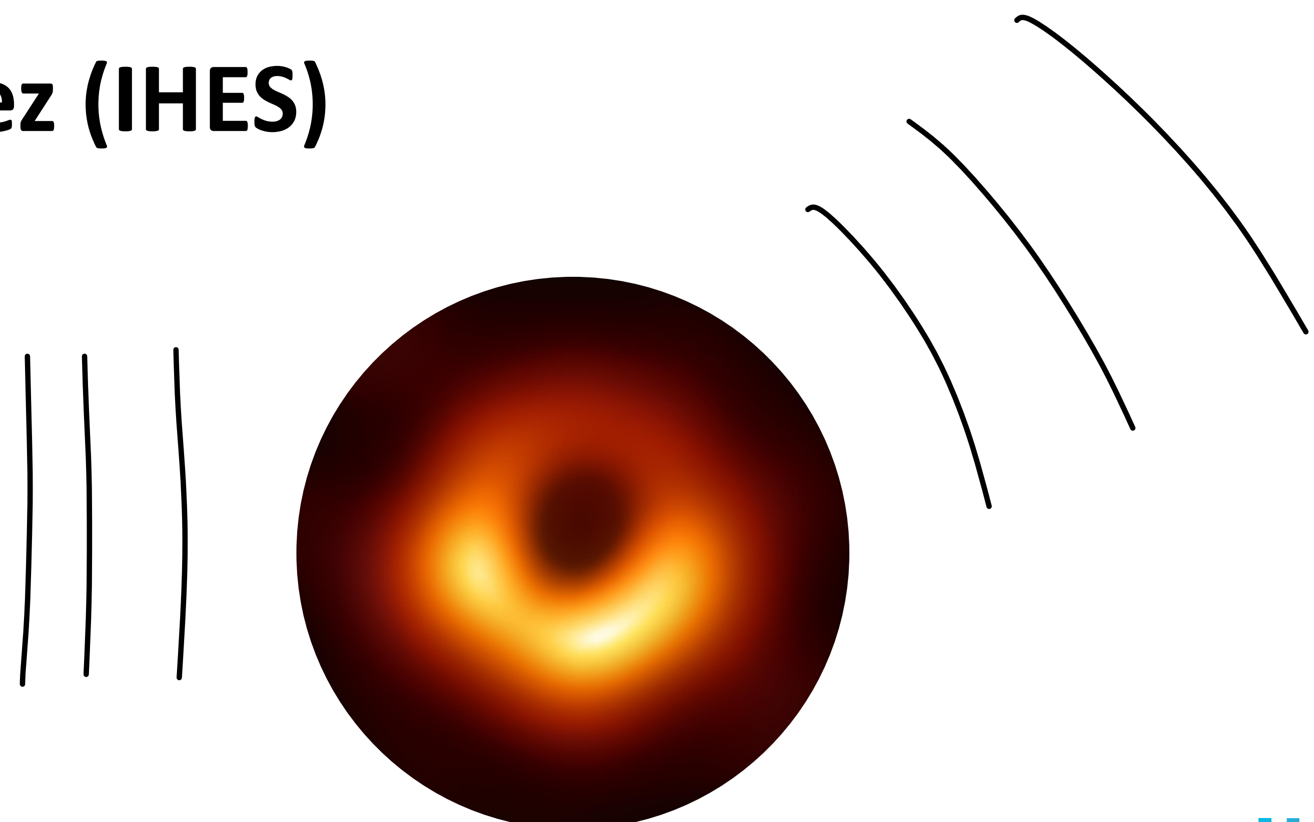
Tidal Love numbers and scattering amplitudes

Julio Parra-Martinez (IHES)

w/ Ivanov, Li, Zhou



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Institut des Hautes Études Scientifiques

