SIMPLIFIED MODELS OF D=7 LEPTON NUMBER VIOLATION

ICHEP 2024, Prague, 17th-24th July 2024

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Based on work done in collaboration with Lukáš Gráf, Chandan Hati and Julia Harz

JHEP 05 (2024) 154 & arXiv:2407:XXXXX

Majorana neutrino masses require violation of lepton number ΔL=2, which can be studied model-independently using SMEFT

$$\mathcal{L}_{\rm EFT} = \sum_i C_i \mathcal{O}_i + \text{h.c.} \qquad \text{Wilson coefficient: } C_i \propto \frac{1}{\Lambda^{(D-4)}},\ \Lambda = \text{New Physics (NP) scale}$$

Babu, Leung, Nucl.Phys.B 619 (2001) 667-689

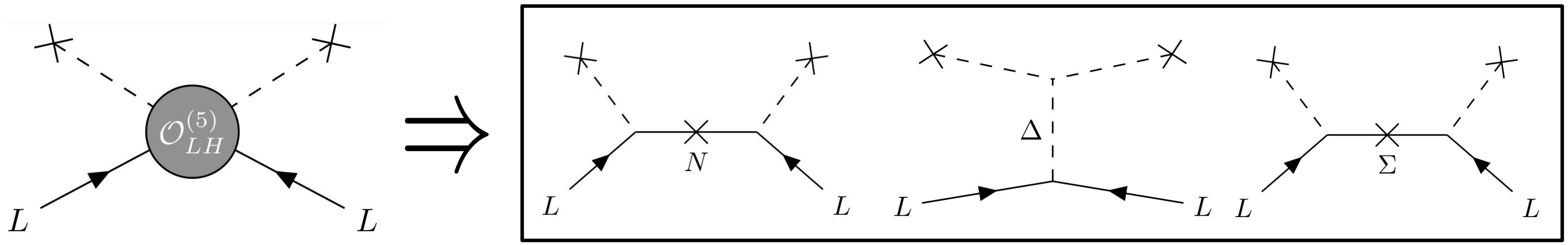
de Gouvêa, Jenkins, Phys.Rev.D 77 (2008) 013008

LNV only occurs at odd mass dimensions:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda_1}\mathcal{O}_1^{(5)} + \sum_i \frac{1}{\Lambda_i^3}\mathcal{O}_i^{(7)} + \sum_i \frac{1}{\Lambda_i^5}\mathcal{O}_i^{(9)} + \cdots$$

Lowest dimension possible is 5, where we only have a single operator:

$$\mathcal{O}_{LH}^{(5)} = L^\alpha L^\beta H^\rho H^\sigma \epsilon_{\alpha\rho}\epsilon_{\beta\sigma}$$



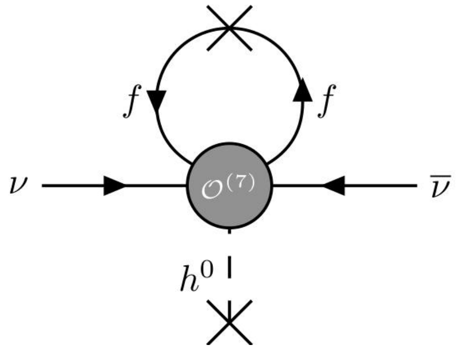
From here we get the neutrino mass: $m_\nu \approx \dfrac{v^2}{\Lambda}$ $\quad m_\nu \approx 0.1\ \text{eV} \ \rightarrow\ \Lambda \approx 10^{14}\ \text{GeV}$

Second-most simple realization: LNV at dimension-7

12 operators instead of one

Some are higher-order corrections to the dim-5 operator

Others lead to radiative neutrino masses



Type	$\mathcal{O}$	Operator
$\Psi^2 H^4$	$\mathcal{O}_{LH}^{pr}$	$\epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{c}}^i L_r^m\right) H^j H^n \left(H^\dagger H\right)$
$\Psi^2 H^3 D$	$\mathcal{O}_{LeHD}^{pr}$	$\epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{c}}^i \gamma_\mu e_r\right) H^j \left(H^m iD^\mu H^n\right)$
$\Psi^2 H^2 D^2$	$\mathcal{O}_{LHD1}^{pr}$	$\epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{c}}^i D_\mu L_r^j\right)\left(H^m D^\mu H^n\right)$
	$\mathcal{O}_{LHD2}^{pr}$	$\epsilon_{im}\epsilon_{jn}\left(\overline{L_p^{c}}^i D_\mu L_r^j\right)\left(H^m D^\mu H^n\right)$
$\Psi^2 H^2 X$	$\mathcal{O}_{LHB}^{pr}$	$g\epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{c}}^i \sigma_{\mu\nu} L_r^m\right) H^j H^n B^{\mu\nu}$
	$\mathcal{O}_{LHW}^{pr}$	$g'\epsilon_{ij}\left(\epsilon\tau^I\right)_{mn}\left(\overline{L_p^{c}}^i \sigma_{\mu\nu} L_r^m\right) H^j H^n W^{I\mu\nu}$
$\Psi^4 D$	$\mathcal{O}_{\bar{d}uLLD}^{prst}$	$\epsilon_{ij}\left(\overline{d_p}\gamma_\mu u_r\right)\left(\overline{L_s^{c}}^i iD^\mu L_t^j\right)$
$\Psi^4 H$	$\mathcal{O}_{\bar{e}LLLH}^{prst}$	$\epsilon_{ij}\epsilon_{mn}\left(\overline{e_p} L_r^i\right)\left(\overline{L_s^{c}}^j L_t^m\right) H^n$
	$\mathcal{O}_{\bar{d}LueH}^{prst}$	$\epsilon_{ij}\left(\overline{d_p} L_r^i\right)\left(\overline{u_s^c} e_t\right) H^j$
	$\mathcal{O}_{\bar{d}LQLH1}^{prst}$	$\epsilon_{ij}\epsilon_{mn}\left(\overline{d_p} L_r^i\right)\left(\overline{Q_s^{c}}^j L_t^m\right) H^n$
	$\mathcal{O}_{\bar{d}LQLH2}^{prst}$	$\epsilon_{im}\epsilon_{jn}\left(\overline{d_p} L_r^i\right)\left(\overline{Q_s^{c}}^j L_t^m\right) H^n$
	$\mathcal{O}_{\bar{Q}uLLH}^{prst}$	$\epsilon_{ij}\left(\overline{Q_p} u_r\right)\left(\overline{L_s^{c}} L_t^i\right) H^j$

Conventional wisdom for m_ν: close the loop and remove two powers of Λ

$$m_\nu \approx \frac{1}{16\pi^2}\int_{\text{loop}} \frac{y_f v^2}{\Lambda^3} \approx \frac{1}{16\pi^2}\frac{y_f v^2}{\Lambda} \qquad m_\nu \approx 0.1\ \text{eV} \quad \rightarrow \quad \Lambda \approx 10^{12}\ \text{GeV}$$

What are the caveats of this estimate?

We consider the full range of underlying tree-level models at dim-7, similar to how the three seesaw models generate the dim-5 operator

UV-completions of four-fermion operators at dim-7 require at least two fields

Example of a complete set:

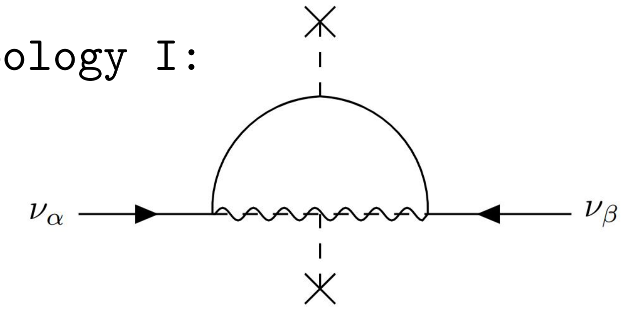
$$\mathcal{O}_{\bar{d}LQLH1} = \epsilon_{ij}\epsilon_{mn}\left(\overline{d_p}L_r^i\right)\left(\overline{Q_s^{cj}}L_t^m\right)H^n$$

	Δ	φ	S_1	$\tilde{R}_2$	S_3	N	Σ	$Q_5^\dagger$	T_2
Δ		I						II	II
φ						○	○		
S_1				I		○		II	
$\tilde{R}_2$					I	○	○		II
S_3							○	II	
N									
Σ									
$Q_5^\dagger$									
T_2									

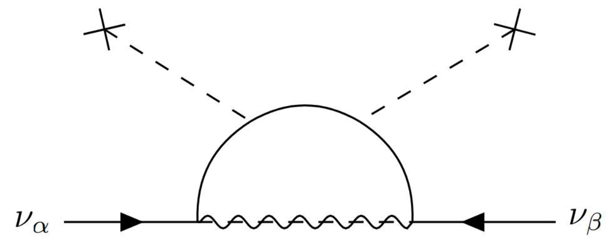
$SU(3)_c \times SU(2)_L \times U(1)_Y$

Field	Rep
Δ	$S(1,3,1)$
φ	$S(1,2,1/2)$
S_1	$S(\bar{3},1,1/3)$
$\tilde{R}_2$	$S(3,2,1/6)$
S_3	$S(\bar{3},3,1/3)$
N	$F(1,1,0)$
Σ	$F(1,3,0)$
Q_5	$F(3,2,-5/6)$

Topology I:



Topology II:



KF, Gráf, Harz, Hati, arXiv:2407:XXXXX

A roman number means that we generate the corresponding radiative neutrino mass topology, given this combination of two fields

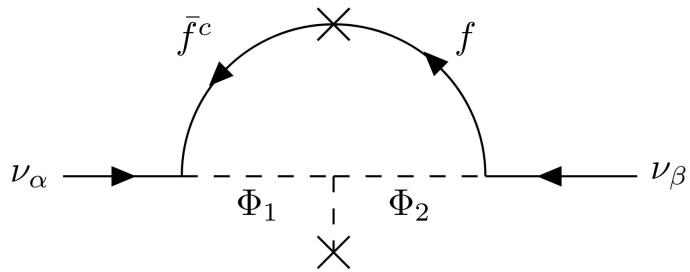
A circle means that we generate the dim-5 operator, which should dominate

