Spinning gravitational waveforms from scattering amplitudes

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with

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ICHEP 2024, Prague, 20th July 2024

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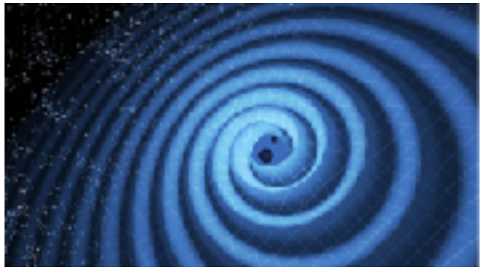

Credit: Tim Pyle (LIGO)

- Increasing experimental precision in gravitational wave observations
  - Need higher perturbative orders (in Newton's constant G)
  - Modifications due to the spin of the black holes
  - Include effects of deformations of Einstein-Hilbert theory
- Apply powerful scattering amplitude methods
  - From the perspective of long-wavelength radiation, black holes appear as point-like particles
  - Use language of Effective Field Theory
- Reliable predictions of the infrared theory
  - non-analytic terms in the amplitudes, arising from long-range propagation of massless particles at low energy
  - Ideal for unitarity-based techniques

Menu

- Scattering waveform of two Kerr black holes
  - Scattering amplitude of two spinning black holes with emission of a graviton
  - Effective field theories of gravity with cubic deformations and General Relativity
  - Spinning waveforms from scalar waveforms, exact in spin expressions

Gravity as an effective field theory

- First relevant deformations are cubic in curvature

$$S = \int d^4x\sqrt{-g}\left[-\frac{2}{\kappa^2}R + \beta_1 I_1 + \beta_2 G_3 + \tilde{\beta}_1 \tilde{I}_1 + \tilde{\beta}_2 \tilde{G}_3 + \cdots \right]$$

$$I_1 := R^{\alpha\beta}{}_{\mu\nu}R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta} \qquad G_3 := I_1 - 2R^{\mu\nu\alpha}{}_{\beta}R^{\beta\gamma}{}_{\nu\sigma}R^{\sigma}{}_{\mu\gamma\alpha}$$

  - Parity odd $\tilde{I}_1, \tilde{G}_3$: replace one Riemann tensor with $\tilde{R}^{\mu\nu\alpha\beta}=(1/2)\,\epsilon^{\mu\nu\rho\sigma}R_{\rho\sigma}{}^{\alpha\beta}$
  - All parity-even/odd independent cubic couplings (Metsaev & Tseytlin)
  - Bosonic string theory: $\beta_1 = -\frac{2}{\kappa^2}\frac{\alpha'^2}{48}\,, \qquad \beta_2 = -\frac{2}{\kappa^2}\frac{\alpha'^2}{24}\,, \qquad \tilde{\beta}_{1,2} = 0$
  - Quadratic in curvature deformations do not contribute to the S-matrix (Tseytlin; Deser, Redlich; Accetulli Huber, Brandhuber, De Angelis, GT)
  - Earlier results for Schwarzschild black holes (i.e. no spin):
    - Newton potential and scattering angle (Brandhuber, GT; Emond, Moynihan)
    - deflection of massless particles and quadrupole moments (Accettulli Huber, Brandhuber, De Angelis, GT)

Spinning waveforms

(Brandhuber, Brown, Chen, Gowdy, GT; Brandhuber, Brown, Chen, GT, Vives Matasan)

- Gravitational field from scattering two black holes
  - Far-field limit, corresponding to large observer distance $r = |\vec{x}|$ and fixed retarded time $u := t - r$

$$\langle h_+ \pm i h_\times \rangle := \langle h^{\text{out}}_{\mu\nu} \rangle \varepsilon^{\mu\nu}_{(\pm\pm)} = \frac{1}{4\pi r}(h^\infty_+ \pm i h^\infty_\times)$$

$$h^\infty_+ \pm i h^\infty_\times = \kappa \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega u}\, W(b, k^\pm)\big|_{k=\omega(1,\hat{\mathbf{x}})}$$

