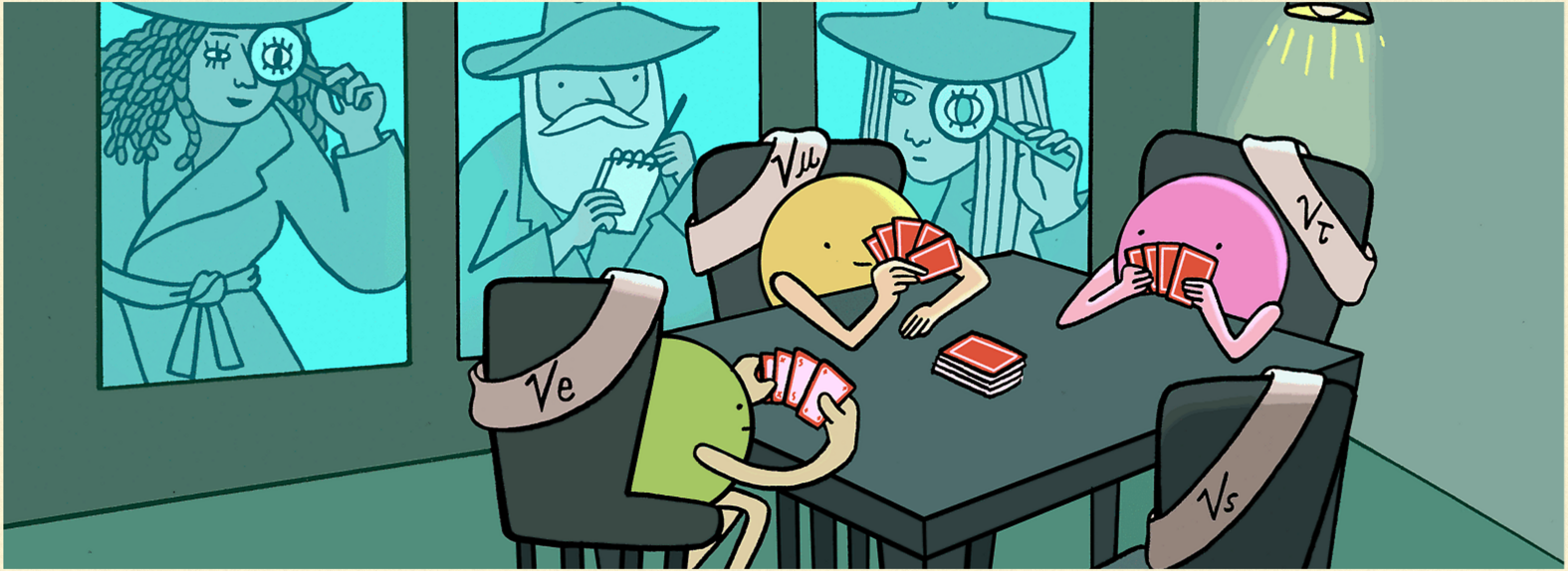
# SYNERGISTIC EFFECTS IN ENHANCING OCTANT AND MASS ORDERING SENSITIVITY IN THE PRESENCE OF A STERILE NEUTRINO





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Phys.Rev.D 108 (2023); Nucl.Phys.B 996 (2023)


Prague, 20th July

ICHEP 2024

# Outline

- Three Flavour Neutrino oscillations framework
- Sterile Neutrinos
- Sensitivity to octant of $\theta_{23}$
- Sensitivity to mass ordering
- Summary

# Three Flavour Neutrino Oscillation

$\nu_e$ $\nu_\mu$ $\nu_\tau$

https://sanfordlab.org/feature/unlocking-mysteries-neutrinos

$$\nu_\alpha = U_{\alpha i}\nu_i$$

$$\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-\imath\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{\imath\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

Solar | Reactor | Atmospheric/LBL

$$i\frac{\partial}{\partial t}\nu_\alpha = [UHU^\dagger + H_{int}]\nu_\alpha \qquad H = \frac{1}{2E_\nu}\begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 \\ 0 & 0 & \Delta_{31} \end{bmatrix}$$

- Mass-squared difference: $\Delta_{ij} = m_i^2 - m_j^2$
- Mixing angles: $\theta_{12}, \theta_{13}, \theta_{23}$, Phase: $\delta_{CP}$
- Baseline: L, Neutrino energy: $E_\nu$

$$H_{int} = \mathrm{diag}(A/2E_\nu, 0, 0) \qquad A = 2\sqrt{2}G_F N_e E_\nu = 7.6\times 10^{-5}\frac{\rho}{\mathrm{g/cc}}\frac{E_\nu}{\mathrm{GeV}}\mathrm{eV}^2$$

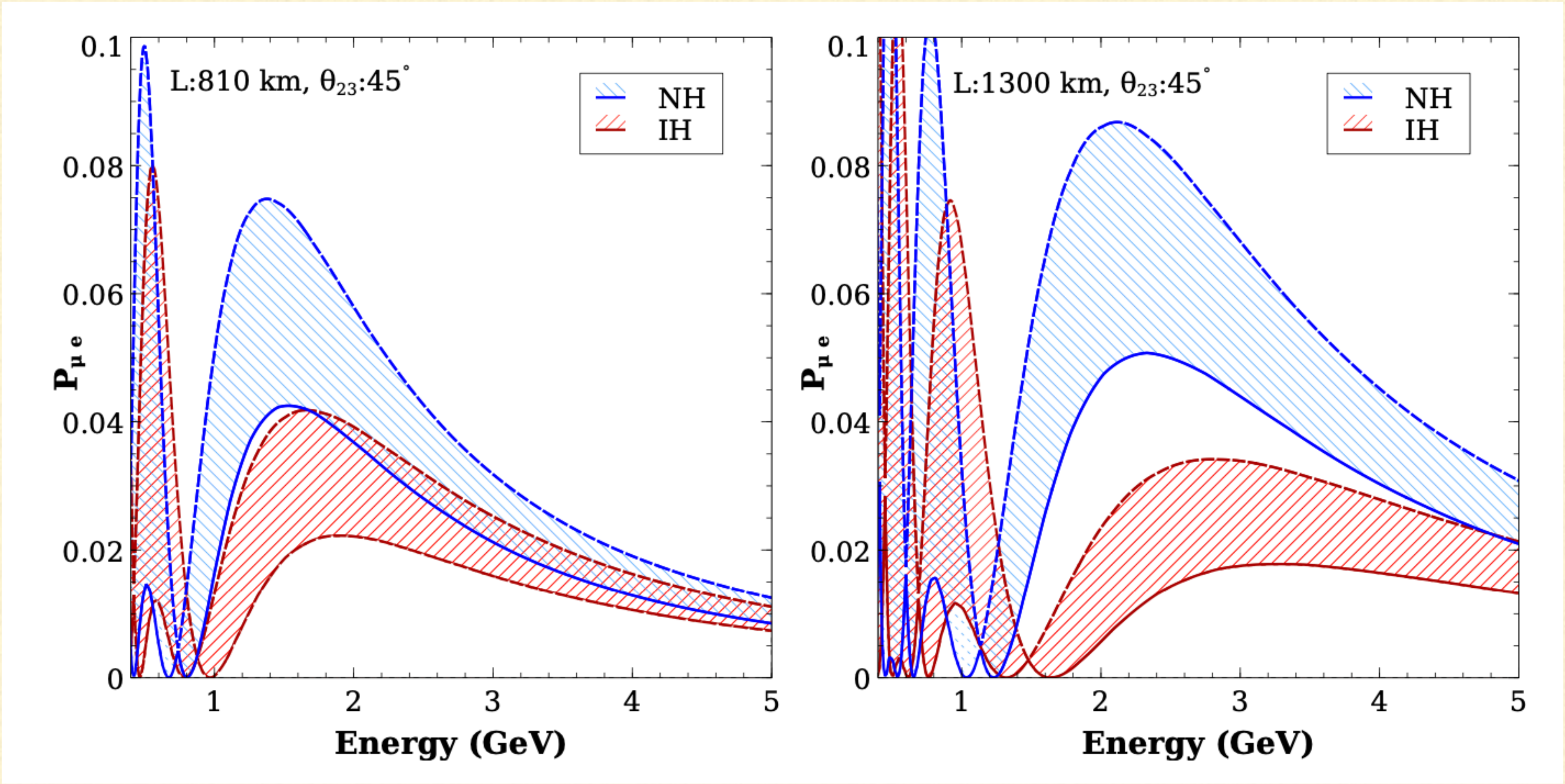
$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i>j}^{3}\mathrm{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*)\sin^2\frac{1.27\Delta_{ij}L}{E_\nu} + 2\sum_{i>j}^{3}\mathrm{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*)\sin\frac{2\times 1.27\Delta_{ij}L}{E_\nu}$$

# Neutrino Oscillation Probability

$$P_{\mu e}^{vac} = \sin^2 2\theta \sin^2 \left[\frac{1.27\Delta L}{E}\right]$$

$$P_{\mu e}^{mat} = \sin^2 2\theta^m \sin^2 \left[\frac{1.27\Delta^m L}{E}\right]$$

Not sensitive to $\theta \to \frac{\pi}{2} - \theta; \Delta \to -\Delta$




$$\Delta^m = \sqrt{\Delta_{21}^2 \sin^2 2\theta + (A - \Delta_{21}\cos 2\theta)^2}$$

$$\tan 2\theta^m = \frac{\Delta_{21}\sin 2\theta}{\Delta_{21}\cos 2\theta - A}; \text{changes} : \theta \to \frac{\pi}{2} - \theta$$

Sensitive to $\theta \to \frac{\pi}{2} - \theta; \Delta \to -\Delta$

$$E_{res} = \frac{\Delta \cos 2\theta}{2\sqrt{2}G_F N_e}$$

[Wolfenstein,PRD 17(1978)]
[Mikhyev, Smirov, SJNP (1985)]

