimal seesaw* case (ZZX, 2306.02362):

→

9 combinations of 3 original CPV phases

So we arrive at … (2)

- **The simplest *3-family interference* term is obtained as follows:**

$$
\begin{aligned}
T_{\mathbf{411}} = \underline{M_4^4 M_5 M_6} I_{44} s_{14} s_{24} s_{34} \Bigg[& - s_{14} s_{24} s_{34} \sum_{i=1}^{3} s_{i5}^2 \Big[s_{i'6}^2 \underline{\sin 2\left(\alpha_i + \gamma_{i'}\right)} - s_{i''6}^2 \underline{\sin 2\left(\alpha_i + \gamma_{i''}\right)} \Big] \\
& + \sum_{i=1}^{3} s_{i4} \left(s_{i'4}^2 - s_{i''4}^2 \right) \Big[s_{i5}^2 s_{i'6} s_{i''6} \underline{\sin\left(2\alpha_i + \gamma_{i'} + \gamma_{i''}\right)} - s_{i6}^2 s_{i'5} s_{i''5} \underline{\sin\left(\alpha_{i'} + \alpha_{i''} + 2\gamma_i\right)} \Big] \\
& + \sum_{i=1}^{3} s_{i4} s_{i'5} s_{i'6} \left(s_{i4}^2 + s_{i''4}^2 \right) \Big[s_{i'5} s_{i''6} \underline{\sin\left(\alpha_{i'} - \beta_{i'} + \gamma_{i''}\right)} - s_{i'6} s_{i''5} \underline{\sin\left(\alpha_{i''} - \beta_{i'} + \gamma_{i'}\right)} \Big] \\
& + \sum_{i=1}^{3} s_{i4} s_{i''5} s_{i''6} \left(s_{i4}^2 + s_{i'4}^2 \right) \Big[s_{i'5} s_{i''6} \underline{\sin\left(\alpha_{i'} - \beta_{i''} + \gamma_{i''}\right)} - s_{i'6} s_{i''5} \underline{\sin\left(\alpha_{i''} - \beta_{i''} + \gamma_{i'}\right)} \Big] \\
& + \sum_{i=1}^{3} 2 s_{i4} s_{i5} s_{i6} \left(s_{i'4}^2 + s_{i''4}^2 \right) \Big[s_{i'5} s_{i''6} \underline{\sin\left(\alpha_{i'} - \beta_i + \gamma_{i''}\right)} - s_{i'6} s_{i''5} \underline{\sin\left(\alpha_{i''} - \beta_i + \gamma_{i'}\right)} \Big] \Bigg] \\
& + \text{term}\Big\{ (4,5,6) \to (5,4,6)\,;\ (\alpha_i,\ \beta_i,\ \gamma_i) \to -(\alpha_i,\ \gamma_i,\ \beta_i) \Big\} \\
& + \text{term}\Big\{ (4,5,6) \to (6,5,4)\,;\ (\alpha_i,\ \beta_i,\ \gamma_i) \to -(\beta_i,\ \alpha_i,\ \gamma_i) \Big\}
\end{aligned}
$$

The terms T_{321} + T_{222} are very complicated and can be found in ZZX, 2406.01142.

Counting the phase combinations (1)

- **After a very tedious survey of all terms of the Jarlskog invariant, we find 240 linear combinations of the 6 original seesaw phase parameters —— 72 of them:**

$$\sin\alpha_1, \quad \sin\alpha_2, \quad \sin\alpha_3; \quad \sin\beta_1, \quad \sin\beta_2, \quad \sin\beta_3; \quad \sin\gamma_1, \quad \sin\gamma_2, \quad \sin\gamma_3;$$

$$\sin 2\alpha_1, \quad \sin 2\alpha_2, \quad \sin 2\alpha_3; \quad \sin 2\beta_1, \quad \sin 2\beta_2, \quad \sin 2\beta_3; \quad \sin 2\gamma_1, \quad \sin 2\gamma_2, \quad \sin 2\gamma_3;$$

$$\sin\left(\alpha_1+\alpha_2\right), \quad \sin\left(\alpha_2+\alpha_3\right), \quad \sin\left(\alpha_3+\alpha_1\right); \quad \sin\left(\beta_1+\beta_2\right), \quad \sin\left(\beta_2+\beta_3\right), \quad \sin\left(\beta_3+\beta_1\right);$$