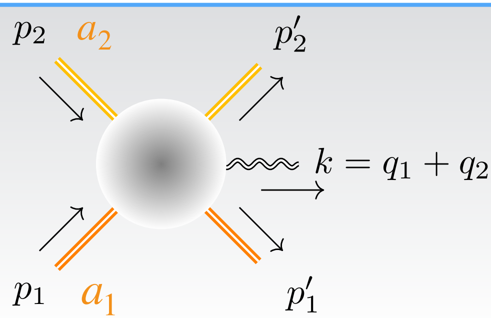
- Spectral waveform W from classical 5-point amplitude $\mathcal{M}_5$

$$W(b,k) = -i \int d\mu^{(4)}(q_1, q_2)\; e^{i(q_1\cdot b_1 + i q_2 \cdot b_2)}\; \mathcal{M}_5$$

$$q_1 = p_1 - p_1' \,, \quad q_2 = p_2 - p_2'$$

  - $b := b_1 - b_2$ = asymptotic impact parameter, $b \cdot p_1 = b \cdot p_2 = 0$
  - Derived from KMOC procedure (Cristofoli, Gonzo, Kosower O'Connell)
  - Leading order (tree level)

5-point amplitude

$\mathcal{M}_5 =$ 

$p_1^2 = (p_1')^2 = m_1^2, \quad p_2^2 = (p_2')^2 = m_2^2$

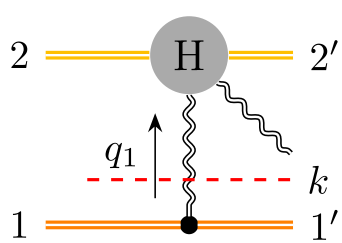
$q_1 = p_1 - p_1'$

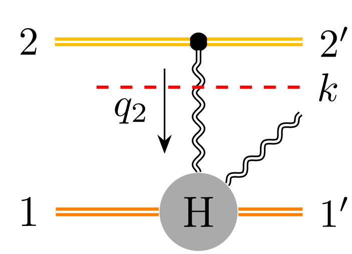
$q_2 = p_2 - p_2'$

- Classical tree-level 5-point amplitude of two spinning particles and a graviton
- Spin encoded in the ring radius $a^\mu = S^\mu / m$ with $S^\mu = 1/(2m)\, \epsilon^{\mu\nu\alpha\beta} p_\nu S_{\alpha\beta}$
- Magnitude of a^μ = radius of ring singularity, and $a(p)\cdot p = 0$

• Only poles contribute to waveform

- built out of classical (HEFT) Compton amplitude — Andi Brandhuber's talk

$\mathcal{M}_{q_1^2} =$ 

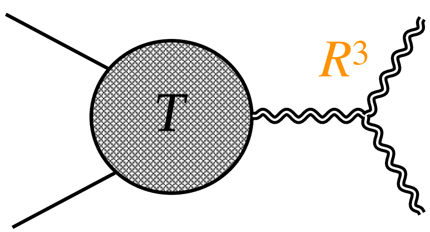
$\mathcal{M}_{q_2^2} =$ 

$$\mathcal{M}_5 = \mathcal{M}_{q_1^2} + \mathcal{M}_{q_2^2} + \text{contact terms}$$

- $h^\infty(u) = h^{\infty,(1)}(u) + h^{\infty,(2)}(u)$ with $\mathcal{M}_{q_2^2} = \mathcal{M}_{q_1^2}\Big|_{1\leftrightarrow 2}$

- Leading-order GR computation:
  - non-spinning (Jakobsen, Mogull, Plefka, Steinhoff)
  - spinning (De Angelis, Gonzo, Novichkov; Brandhuber, Brown, Chen, Gowdy, GT; Aude, Haddad, Heissenberg, Helset)
  - Key ingredient: Compton amplitude, GR state of the art is $\mathcal{O}(a^8)$ Gang Chen's talk
- Waveforms with cubic deformations nicely compact
  - Compton amplitude known to all orders in spin. To lowest order in the cubic deformations:

Glue the stress tensor T of a Kerr black…



…to a cubic vertex

"Cubic" Compton amplitudes

(Brandhuber, Brown, Pichini, GT, Vives Matasan)

- Stress-energy tensor for a Kerr black hole: (Vines; Guevara, Ochirov, Vines)

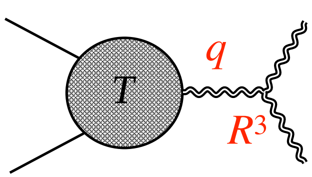
$$T^{\mu\nu} = \left(\frac{\kappa}{2}\right) 2 \left[\cosh(a\cdot q) p^\mu p^\nu - i\frac{\sinh(a\cdot q)}{a\cdot q} p^{(\mu}\epsilon^{\nu)}{}_{\alpha\beta\gamma} p^\alpha a^\beta q^\gamma\right] \qquad (p^2 = m^2)$$

- Cubic vertices:

$$V^{\mu\nu}_{I_1}(k_1^{++}, k_2^{++}) = -i\frac{3!}{4}\left(\frac{\kappa}{2}\right)^3 [1\,2]^4 \Big(\langle 2|\mu|1]\langle 1|\nu|2] + \mu \leftrightarrow \nu\Big)$$

$$V^{\mu\nu}_{I_2}(k_1^{++}, k_2^{++}) = i\frac{3!}{4}\left(\frac{\kappa}{2}\right)^3 [1\,2]^4 (k_1 - k_2)^\mu (k_1 - k_2)^\nu$$

- Results:



✲ $$M_{I_1}(p, k_1^{++}, k_2^{++}) = i\left(\frac{\kappa}{2}\right)^4 3!\frac{[1\,2]^4}{q^2}\Big\{ -4\cosh(a\cdot q)\ (p\cdot k_1)(p\cdot k_2) + \frac{1}{2} p\cdot(k_1 - k_2)\frac{\sinh(a\cdot q)}{a\cdot q}\Big([1|p|2\rangle[2|a|1\rangle - [2|p|1\rangle[1|a|2\rangle\Big)\Big\}$$

✲ $$M_{G_3}(p, k_1^{++}, k_2^{++}) = -3\,i\,m^2 \left(\frac{\kappa}{2}\right)^4 \cosh(a\cdot q)[1\,2]^4$$

✲ $$M_{I_1}(p, k_1^{\pm\pm}, k_2^{\mp\mp}) = M_{G_3}(p, k_1^{\pm\pm}, k_2^{\mp\mp}) = 0$$

- Different strategies to compute waveform integral

$$h^{\infty}(u) = -i\kappa \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega u} \int \frac{d^4 q_1}{(2\pi)^2} \delta(2p_1 \cdot q_1)\delta(2p_2 \cdot (k - q_1))\; e^{i(q_1 \cdot b_1 + q_2 \cdot b_2)}\; \mathcal{M}_5$$

  - direct integration (Jakobsen, Mogull, Plefka, Steinhoff)
  - residues (De Angelis, Gonzo, Novichkov; Brandhuber, Brown, Chen, Gowdy, GT)

- All orders in spin:
  - Instead of expanding in spin, use $\begin{cases} \cosh(a_i \cdot q_j) & = \frac{1}{2}(e^{a_i \cdot q_j} + e^{-a_i \cdot q_j}) \\ \sinh(a_i \cdot q_j) & = \frac{1}{2}(e^{a_i \cdot q_j} - e^{-a_i \cdot q_j}) \end{cases}$
  - Spinning waveform in terms of scalar waveform with shifted impact parameter

- Result in cubic theories (diagram 1):

$$h^{\infty,(1)}(u) = \frac{1}{2}\left[h^{\infty,(1)}(u)\Big|_{a_i=0,\; b\to b+i(\tilde{a}_1+\tilde{a}_2)} + h^{\infty,(1)}(u)\Big|_{a_i=0,\; b\to b+i(\tilde{a}_1-\tilde{a}_2)} \right]$$

$$\tilde{a}_i := a_i - v_i \frac{a_i \cdot \hat{k}}{v_i \cdot \hat{k}}$$

  - Only two "sectors". GR result has similar structure (but more sectors)