# Degeneracies

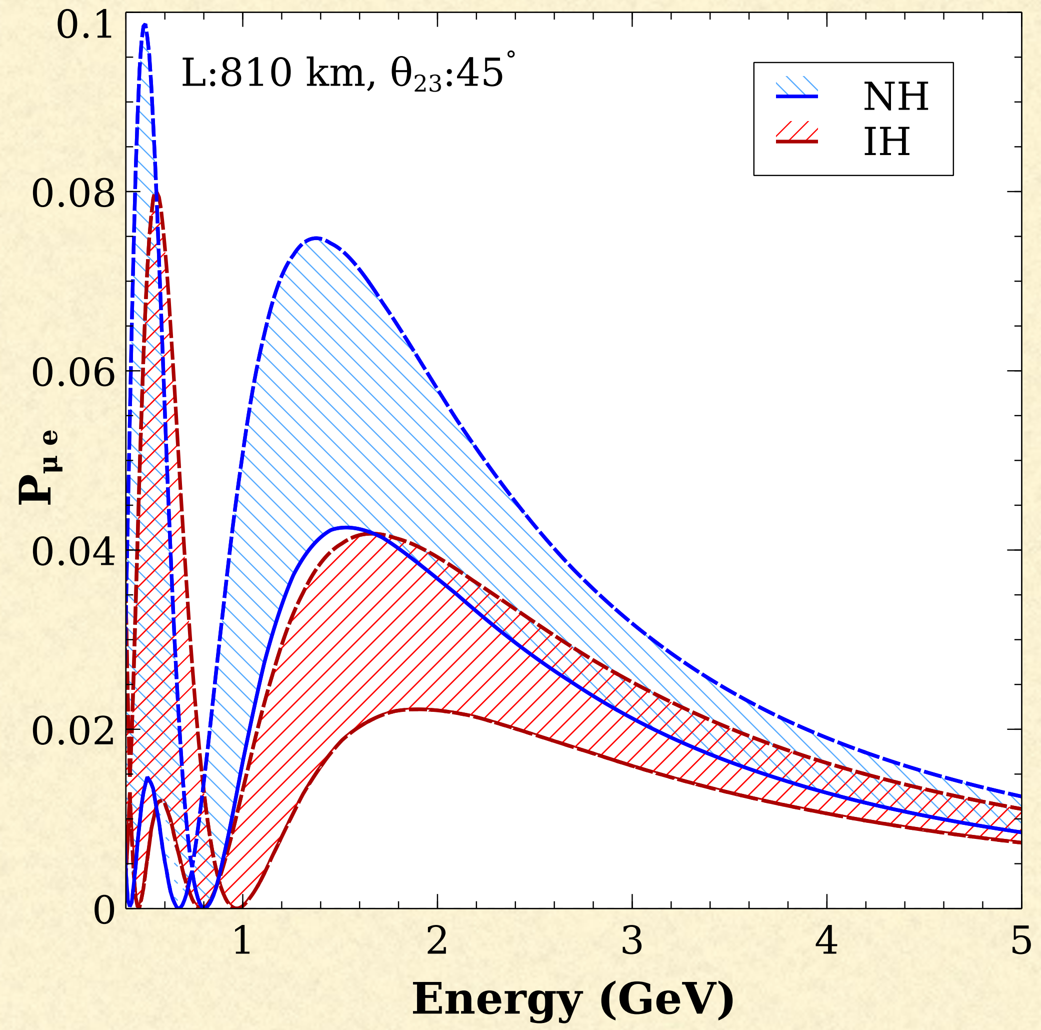
$\theta_{12} = 33.44^\circ, \theta_{13} = 8.57^\circ, \theta_{23} = 49.20^\circ, \delta_{13} = 194^\circ, \Delta_{21} = 7.42 \times 10^{-5}\text{eV}^2, |\Delta_{31}| = 2.52 \times 10^{-3}\text{eV}^2$ NuFIT 5.2 (2022)

**Mass Ordering:**

- $\Delta_{31} > 0$: Normal Ordering
- $\Delta_{31} < 0$: Inverted Ordering

$P_{\mu e}(\Delta_{31}[NO], \delta_{13}) = P_{\mu e}(\Delta_{31}[IO], \delta'_{13})$

[Minakata, Nunokawa, *JHEP* 10 (2001) 001]



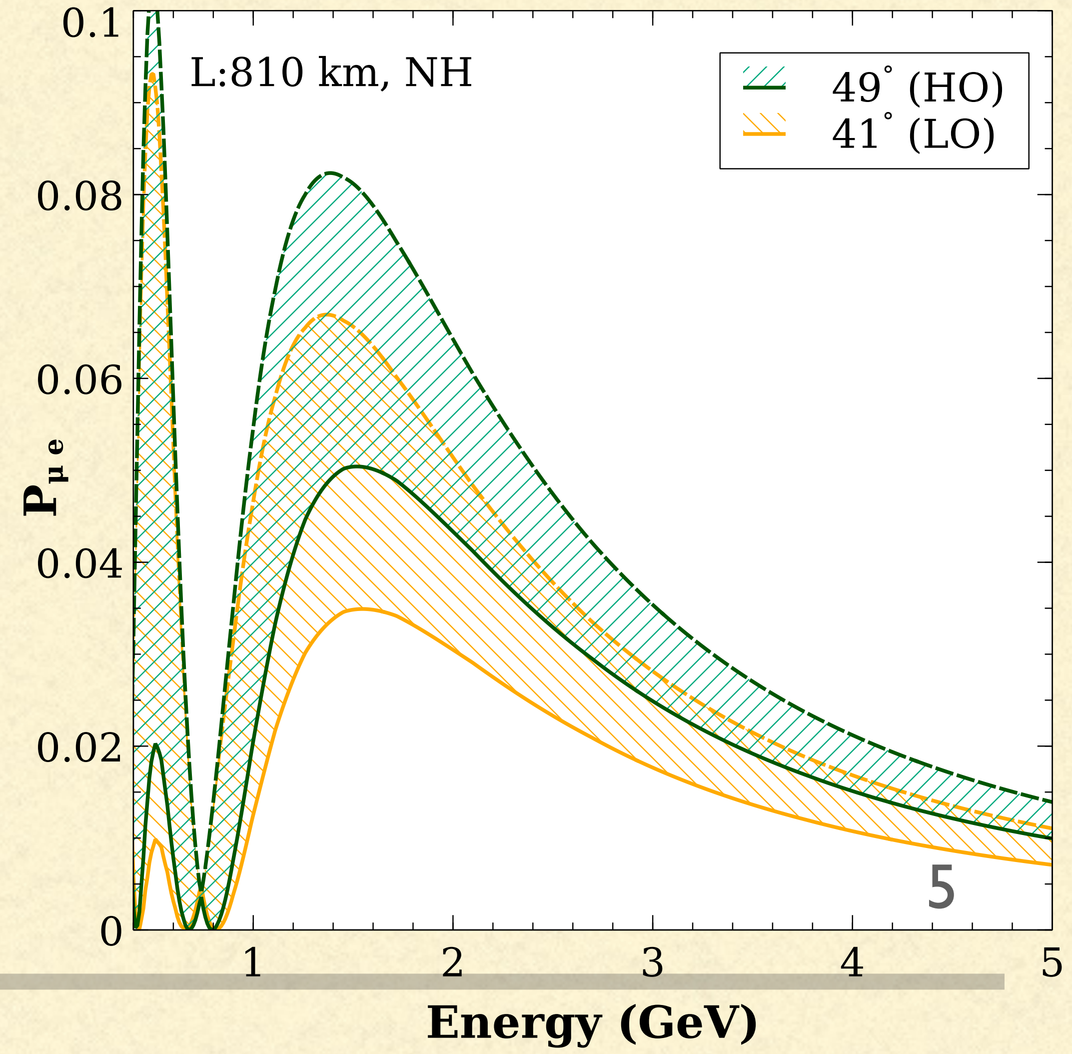

**Value of $\delta_{13}$**

**Octant of $\theta_{23}$**

- Higher Octant : $\theta_{23} > 45^\circ$
- Lower Octant : $\theta_{23} < 45^\circ$

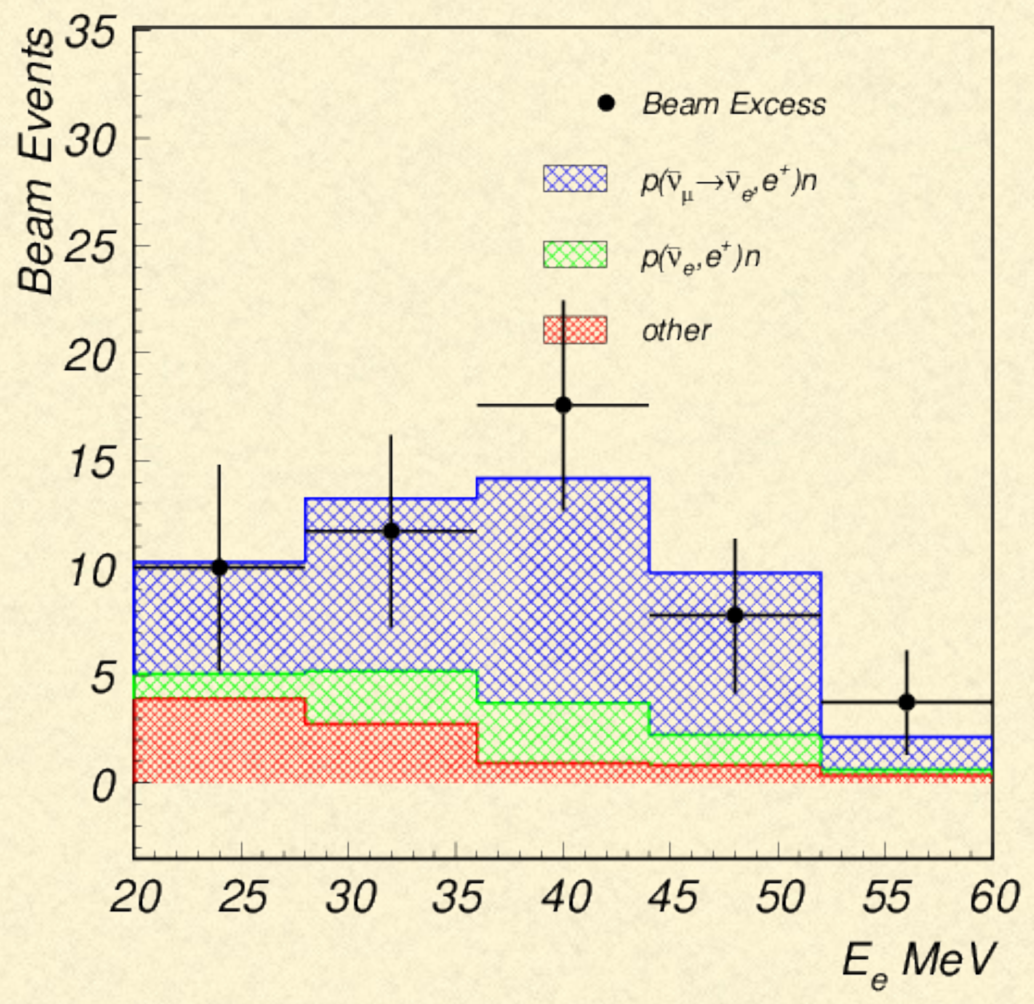
$P_{\mu e}(\theta_{23}[HO], \delta_{13}) = P_{\mu e}(\theta_{31}[LO], \delta'_{13})$

[Gandhi, Ghoshal, Goswami, Shankar, hep-ph/0506145 ]