$$\sin\left(\gamma_1+\gamma_2\right), \quad \sin\left(\gamma_2+\gamma_3\right), \quad \sin\left(\gamma_3+\gamma_1\right);$$

$$\sin\left(\alpha_1-\alpha_2\right), \quad \sin\left(\alpha_2-\alpha_3\right), \quad \sin\left(\alpha_3-\alpha_1\right); \quad \sin\left(\beta_1-\beta_2\right), \quad \sin\left(\beta_2-\beta_3\right), \quad \sin\left(\beta_3-\beta_1\right);$$

$$\sin\left(\gamma_1-\gamma_2\right), \quad \sin\left(\gamma_2-\gamma_3\right), \quad \sin\left(\gamma_3-\gamma_1\right);$$

$$\sin\left(\alpha_1+\beta_2\right), \quad \sin\left(\alpha_1+\beta_3\right), \quad \sin\left(\alpha_2+\beta_1\right), \quad \sin\left(\alpha_2+\beta_3\right), \quad \sin\left(\alpha_3+\beta_1\right), \quad \sin\left(\alpha_3+\beta_2\right);$$

$$\sin\left(\alpha_1+\gamma_2\right), \quad \sin\left(\alpha_1+\gamma_3\right), \quad \sin\left(\alpha_2+\gamma_1\right), \quad \sin\left(\alpha_2+\gamma_3\right), \quad \sin\left(\alpha_3+\gamma_1\right), \quad \sin\left(\alpha_3+\gamma_2\right);$$

$$\sin\left(\beta_1+\gamma_2\right), \quad \sin\left(\beta_1+\gamma_3\right), \quad \sin\left(\beta_2+\gamma_1\right), \quad \sin\left(\beta_2+\gamma_3\right), \quad \sin\left(\beta_3+\gamma_1\right), \quad \sin\left(\beta_3+\gamma_2\right);$$

$$\sin 2\left(\alpha_1+\beta_2\right), \quad \sin 2\left(\alpha_1+\beta_3\right), \quad \sin 2\left(\alpha_2+\beta_1\right), \quad \sin 2\left(\alpha_2+\beta_3\right), \quad \sin 2\left(\alpha_3+\beta_1\right), \quad \sin 2\left(\alpha_3+\beta_2\right);$$

$$\sin 2\left(\alpha_1+\gamma_2\right), \quad \sin 2\left(\alpha_1+\gamma_3\right), \quad \sin 2\left(\alpha_2+\gamma_1\right), \quad \sin 2\left(\alpha_2+\gamma_3\right), \quad \sin 2\left(\alpha_3+\gamma_1\right), \quad \sin 2\left(\alpha_3+\gamma_2\right);$$

$$\sin 2\left(\beta_1+\gamma_2\right), \quad \sin 2\left(\beta_1+\gamma_3\right), \quad \sin 2\left(\beta_2+\gamma_1\right), \quad \sin 2\left(\beta_2+\gamma_3\right), \quad \sin 2\left(\beta_3+\gamma_1\right), \quad \sin 2\left(\beta_3+\gamma_2\right);$$

Counting the phase combinations (2)

- ♦ **Of 240 linear combinations of the 6 original seesaw phase parameters —— 60 of them:**

$$\sin\left(2\alpha_1-\beta_2\right), \quad \sin\left(2\alpha_1-\beta_3\right), \quad \sin\left(2\alpha_2-\beta_1\right), \quad \sin\left(2\alpha_2-\beta_3\right), \quad \sin\left(2\alpha_3-\beta_1\right), \quad \sin\left(2\alpha_3-\beta_2\right);$$

$$\sin\left(2\alpha_1-\gamma_2\right), \quad \sin\left(2\alpha_1-\gamma_3\right), \quad \sin\left(2\alpha_2-\gamma_1\right), \quad \sin\left(2\alpha_2-\gamma_3\right), \quad \sin\left(2\alpha_3-\gamma_1\right), \quad \sin\left(2\alpha_3-\gamma_2\right);$$

$$\sin\left(2\beta_1-\alpha_2\right), \quad \sin\left(2\beta_1-\alpha_3\right), \quad \sin\left(2\beta_2-\alpha_1\right), \quad \sin\left(2\beta_2-\alpha_3\right), \quad \sin\left(2\beta_3-\alpha_1\right), \quad \sin\left(2\beta_3-\alpha_2\right);$$

