Vertex correction to nuclear matrix elements of double-β decays

ID # 828

ICHEP 2024

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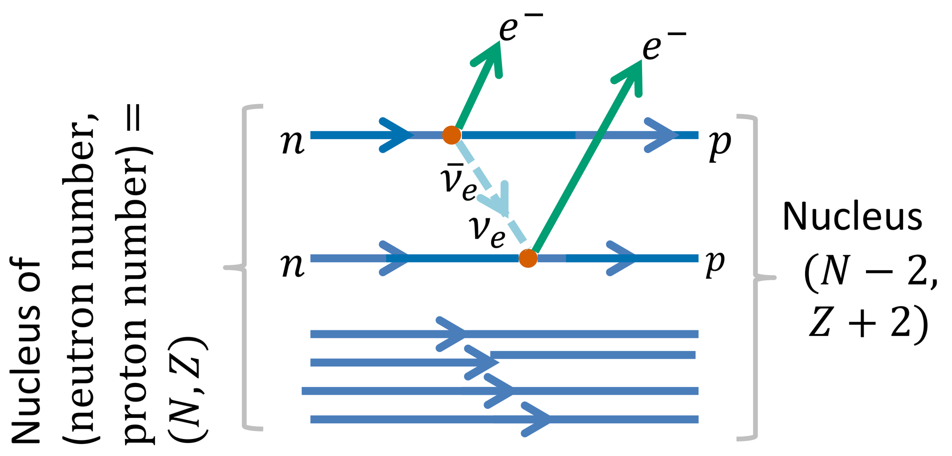
The goal is to determine the effective mass of the neutrino using the neutrinoless double-β (0νββ) decay of the nuclei. The theoretical calculation of the nuclear matrix element (NME) of this decay is necessary; however, the calculated NMEs by different methods and groups are distributed in a broad range of the max-to-min ratio of 2–3 depending on the decay instance. I calculate the NME including the many-body corrections to the transition operator, to solve this problem.



Possible change of two neutrons to two protons in a nucleus emitting two electrons with neutrino exchange [neutrinoless double-β (0νββ) decay]. This decay occurs, if the neutrino（ν） is a Majorana particle （$\nu = \bar{\nu}$）, and the effective neutrino mass can be determined, see the equations below. Determination of the effective neutrino mass is one of the most important subjects in modern physics.

Why nuclei?

Because E(final state) < E(initial state) is necessary.

List of nuclei used in the experiments

^{76}Ge→^{76}Se	^{130}Te→^{130}Xe	^{136}Xe→^{136}Ba
^{150}Nd→^{150}Sm	^{48}Ca→^{48}Ti	^{82}Se→^{82}Kr
^{96}Zr→^{96}Mo	^{100}Mo→^{100}Ru	^{110}Pd→^{110}Cd
^{116}Cd→^{116}Sn	^{124}Sn→^{124}Te	and more

Principle to determine effective neutrino mass

$$\langle m_\nu \rangle = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right|$$

U: Pontecorvo-Maki-Nakagawa-Sakata matrix
m_i: neutrino eigen mass (i =1,2,3)

$$\underbrace{1/T_{0\nu}(\mathrm{g.s.} \to \mathrm{g.s.})}_{\text{Half-life}} = \underbrace{\left|M^{(0\nu)}\right|^2 G_{0\nu}}_{\text{Theoretical calculation}} g_A^4 \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

Half-life ← Experimental measurement

Nuclear matrix element (NME)

Phase-space factor

Effective ν mass

Axial-vector current coupling

Theoretical calculation

g.s.: ground state; m_e: electron mass

Phase-space factor ⟵ Wave functions of emitted electrons

NME ⟵ Nuclear wave functions

Accurate calculation more difficult than the phase-space factor

Approximation is indis-pensable.

NME

$$M_{0\nu} = 4\pi R \sum_B \sum_{\substack{pp' \\ nn'}} \langle pp'|V_\nu(r_{12}, E_B)|nn'\rangle \langle F|c_{p'}^\dagger c_{n'}|B\rangle \times \langle B|c_p^\dagger c_n|I\rangle$$

Final state, ground state of nucleus(N−2,Z+2)

intermediate state,nucleus (N−1,Z+1)

Initial state, ground state of nucleus (N,Z)

R: nuclear root-mean radius

The transition operator used in my calculation is

$$V_\nu(r_{12}, E_B) \cong h_+(r_{12})\{-\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + g_V^2/g_A^2\}\, \tau_1^- \tau_2^-$$