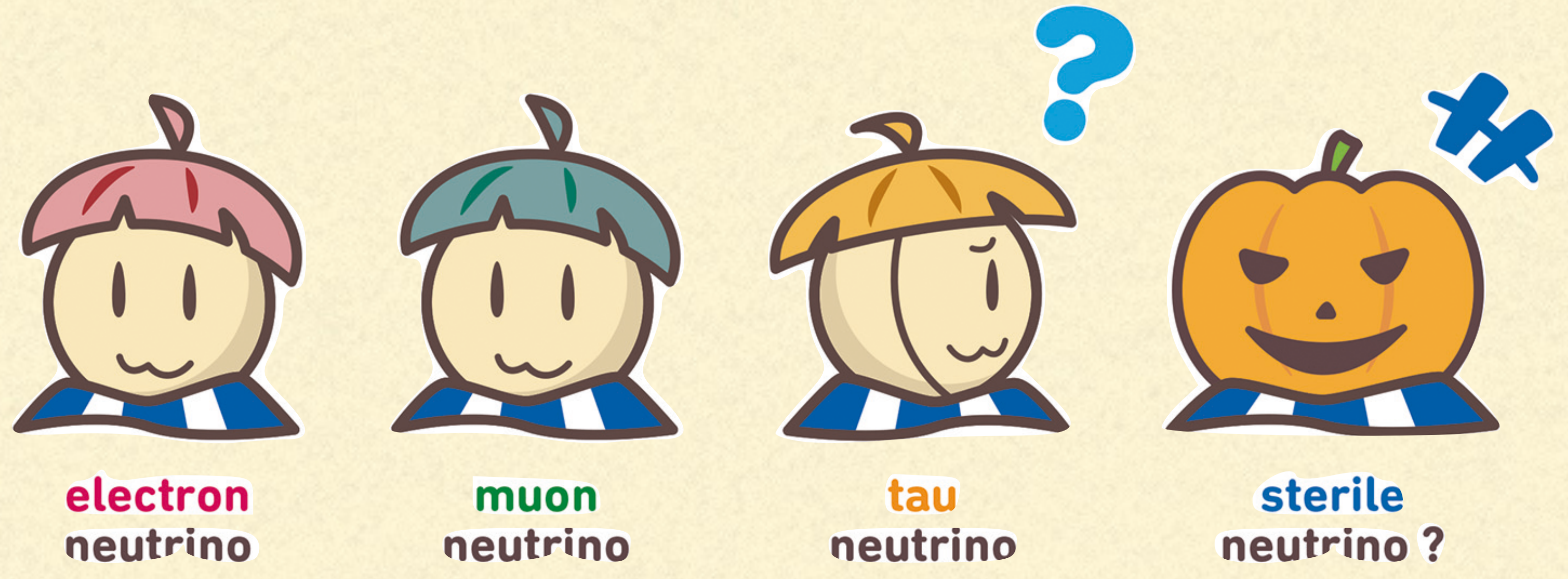
# Sterile Neutrino (eV scale)

**LSND (PRD.64.112007)**

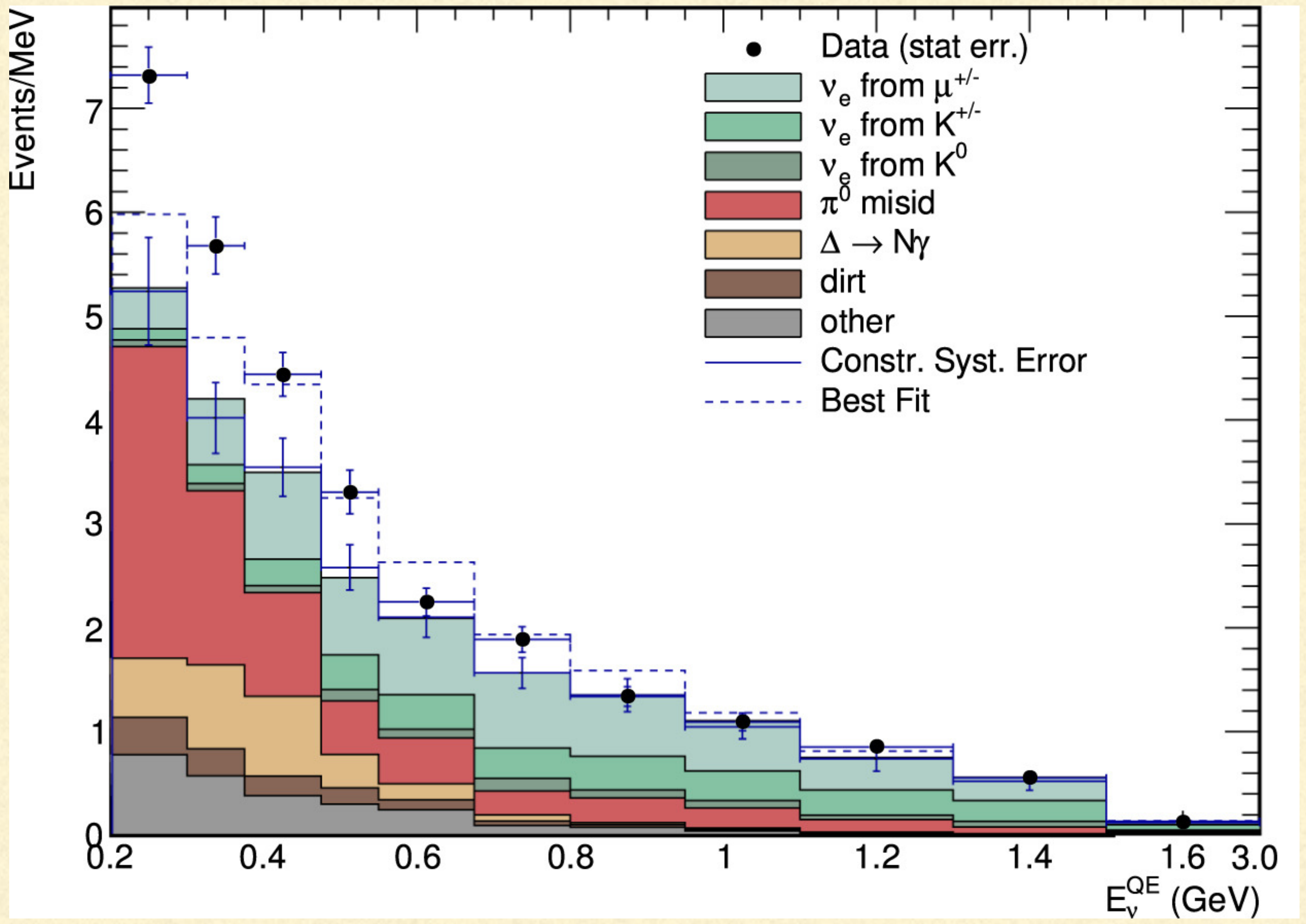



$\bar{\nu}_\mu \to \bar{\nu}_e$ at $3.8\sigma$ (C. Athanassopoulos et al , PRL 1995)

L:30 m; 20 MeV<E<55 MeV



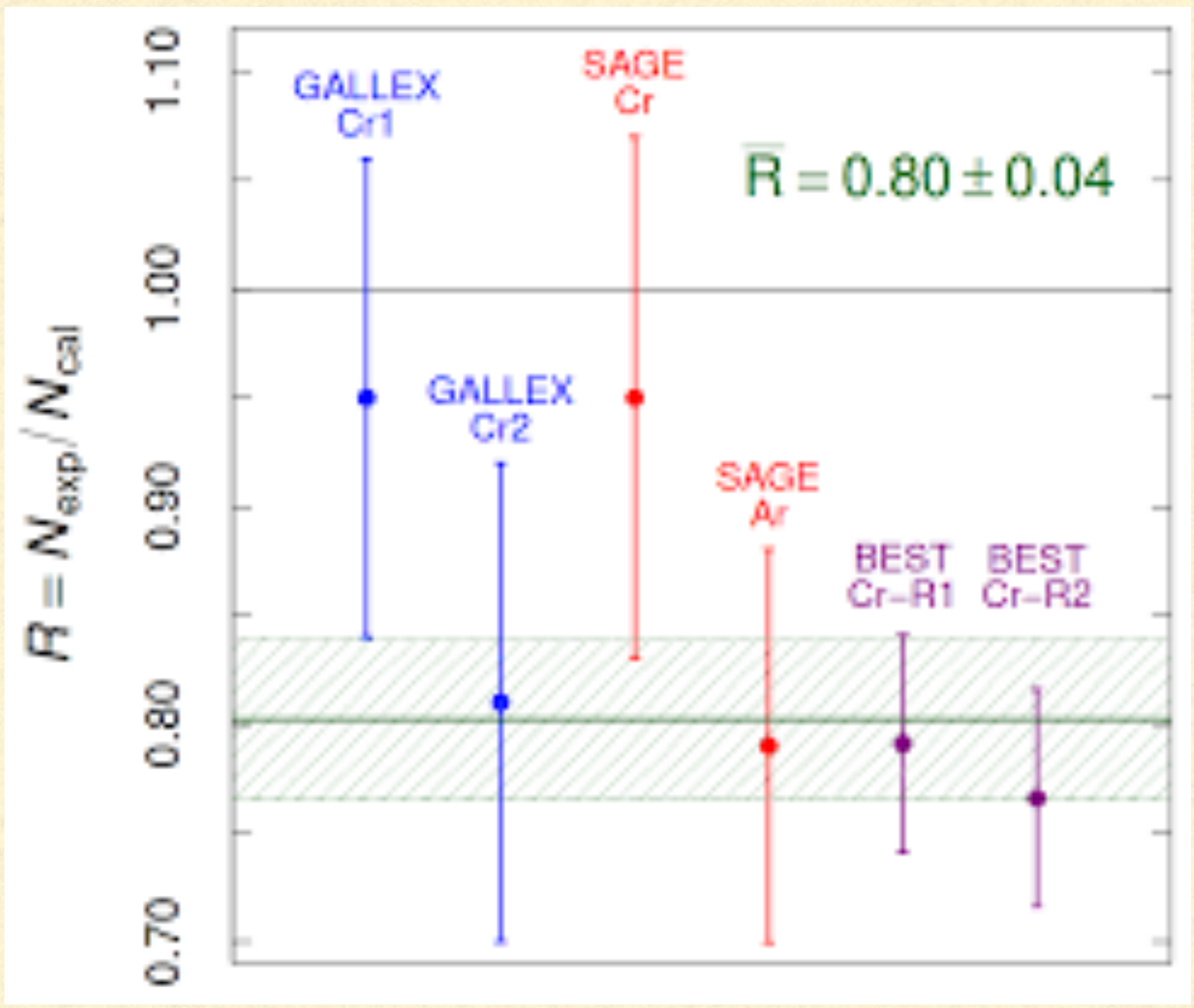

**MiniBooNE(PRD.103.052002)**




$\nu_\mu \to \nu_e$ at $4.8\sigma$ (Aguilar-Arevalo et al.)