$$\sin\left(2\beta_1-\gamma_2\right), \quad \sin\left(2\beta_1-\gamma_3\right), \quad \sin\left(2\beta_2-\gamma_1\right), \quad \sin\left(2\beta_2-\gamma_3\right), \quad \sin\left(2\beta_3-\gamma_1\right), \quad \sin\left(2\beta_3-\gamma_2\right);$$

$$\sin\left(2\gamma_1-\alpha_2\right), \quad \sin\left(2\gamma_1-\alpha_3\right), \quad \sin\left(2\gamma_2-\alpha_1\right), \quad \sin\left(2\gamma_2-\alpha_3\right), \quad \sin\left(2\gamma_3-\alpha_1\right), \quad \sin\left(2\gamma_3-\alpha_2\right);$$

$$\sin\left(2\gamma_1-\beta_2\right), \quad \sin\left(2\gamma_1-\beta_3\right), \quad \sin\left(2\gamma_2-\beta_1\right), \quad \sin\left(2\gamma_2-\beta_3\right), \quad \sin\left(2\gamma_3-\beta_1\right), \quad \sin\left(2\gamma_3-\beta_2\right);$$

$$\sin\left(\alpha_1+\alpha_2+2\beta_3\right), \quad \sin\left(\alpha_1+\alpha_2+2\gamma_3\right), \quad \sin\left(\alpha_1+\alpha_3+2\beta_2\right), \quad \sin\left(\alpha_1+\alpha_3+2\gamma_2\right),$$

$$\sin\left(\alpha_2+\alpha_3+2\beta_1\right), \quad \sin\left(\alpha_2+\alpha_3+2\gamma_1\right); \quad \sin\left(\beta_1+\beta_2+2\alpha_3\right), \quad \sin\left(\beta_1+\beta_2+2\gamma_3\right),$$

$$\sin\left(\beta_1+\beta_3+2\alpha_2\right), \quad \sin\left(\beta_1+\beta_3+2\gamma_2\right), \quad \sin\left(\beta_2+\beta_3+2\alpha_1\right), \quad \sin\left(\beta_2+\beta_3+2\gamma_1\right);$$

$$\sin\left(\gamma_1+\gamma_2+2\alpha_3\right), \quad \sin\left(\gamma_1+\gamma_2+2\beta_3\right), \quad \sin\left(\gamma_1+\gamma_3+2\alpha_2\right), \quad \sin\left(\gamma_1+\gamma_3+2\beta_2\right),$$

$$\sin\left(\gamma_2+\gamma_3+2\alpha_1\right), \quad \sin\left(\gamma_2+\gamma_3+2\beta_1\right);$$

$$\sin\left(\alpha_1+\beta_2+\gamma_3\right), \quad \sin\left(\alpha_1+\beta_3+\gamma_2\right), \quad \sin\left(\alpha_2+\beta_1+\gamma_3\right), \quad \sin\left(\alpha_2+\beta_3+\gamma_1\right),$$

$$\sin\left(\alpha_3+\beta_1+\gamma_2\right), \quad \sin\left(\alpha_3+\beta_2+\gamma_1\right).$$

Counting the phase combinations (3)

- **Of 240 linear combinations of the 6 original seesaw phase parameters —— 54 of them:**

$$\sin\left(\alpha_1+\alpha_2-\beta_1\right)\,,\quad \sin\left(\alpha_1+\alpha_2-\beta_2\right)\,,\quad \sin\left(\alpha_1+\alpha_2-\beta_3\right)\,,\quad \sin\left(\alpha_1+\alpha_3-\beta_1\right)\,,$$

$$\sin\left(\alpha_1+\alpha_3-\beta_2\right)\,,\quad \sin\left(\alpha_1+\alpha_3-\beta_3\right)\,,\quad \sin\left(\alpha_2+\alpha_3-\beta_1\right)\,,\quad \sin\left(\alpha_2+\alpha_3-\beta_2\right)\,,$$

$$\sin\left(\alpha_2+\alpha_3-\beta_3\right)\,;\quad \sin\left(\alpha_1+\alpha_2-\gamma_1\right)\,,\quad \sin\left(\alpha_1+\alpha_2-\gamma_2\right)\,,\quad \sin\left(\alpha_1+\alpha_2-\gamma_3\right)\,,$$

$$\sin\left(\alpha_1+\alpha_3-\gamma_1\right)\,,\quad \sin\left(\alpha_1+\alpha_3-\gamma_2\right)\,,\quad \sin\left(\alpha_1+\alpha_3-\gamma_3\right)\,,\quad \sin\left(\alpha_2+\alpha_3-\gamma_1\right)\,,$$