- G_3 **waveform:** (Brandhuber, Brown, Chen, GT, Vives Matasan, to appear)

$$h^{\infty,(1)}_{G_3}(u) = -\frac{3}{2}\left(\frac{\kappa}{2}\right)^6 \frac{m_1 m_2}{(\hat{k}\cdot v_2)^3}\left[\mathcal{C}^{(1)}\Big|_{\tilde{b}\to\tilde{b}+i(\tilde{a}_1-\tilde{a}_2)} + \mathcal{C}^{(1)}\Big|_{\tilde{b}\to\tilde{b}+i(\tilde{a}_1+\tilde{a}_2)}\right]$$

  - with

$$\mathcal{C}^{(1)} = -\frac{3}{8\pi|\tilde{b}_{(1)}|^5}\Bigg[2[\hat{k}|v_1 v_{2(1)}|\hat{k}]^2 + \frac{5}{|\tilde{b}_{(1)}|^2}\left(v_{2(1)}^2[\hat{k}|v_1\tilde{b}_{(1)}|\hat{k}]^2 + 4[\hat{k}|v_1 v_{2(1)}|\hat{k}][\hat{k}|v_1\tilde{b}_{(1)}|\hat{k}](\tilde{b}_{(1)}\cdot v_{2(1)}) + 7\frac{(\tilde{b}_{(1)}\cdot v_2)^2}{|\tilde{b}_{(1)}|^2}[\hat{k}|v_1\tilde{b}_{(1)}|\hat{k}]^2\right)\Bigg]$$

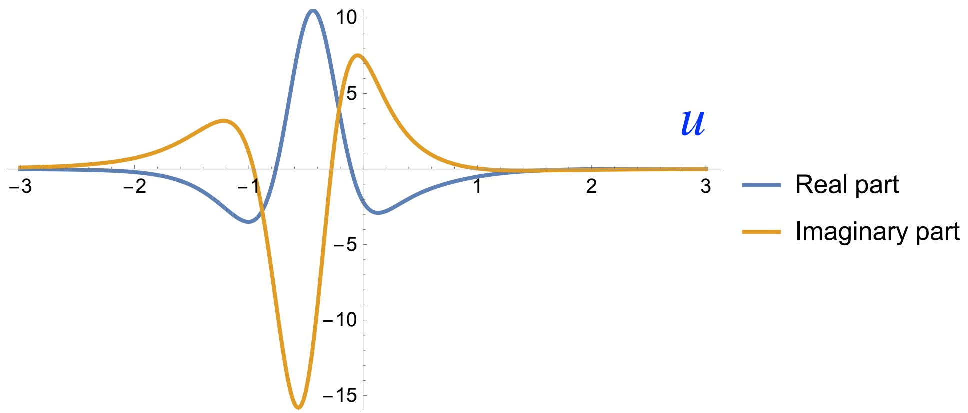
  * $\tilde{b}_{(1)} = P_1^{\mu\nu}\tilde{b}_\nu = (\eta^{\mu\nu} - v_1^\mu v_1^\nu)\tilde{b}_\nu$
  * $\hat{k} = (1, \hat{\mathbf{x}}), \qquad p_i = m_i v_i, \quad i = 1, 2$
  * $\tilde{b} := \tilde{b}_1 - \tilde{b}_2\,, \qquad \tilde{b}_i := b_i + u_i v_i\,, \qquad u_i := \frac{u - \hat{k}\cdot b_i}{\hat{k}\cdot v_i}$

  - For the emission of a graviton with positive helicity
  - Complete waveform $h^\infty(u) = h^{\infty,(1)}(u) + h^{\infty,(1)}(u)\Big|_{1\leftrightarrow 2}$
  - Similar result for the I_1 deformation