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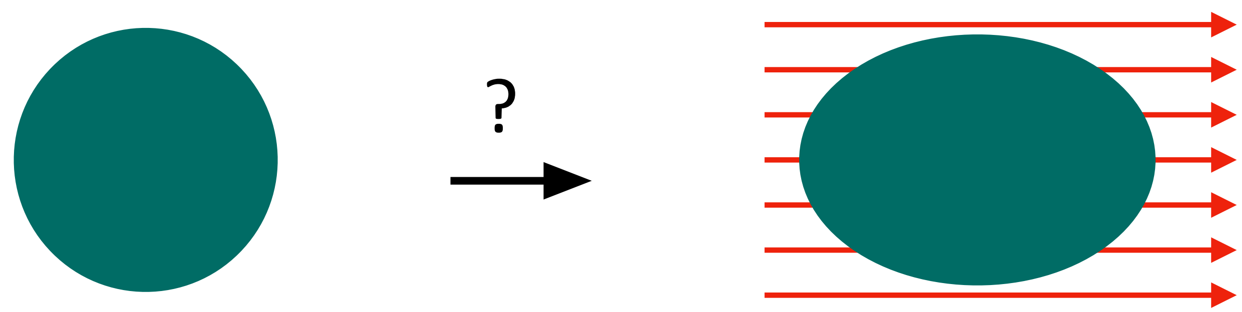
Tidal properties of compact objects

Love number = “gravitational polarizability”



$$I_{ij} = \lambda E_{ij}$$

Linear response to applied external gravitational field.

They distinguish different compact objects (BH, NS).

Generically can be a function of frequency, “dynamical tides”

$$\lambda(\omega) = \lambda_0 + \lambda_1\omega^1 + \lambda_2\omega^2 + \cdots$$

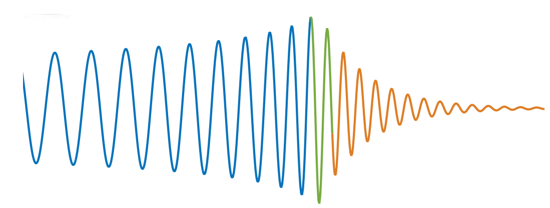
Static

Dynamical

Why are these interesting?

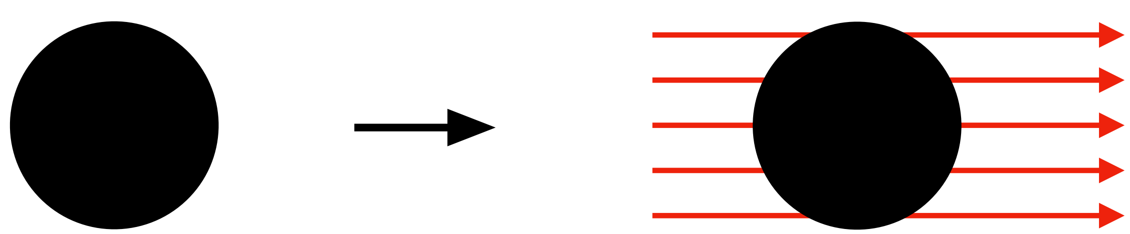
- The modify the post-Newtonian potential [Damour; Bern, **JPM**, Roiban, Sawyer, Shen;...]

$$\Delta V(r) \sim \lambda \left(\frac{GM}{r}\right)^6$$ “5PN”



So we should be able to measure with gravitational wave detectors. Equation of state of NS? [Flanagan, Hinderer; Chia, Ivanov, Zhou]

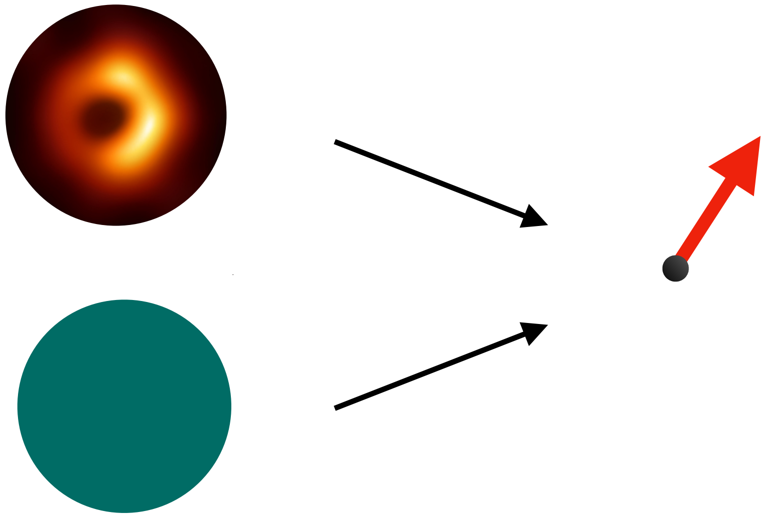
- Static responses are zero for BH in D=4 GR! [Damour, Nagar; Binnington, Poisson]



Excellent window into possible new physics!