$$\sin\left(\alpha_2+\alpha_3-\gamma_2\right)\,,\quad \sin\left(\alpha_2+\alpha_3-\gamma_3\right)\,;\quad \sin\left(\beta_1+\beta_2-\alpha_1\right)\,,\quad \sin\left(\beta_1+\beta_2-\alpha_2\right)\,,$$

$$\sin\left(\beta_1+\beta_2-\alpha_3\right)\,,\quad \sin\left(\beta_1+\beta_3-\alpha_1\right)\,,\quad \sin\left(\beta_1+\beta_3-\alpha_2\right)\,,\quad \sin\left(\beta_1+\beta_3-\alpha_3\right)\,,$$

$$\sin\left(\beta_2+\beta_3-\alpha_1\right)\,,\quad \sin\left(\beta_2+\beta_3-\alpha_2\right)\,,\quad \sin\left(\beta_2+\beta_3-\alpha_3\right)\,;\quad \sin\left(\beta_1+\beta_2-\gamma_1\right)\,,$$

$$\sin\left(\beta_1+\beta_2-\gamma_2\right)\,,\quad \sin\left(\beta_1+\beta_2-\gamma_3\right)\,,\quad \sin\left(\beta_1+\beta_3-\gamma_1\right)\,,\quad \sin\left(\beta_1+\beta_3-\gamma_2\right)\,,$$

$$\sin\left(\beta_1+\beta_3-\gamma_3\right)\,,\quad \sin\left(\beta_2+\beta_3-\gamma_1\right)\,,\quad \sin\left(\beta_2+\beta_3-\gamma_2\right)\,,\quad \sin\left(\beta_2+\beta_3-\gamma_3\right)\,;$$

$$\sin\left(\gamma_1+\gamma_2-\alpha_1\right)\,,\quad \sin\left(\gamma_1+\gamma_2-\alpha_2\right)\,,\quad \sin\left(\gamma_1+\gamma_2-\alpha_3\right)\,,\quad \sin\left(\gamma_1+\gamma_3-\alpha_1\right)\,,$$

$$\sin\left(\gamma_1+\gamma_3-\alpha_2\right)\,,\quad \sin\left(\gamma_1+\gamma_3-\alpha_3\right)\,,\quad \sin\left(\gamma_2+\gamma_3-\alpha_1\right)\,,\quad \sin\left(\gamma_2+\gamma_3-\alpha_2\right)\,,$$

$$\sin\left(\gamma_2+\gamma_3-\alpha_3\right)\,;\quad \sin\left(\gamma_1+\gamma_2-\beta_1\right)\,,\quad \sin\left(\gamma_1+\gamma_2-\beta_2\right)\,,\quad \sin\left(\gamma_1+\gamma_2-\beta_3\right)\,,$$

$$\sin\left(\gamma_1+\gamma_3-\beta_1\right)\,,\quad \sin\left(\gamma_1+\gamma_3-\beta_2\right)\,,\quad \sin\left(\gamma_1+\gamma_3-\beta_3\right)\,,\quad \sin\left(\gamma_2+\gamma_3-\beta_1\right)\,,$$

$$\sin\left(\gamma_2+\gamma_3-\beta_2\right)\,,\quad \sin\left(\gamma_2+\gamma_3-\beta_3\right)\,;$$

Counting the phase combinations (4)

- **Of 240 linear combinations of the 6 original seesaw phase parameters —— 54 of them:**