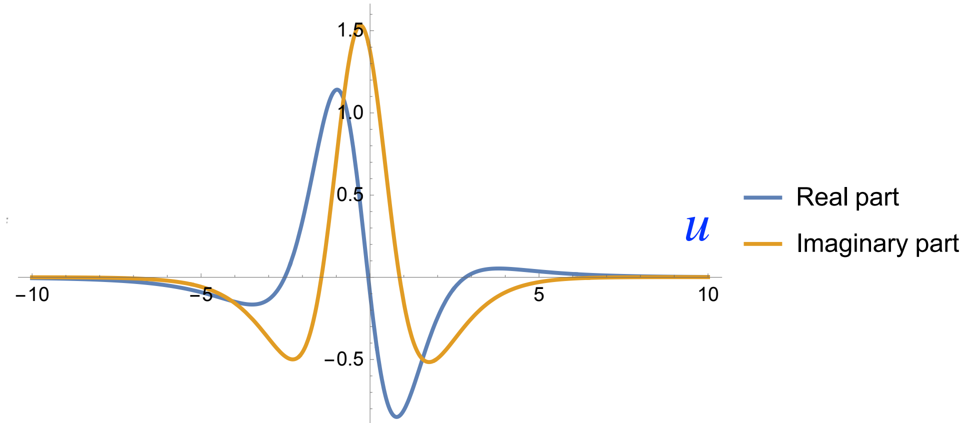
- **Parity-odd waveforms:** $h_+^{\infty,\mathrm{P.O.}} = -h_\times^{\infty,\mathrm{P.E.}}\,, \qquad h_\times^{\infty,\mathrm{P.O.}} = h_+^{\infty,\mathrm{P.E.}}$

Sample waveforms

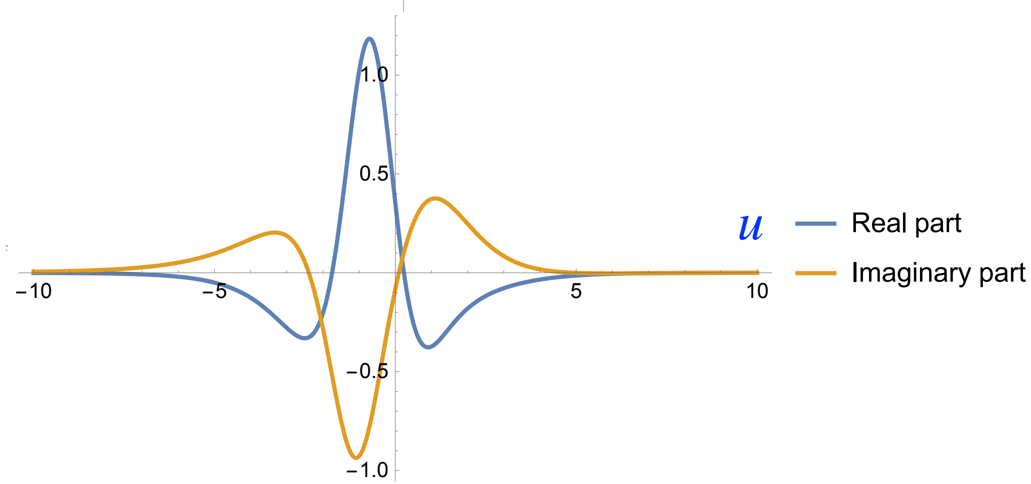
$b_1 = 1,\ b_2 = 0$



G_3 deformation
no spin, equal masses
$v_1 \cdot v_2 = 5/4$



G_3 deformation
no spin, equal masses
$v_1 \cdot v_2 = 25/24$



G_3 deformation
spinning, equal masses

Note absence of memory

Conclusions

- Efficient computation of spinning waveforms
  - Exact-in-spin expressions
  - Leading (tree-level) order so far
- Treat equally Einstein-Hilbert and deformed theories
  - Parity-even and -odd
  - Recent interest in parity violation (CMB, large-scale structure of galaxies)
  - "Agnostic" Effective Field Theory approach