We can estimate the neutrino mass from the generic topology

E.g. from Topology I:

1-Loop neutrino mass

$$m_\nu \propto \frac{1}{16\pi^2}\frac{v^2\mu}{\max(m_{\Phi_1}^2, m_{\Phi_2}^2)}$$



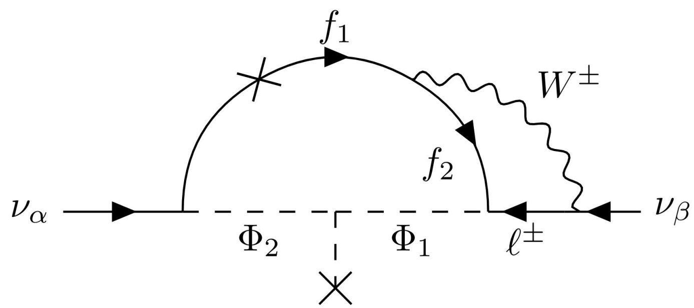
Some dim-7 LNV operators do not lead to radiative neutrino masses at 1-loop

$$\mathcal{O}_{\bar{d}LueH} = \epsilon_{ij}\left(\overline{d_p}L_r^i\right)\left(\overline{u_s^c}e_t\right)H^j$$

	S_1	$\tilde{R}_2$	N	$\Delta_1^\dagger$	$Q_5^\dagger$	Q_7	W_1'	V_3	U_1	$\bar{V}_2^\dagger$
S_1		●	○		●					
$\tilde{R}_2$				●		●				
N							○		○	
$\Delta_1^\dagger$							●			●
$Q_5^\dagger$								●		●
Q_7								●	●	
W_1'								●		
V_3										
U_1										●
$\bar{V}_2^\dagger$										

Field	Rep
S_1	$S(\bar{3},1,1/3)$
$\tilde{R}_2$	$S(3,2,1/6)$
N	$F(1,1,0)$
Δ_1	$F(1,2,-1/2)$
Q_5	$F(3,2,-5/6)$
Q_7	$F(3,2,7/6)$
W_1'	$V(1,1,1)$
V_3	$V(1,2,3/2)$
U_1	$V(3,1,2/3)$
$\bar{V}_2$	$V(\bar{3},2,-1/6)$

2-Loop neutrino mass



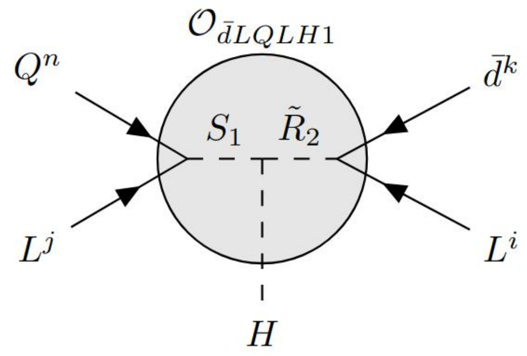
KF, Gráf, Harz, Hati, arXiv:2407:XXXXX

Filled circle: 2-loop neutrino mass

$$m_\nu \propto \frac{1}{(16\pi^2)^2}\frac{m_W}{\min(m_{\Phi_1}, m_{\Phi_2})}\frac{v^2\mu}{\max(m_{\Phi_1}^2, m_{\Phi_2}^2)}$$

To determine the accuracy of these approximations we can compare with a model

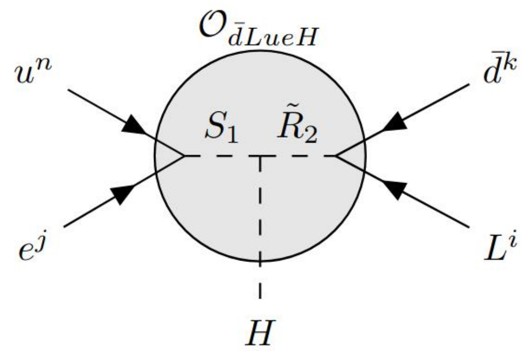
$$\mathcal{L} = \mathcal{L}_{\rm SM} - \tilde{R}_2^\dagger(\Box + m^2_{\tilde{R}_2})\tilde{R}_2 - S_1^*(\Box + m^2_{S_1})S_1 + \mu S_1 H^\dagger \tilde{R}_2$$
$$-g_1^{ik}\bar{L}_i i\sigma_2 \tilde{R}_2^* \bar{d}_k^c - g_2^{jn} Q_n \epsilon L_j S_1 - g_3^{jn}\bar{u}_n^c e_j S_1 + \text{h.c.}$$



See e.g. Catà, Mannel, arXiv:1903.01799 [hep-ph], Doršner et. al. Phys. Rept. 641 (2016) 1–68

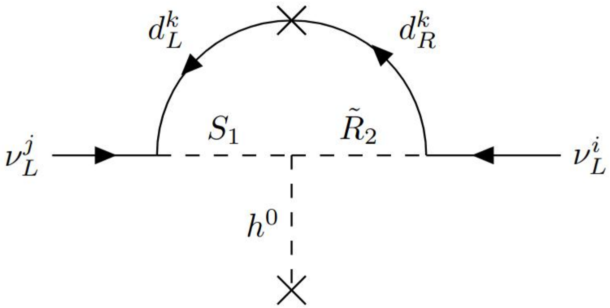
Scalar leptoquarks: $S_1 \in S(\overline{3}, 1, 1/3), \quad \tilde{R}_2 \in S(3, 2, 1/6)$

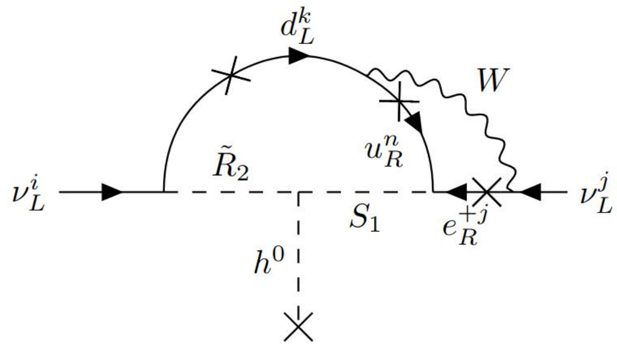
Generates both operators that we used as examples:



$$C^{ijkn}_{\bar{d}LQLH1} = -\frac{\mu g_1^{ik} g_2^{jn}}{2m^2_{\tilde{R}_2} m^2_{S_1}}\,, \qquad C^{ijkn}_{\bar{d}LueH} = \frac{\mu g_1^{ik} g_3^{jn}}{4m^2_{\tilde{R}_2} m^2_{S_1}}$$

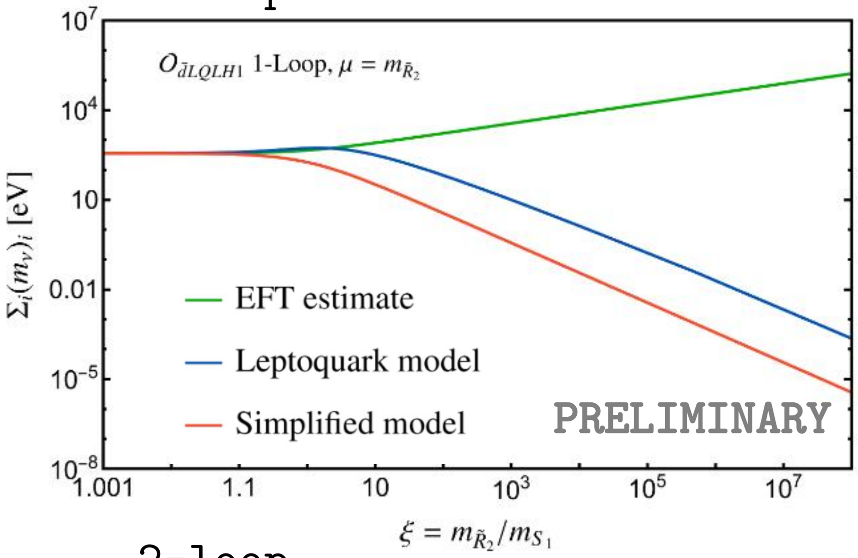
And consequently also 1- and 2-loop neutrino masses:



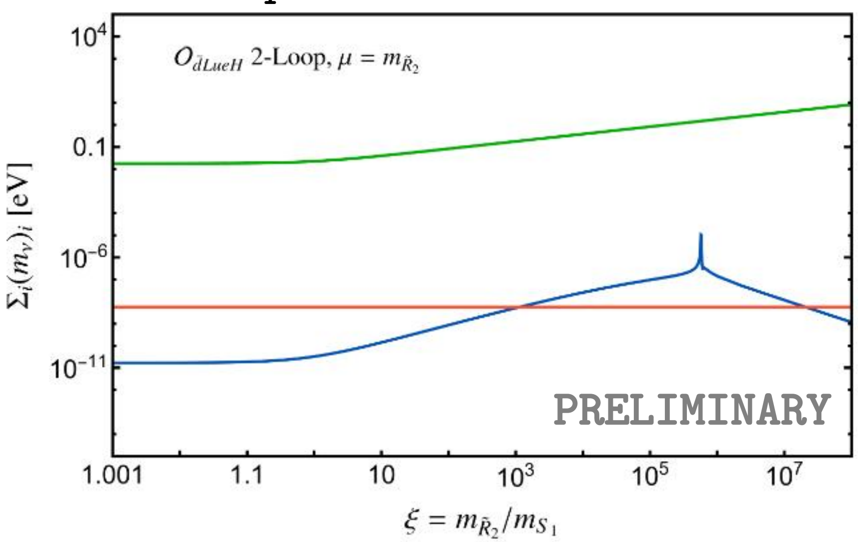


How do the different expressions compare?