$$
\begin{array}{llll}
\sin\left(\alpha_1+\beta_2-\gamma_1\right), & \sin\left(\alpha_1+\beta_2-\gamma_2\right), & \sin\left(\alpha_1+\beta_2-\gamma_3\right), & \sin\left(\alpha_1+\beta_3-\gamma_1\right), \\
\sin\left(\alpha_1+\beta_3-\gamma_2\right), & \sin\left(\alpha_1+\beta_3-\gamma_3\right), & \sin\left(\alpha_2+\beta_1-\gamma_1\right), & \sin\left(\alpha_2+\beta_1-\gamma_2\right), \\
\sin\left(\alpha_2+\beta_1-\gamma_3\right), & \sin\left(\alpha_2+\beta_3-\gamma_1\right), & \sin\left(\alpha_2+\beta_3-\gamma_2\right), & \sin\left(\alpha_2+\beta_3-\gamma_3\right), \\
\sin\left(\alpha_3+\beta_1-\gamma_1\right), & \sin\left(\alpha_3+\beta_1-\gamma_2\right), & \sin\left(\alpha_3+\beta_1-\gamma_3\right), & \sin\left(\alpha_3+\beta_2-\gamma_1\right), \\
\sin\left(\alpha_3+\beta_2-\gamma_2\right), & \sin\left(\alpha_3+\beta_2-\gamma_3\right); & \sin\left(\alpha_1-\beta_1+\gamma_2\right), & \sin\left(\alpha_1-\beta_1+\gamma_3\right), \\
\sin\left(\alpha_1-\beta_2+\gamma_2\right), & \sin\left(\alpha_1-\beta_2+\gamma_3\right), & \sin\left(\alpha_1-\beta_3+\gamma_2\right), & \sin\left(\alpha_1-\beta_3+\gamma_3\right), \\
\sin\left(\alpha_2-\beta_1+\gamma_1\right), & \sin\left(\alpha_2-\beta_1+\gamma_3\right), & \sin\left(\alpha_2-\beta_2+\gamma_1\right), & \sin\left(\alpha_2-\beta_2+\gamma_3\right), \\
\sin\left(\alpha_2-\beta_3+\gamma_1\right), & \sin\left(\alpha_2-\beta_3+\gamma_3\right), & \sin\left(\alpha_3-\beta_1+\gamma_1\right), & \sin\left(\alpha_3-\beta_1+\gamma_2\right), \\
\sin\left(\alpha_3-\beta_2+\gamma_1\right), & \sin\left(\alpha_3-\beta_2+\gamma_2\right), & \sin\left(\alpha_3-\beta_3+\gamma_1\right), & \sin\left(\alpha_3-\beta_3+\gamma_2\right); \\
\sin\left(\alpha_1-\beta_1-\gamma_2\right), & \sin\left(\alpha_1-\beta_1-\gamma_3\right), & \sin\left(\alpha_1-\beta_2-\gamma_1\right), & \sin\left(\alpha_1-\beta_2-\gamma_3\right), \\
\sin\left(\alpha_1-\beta_3-\gamma_1\right), & \sin\left(\alpha_1-\beta_3-\gamma_2\right), & \sin\left(\alpha_2-\beta_1-\gamma_2\right), & \sin\left(\alpha_2-\beta_1-\gamma_3\right), \\
\sin\left(\alpha_2-\beta_2-\gamma_1\right), & \sin\left(\alpha_2-\beta_2-\gamma_3\right), & \sin\left(\alpha_2-\beta_3-\gamma_1\right), & \sin\left(\alpha_2-\beta_3-\gamma_2\right), \\
\sin\left(\alpha_3-\beta_1-\gamma_2\right), & \sin\left(\alpha_3-\beta_1-\gamma_3\right), & \sin\left(\alpha_3-\beta_2-\gamma_1\right), & \sin\left(\alpha_3-\beta_2-\gamma_3\right), \\
\sin\left(\alpha_3-\beta_3-\gamma_1\right), & \sin\left(\alpha_3-\beta_3-\gamma_2\right). & &
\end{array}
$$

CPV in heavy Majorana neutrino decays

- **The flavor-dependent CP-violating asymmetries in LNV decays of heavy Majorana neutrinos:**

$$\varepsilon_{j\alpha} \equiv \frac{\Gamma(N_j \to \ell_\alpha + H) - \Gamma(N_j \to \overline{\ell_\alpha} + \overline{H})}{\sum\limits_\alpha \left[\Gamma(N_j \to \ell_\alpha + H) + \Gamma(N_j \to \overline{\ell_\alpha} + \overline{H})\right]}$$

$$\simeq \frac{1}{8\pi\langle H\rangle^2 \sum\limits_\beta \left|R_{\beta j}\right|^2} \sum_{k=4}^{6} \left\{ M_k^2 \,\mathrm{Im}\left[\left(R^*_{\alpha j} R_{\alpha k}\right) \sum_\beta \left[\left(R^*_{\beta j} R_{\beta k}\right) \xi(x_{kj}) + \left(R_{\beta j} R^*_{\beta k}\right) \zeta(x_{kj}) \right] \right] \right\}$$

- **Baryogenesis via leptogenesis in the early Universe:**

A net lepton number asymmetry

$$\boxed{Y_{\rm L} \equiv \frac{n_{\rm L} - n_{\overline{\rm L}}}{s} = \frac{1}{g_*} \sum_{j,\alpha} \kappa_{j\alpha} \varepsilon_{j\alpha}}$$