Extra slides

(Other) observables from KMOC

(Brandhuber, Brown, Pichini, GT, Vives Matasan)

- Compute variation $\langle \Delta \mathbb{O} \rangle_\psi = {}_{\text{out}}\langle \psi | \mathbb{O} | \psi \rangle_{\text{out}} - {}_{\text{in}}\langle \psi | \mathbb{O} | \psi \rangle_{\text{in}} = {}_{\text{in}}\langle \psi | S^\dagger \mathbb{O} S | \psi \rangle_{\text{in}} - {}_{\text{in}}\langle \psi | \mathbb{O} | \psi \rangle_{\text{in}}$
  - $\mathbb{O}$ is an observable, and $|\psi\rangle_{\text{in}}$ describes the initial state of the two black holes
  - Connection to amplitudes via $|\psi\rangle_{\text{out}} = S|\psi\rangle_{\text{in}}$
- Momentum kick and spin kick from 4-point amplitudes:

$$\langle \Delta \mathbb{P}_1^\mu \rangle_\psi = \int d\mu^{(D)}\, e^{iq\cdot b}\, q^\mu\, \mathcal{M}_4$$

$$\langle \Delta \mathbb{S}_1^\mu \rangle_\psi = \int d\mu^{(D)} e^{iq\cdot b} \left([S_1^\mu(p_1), \mathcal{M}] - \frac{q \cdot S_1(p_1)}{m_1^2} p_1^\mu\, \mathcal{M} \right)$$

  - $d\mu^{(D)} := \frac{d^D q}{(2\pi)^{D-2}} \delta(2p_1\cdot q)\delta(2p_2\cdot q)$ with $q = p_1 - p_1'$
  - Observables written as Fourier transforms to impact parameter space

Results with cubic deformation

- Impulse (momentum kick) to all orders in spin:

$$\langle \Delta \mathbb{P}_1^\mu \rangle_{I_1} = i \left(\frac{\kappa}{2}\right)^6 \frac{3}{64} \frac{m_1^2 m_2}{\sqrt{\sigma^2-1}} \Big((\sigma^2-1)\mathcal{B}_1^\mu - i\sigma \mathcal{B}_2^\mu \Big) + (1 \leftrightarrow 2)\,,$$

$$\langle \Delta \mathbb{P}_1^\mu \rangle_{G_3} = -i \left(\frac{\kappa}{2}\right)^6 \frac{3}{64} \frac{m_1^2 m_2}{\sqrt{\sigma^2-1}} \mathcal{B}_1^\mu + (1 \leftrightarrow 2)\,,$$

$$\langle \Delta \mathbb{P}_1^\mu \rangle_{\tilde{I}_1} = -\left(\frac{\kappa}{2}\right)^6 \frac{3}{64} \frac{m_1^2 m_2}{\sqrt{\sigma^2-1}} \Big((\sigma^2-1)\tilde{\mathcal{B}}_1^\mu - i\sigma \tilde{\mathcal{B}}_2^\mu \Big) - (1 \leftrightarrow 2)\,,$$

$$\langle \Delta \mathbb{P}_1^\mu \rangle_{\tilde{G}_3} = \left(\frac{\kappa}{2}\right)^6 \frac{3}{64} \frac{m_1^2 m_2}{\sqrt{\sigma^2-1}} \tilde{\mathcal{B}}_1^\mu - (1 \leftrightarrow 2)\,.$$

$$\mathcal{B}_1^\mu = -\frac{45 i}{8\pi} \sum_{\sigma_1=\pm 1} \sum_{\sigma_2=\pm 1} \frac{b^\mu_{\sigma_1\sigma_2}}{|b_{\sigma_1\sigma_2}|^7}\,,$$

