Classical Observables in General Relativity from Heavy-Mass Effective Theory

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with

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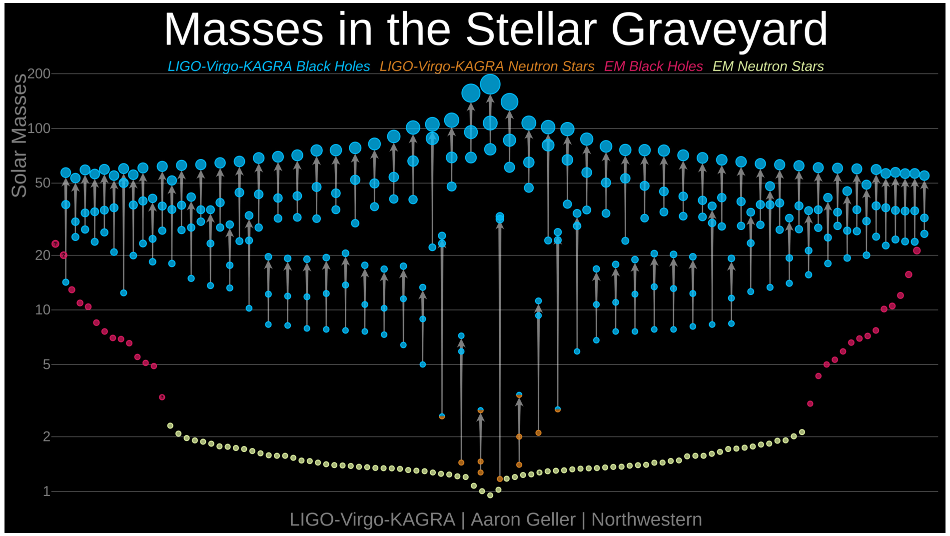
Gravitational Waves

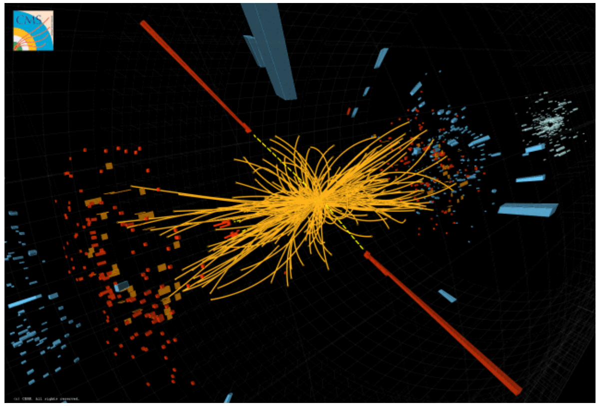


- A New Observational Era:
- LIGO-Virgo-KAGRA
  O1,2,3 ~100 binary mergers
  O4 2-3 days per merger
- Future GW observatories to start in 2030's:
  Advanced LIGO, Cosmic Explorer, Einstein Telescope, LISA ⇒ increased sensitivity + frequency range
- Astrophysics: Black hole formation/evolution, neutron stars, new sources of GWs
- Fundamental physics: Precision tests of GR. Modified GR? DM?

Need for high precision theory predictions

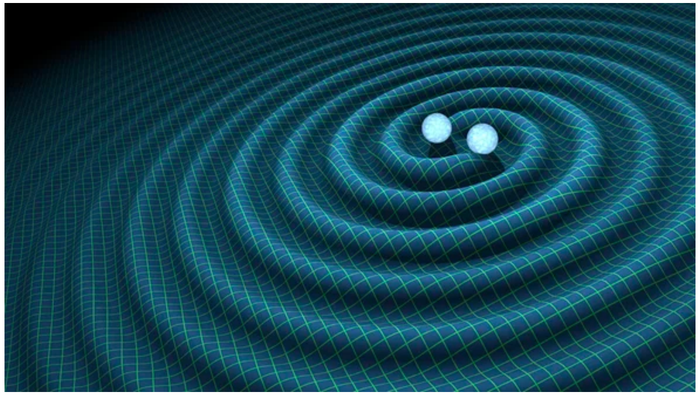
Why can amplitudes help us with this?