1-loop neutrino mass



2-loop



KF, Gráf, Harz, Hati, arXiv:2407:XXXXX

Assuming a given hierarchy we can study the neutrino mass as a function of the mass difference

$$\xi \equiv \frac{m_{\tilde{R}_2}}{m_{S_1}} \qquad m_{\tilde{R}_2} > m_{S_1}$$

Green lines show the conventional EFT-based estimate

$$\boxed{m_\nu \propto \frac{1}{16\pi^2}\frac{v^2}{\Lambda}}$$

Blue lines show the expression from the full model

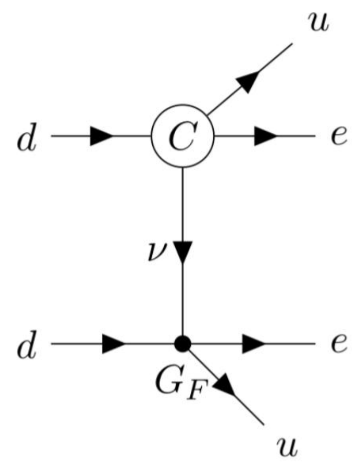
$$\boxed{m_\nu \propto \frac{3v\sin 2\theta}{32\pi^2}\log\frac{m_{\tilde{R}_2}^2+m_{S_1}^2+\sqrt{(m_{\tilde{R}_2}^2-m_{S_1}^2)^2+\mu^2v^2}}{m_{\tilde{R}_2}^2+m_{S_1}^2-\sqrt{(m_{\tilde{R}_2}^2-m_{S_1}^2)^2+\mu^2v^2}}}$$

Red lines show our estimate based on generic simplified models

$$\boxed{m_\nu \propto \frac{1}{16\pi^2}\frac{v^2}{\max(m_{\tilde{R}_2}, m_{S_1})}}$$

Other observabes of dim-7 lepton number violation:

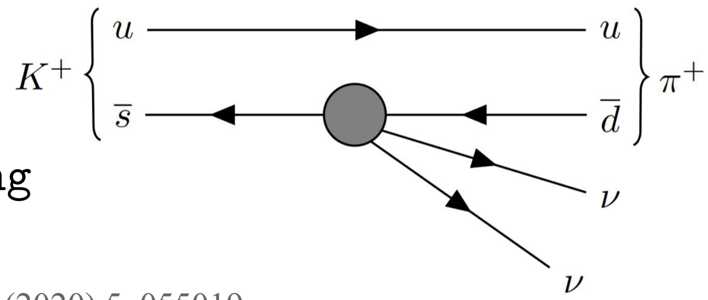
Neutrinoless double beta (0νββ) decay



- Generally most sensitive probe of dim-7 LNV
- Only sensitive to LNV in the 1st generation fermions
- Triggered by most dim-7 operators

Cirigliano et. al. JHEP 12 (2017) 082

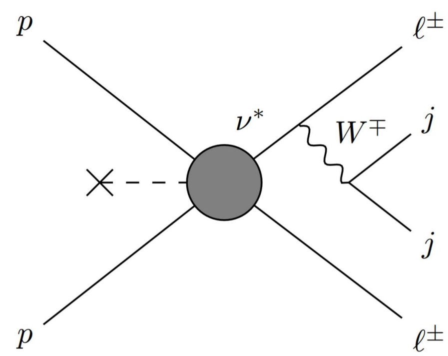
Rare kaon decay



- Lepton flavour universal
- Only a single dim-7 operator
- Can be distinguished from the SM mode using pion kinematics

Deppisch, KF, Harz, JHEP 12 (2020) 186 Li, Ma, Schmidt, Phys.Rev.D 101 (2020) 5, 055019

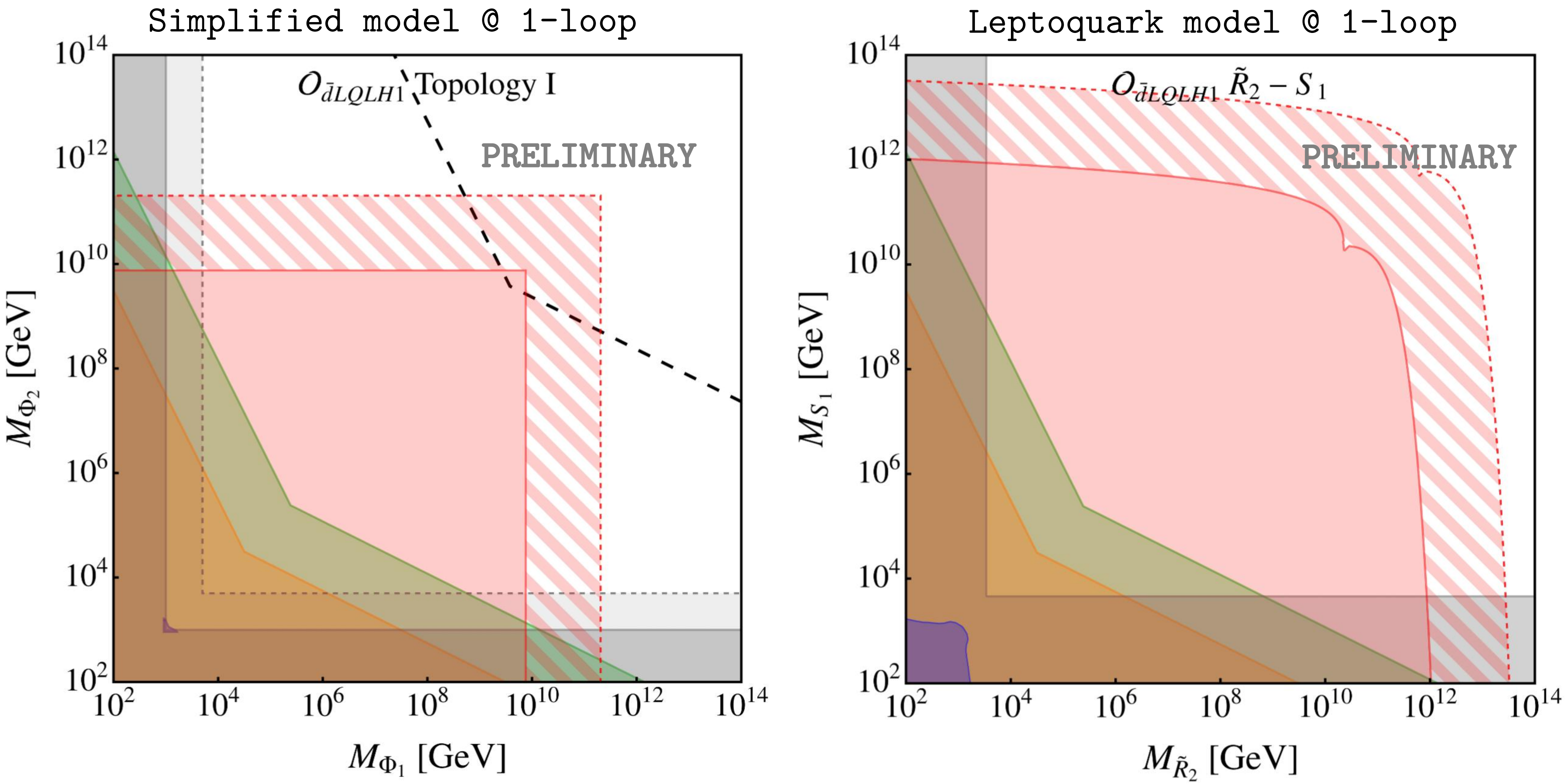
LHC



- Most sensitive probe for the muon flavour
- Includes most of the dim-7 operators

KF, Gráf, Harz, Hati, JHEP 05 (2024) 154

\+ Long-baseline neutrino oscillation, rare muon decay, CEνNS, neutrino magnetic moment, ...



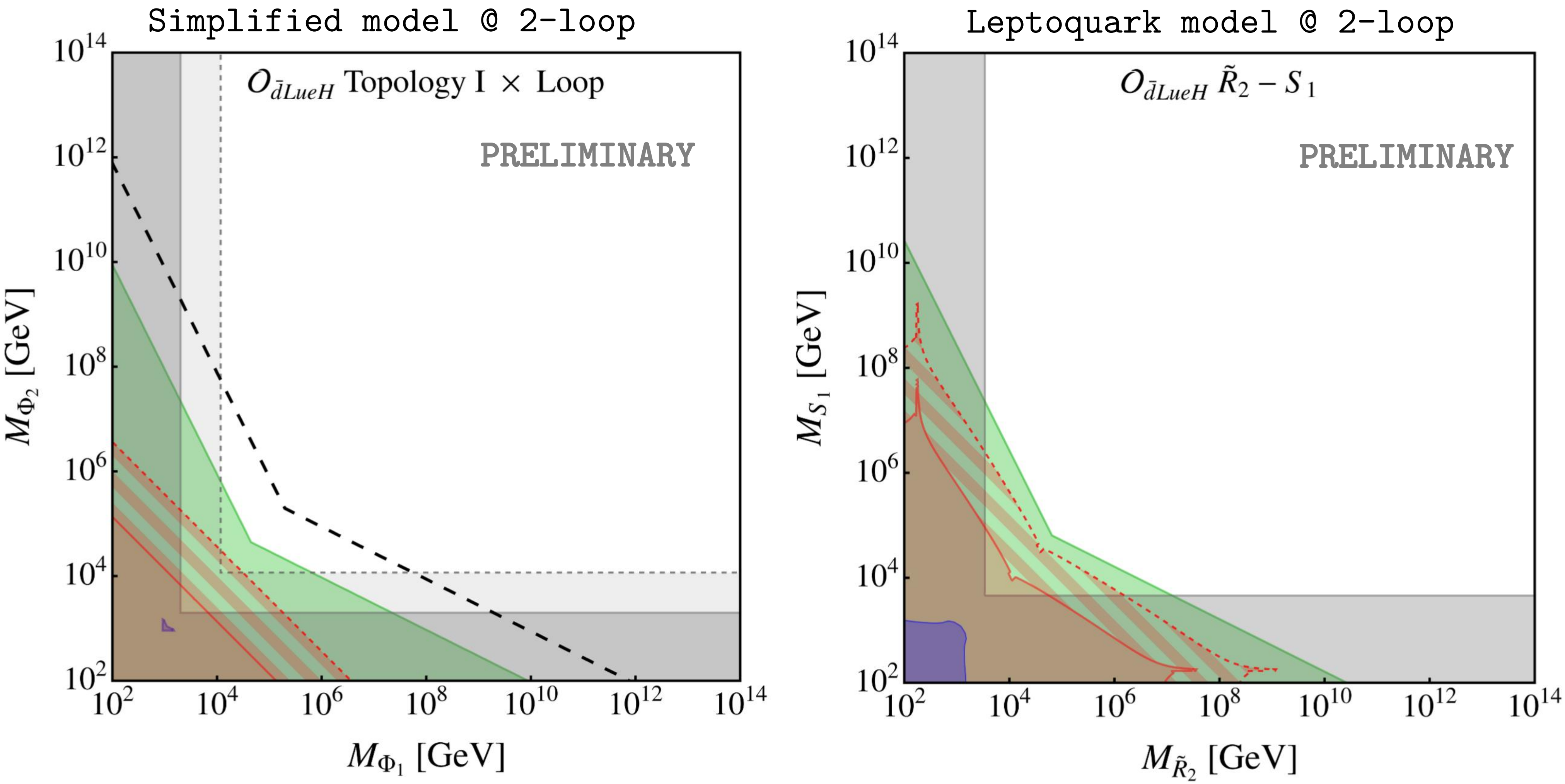
KF, Gráf, Harz, Hati, arXiv:2407:XXXXX

We see a significant effect in varying the hierarchy between the two masses

Red stripes: observed neutrino mass, Red solid: neutrino is too massive

LHC in grey, $0\nu\beta\beta$ decay in green, kaon decay in orange, LNV@LHC in purple