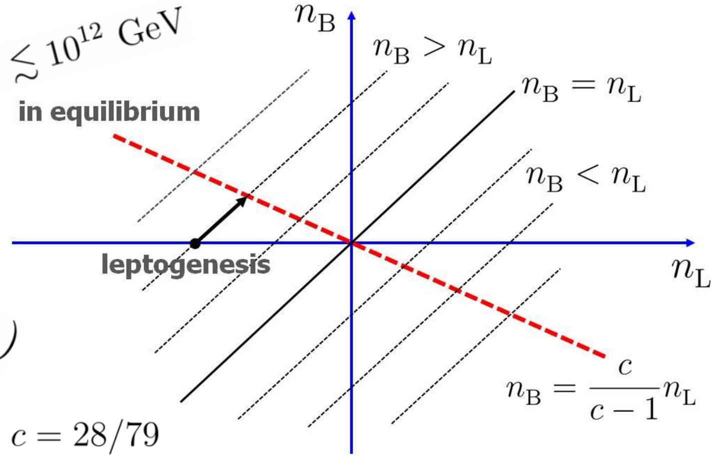
sphaleron-induced (B-L)-conserving process in thermal equilibrium

$$\boxed{Y_{\rm B} \equiv \frac{n_{\rm B} - n_{\overline{\rm B}}}{s} = -c Y_{\rm L}}$$

A net baryon number asymmetry

$$\Gamma(N_j \to \ell_\alpha + H) < \mathcal{H}(T = M_j)$$

$$10^2~\text{GeV} \lesssim T \lesssim 10^{12}~\text{GeV}$$



How many phase combinations (1)

- A **flavor-dependent** CP-violating asymmetry of the first ($j = 4$) heavy Majorana neutrino decays:

$$\begin{aligned}\varepsilon_{4e} = \frac{1}{8\pi\langle H\rangle^2 I_{44}} \Bigg\{ & \underset{\uparrow}{M_5^2} s_{14} s_{15} \left[\xi(x_{54}) \sum_{i=1}^{3} s_{i4} s_{i5} \underline{\sin(\alpha_1 + \alpha_i)} + \zeta(x_{54}) \Big[s_{24} s_{25} \underline{\sin(\alpha_1 - \alpha_2)} \right. \\ & \left. + s_{34} s_{35} \underline{\sin(\alpha_1 - \alpha_3)} \Big] \right] - \underset{\uparrow}{M_6^2} s_{14} s_{16} \left[\xi(x_{64}) \sum_{i=1}^{3} s_{i4} s_{i6} \underline{\sin(\gamma_1 + \gamma_i)} \right. \\ & \left. + \zeta(x_{64}) \Big[s_{24} s_{26} \underline{\sin(\gamma_1 - \gamma_2)} + s_{34} s_{36} \underline{\sin(\gamma_1 - \gamma_3)} \Big] \right] \Bigg\}\end{aligned}$$

The formulas for "5" and "6" can be similarly written out.

$$\begin{aligned}\varepsilon_{4\mu} = \frac{1}{8\pi\langle H\rangle^2 I_{44}} \Bigg\{ & \underset{\uparrow}{M_5^2} s_{24} s_{25} \left[\xi(x_{54}) \sum_{i=1}^{3} s_{i4} s_{i5} \underline{\sin(\alpha_2 + \alpha_i)} + \zeta(x_{54}) \Big[s_{14} s_{15} \underline{\sin(\alpha_2 - \alpha_1)} \right. \\ & \left. + s_{34} s_{35} \underline{\sin(\alpha_2 - \alpha_3)} \Big] \right] - \underset{\uparrow}{M_6^2} s_{24} s_{26} \left[\xi(x_{64}) \sum_{i=1}^{3} s_{i4} s_{i6} \underline{\sin(\gamma_2 + \gamma_i)} \right. \\ & \left. + \zeta(x_{64}) \Big[s_{14} s_{16} \underline{\sin(\gamma_2 - \gamma_1)} + s_{34} s_{36} \underline{\sin(\gamma_2 - \gamma_3)} \Big] \right] \Bigg\}\end{aligned}$$

The formulas for "5" and "6" can be similarly written out.