Scattering problem



?

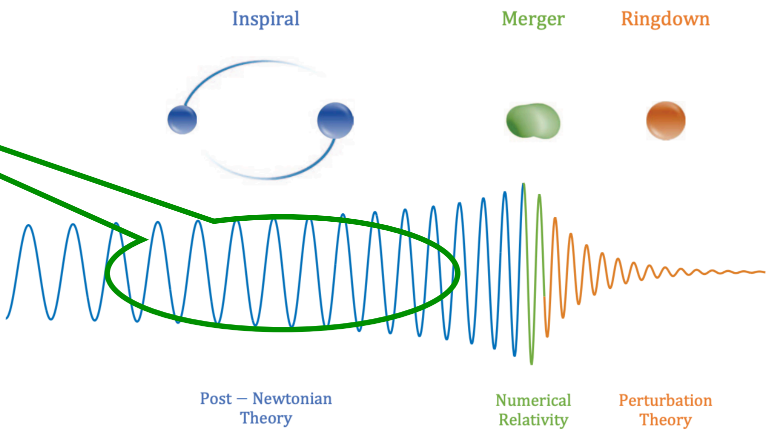
Bound state problem



QCD, EW theory
Quantum Field Theory

Classical GR, Einstein equations

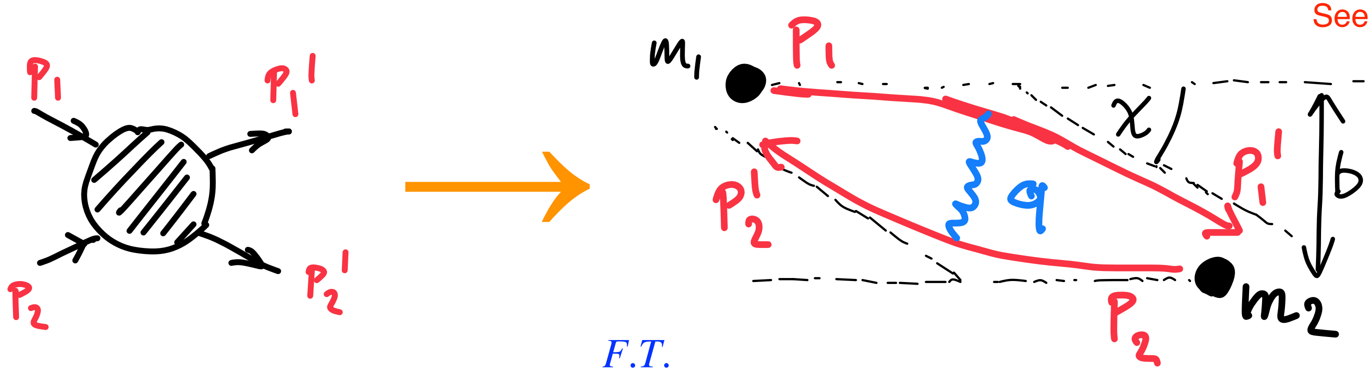
- Black holes/neutron stars are effectively point particles (EFT)
- Unleash the full arsenal of advanced amplitudes techniques in the perturbative regime
- Post-Minkowskian (PM) exp.
  - nPM $\rightarrow G^n \rightarrow (n-1)$ loops