Conventional EFT-estimate in black dashed line, misses the effect of hierarchy



KF, Gráf, Harz, Hati, arXiv:2407:XXXXX

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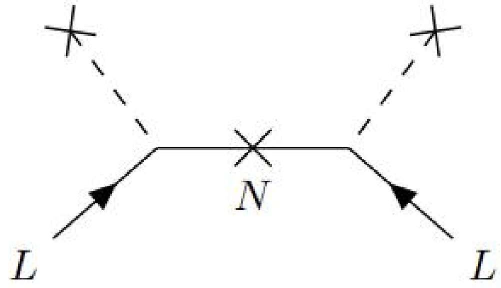
Conclusions

- EFT-based estimates of the neutrino mass in dim-7 models completely misses the effect of a hierarchy in the internal degrees of freedom
- Such a hierarchy can alter the size of the neutrino mass by several orders of magnitude
- We provide model-independent expressions that more accurately describes the neutrino mass at both 1- and 2-loop
- Using this framework we re-open large areas of parameter space that were previously thought to be excluded

Back-up

Type-i seesaw:

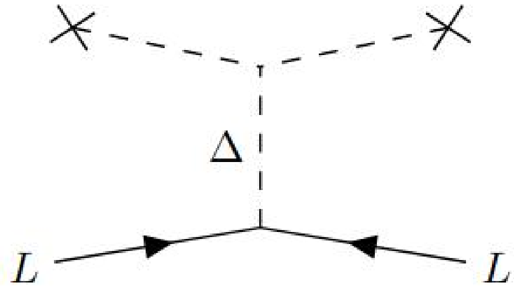
$$\mathcal{L} \supset -y^{\nu}_{ij}\tilde{H}\bar{L}_i N_j - M_{ij}\bar{N^c}_i N_j + \mathrm{h.c.}$$



Type-ii seesaw:

$$\mathcal{L} \supset -h^{\nu}_{ij}\Delta\bar{L^c}_i L_j - \mu H^{\dagger} H \Delta + \mathrm{h.c.}$$

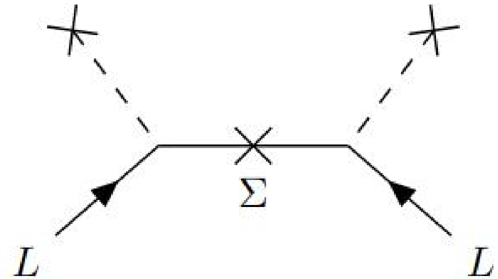
The $SU(2)_L$ triplet Δ obtains a vev in the neutral component



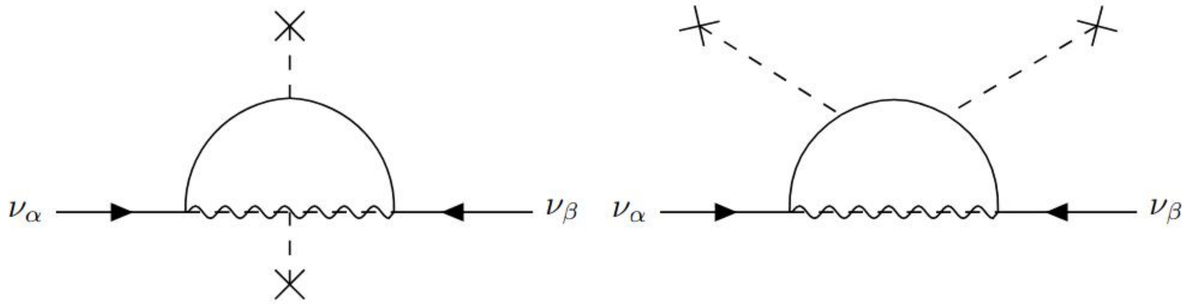
Type-iii seesaw:

Same as type-i except that the fermion is a triplet

$$\mathcal{L} \supset -y^{\nu}_{ij}\tilde{H}\bar{L}_i \Sigma_j - M_{ij}\bar{\Sigma^c}_i \Sigma_j + \mathrm{h.c.}$$



Radiative neutrino mass models:



Radiative = has loops

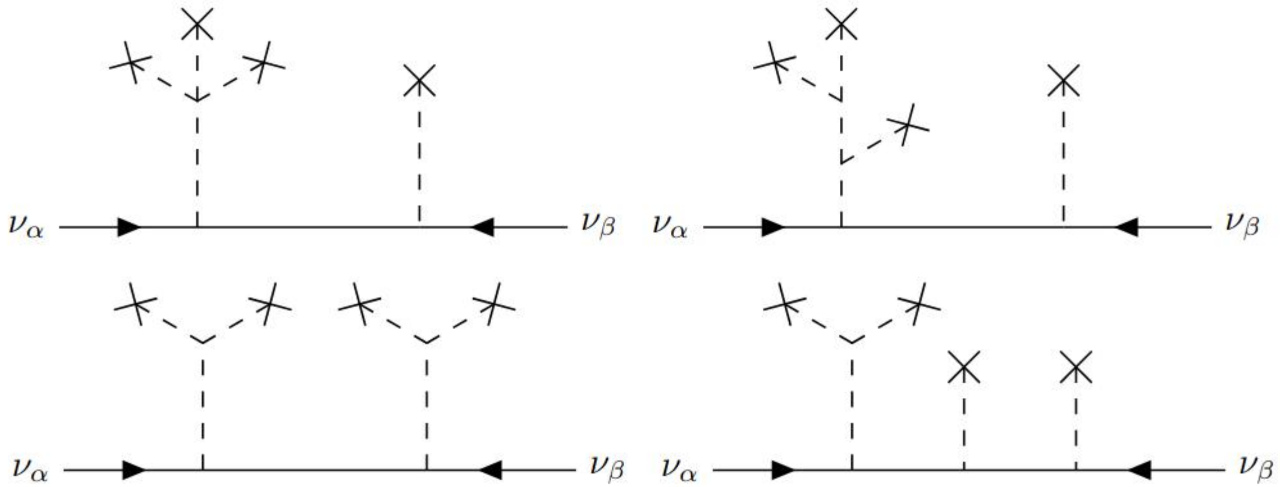
Examples:

- Zee model
- Scotogenic model
- ...

Zee, Phys.Lett.B 93 (1980) 389

Ma, Phys.Rev.D 73 (2006) 077301

More complicated seesaws:

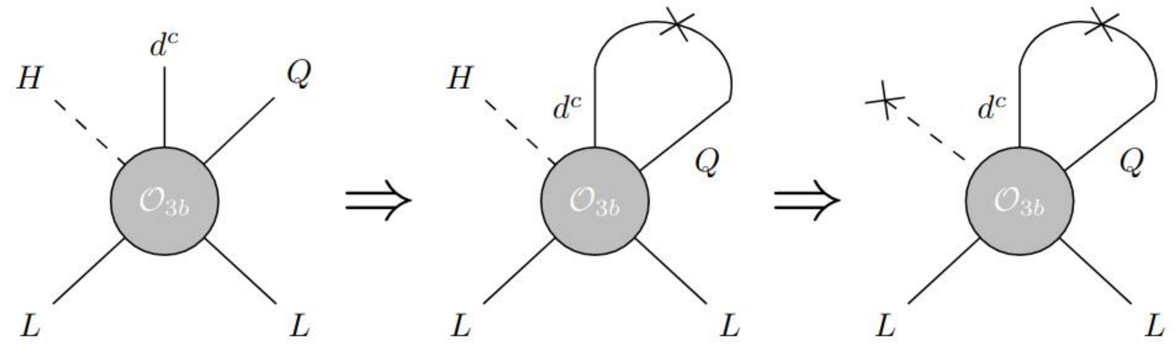


E.g. inverse seesaw

Deppisch, Valle, Phys.Rev.D 72 (2005) 036001

Gets complicated very fast: can we classify LNV in a different way?

