Measure of Quantum Complexity in Neutrino Oscillations

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Abstract

Neutrino flavor oscillation, a crucial phenomenon in particle physics, explores the interplay between flavor and mass eigenstates, revealing insights beyond the standard model. Probabilistic measures traditionally study these transitions, while the quantum features of neutrinos, such as entanglement, open avenues for quantum information tasks. Quantum complexity, an evolving field, finds application in understanding neutrino oscillations, particularly through quantum spread complexity, offering insights into charge-parity symmetry violations. Our results suggest that complexity favors the maximum violation of charge-parity, which is consistent with recent experimental data. This approach enhances our grasp of neutrino behavior, connecting quantum information theory with particle physics.

Motivation

- Quantum computational complexity → Estimates the difficulty of constructing quantum states from elementary operations.
- It can also serve to study Information processing inside black holes.
- Extends the connection between geometry and information → Growth of complexity ≡ growth of black hole interiors [1].
- The highest rate of complexity growth has been observed for de Sitter space, most popular model for inflation [2].
- *What characteristics complexity shows in other natural processes of evolution?*
- Neutrinos have shown features such as entanglement and nonlocal correlations [3,4].
- How complex is an evolution of neutrino system and if complexity can also probe any open issue in the neutrino sector.
- *Whether this maximization of complexity occurs in neutrino oscillations?*

Neutrino-System

- Flavor states are superposition of mass eigenstates $|\nu_\alpha\rangle = U^*|\nu_i\rangle$
- Time evolution of the flavor states is given by $i\frac{\partial}{\partial t}|\nu_\alpha(t)\rangle = Hf|\nu_\alpha(t)\rangle$
  Flavor Hamiltonian $H_f = UHmU^{-1}$; Mass Hamiltonian $H_m = diag(Ei)$
- Time-evolved states $|\nu_\alpha(t)\rangle = e^{-iH_f t}|\nu_\alpha(0)\rangle$ for an initial flavor ν_α
  Unitary operator
- **2-Flavor Scenario:** $H_m = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$; $U = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$ (mixing matrix)

Osc. Probability $P_{\alpha\beta} = 1 - P_{\alpha\alpha} = sin^2 2\theta\, sin^2(\frac{\Delta m^2_{21} L}{4E})$; $E_2 - E_1 \approx \Delta m^2_{21}/2E$

3-Flavor Scenario:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{13}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