Point particle Effective Theory

- Black holes, Neutron stars, … $\longrightarrow$ point particle theory



with given mass, spin

In the language of effective field theory, point particle to first approximation [Goldberger, Rothstein; Porto]

$$S = m\int d\tau\sqrt{g_{\mu\nu}(x)\dot{x}^{\mu}x^{\nu}} \quad + \quad \text{spin}$$

(can also describe as quantum field, massive higher spin)

Tidal effects in the EFT

- Tidal effects = non-minimal couplings

$$\Delta S = \lambda_{1\omega^0}\int d\tau(\nabla\phi)^2 + \lambda_{0\omega^2}\int d\tau\dot{\phi}^2 + \cdots$$

Scalar toy model

static dynamical

Similar for gravitons & photons, e.g.,

$$\Delta S = \lambda^E_{0\omega^0}\int d\tau E^2_{\mu\nu} + \lambda^E_{0\omega^2}\int d\tau(\dot{E}_{\mu\nu})^2 + \cdots$$

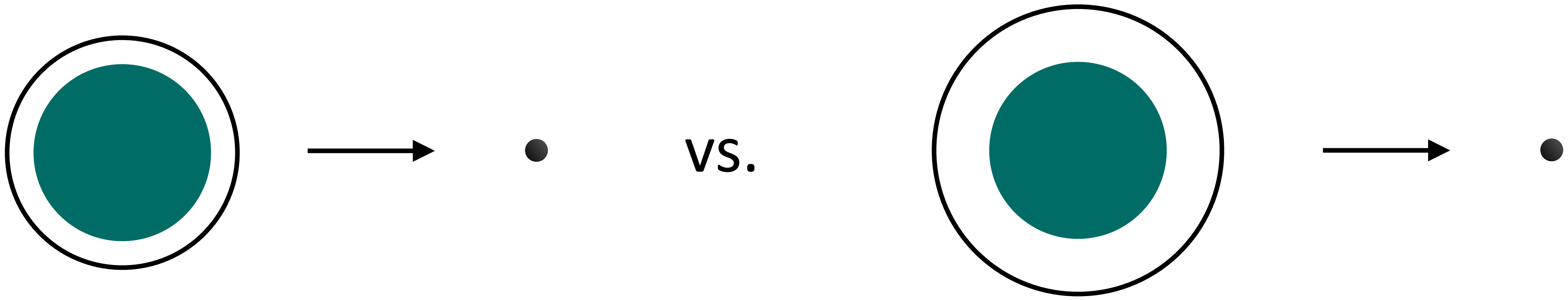
+ magnetic

- We can power-count as in any EFT $\lambda_{\ell\omega^n} \sim mR^{n+2\ell+2s+1}$

- We should expect UV divergences! Classical GR coupled to a point particle is not a UV complete theory.

Dynamical tides run!

A compact object cannot be cleanly separated from its gravitational field



Hence, tidal effects depend on “how much of the spacetime” is integrated out together with the microscopic dynamics of the object.

$$\lambda(\omega, R) = \lambda_0 + \lambda_1(R)\,\omega + \lambda_2(R)\,\omega^2 + \cdots$$

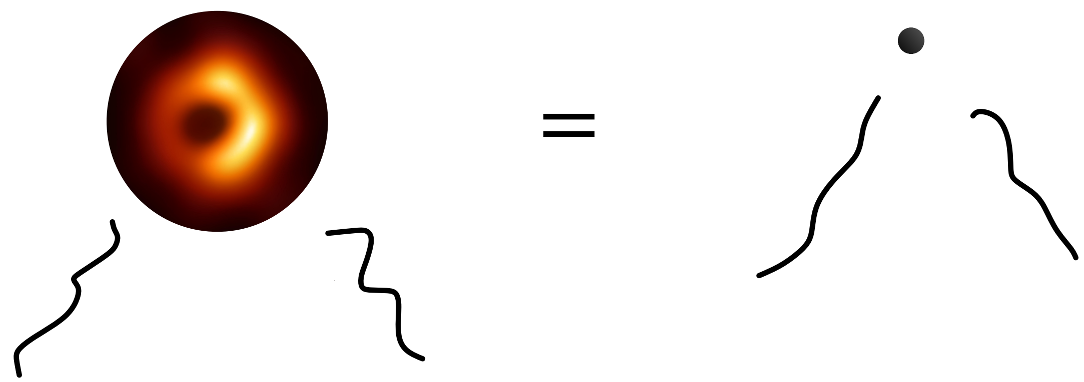
In the EFT this manifests itself in UV divergences, even classically!