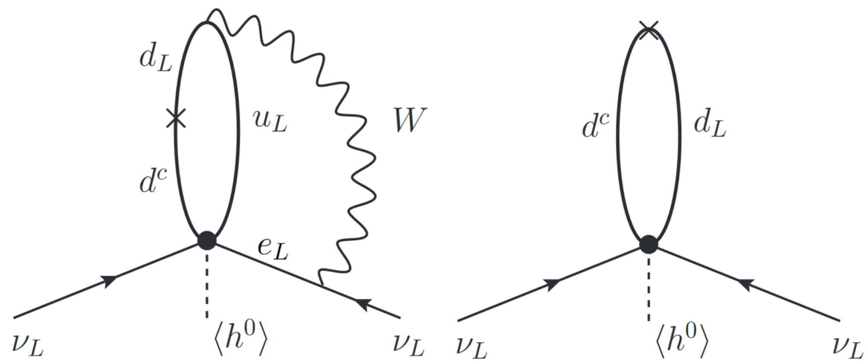
We can get the neutrino mass directly from the operator:

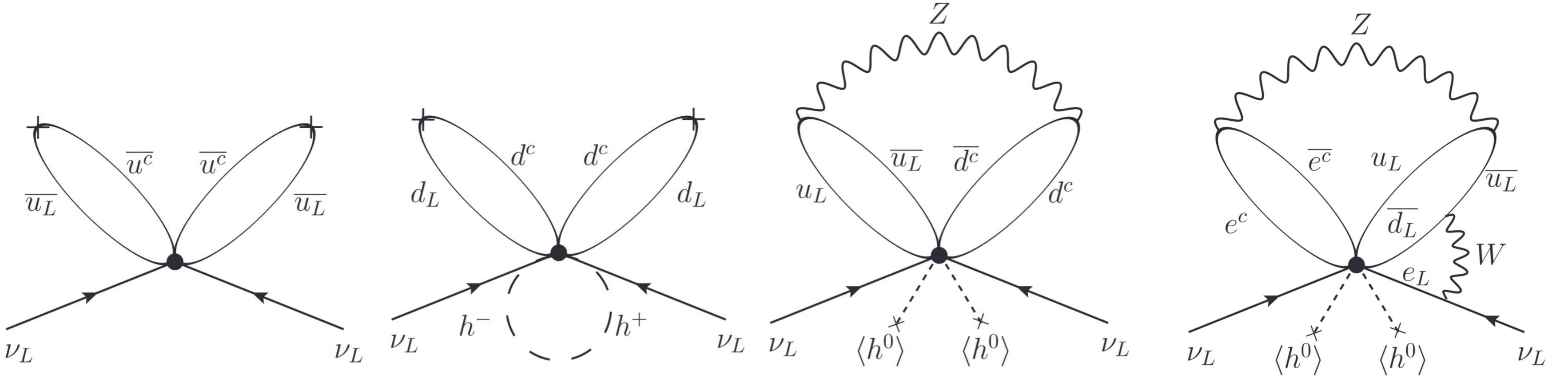


$$\Rightarrow\ m_\nu \approx \frac{1}{16\pi^2}\frac{\sum_i y_i^d v^2}{\Lambda}$$

The diagrams can in principle be made very complex.

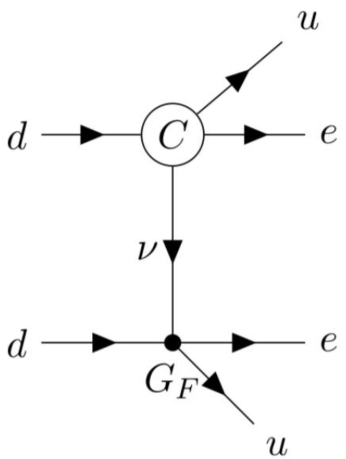
The smallness of neutrino masses can be explained as resulting from loop suppression.

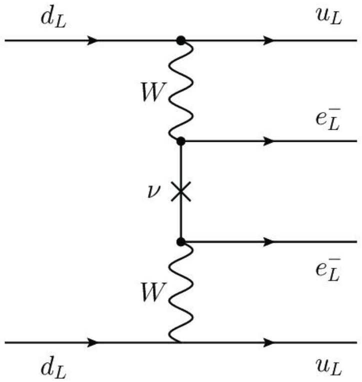




Deppisch, Graf, Harz, Huang, Phys.Rev.D 98 (2018) 5, 055029

Neutrinoless double beta decay (0νββ):



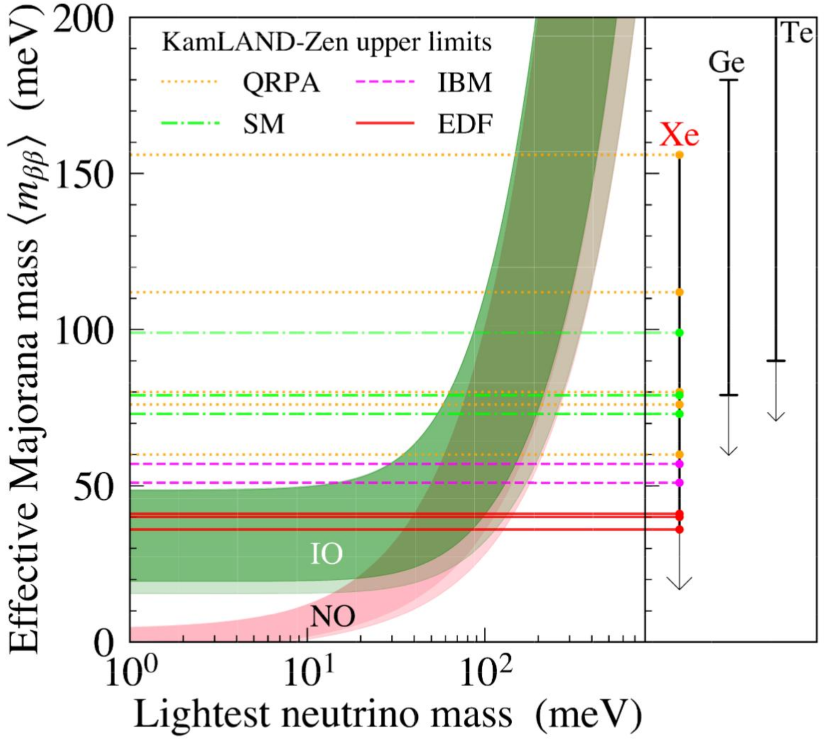


Cirigliano et al., JHEP 12 (2017) 082

Dimension-7 LNV operators give rise to long-range contributions to 0νββ decay

Currently most stringent limit:

$$T_{1/2}^{^{136}\mathrm{Xe}} \leq 2.3 \times 10^{26} \text{ yrs, } 90\% \text{ C.L.}$$



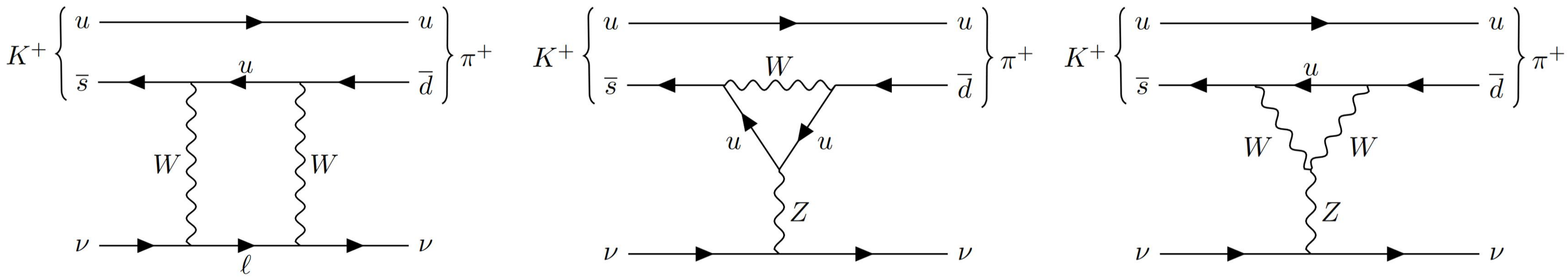
KamLAND-Zen collaboration, Phys.Rev.Lett. 130 (2023) 5, 051801

0νββ decay is (by far) the most sensitive probe of LNV.

However, this probe is only sensitive to electron flavor component.

Rare kaon decays: $K^+ \to \pi^+ \nu\bar{\nu}$ $K_L \to \pi^0 \nu\bar{\nu}$

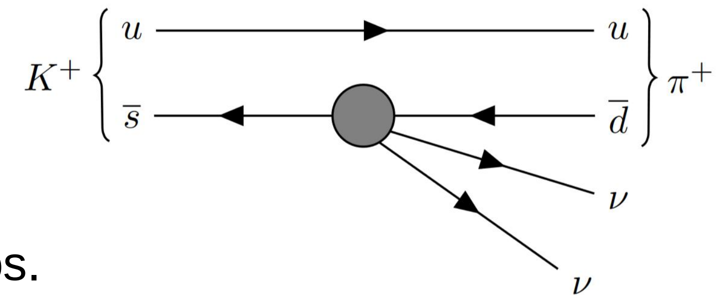
Great probe of New physics since the SM mode is very suppressed



Final state neutrinos are not observed: could be a LNV process

Li, Ma, Schmidt, Phys.Rev.D 101 (2020) 5, 055019

Deppisch, KF, Harz, JHEP 12 (2020) 186



Experiments also cannot determine flavor of neutrinos.

Experiment:

$$\mathrm{BR}(K^+ \to \pi^- \nu\bar{\nu}) = (10.6^{+4.9}_{-4.3}) \times 10^{-11}\,,\ 68\%\ \mathrm{C.L.}$$