Binary dynamics from (HEFT) amplitudes

- 1) Conservative: e.g. bending angle χ from 4-point amplitudes

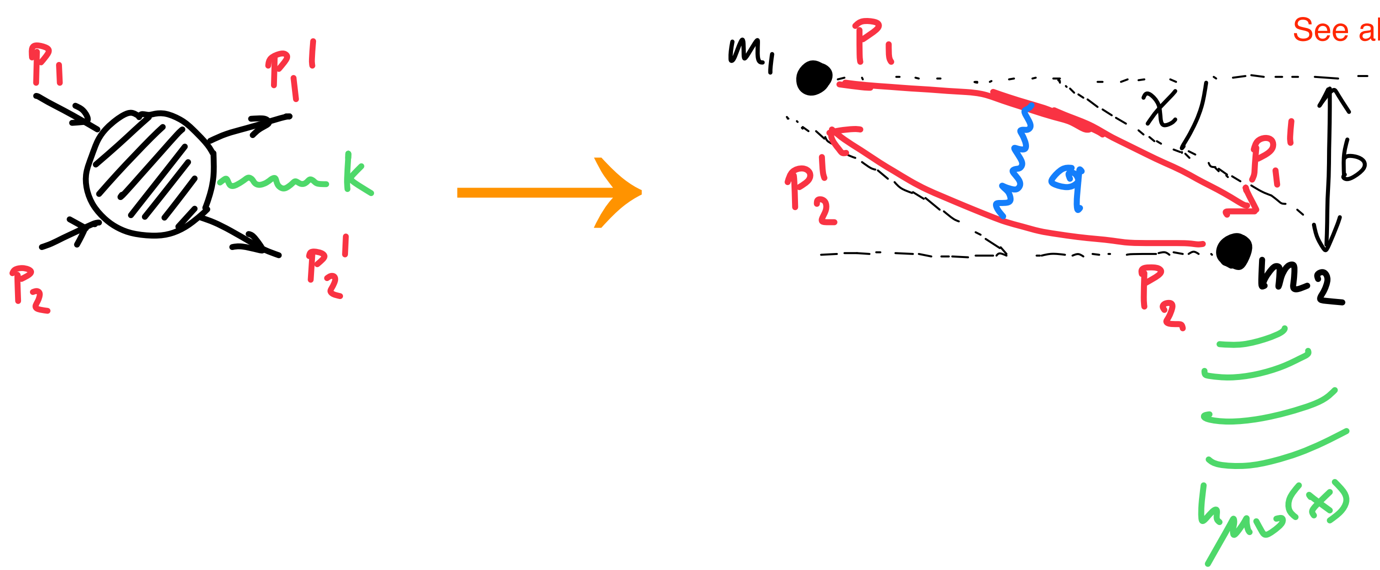
See also Gang's talk



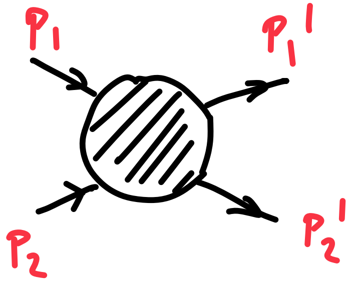
Impact Parameter: $b \overset{F.T.}{\leftrightarrow} q$ momentum transfer

- 2) Dissipation: e.g. bremsstrahlung/waveforms from 5-point amplitudes

See also Gab's talk



Bending angle from amplitudes



$$p_{1,2} = m_{1,2} u_{1,2}$$
$$y = u_1 \cdot u_2 \geq 1$$
$$\bar{p}_i = (p_i + p_i')/2$$

$$p_1 - p_1' = q = p_2' - p_2$$
$$\bar{p}_{1,2} \cdot q = 0$$

- Model blackholes (spin=0) with massive scalars $\Rightarrow$ effective coupling $\dfrac{Gm^2}{\hbar c} \sim 10^{75}$, nonsensical!?
- Exponentiation in Impact Parameter Space (IPS)

$$\widetilde{M}(b) = \int d^4q\, \delta(2\bar{p}_1 \cdot q)\delta(2\bar{p}_2 \cdot q)e^{iq\cdot b}M$$
$$= e^{\frac{i}{\hbar}\delta} = e^{\frac{i}{\hbar}(\delta_0+\delta_1+\delta_2+\ldots)} \quad , \quad \delta_{n+1}/\delta_n \propto Gm/b$$

- Actual expansion parameter: $\dfrac{Gm}{b} \sim \dfrac{R_s}{b} \ll 1$
- Bending Angle: $\chi = -\dfrac{\partial Re(\delta)}{\partial J}$ with $J = |\vec{p}\,|b$