These will appear as subdivergences in the EFT description of a binary of compact objects!

[Bern, Barack, Herrmann, Long, **JPM**
Roiban, Ruf, Shen, Solon, Teng, Zeng]

Q: How do we compute these “running” tides?

Idea: Matching using scattering amplitudes



$$\mathscr{M}^{BHPT} = \mathscr{M}^{EFT}$$

Previous attempts using "near-zone/far-zone" factorization of BHPT answer, which breaks for dynamical tides or in D>4.

[Ivanov, Zhou; + Saketh]

The calculation in BHPT

Regge-Wheeler equation with wave boundary conditions $\Box_{Schw.} h_{\mu\nu} = 0$

$$f_s(\theta) = \frac{2\pi}{i\omega} \sum_{\ell=s}^{\infty} {}_{-s}S_\ell^s(1, a\omega)\,{}_{-s}S_\ell^s(\cos\theta, a\omega)(\eta_{\ell s} e^{2i\delta_{\ell s}} - 1)$$

Phase shifts receive contributions from "near zone" ($r \sim R_s$) and "far zone" $r \gg R_s$

$$\delta_{\ell s} = \delta_{\ell s}\big|^{NZ} + \delta_{\ell s}\big|^{FZ}$$

Contains information about Love numbers

Computable in EFT, mod counterterms

Several methods for solution (MST, Nekrasov Partition function), known to all orders!

The phase shift in BHPT

Result for scalar wave toy model in the far zone

$$\delta_\ell^{G^3}\Big|_{\rm BHPT}^{FZ} = (R_s\omega)^3\left[\frac{1}{6}\psi^{(2)}(1+\ell) + \frac{-11+15\ell+15\ell^2}{2(-1+2\ell)(1+2\ell)(3+2\ell)}\psi^{(1)}(1+\ell) + \frac{3\left(3\ell^2+3\ell-2\right)}{2\ell(\ell+1)(2\ell-1)(2\ell+3)}\right]$$

Note pole at $\ell = 0$, related to a counter-term ambiguity in EFT, $\lambda_2\dot{\phi}^2$.
Near and far zone individually not well defined.

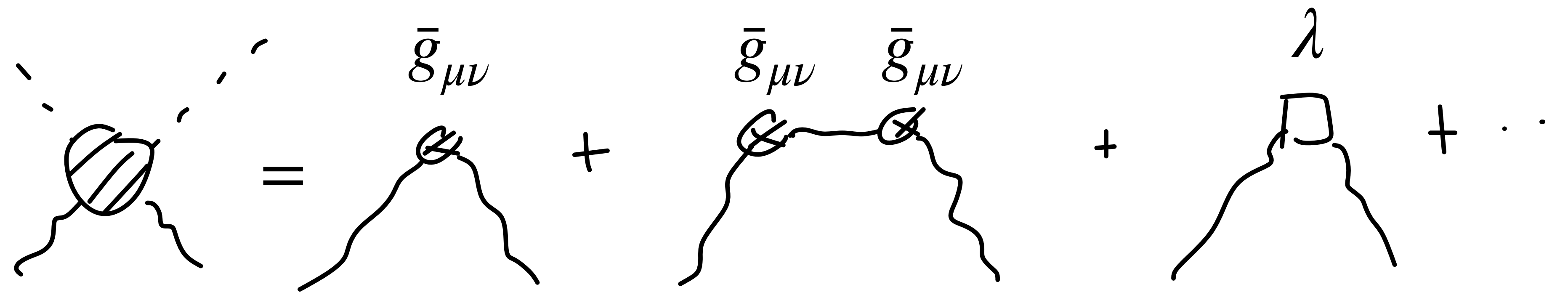
Result after adding the “near zone”

$$\delta_{\ell=0}^{G^3}\Big|_{\rm BHPT} = (R_s\omega)^3\left[\frac{7}{12} - \gamma_E - \ln(2R_s\omega)\right]$$