Double-Gamow-Teller + Double-Fermi

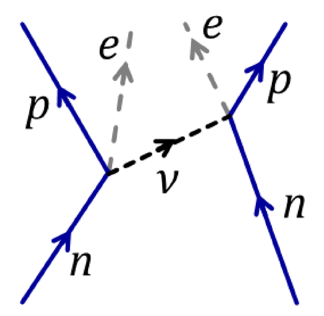
Neutrino potential

g_V: vector current coupling = 1

Status:
The calculated NMEs by various approximation methods and groups are distributed in a range of the max-min ratio of 2–3. The NME cannot be obtained by experiment. Thus, examination and improvement of the calculation are essential.

Principle to derive the eq. of the NME

Apply the quantum field theory to the leptons and the Rayleigh-Schrödinger perturbation theory to the nuclei ⇒ $M_{0\nu}$ in Sec. NME.



The lowest-order diagram in terms of the vertex; this is the usually calculated one. The Majorana neutrino is used. Other nucleons are omitted in this diagram.

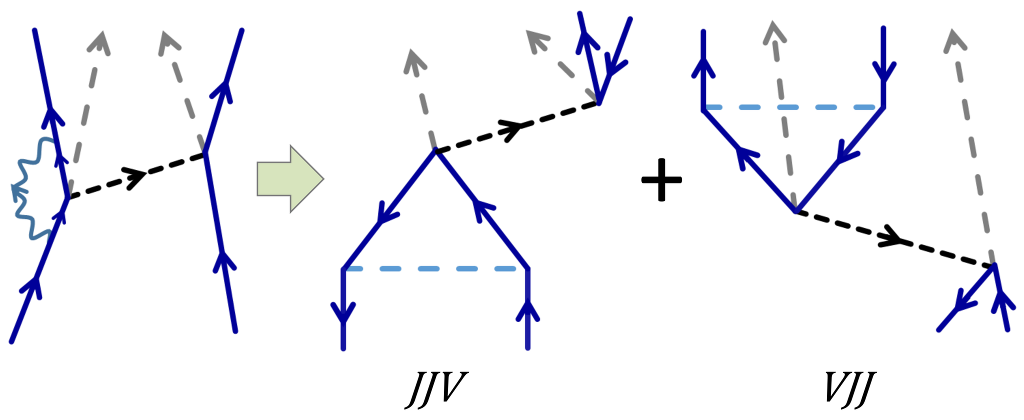
Neutrino potential

$$h_+(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} \frac{1}{|\boldsymbol{q}|} \frac{\exp[i\boldsymbol{q}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)]}{\frac{1}{2}(M_I + M_F) - \bar{E}_C - |\boldsymbol{q}|}$$

$\boldsymbol{r}_1, \boldsymbol{r}_2$: the coordinates of the two nucleons.
M_I, M_F: the masses of the initial and final nuclei, respectively; the nucleon rest mass is excluded.
$\bar{E}_C$: the average energy of the intermediate states.
$\boldsymbol{q}$: the momentum of the Majorana neutrino.

The intermediate nuclear states are obtained by the quasiparticle random-phase approximation (QRPA). The initial and final ground states have many-body correlations.

The first-order vertex correction



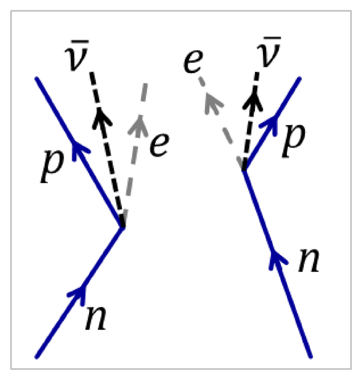
The meson propagator (wavy line) is replaced by the nuclear interaction (horizontal dashed line).

$$M_{0\nu}^{(VJJ)} = 4\pi R \sum_{j'l'mn} \langle j'm|V_\nu(r_{12}, \bar{E}_C)|l'n\rangle \sum_{\substack{\nu'\mu'\lambda\kappa \\ \text{all different}}} \frac{1}{\varepsilon_{-\kappa} + \varepsilon_{-\mu'} + \varepsilon_\lambda + \varepsilon_{\nu'}} \frac{1}{4}\sum_{ijkl} \langle ij|{:}V{:}|lk\rangle$$
$$V_{j-\mu'}\, V_{i-\kappa}\, U_{l\lambda}\, U_{k\nu'} \left(U_{j'-\mu'}^* V_{l'\nu'}^* - U_{j'\nu'}^* V_{l'-\mu'}^* \right)$$
$$\sum_{C_F C_I} \langle F|a_{-\kappa} a_\lambda|C_F\rangle \sum_{\mu\nu} U_{m\mu}^* V_{n-\nu}^* \langle C_F|C_I\rangle\langle C_I|a_\mu^\dagger a_{-\nu}^\dagger|I\rangle.$$