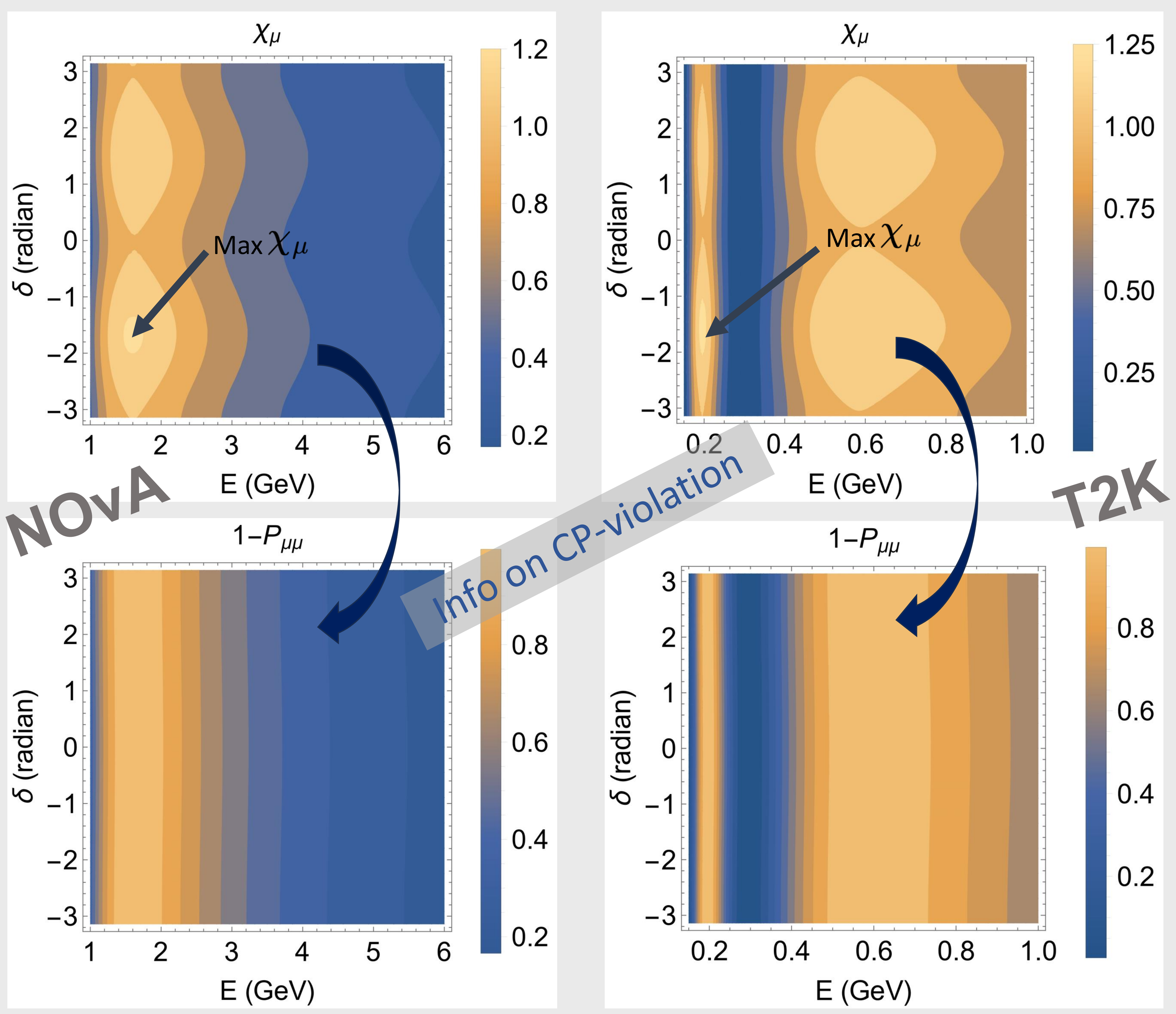

Figure 1. χ_μ mimicking $1-P_{\mu\mu}$ and containing Information on CP-phase.

Spread Complexity

- *Complexity:* For a system [5,6],
  $U_1U_2U_3U_2|\varphi(s)\rangle = U_3U_1U_2U_1(U_1)^3U_2|\varphi(s)\rangle$
  Complexity = Min number of unitaries = 4
- ***Spread Complexity:*** *spread of the target state $|\psi(t)\rangle$ in the Hilbert space relative to the reference state $|\psi(0)\rangle$ through unitary transformations, and the spread minimized over all possible bases [7,8].*
- General time evolution of a system with Hamiltonian
  $|\psi(t)\rangle = \sum_{n=0}^{\infty}\frac{(-it)^n}{n!}H^n|\psi(0)\rangle = \sum_{n=0}^{\infty}\frac{(-it)^n}{n!}|\psi_n\rangle$
- *Krylov basis:* *Gram-Schmidt* procedure on $\{|\psi_n\rangle = H^n|\psi(0)\rangle\}$ gives ordered-orthonormal basis, $\mathcal{K} = \{|K_n\rangle, n = 0, 1, 2\ldots\}$ that minimizes the *Cost Fun:*
  $$\chi = \sum_{n=0}^{\infty} n|\langle K_n|\psi(t)\rangle|^2$$
  Cost fun quantifies complexity in the state evolution [7].

Complexity in Neutrino Oscillations

2-Flavor Case: 2 initial states- $|\nu_e(0)\rangle = \binom{1}{0}, |\nu_\mu(0)\rangle = \binom{0}{1}$

Krylov states ≡ Flavor states

$\{|Kn\rangle\} =$
$\{|K_0\rangle = \binom{1}{0}, |K_1\rangle = \binom{0}{1}\} = \{|\nu_e\rangle, |\nu_\mu\rangle\}$ *for initial* ν_e
$\{|K_1\rangle = \binom{0}{1}, |K_1\rangle = \binom{0}{1}\} = \{|\nu_\mu\rangle, |\nu_e\rangle\}$ *for initial* ν_μ

→ $\chi_e = P_{e\mu}$, $\chi_\mu = P_{\mu e}$ ⇒ Complexity contains same information as probabilities.

3-Flavor case:

3 initial states $-|\nu_e(0)\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}, |\nu_\mu(0)\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}, |\nu_\tau(0)\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$

→ Krylov states ≠ Flavor states
→ $\chi_e, \chi_\mu, \chi_\tau$ have cross terms along with probabilities, and ***hence, contain more information than the probabilities [9].***