How many phase combinations (2)

- **A flavor-dependent CP-violating asymmetry of the first (j = 4) heavy Majorana neutrino decays:**

$$\begin{aligned}\varepsilon_{4\tau} = \frac{1}{8\pi\langle H\rangle^2 I_{44}} \Bigg\{ & \underset{\uparrow}{M_5^2} s_{34}s_{35}\left[\xi(x_{54})\sum_{i=1}^{3} s_{i4}s_{i5}\,\underline{\sin\left(\alpha_3+\alpha_i\right)} + \zeta(x_{54})\Big[s_{14}s_{15}\,\underline{\sin\left(\alpha_3-\alpha_1\right)}\right. \\ & \left.+\, s_{24}s_{25}\,\underline{\sin\left(\alpha_3-\alpha_2\right)}\Big]\right] - \underset{\uparrow}{M_6^2} s_{34}s_{36}\left[\xi(x_{64})\sum_{i=1}^{3} s_{i4}s_{i6}\,\underline{\sin\left(\gamma_3+\gamma_i\right)}\right. \\ & \left.+\, \zeta(x_{64})\Big[s_{14}s_{16}\,\underline{\sin\left(\gamma_3-\gamma_1\right)} + s_{24}s_{26}\,\underline{\sin\left(\gamma_3-\gamma_2\right)}\Big]\right]\Bigg\}\end{aligned}$$

The formulas for "5" and "6" can be similarly written out.

- **The flavor-independent CP-violating asymmetry** $\underline{\varepsilon_4 \equiv \varepsilon_{4e} + \varepsilon_{4\mu} + \varepsilon_{4\tau}}$ **, for example:**

$$\varepsilon_4 = \frac{1}{8\pi\langle H\rangle^2 I_{44}}\left[\sum_{i=1}^{3}\sum_{i'=1}^{3} s_{i4}s_{i'4}\Big[\underset{\uparrow}{M_5^2}\xi(x_{54})s_{i5}s_{i'5}\,\underline{\sin\left(\alpha_i+\alpha_{i'}\right)} - \underset{\uparrow}{M_6^2}\xi(x_{64})s_{i6}s_{i'6}\,\underline{\sin\left(\gamma_i+\gamma_{i'}\right)}\Big]\right]$$

- **Totally 27 linear combinations of the 6 original seesaw phase parameters in CP violation of three heavy Majorana neutrino decays (** $i, i' = 1, 2, 3$ **):**

$$\sin\left(\alpha_i \pm \alpha_{i'}\right),\ \sin\left(\beta_i \pm \beta_{i'}\right),\ \sin\left(\gamma_i \pm \gamma_{i'}\right)$$

The connection can be more direct!

- **The analytical results obtained above imply that the *CP-violating asymmetries* can be expressed as a linear combinations of the sines of the 6 original seesaw phase parameters:**

$$\boxed{\begin{aligned}\varepsilon_{j\alpha} &= \sum_{i=1}^{3}\left(C'_{\alpha i}\sin\alpha_i + C'_{\beta i}\sin\beta_i\right)\\ \varepsilon_{j} &= \sum_{i=1}^{3}\left(C''_{\alpha i}\sin\alpha_i + C''_{\beta i}\sin\beta_i\right)\end{aligned}}$$

It's then straightforward to extract the coefficients from the formulas of CP violating asymmetries.

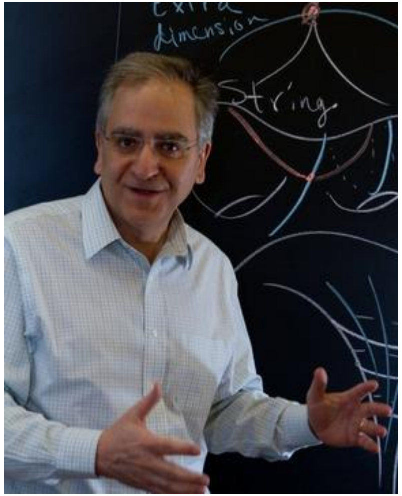
- **In comparison, the achieved result of the Jarlskog invariant implies that it can also be expressed as a linear combinations of the sines of the 6 original seesaw phase parameters:**

$$\boxed{\mathcal{J}_\nu = \sum_{i=1}^{3}\left(C_{\alpha i}\sin\alpha_i + C_{\beta i}\sin\beta_i\right)}$$