Heavy-mass Effective Field Theory (HEFT)

Damgaard, Haddad, Helset; AB, Chen, Travaglini, Wen

- Classical Limit: set $\hbar \to 0$, with:
- Couplings: $G \to G/\hbar$ or $\kappa \to \kappa/\sqrt{\hbar}$ with $32\pi^2 G = \kappa^2$
  Graviton momenta: $q \to \bar{q}\hbar$, $\ell_i \to \bar{\ell}_i \hbar$ keeping $\bar{q}, \bar{\ell}_i$ fixed
- Expand massive propagators
  $$\frac{1}{(p \pm \ell)^2 - m^2} = \frac{1}{(p \pm \hbar \bar{\ell})^2 - m^2} = \frac{1}{\pm \hbar 2p \cdot \bar{\ell} + i\epsilon} - \frac{\bar{\ell}^2}{(2p \cdot \bar{\ell})^2} + \mathcal{O}(\hbar)$$
- Heavy mass expansion $(\ell/m \ll 1)$ is equivalent to $\hbar$-expansion
  Reminiscent of Heavy Quark Effective Theory (Georgi)
- Take classical limit as early as possible $\Rightarrow$
  Heavy-Mass Effective Field Theory (HEFT)

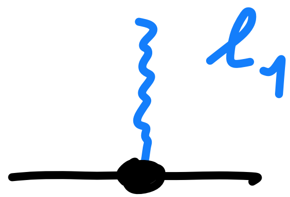
HEFT expansion

- In Unitarity cuts we need trees with 2 massive scalars and n gravitons $M_{n+2}(p = mu, \ell_1 \ldots \ell_n)$

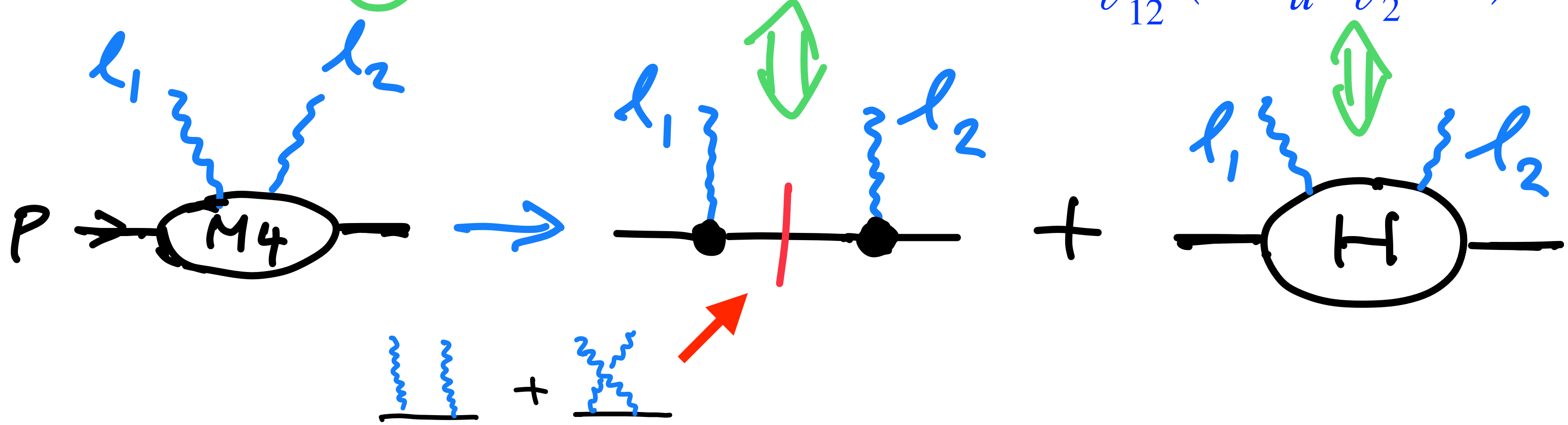
- Expand in terms of HEFT amplitudes and delta functions!