L:540 m; 200 MeV<E<3 GeV

**Gallium Anomaly**




Deficit in $\nu_e$ at GALLEX, SAGE, BEST(Barinov et al.,2021)



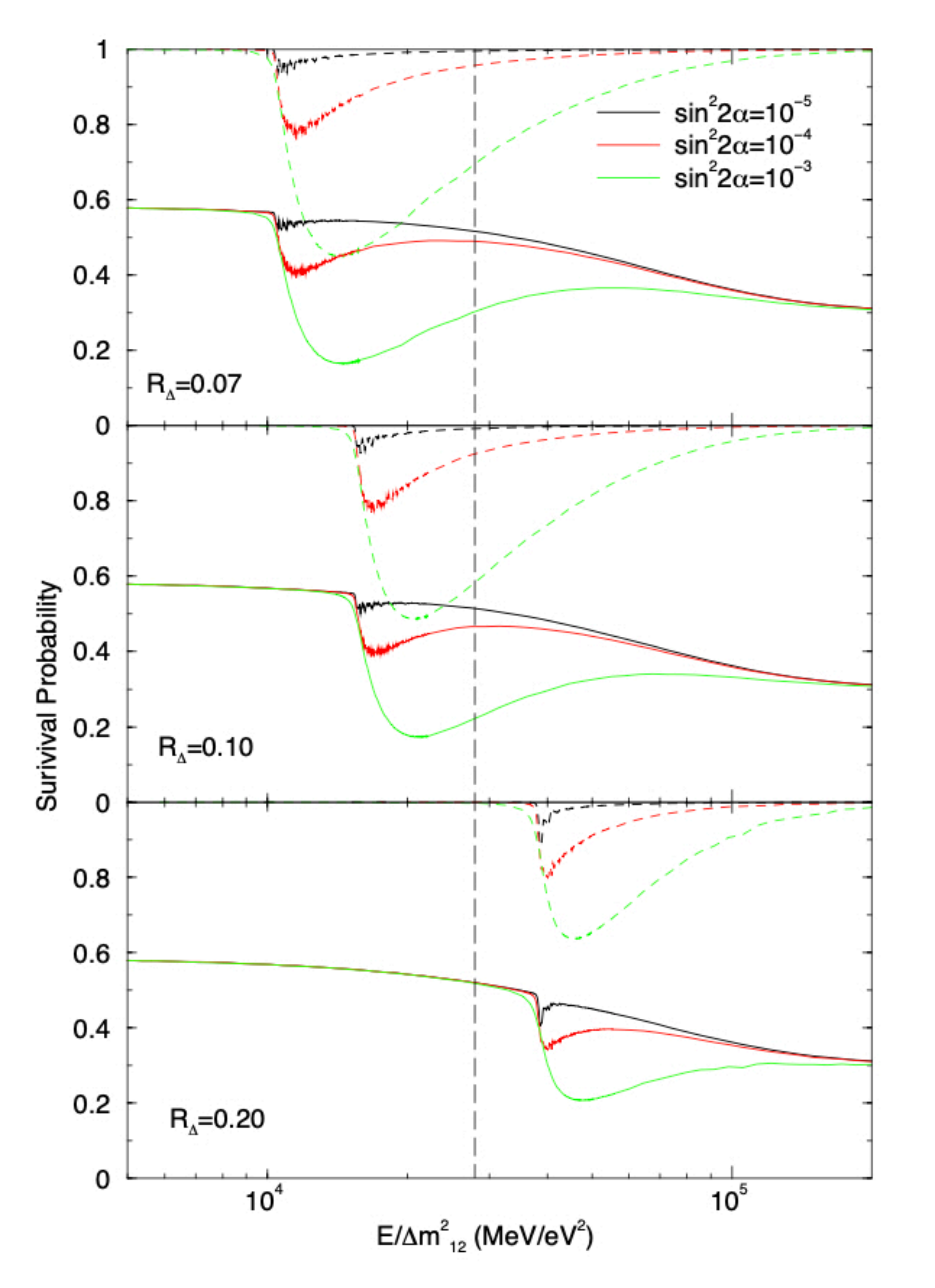
L/E ~ $1$ suggests $\Delta \sim 1\mathrm{eV}^2$ , sterile neutrino with mass ~ eV can explain these.

# Can sterile neutrino be lighter? (< eV scale)

Results from SNO, Super-K, Borexino doesn't show signatures of upturn of energy spectrum of events $(N^{obs}/N^{SSM})$below 8 MeV expected from SSM. [Phys. Rev. C 81, 055504; Phys. Rev. D 82, 033006; Phys.Rev.D 83, 052010; Phys.Rev.D 83, 113011]

- ✓ $\Delta_s \sim 10^{-5}\,\mathrm{eV}^2, R_\Delta = \Delta_s/\Delta m^2_{12}, \sin^2 2\alpha \sim 10^{-5} : 10^{-3}$
- ✓ Mass Ordering : $m_1 < m_s < m_2$ [Phys. Rev. D 69, 113002]

$[\Delta_{41} : 10^{-5} - 0.1\mathrm{eV}^2]$ alleviates the tension between results of T2K and NOvA for $\delta_{cp}$ value. *[deGouvea, PRD 106, 055025(2022)]*



# The 3+1 Oscillation Framework

$$U = \tilde{R}_{34}(\theta_{34}, \delta_{34}) R_{24}(\theta_{24}) \tilde{R}_{14}(\theta_{14}\delta_{14}) R_{23}(\theta_{23}) \tilde{R}_{13}(\theta_{13}, \delta_{13}) R_{12}(\theta_{12})$$

$$H = \frac{1}{2E_\nu} U \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 & 0 \\ 0 & 0 & \Delta_{31} & 0 \\ 0 & 0 & 0 & \Delta_{41} \end{bmatrix} U^\dagger + \frac{1}{2E_\nu} \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{A}{2} \end{bmatrix}$$

More parameters means more possible degeneracies

Agarwalla et al., PRL, 2017 showed that with an eV scale sterile neutrino the octant sensitivity reduces in the context of DUNE using beam neutrinos.

We explored the octant sensitivity with combined beam+atmospheric baselines where the matter effect is essential and also explored sensitivity to mass ordering

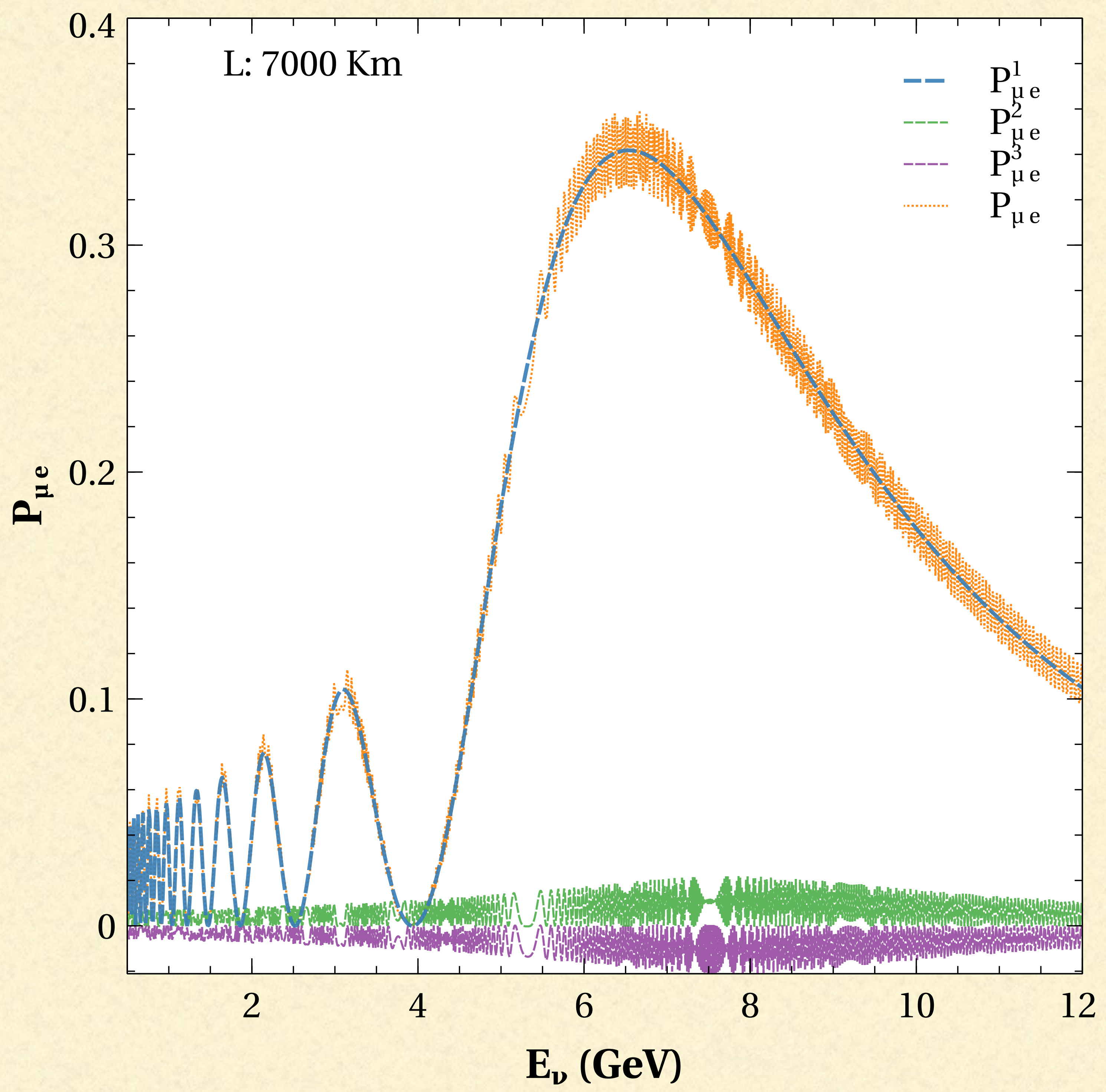
For our calculation: $\Delta_{21} = 0$ , $\theta_{34} = 0$ approximation is used, with perturbation theory

# Analytical Probability

$$E_{res} = \frac{\Delta_{31}\cos 2\theta_{13}}{\sqrt{2}G_F N_e(1+\cos^2\theta_{14}+\cos^2\theta_{14}\sin^2\theta_{24}-2\sin^2\theta_{24})}$$

$$E_{res} \sim 7.5\text{GeV}$$

$$\Delta_{31}^m = \sqrt{(\Delta_{31}\cos 2\theta_{13} - A)^2 + (\Delta_{31}\sin 2\theta_{13})^2}$$