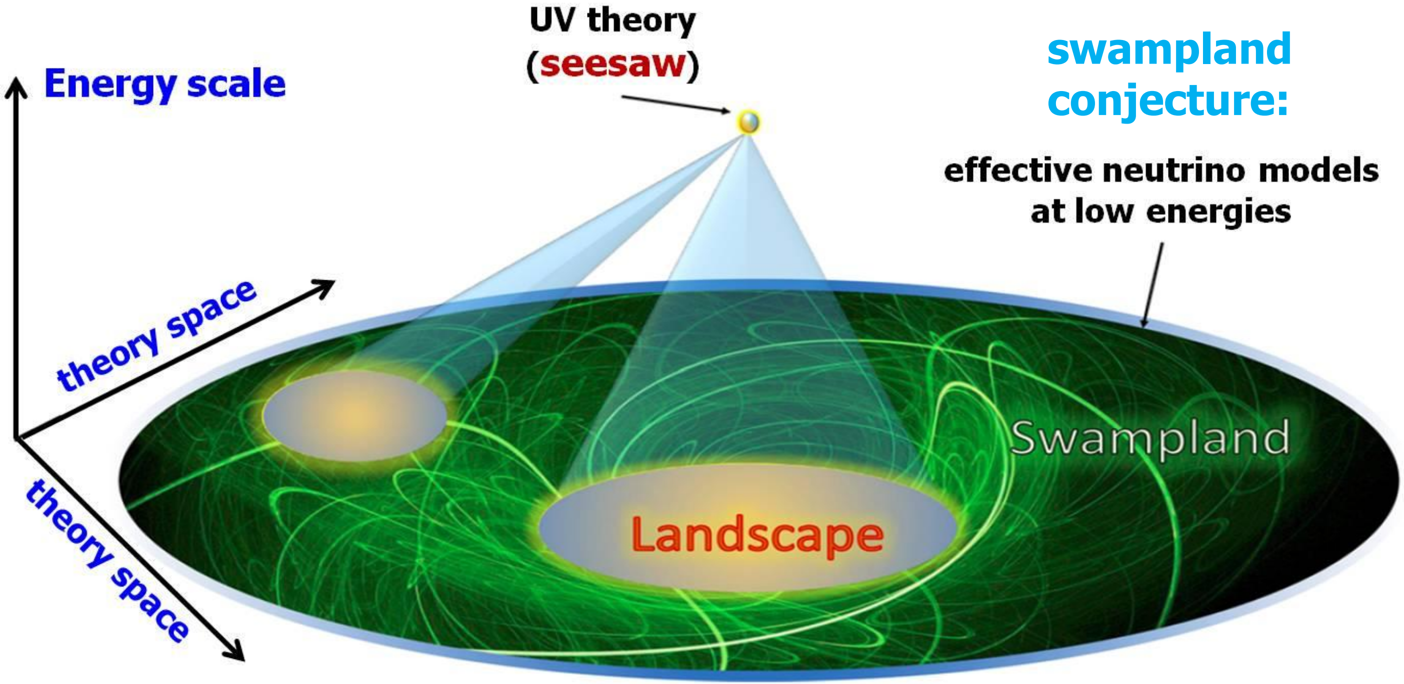
It is straightforward to extract the lengthy coefficients from the $T_{33} + T_{42} + T_{411} + T_{321} + T_{222}$ terms, but the expressions are so complicated that they cannot be presented here.

Concluding remark (1)

- ♦ **Extending the SM framework in a way that is as natural and economical as possible, we have argued that the canonical seesaw mechanism is most convincing to give mass to the active neutrinos. It is fully consistent with the spirit of Weinberg's EFT and thus should be located in Vafa's landscape of particle physics.**



Cumrun Vafa 2005



Concluding remark (2)

♦ The new era of precision measurements, as characterized by **JUNO**, **DUNE** and **T2HK**, is coming. It is high time to experimentally test the canonical seesaw in a systematical and model-independent way at low energies.

♦ This becomes possible, with the help of a complete Euler-like block parametrization of the seesaw flavor structure, since it makes *analytical* calculations of all observables possible. The present talk give a *PoC* example by clarifying the **Buchmüller**-**Plümacher** claim. For the first time, we have shown that a direct, explicit and model-independent connection exists between CP violation at high and low energy scales.

♦ **A take-home message:** to really test the seesaw, you should calculate everything by using the original seesaw parameters instead of the derivational ones or a mixture.

♦ We are making progress in calculating all the other observables of 3 active neutrino oscillations (in collaboration with **Jing-yu Zhu**).

THANK YOU