3 points: $M_3 \to \bar{m}^2(\bar{u}\cdot\varepsilon_1)^2$

HEFT Amplitudes

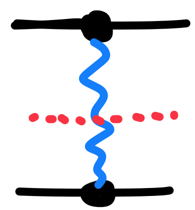


4 points: $M_4 \to \bar{m}^3(-i\pi)\delta(\bar{u}\cdot\ell_1)(\bar{u}\cdot\varepsilon_1)^2(\bar{u}\cdot\varepsilon_2)^2 + \frac{\bar{m}^2}{\ell_{12}^2}\left(\frac{\bar{u}\cdot F_1\cdot F_2\cdot\bar{u}}{\bar{u}\cdot\ell_2}\right)^2 + \cdots$



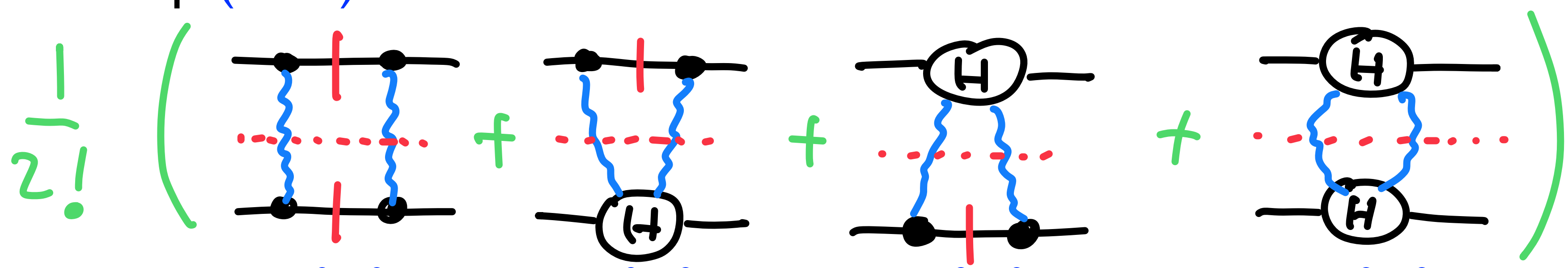
HEFT diagram expansion

Tree level (1PM):



$$= -i\frac{16\pi G m_1^2 m_2^2 (2y^2-1)}{\hbar^3 \overline{q}^2}\,, \quad y = u_1 \cdot u_2$$

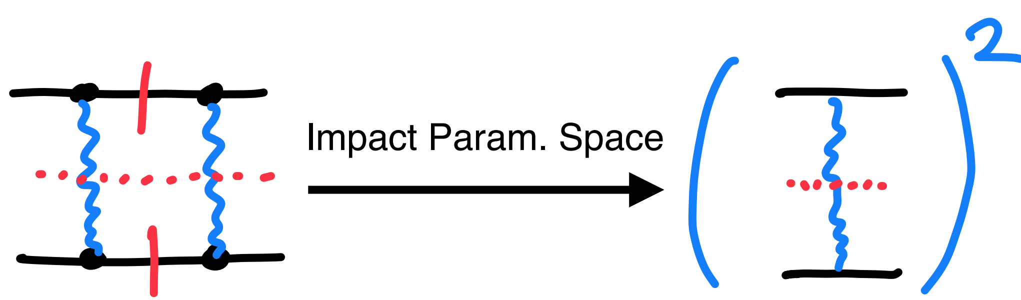
One loop (2PM):



$m_1^3 m_2^3$ — $m_1^3 m_2^2$ — $m_1^2 m_2^3$ — $m_1^2 m_2^2$

$\hbar^{-4}$(hyperclassical) — $\hbar^{-3}$ (classical) — $\hbar^{-2}$ (quantum)