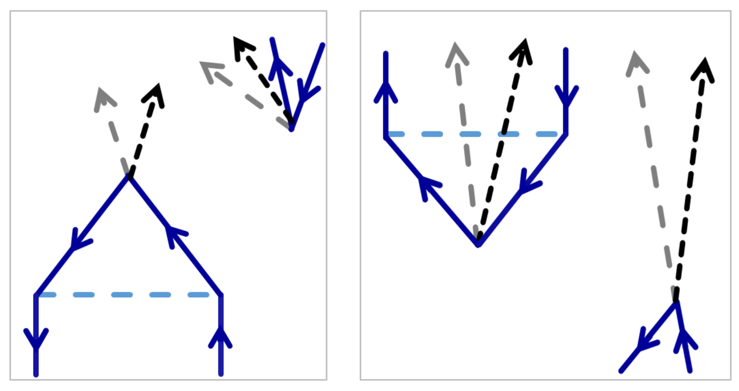
ε_λ and others: quasiparticle energy.
V: nuclear interaction.
$V_{i-\kappa}$, $U_{l\lambda}$ and others: transformation matrix elements between single particles (i,j,...) and quasiparticles (μ, ν,...) created by $a_\mu^\dagger$ and others.
C_I and C_F: intermediate nuclear states obtained by the QRPA

For numerical calculation, the ground states (GS) are approximated by the Hartree-Fock-Bogoliubov states. The GS correlations are partially neglected in the other parts.
$M_{0\nu}^{(JJV)}$ is calculated analogously.

NME of two-neutrino double-β (2νββ) decay



The lowest-order diagram; usually calculated one. Note that this decay is different from sequential two β decays. The Dirac neutrino is used.



The first-order vertex corrections to the 2νββ NME.

The NME of the above diagrams can be calculated in the same manner as the 0νββ NME, except for the lack of the neutrino potential and a few other details.

Calculation result and discussion

The calculation was performed for ^{136}Xe→^{136}Ba with the phenomenological Skyrme interaction (SkM*) and the contact pairing interactions; For the calculation procedure , see J.T., Phys. Rev. C **102**, 044303 (2019).

Term		GT	Fermi	GT − g_V^2/g_A^2 × Fermi
The lowest order		3.095	−0.467	3.384
VC, compo-nents	*JJV*	−0.048	0.025	−0.064
	VJJ	−0.715	0.519	−1.037
Total VC		−0.763	0.544	−1.100
The lowest + VC		2.332	0.077	2.284
VC/lowest (%)		−24.6	−116.5	−32.5

The lowest-order NME and vertex corrections (VC) for the 0νββ decay of ^{136}Xe. The values of g_A = 1.27 (bare value) and g_V = 1 are used. GT: Gamow-Teller

The *VJJ* term is the major term. This is because the NME is sensitive to the tail of the charge-change strength function of ^{136}Ba→^{136}Cs (intermediate nucleus).

Phenomenological effective g_A with the lowest-order NME is often used to reproduce the decay rate if it is measured. A simulation of the effective g_A^{eff} [g_A^{eff}(Sim)] is made using the perturbed NME.

	g_A^{eff}(Sim)
2νββ	0.891
0νββ	1.019

The g_A^{eff}(Sim) of the 2νββ and the 0νββ NME are close, indicating a possibility that the phenomenological effective g_A for the 2νββ decay [g_A^{eff}(phen.,2νββ)] can be used approximately for the 0νββ NME [NME(est.,0νββ)].

	g_A^{eff}(phen., 2νββ)	NME(est, 0νββ)	0νββ Half-life (10^{30} yr), $\langle m_\nu \rangle$ = 10 meV
0th	0.4216	5.104	3.0
0th + VC	0.7557	2.228	1.5

The 0νββ half-life can be estimated with an assumed effective neutrino mass $\langle m_\nu \rangle$. The experimental half-life of the 2νββ decay was taken from A.S. Barabash, Proc. of MEDEX'19 for obtaining g_A^{eff}(phen.,2νββ).

The important step has been taken towards solving the long-standing problem of the uncertainty of the 0νββ NME.