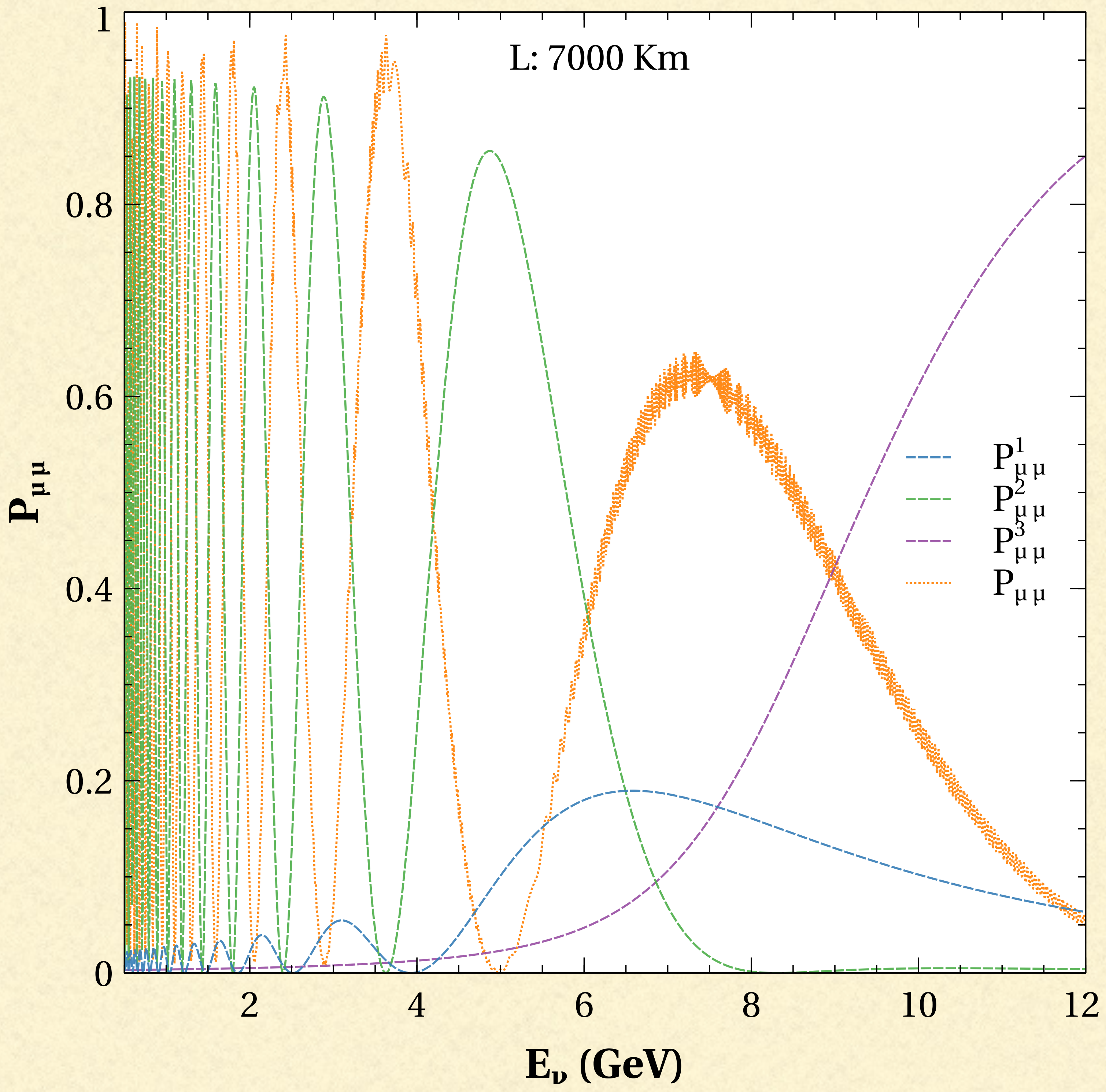



$$\sin 2\theta_{13m} = \frac{\Delta_{31}\sin 2\theta_{13} + A^{'}\cos\delta\sin\theta_{14}\sin\theta_{23}\sin 2\theta_{24}}{f}$$

$$f = \left([\Delta_{31}\sin 2\theta_{13} + A^{'}s_{14}s_{23}\sin 2\theta_{24}\cos\delta]^2 + [\Delta_{31}\cos 2\theta_{13} - A^{'}(1+c_{14}^2+c_{14}^2 s_{24}^2 - 2s_{24}^2)]^2\right)^{1/2}$$

$$\cos\theta_{14m} = \cos\theta_{14}[1 - \frac{A^{'}}{\Delta_{41}}\sin^2\theta_{14}(1+\sin^2_{24})]$$

$$\cos\theta_{24m} = \cos\theta_{24}[1 + \frac{A^{'}}{\Delta_{41}}\cos^2\theta_{14}\sin^2\theta_{24}]$$




$$P^1_{\mu e} \sim \cos^2\theta_{14m}\sin^2 2\theta_{13m}\cos^2\theta_{24m}\sin^2\theta_{23}\sin^2\frac{1.27\Delta_{31}^m L}{E}$$

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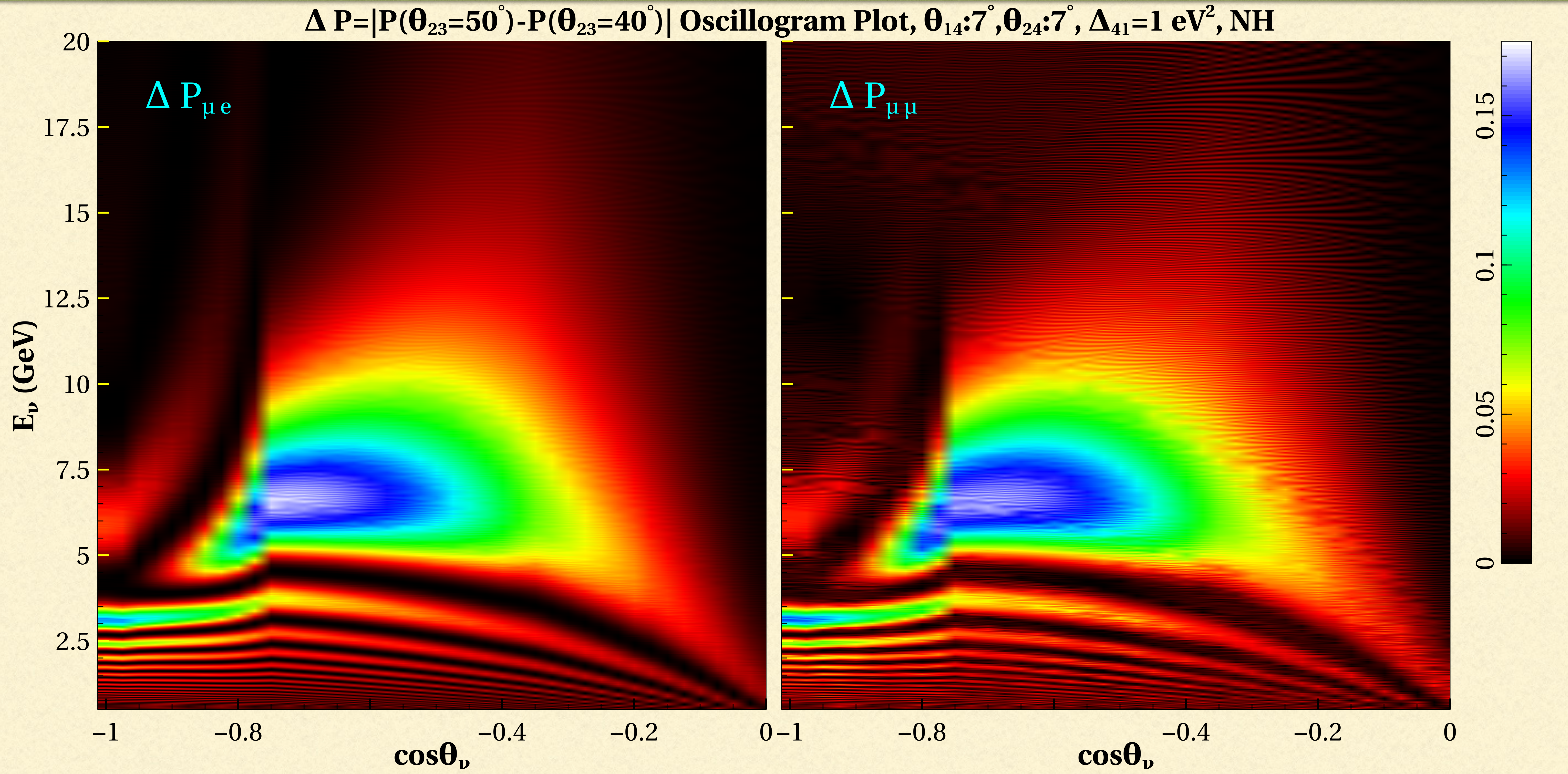
$$P^1_{\mu\mu} \sim \cos^4\theta_{24m}\sin^2 2\theta_{13m}\sin^4\theta_{23}\sin^2\frac{1.27\Delta_{31}^m L}{E}$$

# Probability Oscillogram

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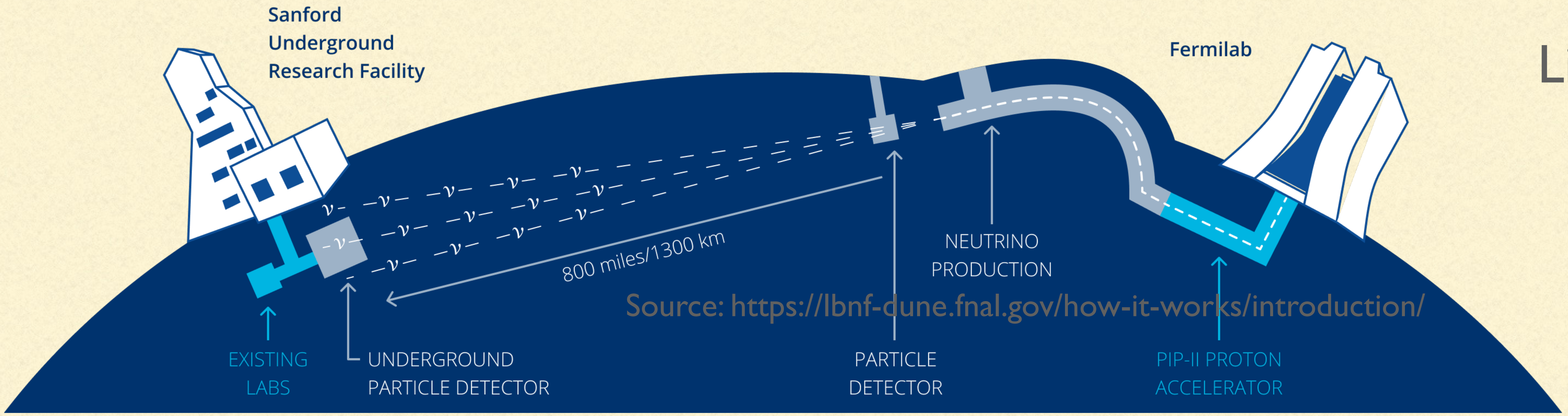
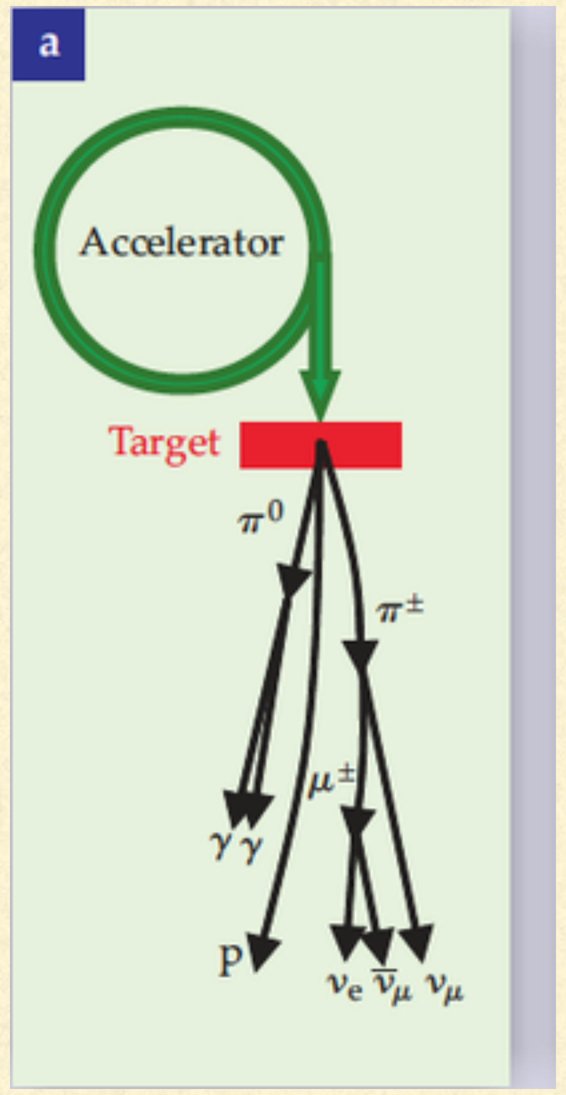
- For $L : [5 - 10] \times 10^3$ km, and $E_\nu : 5 - 10$ GeV, $\Delta P$ is higher
- These higher baselines correspond to atmospheric $\nu$.