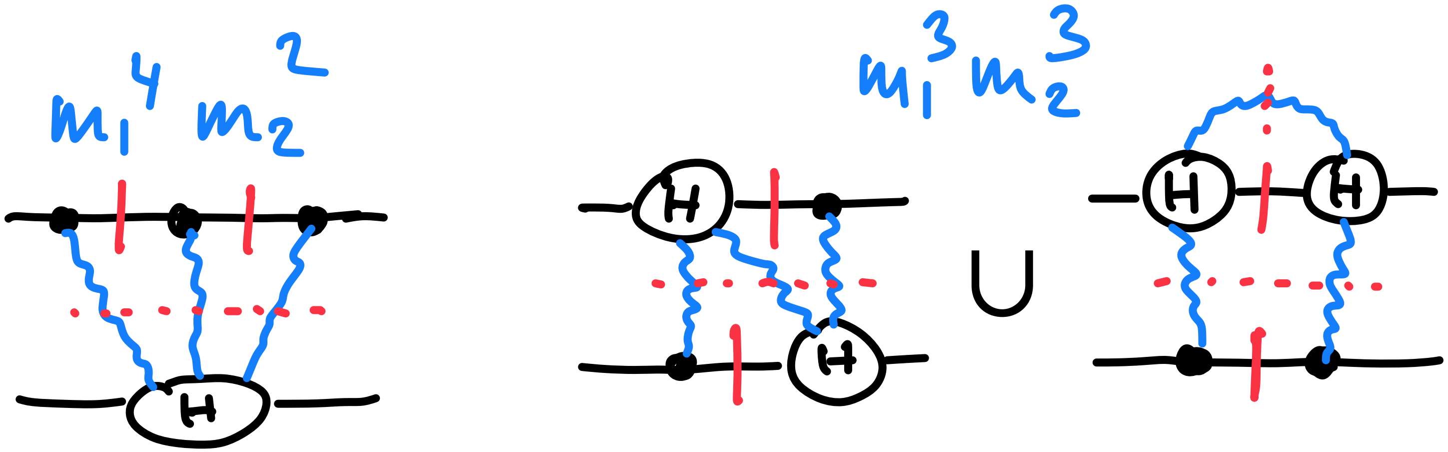
UV-divergent



$$= \left(\widetilde{M}_{1PM}(b)\right)^2$$

- 2-massive-particle-reducible diagrams factorise in IPS → Exponentiation

2 loop (3PM)



probe limit | conservative | radiation reaction

- Advantages of HEFT
  - $\hbar$-expansion done at the earliest stage
  - HEFT trees very compact, any multiplicity, double copy, Hopf-algebra

    See also Laurentiu's talk
  - 2-Mass-Particle-Irreducible diagrams!

Scattering angle up to 3PM

$$\chi = -\frac{\partial}{\partial J}\,\mathrm{Re}\,\delta_{\mathrm{HEFT}} \;\;,\;\; J = Pb\,,\;\; y = u_1\cdot u_2\,,\;\; s = (p_1+p_2)^2$$

$$\chi = \frac{G}{J}\,\frac{2m_1m_2\left(2y^2-1\right)}{\sqrt{y^2-1}} + \frac{G^2}{J^2}\,\frac{3\pi}{4\sqrt{s}}m_1^2m_2^2(m_1+m_2)\left(5y^2-1\right)$$

1PM + 2PM

$$+\frac{G^3}{J^3}\,\frac{m_1m_2\sqrt{y^2-1}}{\pi s}\Bigg\{m_1^2m_2^2(m_1^2+m_2^2)\frac{2\pi\left(64y^6-120y^4+60y^2-5\right)}{3\left(y^2-1\right)^2}$$

Probe limit

$$+m_1^3m_2^3(-8\pi)\Bigg[\frac{\left(5y^2-8\right)\left(1-2y^2\right)^2}{6\left(y^2-1\right)^{3/2}} - \frac{y\left(2y^2-3\right)\left(1-2y^2\right)^2\mathrm{arccosh}(y)}{2\left(y^2-1\right)^2}$$

Radiation reaction

$$+\frac{y\left(55-6y^2\left(6y^4-19y^2+22\right)\right)}{6\left(y^2-1\right)^2} + \frac{\left(4y^4-12y^2-3\right)\mathrm{arccosh}(y)}{\sqrt{y^2-1}}\Bigg]\Bigg\}$$