The calculation in the EFT

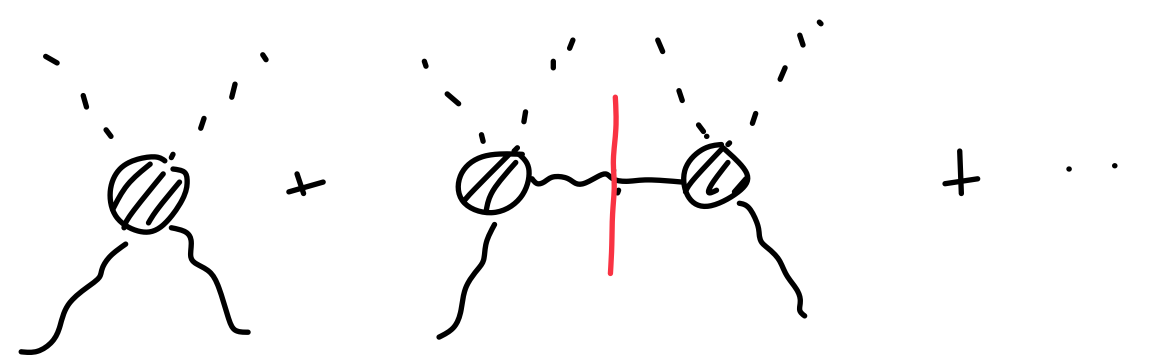
- Background field method for "integrand"

$$i\mathscr{M} =$$



- Logarithm of amplitude to directly compute phase shift

$$i\Delta = \log(1 + i\mathscr{M}) =$$



- Differential equation in angle for integrals

$$\mathscr{M}(\omega, x = \sin^2\frac{\theta}{2}) = (R_s\omega)\mathscr{M}(x)^{1\mathrm{PM}} + (R_s\omega)^2\mathscr{M}(x)^{2\mathrm{PM}} + (R_s\omega)^3\mathscr{M}(x)^{3\mathrm{PM}} + \cdots$$

The phase shift in the EFT

Result matches BHPT in the far zone for $\ell \neq 0$

$$\delta_{\ell}^{G^3}\Big|_{\text{EFT}}^{FZ} = (R_s\omega)^3\left[\frac{1}{6}\psi^{(2)}(1+\ell) + \frac{-11+15\ell+15\ell^2}{2(-1+2\ell)(1+2\ell)(3+2\ell)}\psi^{(1)}(1+\ell) + \frac{3\left(3\ell^2+3\ell-2\right)}{2\ell(\ell+1)(2\ell-1)(2\ell+3)}\right]$$

Result for $\ell = 0$ has UV divergence that is renormalized by dynamical love number

$$\delta_{\ell=0}^{G^3}\Big|_{\text{EFT}} = (R_s\omega)^3\left[\frac{1}{4\epsilon_{\text{UV}}} + \frac{49}{24} - \frac{1}{2}\ln\left(\frac{4\omega^2}{\bar{\mu}^2}\right)\right] + \frac{\lambda_2}{4\pi}\omega^3$$

Comparing to BHPT result we can extract dynamical Love number!

$$\lambda_{0\omega^2}(\bar{\mu} \sim 1/R) = -4\pi R_s^3\left[\frac{1}{4\epsilon_{\text{UV}}} + \ln(\bar{\mu}R_s) + \frac{35}{24} + \gamma_E\right]$$

Our results

- Verified that the static $\ell = 0{,}1$ Love numbers vanish for scalar waves

- Matched the leading dynamical Love number ($\ell = 0$) by comparing BHPT to EFT calculation

To appear:

- Verified that for gravitons the EFT matches precisely BHPT up to 3PM, as expected by power counting (static tides start at 5PM)

- Verified that the static $\ell = 1$ Love number vanishes for photons (leading dynamical tide at 5PM)

- Extracted running of the lowest tides in D=5, 7. New prediction from EFT!

We have also computed dissipative tides (ask me later!)

Conclusions

- We can compute scattering of waves off compact objects using effective field theory and tools from scattering amplitudes.
- These can be used to extract the tidal properties of compact objects. Demonstrated for black holes and also neutron stars.
- New arena for calculation of “classical” scattering amplitudes. Much simpler than massive scattering! Only ladder integrals, hints of all orders structure.
- Matching the leading (non-zero) conservative dynamical tides for gravity will require a 7PM (6-loop) calculation.
- Q: Do tides of supersymmetric black holes also run? UV divergences?