# Deep Underground Neutrino Experiment (DUNE)

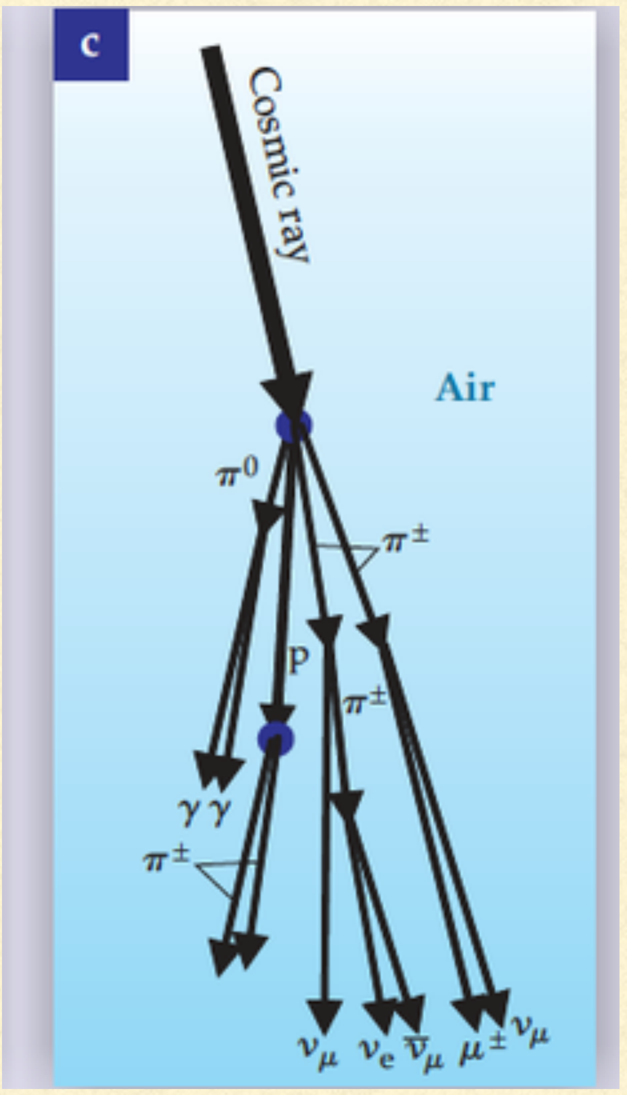



Source: https://lbnf-dune.fnal.gov/how-it-works/introduction/

Liquid Ar: 40 kton

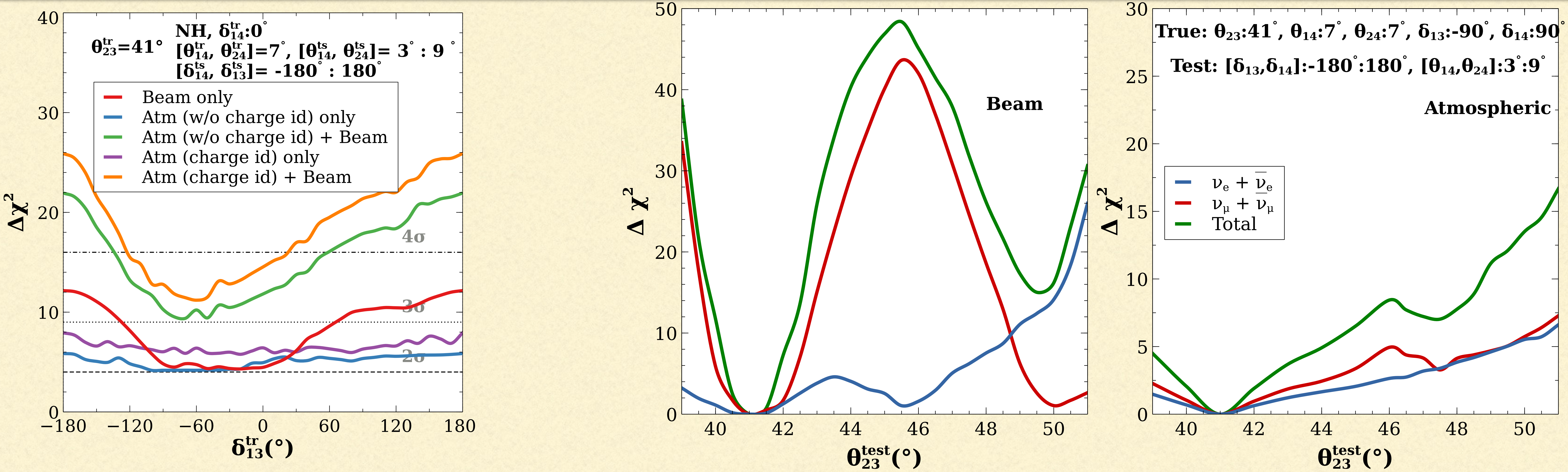



$L^{beam}$ :1300 km
$E_{\nu}^{beam} = 2 - 3\text{GeV}$




$L^{atm}$: $[5 - 12] \times 10^3$ km
$E_{\nu}^{atm}$ : upto 20 GeV

# Octant Sensitivity ($\Delta_{41} = 1 eV^2$)




- The sensitivity is prominently higher for combined analysis; over $3\sigma$ .
- Sensitivity for atmospheric data: $2\sigma - 2.5\sigma$.
- The synergy between different $\nu_e, \nu_\mu$ channels.

$$\chi^2 = \frac{[N^{Th}(\theta_{23}^{tr}) - N^{test}(\theta_{23}^{test})]^2}{N^{test}}; \theta_{23}^{tr} = 41^\circ$$

Marginalised over: $\theta_{23}^{ts}$ in range $(45^\circ, 51^\circ)$

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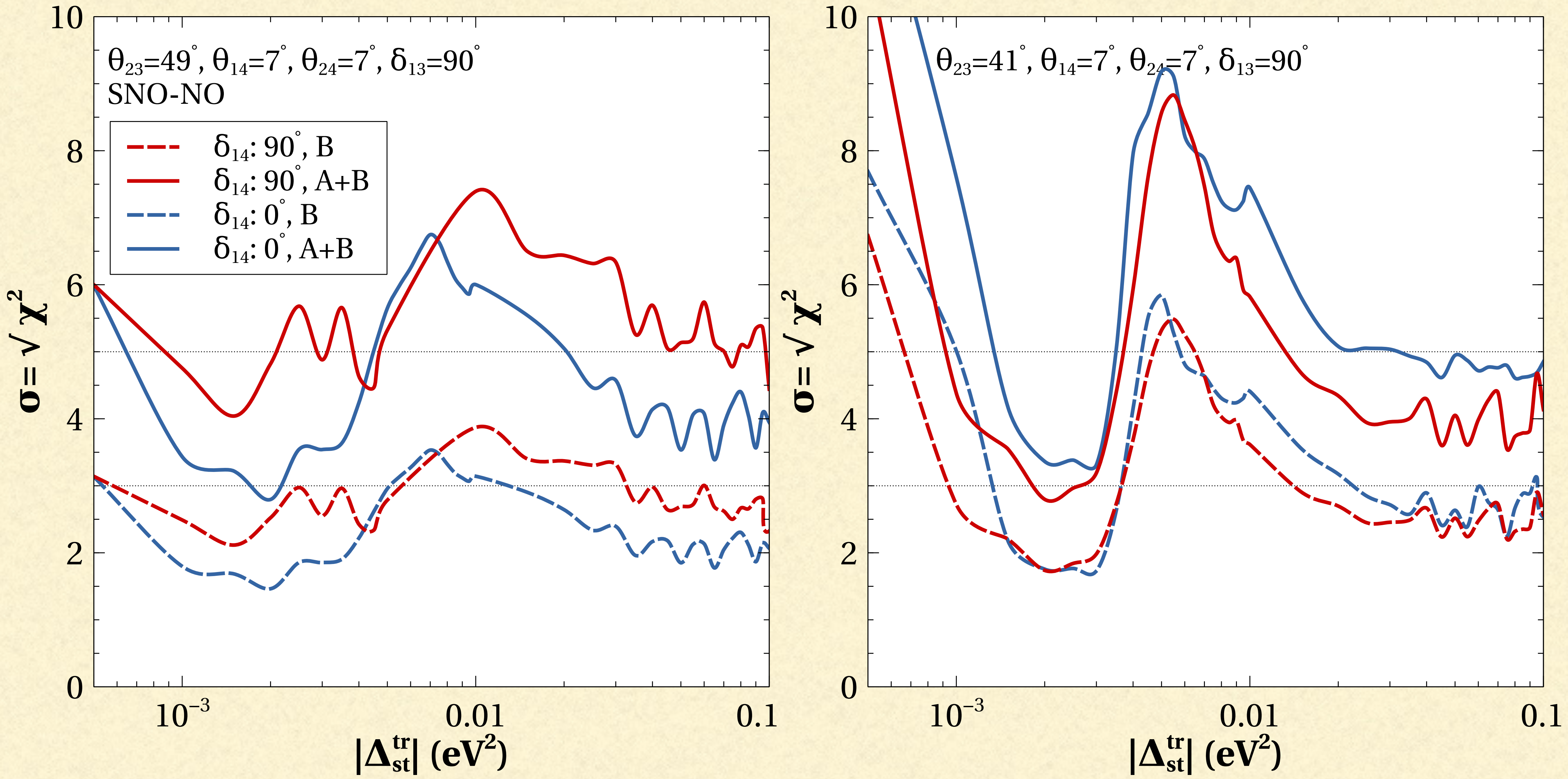
# Octant sensitivity as a function of $\Delta_{41}$

- For (SNO-NO), (SNO-IO), at $\Delta_{41} \approx 2.5 - 3 \times 10^{-3} \mathrm{eV}^2$ , sensitivity dips as $|\Delta_{41}| \sim |\Delta_{31}|$

- Sensitivity peak at $\Delta_{41} \approx 5 \times 10^{-3} \mathrm{eV}^2$ for SNO-IO, $10^{-2} \mathrm{eV}^2$ for SNO-NO.

- Atmospheric neutrinos increase sensitivity helps in the combined analysis to get increased sensitivity.