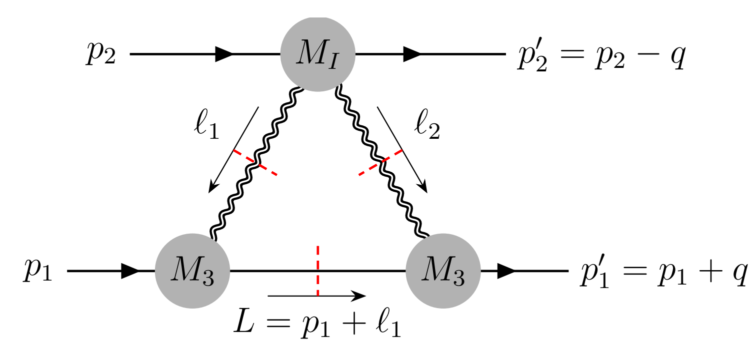
$$\tilde{\mathcal{B}}_1^\mu = -\frac{45 i}{8\pi} \sum_{\sigma_1=\pm 1} \sum_{\sigma_2=\pm 1} \frac{(-\sigma_1)\, b^\mu_{\sigma_1\sigma_2}}{|b_{\sigma_1\sigma_2}|^7}\,,$$

$$\mathcal{B}_2^\mu = \frac{3 i}{8\pi} \sum_{\sigma_1=\pm 1} \sum_{\sigma_2=\pm 1} \frac{\sigma_2 c^\mu_{\sigma_1\sigma_2}}{|b_{\sigma_1\sigma_2}|^7}\,,$$

$$\tilde{\mathcal{B}}_2^\mu = \frac{3 i}{8\pi} \sum_{\sigma_1=\pm 1} \sum_{\sigma_2=\pm 1} \frac{(-\sigma_1)\sigma_2 c^\mu_{\sigma_1\sigma_2}}{|b_{\sigma_1\sigma_2}|^7}$$

$$\sigma := v_1 \cdot v_2$$

  - $b^\mu_{\sigma_1\sigma_2}, c^\mu_{\sigma_1\sigma_2}$ are formed out of the impact parameters and spin (not particularly illuminating)
  - Results are rather compact!
  - Leading corrections appear at one loop



- Spin kick of particle 1, expanded to first order in the spins:

$$\langle \Delta \mathbb{S}_1^\mu \rangle_{I_1} = \left(\frac{\kappa}{2}\right)^6 \frac{135}{128\pi} \frac{1}{\sqrt{\sigma^2-1}} \frac{m_1 m_2}{|b_\perp|^7} \Big[m_2 \sigma \, a_1 \cdot u_2 b_\perp^\mu$$
$$+ a_{1\perp} \cdot b_\perp \big[-(\sigma^2-1) m_1 u_1^\mu + m_2 (u_1^\mu - \sigma u_2^\mu) \big] \Big] ,$$

$$\langle \Delta \mathbb{S}_1^\mu \rangle_{G_3} = \left(\frac{\kappa}{2}\right)^6 \frac{135}{128\pi} \frac{1}{\sqrt{\sigma^2-1}} \frac{m_1 m_2}{|b_\perp|^7} (m_1 + m_2)(a_{1\perp} \cdot b_\perp) u_1^\mu ,$$

$$\langle \Delta \mathbb{S}_1^\mu \rangle_{\tilde{I}_1} = \left(\frac{\kappa}{2}\right)^6 \frac{135}{128\pi} \sqrt{\sigma^2-1} \frac{m_1^2 m_2}{|b_\perp|^7} \epsilon(u_1 a_1 b_\perp \mu) ,$$

$$\langle \Delta \mathbb{S}_1^\mu \rangle_{\tilde{G}_3} = -\left(\frac{\kappa}{2}\right)^6 \frac{135}{128\pi} \frac{1}{\sqrt{\sigma^2-1}} \frac{m_1^2 m_2}{|b_\perp|^7} \epsilon(u_1 a_1 b_\perp \mu)$$

  - $b_\perp^\mu = \Pi^{\mu\nu} b_\nu$ and $a_{1\perp}^\mu = \Pi^{\mu\nu} a_{1\nu}$
  - $\Pi^{\mu\nu}$ projects on the plane orthogonal to the two velocities