NA62 collaboration, JHEP 06 (2021) 093

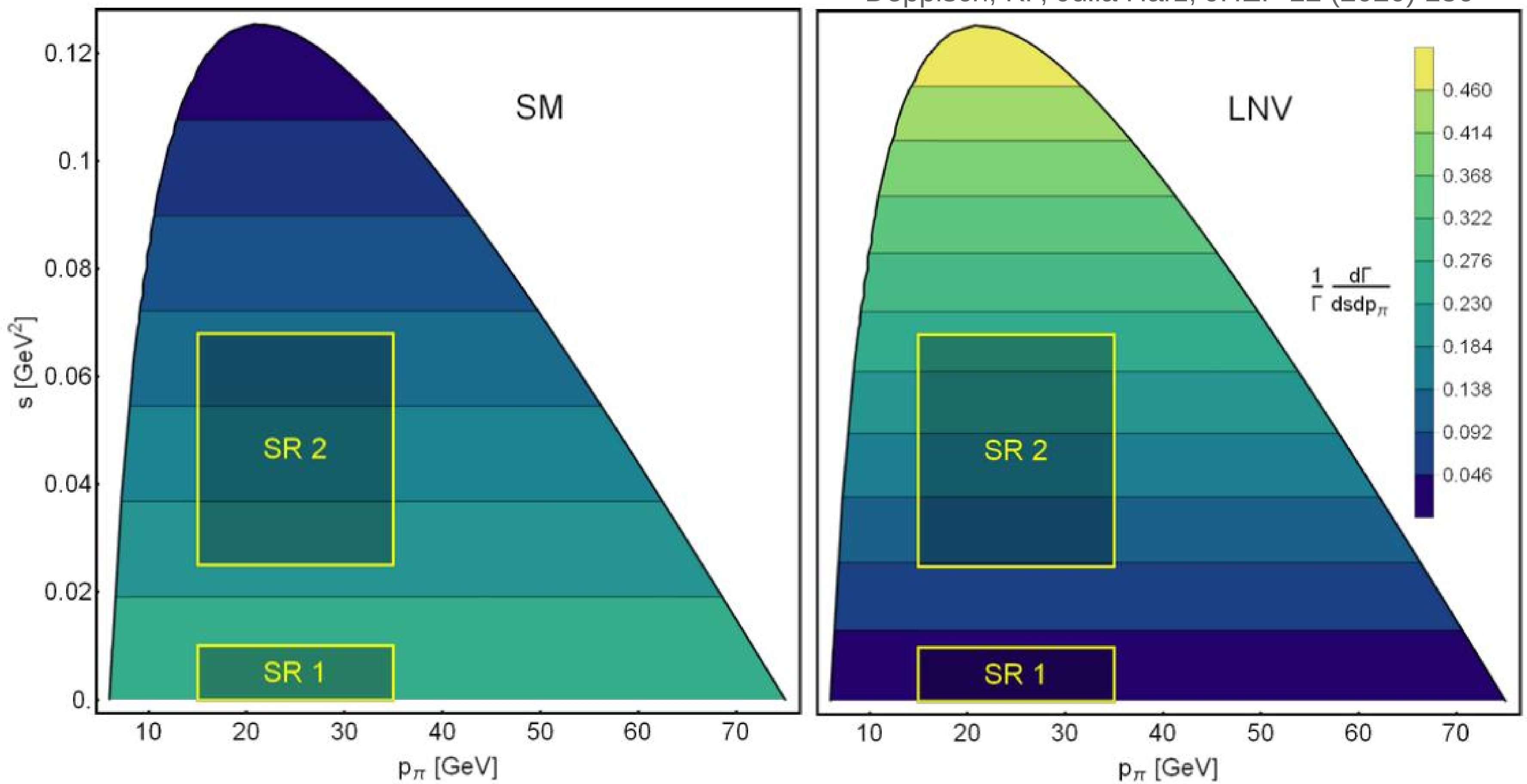
SM expectation:

$$\mathrm{BR}(K^+ \to \pi^- \nu\bar{\nu}) = (8.4 \pm 1.0) \times 10^{-11}$$

Buras et al., JHEP 11 (2015) 033

$s = (E_K - E_\pi)^2$

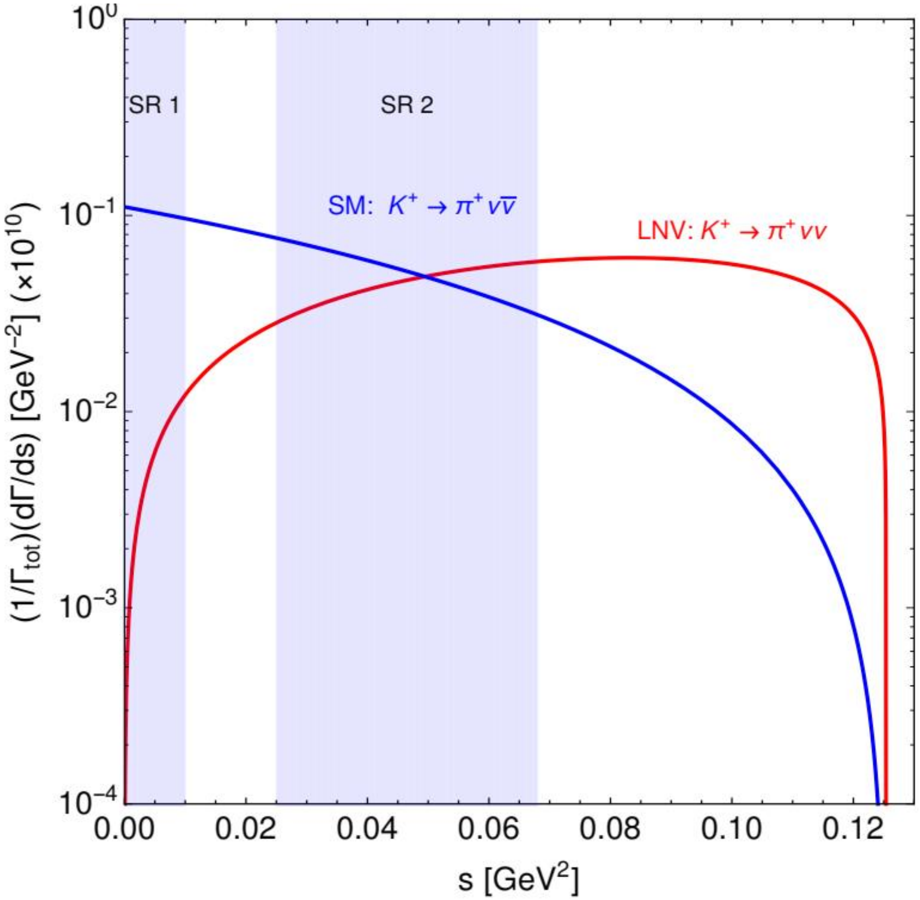
Deppisch, KF, Julia Harz, JHEP 12 (2020) 186



At NA62 the signal regions are defined in terms of the missing energy as well as the pion momentum.

More events in SR 2 could be a sign of LNV.

At NA62:



Deppisch, KF, Julia Harz, JHEP 12 (2020) 186

SM decay is mediated by a vector current

$$\mathcal{L}_{\rm SM}^{K\to\pi\nu\bar{\nu}} = \frac{1}{\Lambda_{\rm SM}^2}\sum_{i=1}^{3}(\bar{\nu}_i\gamma^\mu\nu_i)(\bar{d}\gamma_\mu s)$$

LNV decay via a scalar current

$$\mathcal{L}_{\rm BSM}^{K\to\pi\nu\nu} = \langle\pi\nu\nu|\mathcal{O}_{3b}|K\rangle = \frac{v}{\Lambda_{\rm BSM}^3}\bar{s}d\nu_i\nu_j$$

Percentage of phase space visible:

Experiment	SM (Vector)	LNV (Scalar)
NA62 SR 1	6%	0.3%
NA62 SR 2	17%	15%
E949 $\pi\nu\bar{\nu}(1)$	29%	2%
E949 $\pi\nu\bar{\nu}(2)$	45%	38%
KOTO	64%	30%

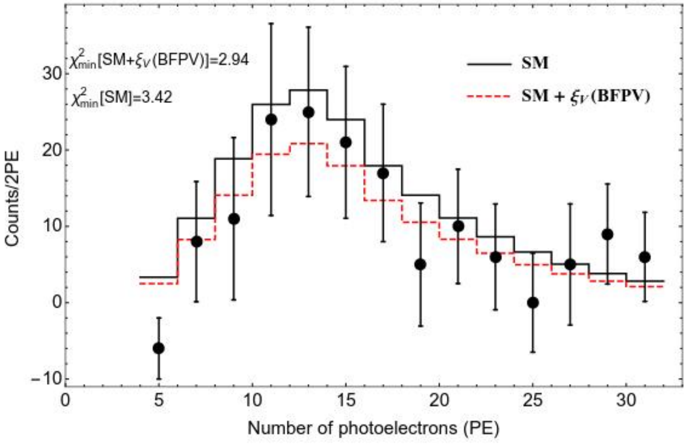
Coherent elastic neutrino-nucleus scattering

COHERENT collaboration, Science 357 (2017) 6356, 1123-1126

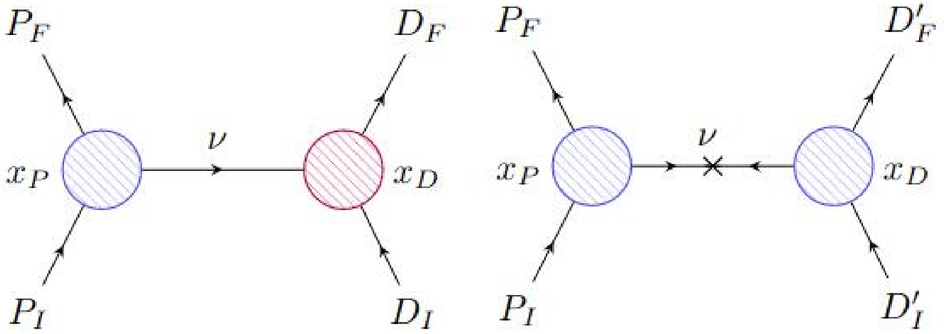
Sierra, De Romeri, Rojas, Phys.Rev.D 98 (2018) 075018

Neutrinos coherently scattering with nuclei

We can look for LNV contributions appearing as deviations from the expected SM result coming from **d=7** operators



Long baseline neutrino oscillations



Bolton, Deppisch Phys.Rev.D 99 (2019) 11, 115011

Both **d=5** and **d=7**

Two possible LNV mechanisms: an LNV interaction with the receiving quarks or a chirality flip along the propagation

The precense of LNV leads to the poduction of an opposite sign final state lepton

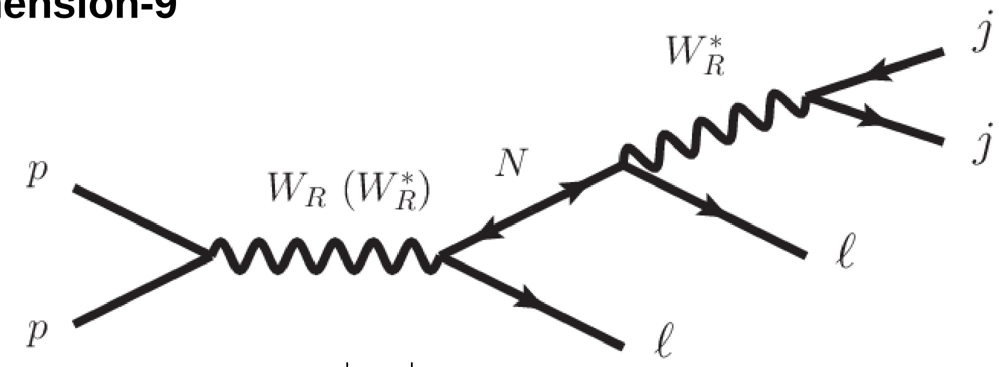
LNV at the LHC:

Keung-Senjanović process: leads to LNV at **dimension-9** when integrating out the heavy fields

Probes $5\ \mathrm{GeV} \gtrsim m_N \gtrsim 50\ \mathrm{GeV}$

for couplings $|U_{e/\mu}| \gtrsim 10^{-5}$

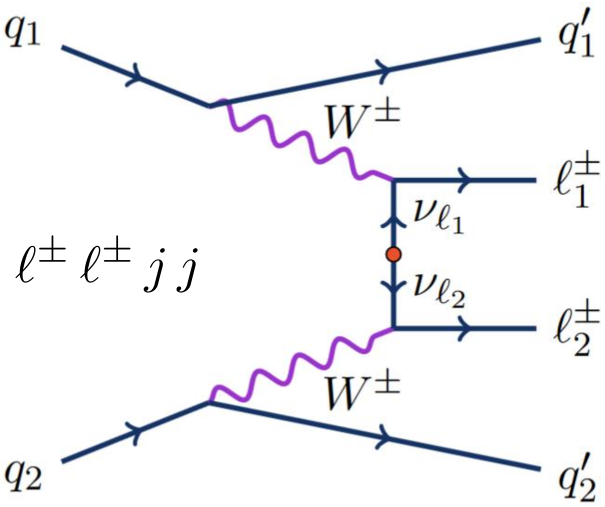
ATLAS collaboration (2019)



$$p\,p \to \ell^{\pm}\,\ell^{\pm}\,j\,j$$

Keung, Senjanovic, Phys.Rev.Lett. 50 (1983) 1427

$$p\,p \to \ell^{\pm}\,\ell^{\pm}\,j\,j$$



Fuks, Neundorf, Peters, Ruiz, Saimpert, Phys.Rev.D 103 (2021) 11, 115014

Possible to probe the **dimension-5** LNV operator at colliders in vector boson fusion

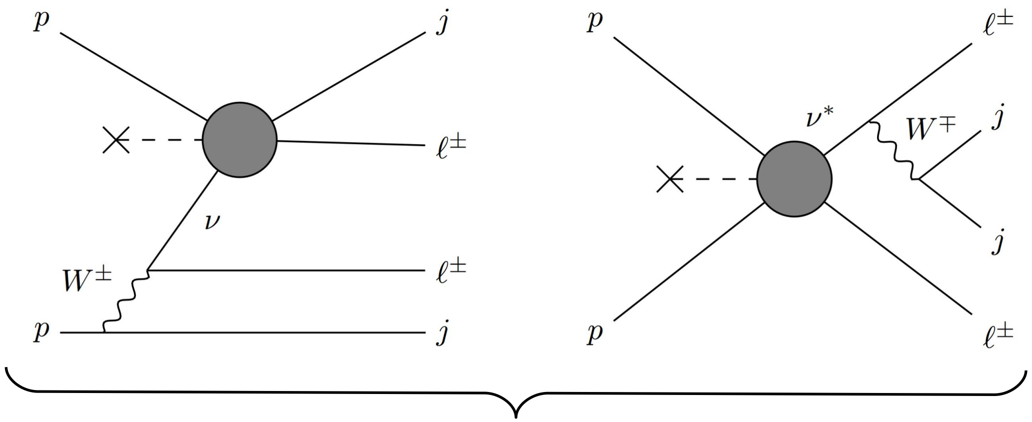
$$m_{\mu\mu} < 10.8\ \mathrm{GeV\ at\ 95\%\ C.L.}$$

$$m_{\ell\ell'} = \frac{C_{\ell\ell'} v^2}{\Lambda}$$

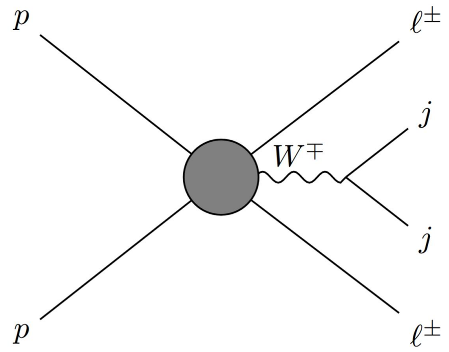
CMS collaboration, Phys.Rev.Lett. 131 (2023) 1, 011803

Dimension-7 operators:

$$p\,p \to \ell^{\pm}\,\ell^{\pm}\,j\,j$$



$$\mathcal{O}^{prst}_{\bar{Q}uLLH} = \epsilon_{ij}\overline{Q_p}u_r\overline{L^c_s}L^i_t H^j$$



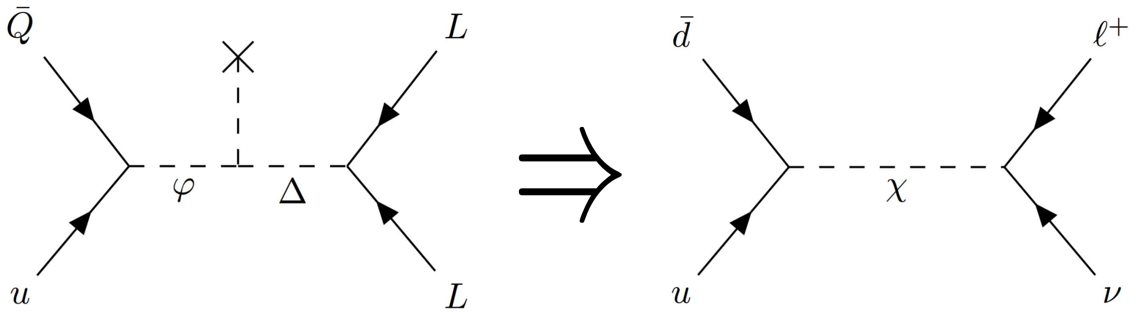
$$\mathcal{O}^{prst}_{\bar{d}uLLD} = \epsilon_{ij}\overline{d_p}\gamma_\mu u_r\overline{L^c_s}^i iD^\mu L^j_t$$

Leads to the same signal as for dimension-9 processes: we can use existing LHC results and compare with EFT cross section obtained using MadGraph5_aMC@NLO

Absence of events in 139 fb^{-1} of data at ATLAS leads to contraints on the dimension-7 Wilson coefficients.

ATLAS collaboration (2023) Eur. Phys. J. C 83 1164

One has to be careful using EFTs for high-energy experiments such as the LHC

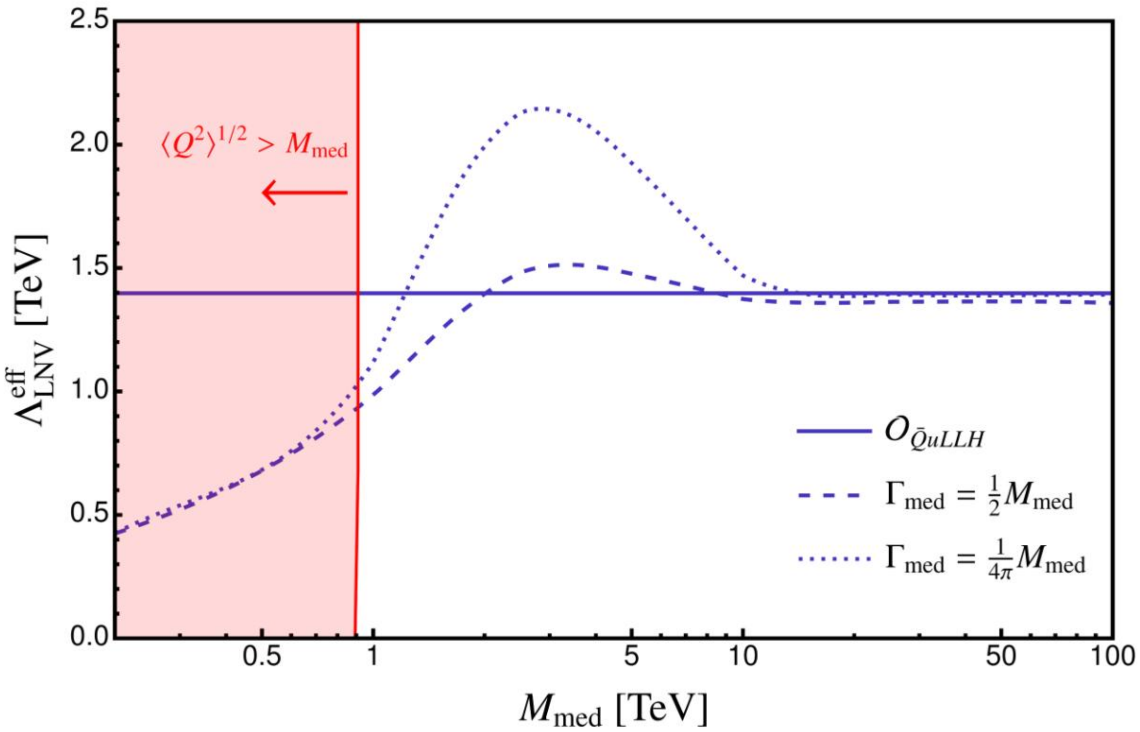


We can compare the EFT approach with a simplified model example to see how well it performs