- Combined analysis: sensitivity $> 3\sigma$ for $|\Delta_{41}| = [10^{-4}, 10^{-1}] \mathrm{eV}^2$.




$$\chi^2 = \frac{[N^{Th}(\theta_{23}^{tr}) - N^{test}(\theta_{23}^{test})]^2}{N^{test}}$$

Marginalised over: $\theta_{23}^{ts}$ in opposite octant, $\theta_{14}, \theta_{24}, \delta_{13}, \delta_{14}$

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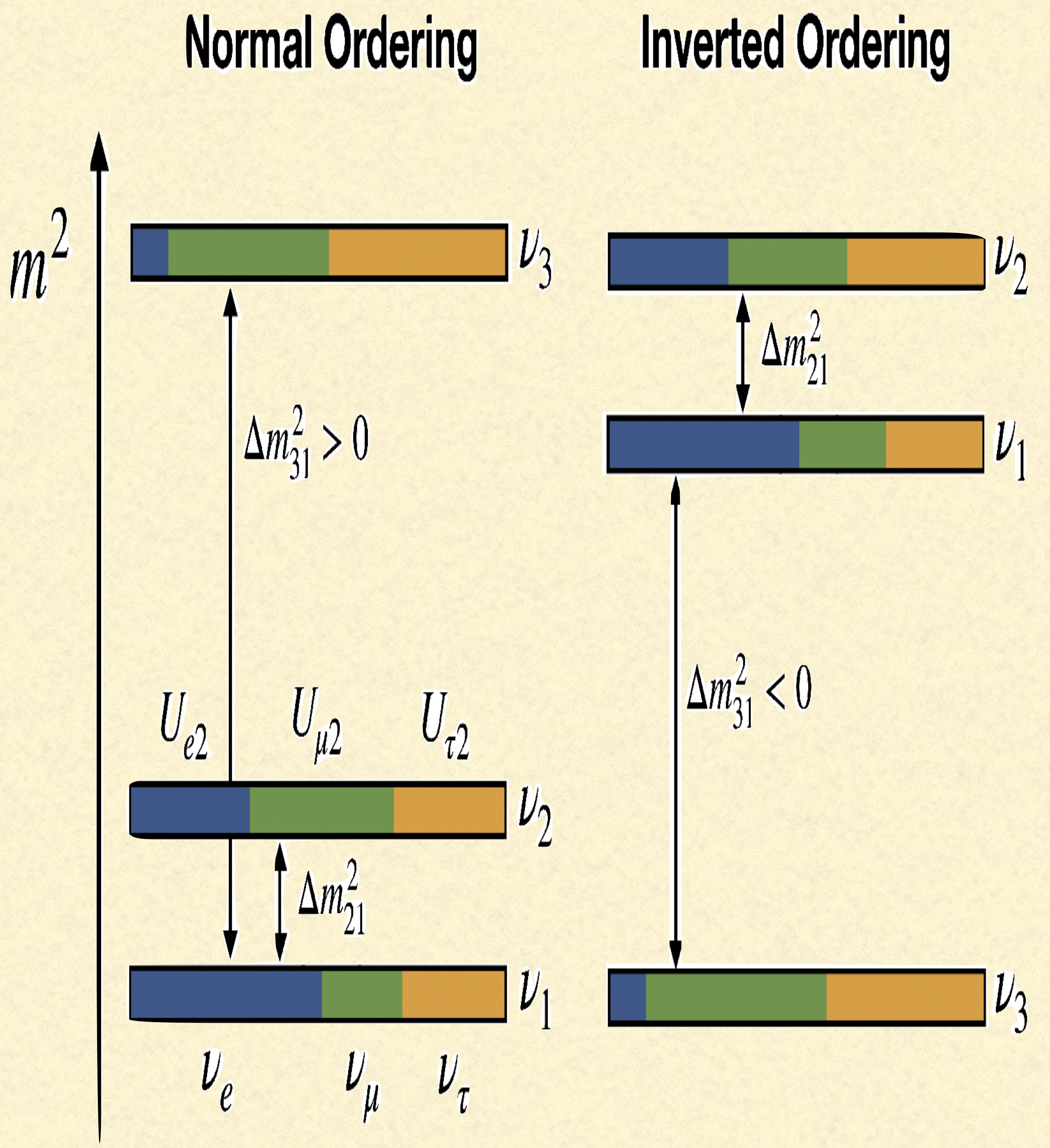
# Mass ordering in presence of a light sterile $\nu_s$

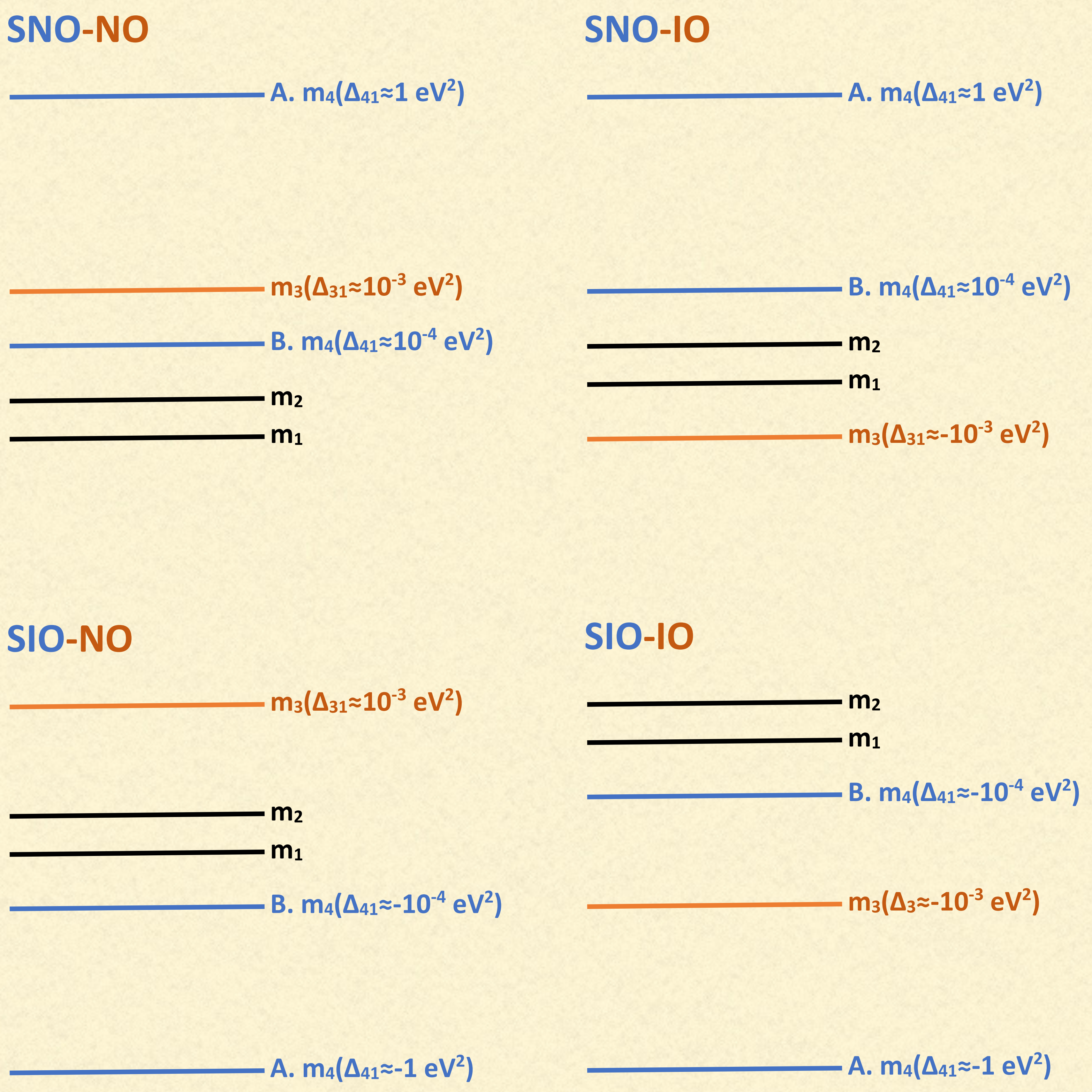
- Sterile mass ordering: the sign of $\Delta_{41}$(SMO)
- Atmospheric mass ordering: the sign of $\Delta_{31}$




https://doi.org/10.3390/psf2023008007

Four possible mass-spectra:

- SNO-NO: $\Delta_{41} > 0, \Delta_{31} > 0$
- SNO-IO: $\Delta_{41} > 0, \Delta_{31} < 0$
- SIO-NO: $\Delta_{41} < 0, \Delta_{31} > 0$
- SIO-IO: $\Delta_{41} < 0, \Delta_{31} < 0$

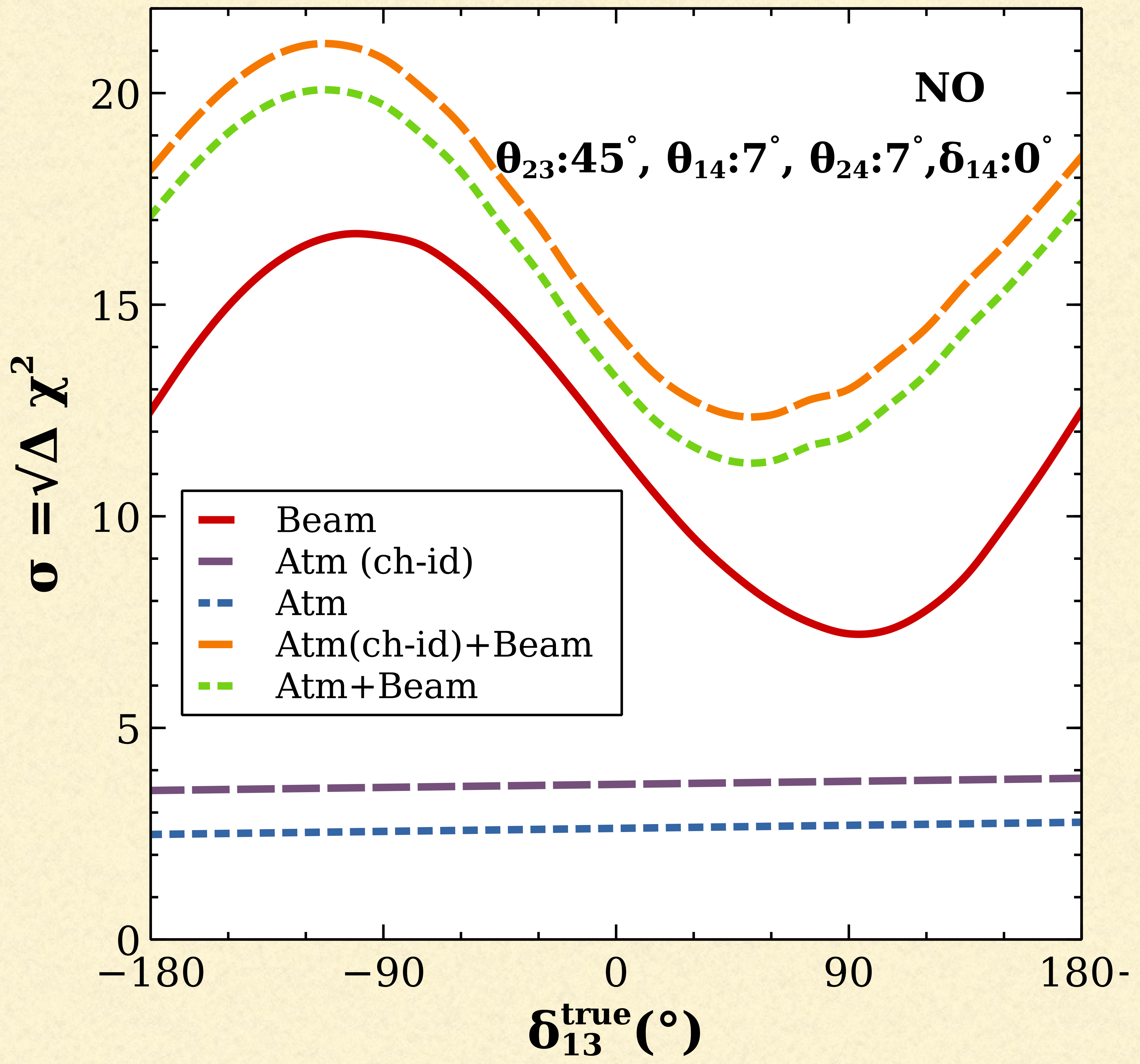



The 3+1 mass spectrum

# Mass Ordering sensitivity ($\Delta_{41} = 1\text{eV}^2$)

- Sensitivity for atmospheric neutrinos doesn't significantly depend on $\delta_{13}$.