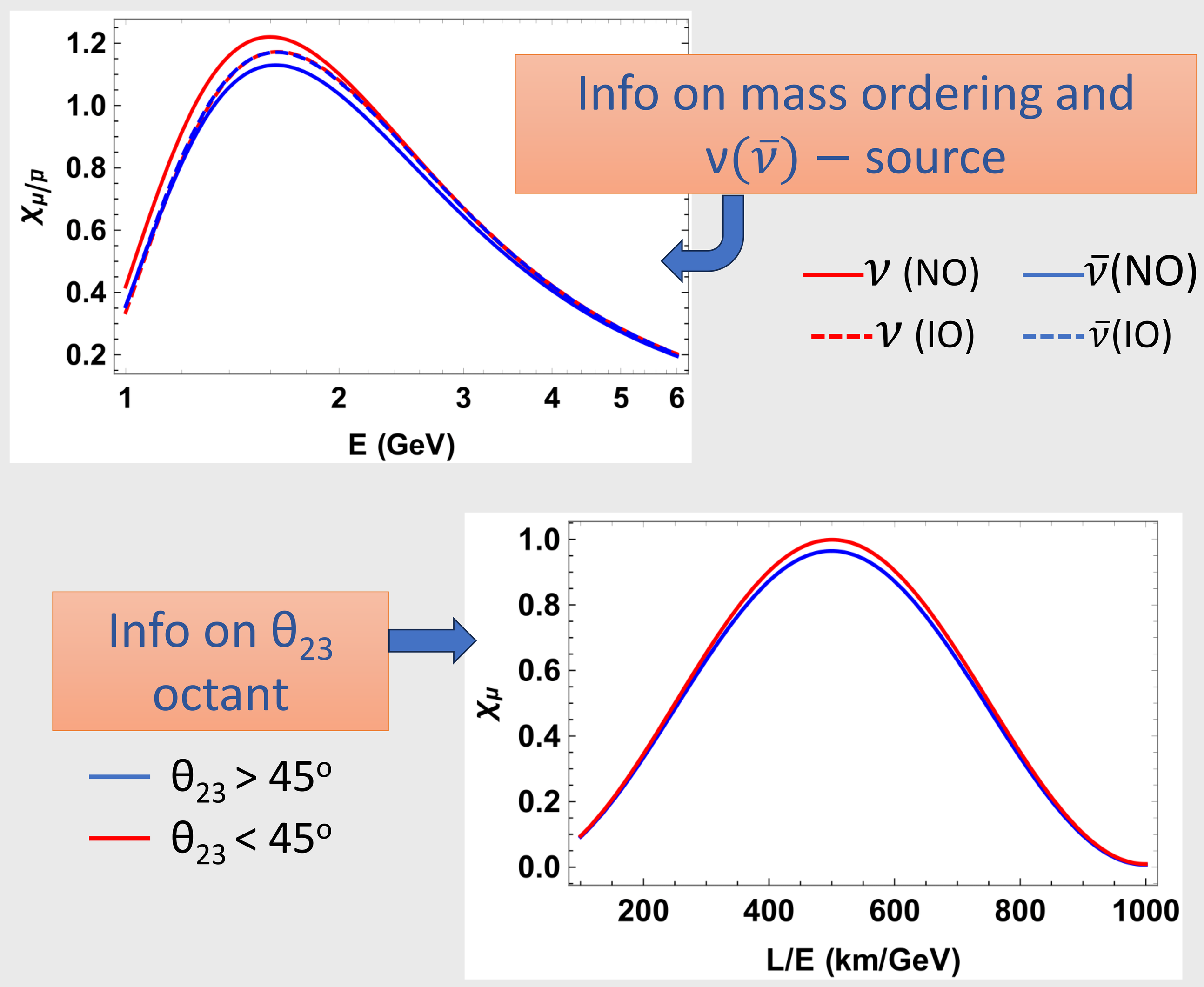

Figure 2. χ_μ containing Information on mass ordering & θ_{23} octant.

What Complexity tells us about Neutrino Oscillations?

- In the T2K and NOvA experimental setups, where only ν_μ-beams are produced, the only relevant complexity is χ_μ.
- *For both the T2K and NOvA, χ_μ is maximized at $\delta \approx -1.5$ radian at the relevant experimental energies. The T2K best-fit value of $\delta = -2.14^{+0.90}_{-0.69}$ radian is consistent with this expectation.*
- The NOvA best-fit, is at $\delta \approx 2.58$ radian which is far away from the maximum χ_μ in the lower-half plane of δ but is still within a region of high χ_μ value in the upper-half plane of δ.
- $P_{\mu e}$, the only oscillation probability accessible to the T2K and NOvA setups, becomes maximum at $\delta \approx -1.5$ radian and is compatible with T2K best-fit but in odd with the NOvA best-fit.
- *Complexity provides correct prediction for δ in experimental setups.*

Conclusions

We examined the spread complexity of neutrino states in 2- and 3-flavor oscillation scenarios:

- *In 2-flavor scenario*, complexity and transition probabilities yield equivalent information. *In 3-flavor oscillation*, complexity contains additional information regarding open issues related to neutrinos, compared to the total oscillation probability.
- *Remarkably, complexity is maximized for a value of the phase angle for which CP is also maximally violated. T2K data also favors this phase angle, which is obtained in terms of flavor transition.*

Quantum spread complexity emerges as a potent and novel quantity for investigating neutrino oscillations, successfully reproducing existing results, also demonstrating the potential to serve as a theoretical tool for predicting new outcomes in future experiments.

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