KF, Gráf, Harz, Hati, JHEP 05 (2024) 154

$$\mathcal{O}_{\bar{Q}uLLH} = \epsilon_{ij}\overline{Q}u\overline{L^c}L^iH^j$$

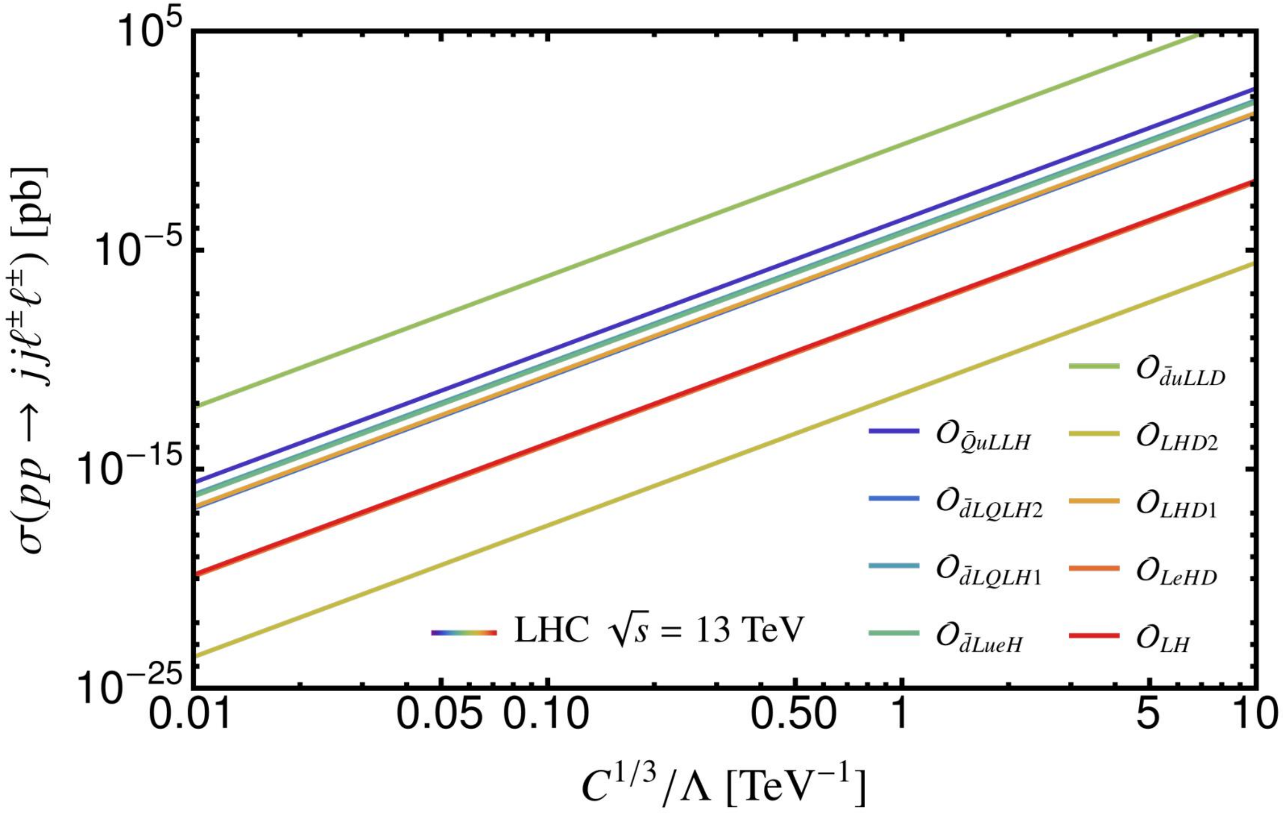
$$\mathcal{L} \supset \lambda_1 \bar{d}_L u_R \chi^* + \lambda_2 \bar{e}^c_L \nu_L \chi + \text{h.c.}$$



Using the relation

$$\frac{\lambda_1\lambda_2}{M^2_{\rm med}} = \frac{v}{(\Lambda^{\rm eff}_{\rm LNV})^3}$$

we naively expect the same cross section for the simplified model and EFT operator



KF, Gráf, Harz, Hati, JHEP 05 (2024) 154

LNV scales in TeV

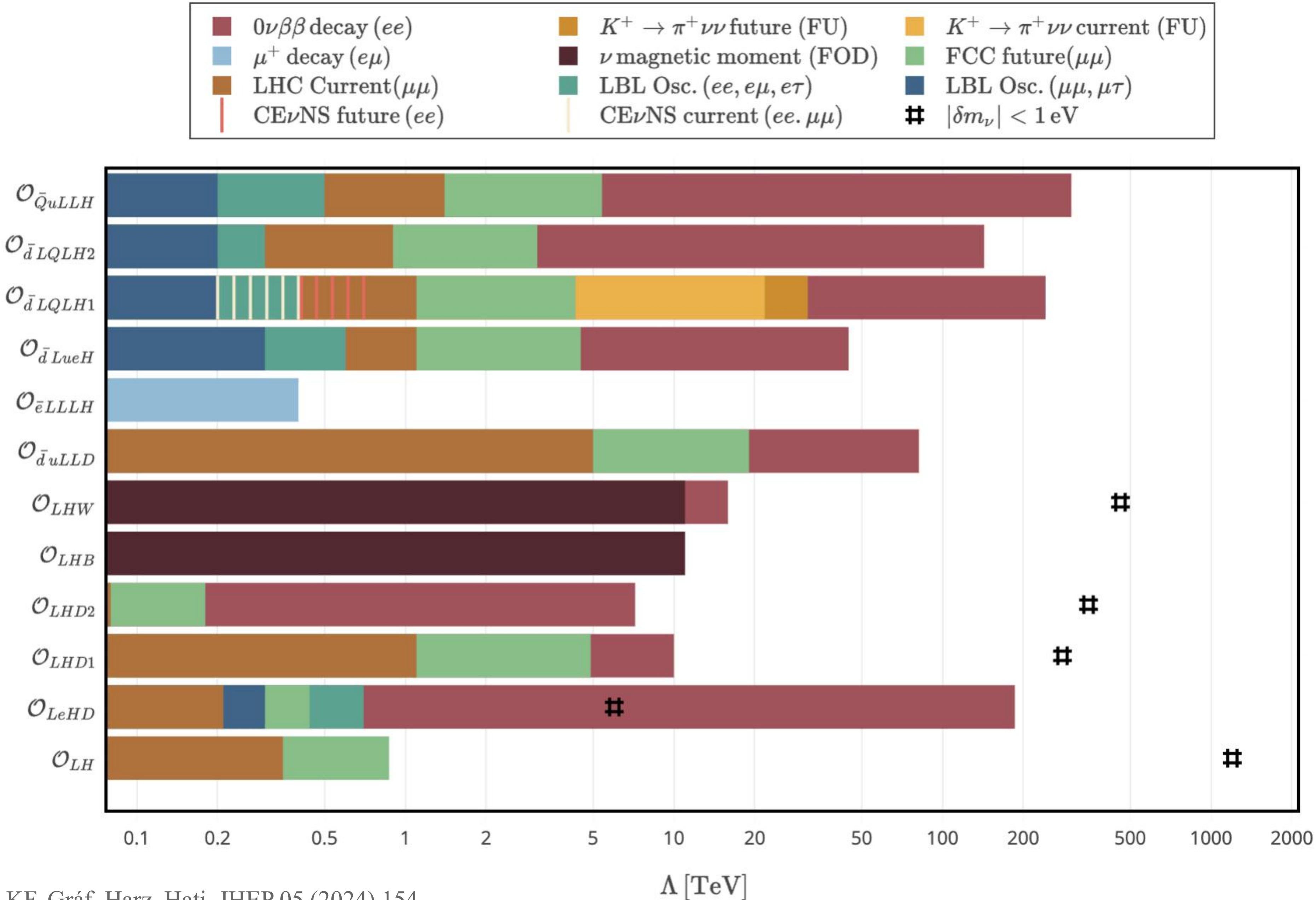
Operator	Λ_{LNV} [TeV]	$\Lambda_{\text{LNV}}^{\text{future}}$ [TeV]
$\mathcal{O}_{\bar{Q}uLLH}$	1.4	5.4
$\mathcal{O}_{\bar{d}LQLH2}$	0.90	3.1
$\mathcal{O}_{\bar{d}LQLH1}$	1.1	4.3
$\mathcal{O}_{\bar{d}LueH}$	1.1	4.5
$\mathcal{O}_{\bar{d}uLLD}$	5.0	19
$\mathcal{O}_{LHD2}$	0.075*	0.18
$\mathcal{O}_{LHD1}$	1.1	4.9
$\mathcal{O}_{LeHD}$	0.21*	0.44
$\mathcal{O}_{LH}$	0.35*	0.87

Constraints at 95% C.L.

* = not within EFT validity

LNV scales ~*O* (1 TeV) are constrained by same-sign dilepton plus dijet searches at the LHC

For FCC the constraints could reach ~*O* (few TeV) up to ~20 TeV



KF, Gráf, Harz, Hati, JHEP 05 (2024) 154

What about neutrino masses?

We can “explode“ an operator to find all possible tree-level UV completions using combinatorics

$$\mathcal{O}_{\bar{Q}uLLH} = \epsilon_{ij}(\overline{Q}u)(\overline{L^c}L^i)H^j \quad \rightarrow \quad \epsilon_{ij}\overline{Q}^k u L^k L^i H^j$$

$$\mathcal{O}_{\bar{Q}uLLH} \quad \rightarrow \quad \epsilon_{ij}\chi_1{}^k L^k L^i H^j \;\Big/\; \epsilon_{ij}\psi_1{}^k u L^i H^j \;\Big/\; \epsilon_{ij}\phi_1{}^k u L^k H^j \;\Big/\; \epsilon_{ij}\omega_1{}^k u L^k L^i$$

Pair field with index “1“ together with either “2“, “3“, or “4“ to get a model. This leads to several different models, but not all of them are unique.

$$\begin{array}{llll}
\chi_1 \sim S(1,2,1/2) & \psi_1 \sim V(\overline{3},1,-2/3) & \phi_1 \sim V(\overline{3},3,-2/3) & \omega_1 \sim F_L(\overline{3},3,1/3) \\
\chi_2 \sim F_R(1,1,0) & \psi_2 \sim F_R(1,1,0) & \phi_2 \sim F_R(1,3,0) & \omega_2 \sim S(1,3,1) \\
\chi_3 \sim F_R(1,3,0) & \psi_3 \sim F_L(\overline{3},2,-7/6) & \phi_3 \sim F_L(\overline{3},2,-7/6) & \omega_3 \sim V(\overline{3},2,-1/6) \\
\chi_4 \sim S(1,3,1) & \psi_4 \sim V(\overline{3},2,-1/6) & \phi_4 \sim V(\overline{3},2,-1/6) & \omega_4 \sim V(\overline{3},2,-1/6)
\end{array}$$

For the dim-5 operator this method leads to the three seesaw models (as mentioned)