- The combined sensitivity of the beam and atmospheric is greater than the sum of individual sensitivities, demonstrating the synergy between these channels.




$$\chi^2 = \frac{[N^{Th}(\Delta_{31}(NO)) - N^{test}(\Delta_{31}(IO))]^2}{N^{test}}$$

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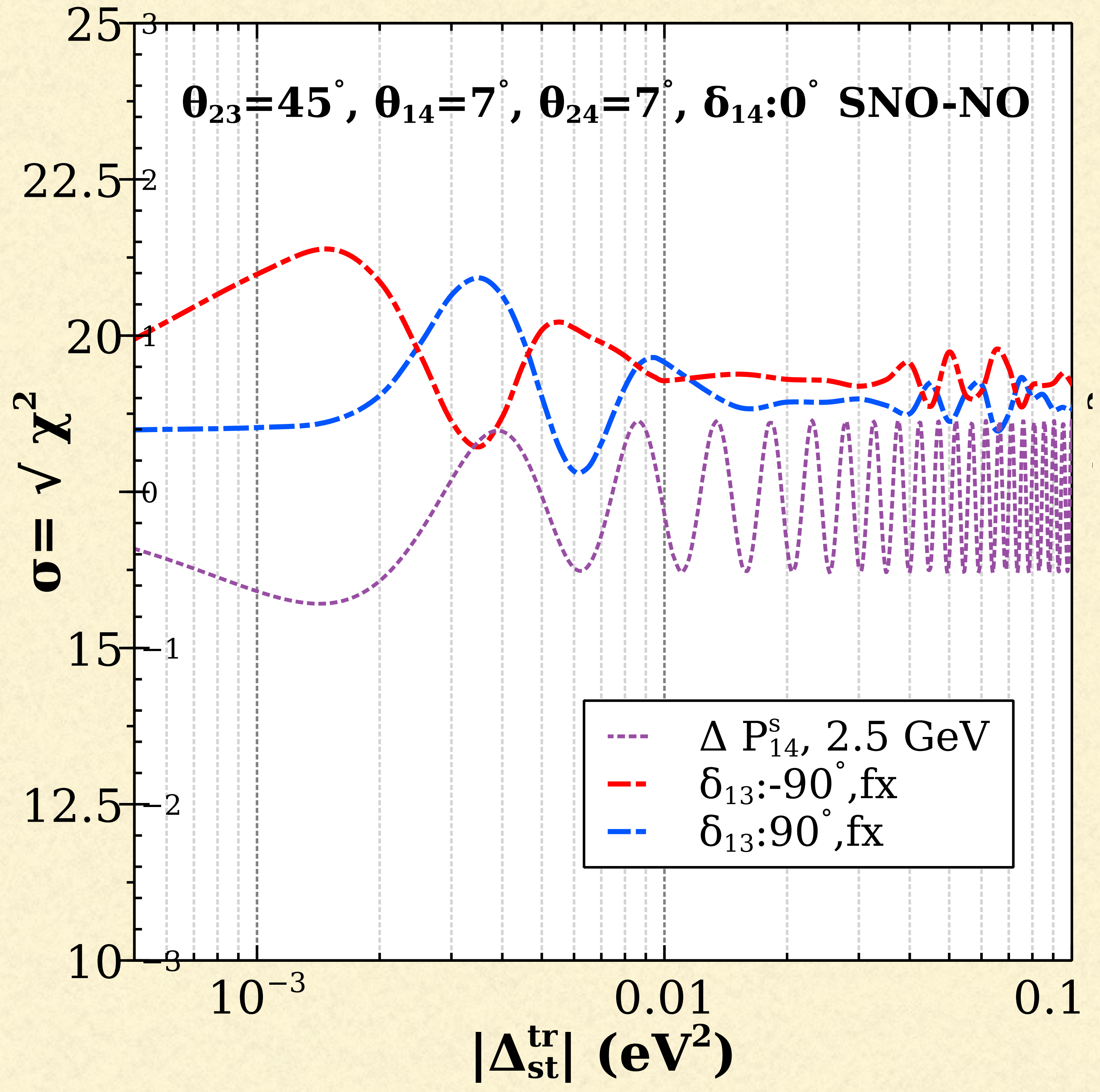
# MO sensitivity as a function of $\Delta_{41}$

$$\Delta P^{st}_{\mu e} = P^{m,true}_{\mu e}(i_1\Delta_{41}, j_1\Delta_{31}) - P^{m,test}_{\mu e}(i_2\Delta_{41}, j_2\Delta_{31}) = 4 s_{13} s_{14} s_{24} s_{23} \Delta P^{s}_{14} \sin[\delta_{13} + \delta_{14}]$$

$i_1, j_1, i_2, j_2$ signify signs of $\Delta_{31}, \Delta_{41}$




- Sensitivity shows maxima/minima in range of $\Delta_{41}$ : $(10^{-3}, 10^{-2})\text{eV}^2$
- $\Delta P^{st}_{\mu e}$ flips sign for $\delta_{13} = 90°$ and $-90°$ leading to the opposite nature of $\chi^2$.
- $\Delta P^{st}_{\mu e}|_{SNO-NO} = -\Delta P^{st}_{\mu e}|_{SNO-IO}$

$$\chi^2 = \frac{[N^{Th}(\Delta^{tr}_{31}) - N^{test}(\Delta^{test}_{31})]^2}{N^{test}}$$

$\chi^2$ : only $\Delta_{31}$ marginalised over opposite MO

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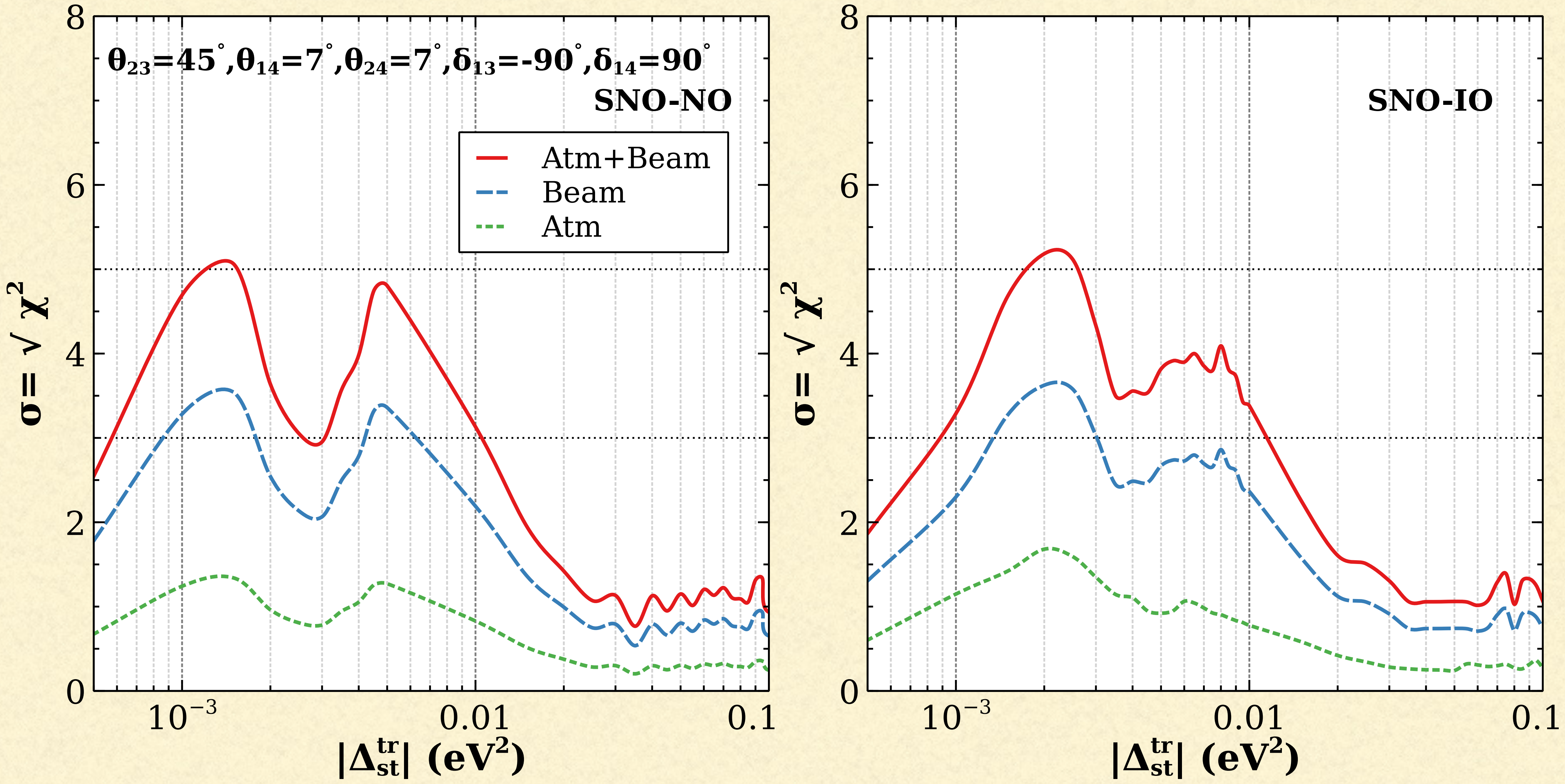
# SMO sensitivity as a function of $\Delta_{41}$

- For (SNO-NO), (SNO-IO), at $\Delta_{41} \approx 2.5 - 3 \times 10^{-3} \mathrm{eV}^2$ , sensitivity dips(rises) as $|\Delta_{41}| \sim |\Delta_{31}|$
- Atmospheric sensitivity is very low but helps in the combined analysis.
- Combined analysis: sensitivity $> 3\sigma$ for $|\Delta_{41}| < 10^{-2} \mathrm{eV}^2$. At higher $|\Delta_{41}|$ sensitivity falls to $< 1.5\sigma$




$$\chi^2 = \frac{[N^{Th}(\Delta_{41}^{tr}) - N^{test}(\Delta_{41}^{test})]^2}{N^{test}}$$

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# Summary

- Analytic results can explain the sensitivity to octant and mass ordering
- Sensitivity to octant of $\theta_{23}$: above $3\sigma$ for beam+atmospheric analysis
- Sensitivity to MO: above $5\sigma$ for beam + atmospheric analysis
- Sensitivity to SMO: above $3\sigma$ at $|\Delta_{41}| < 0.01\mathrm{eV}^2$ for beam + atmospheric analysis
- Sensitivity to octant of $\theta_{23}$ shows dips at $|\Delta_{41}| \sim |\Delta_{31}|$
- Sensitivity to SMO has maxima/minima at $|\Delta_{41}| \sim |\Delta_{31}|$


THANK YOU