Conservative

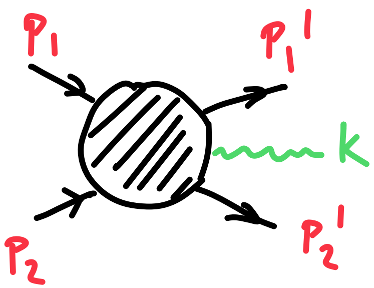
- Agrees with previous 3PM calculations:
- Conservative: Bern, Cheung, Roiban, Shen, Solon, Zeng (2019); Cheung, Solon (2020); Kaelin, Porto
- Radiation reaction: Damour…; Di Vecchia, Heissenberg, Russo, Veneziano; Bjerrum-Bohr, Damgaard, Plante, Vanhove

Waveform/Radiation

AB, Brown, Chen, De Angelis, Gowdy, Travaglini; Feng, Herderschee, Roiban; Georgoudis, Heissenberg, Russo, Vazquez-Holm, Elkhidir, O'Connell, Sergola, Bini, Damour, Geralico; Blumenbust, Ita, Kraus, Schlenk

- Key statement: extract waveforms and other observables like radiated energy (energy loss) from 5-point amplitudes

See also Gab's talk



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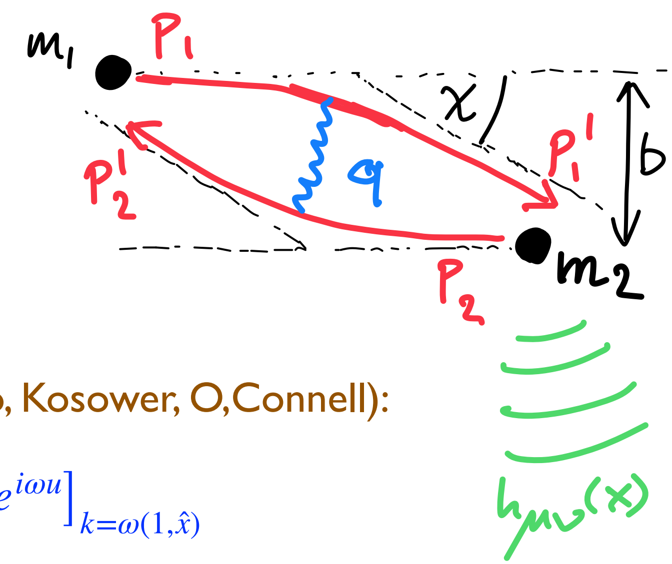
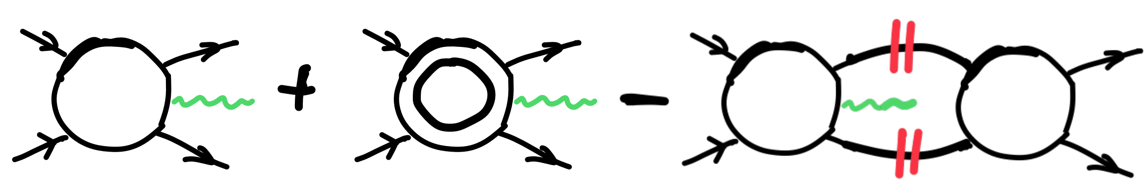


- Time-domain waveform (Cristofoli, Gonzo, Kosower, O,Connell):

$$h(x) = -\frac{\kappa}{8\pi r}\int_0^\infty \frac{d\omega}{2\pi}\left[W(b,k^-)e^{-i\omega u} + W(b,k^+)^\star e^{i\omega u}\right]_{k=\omega(1,\hat{x})}$$

$$W(b,k^h) = -i\int d\mu\, e^{iq_1\cdot b_1 + iq_2\cdot b_2} R\ ,\ d\mu = \frac{d^4q_1 d^4q_2}{(2\pi)^2}\delta^{(4)}(q_1+q_2-k)\delta(2p_1\cdot q_1)\delta(2p_2\cdot q_2)$$

Up to 1-loop order, $R =$

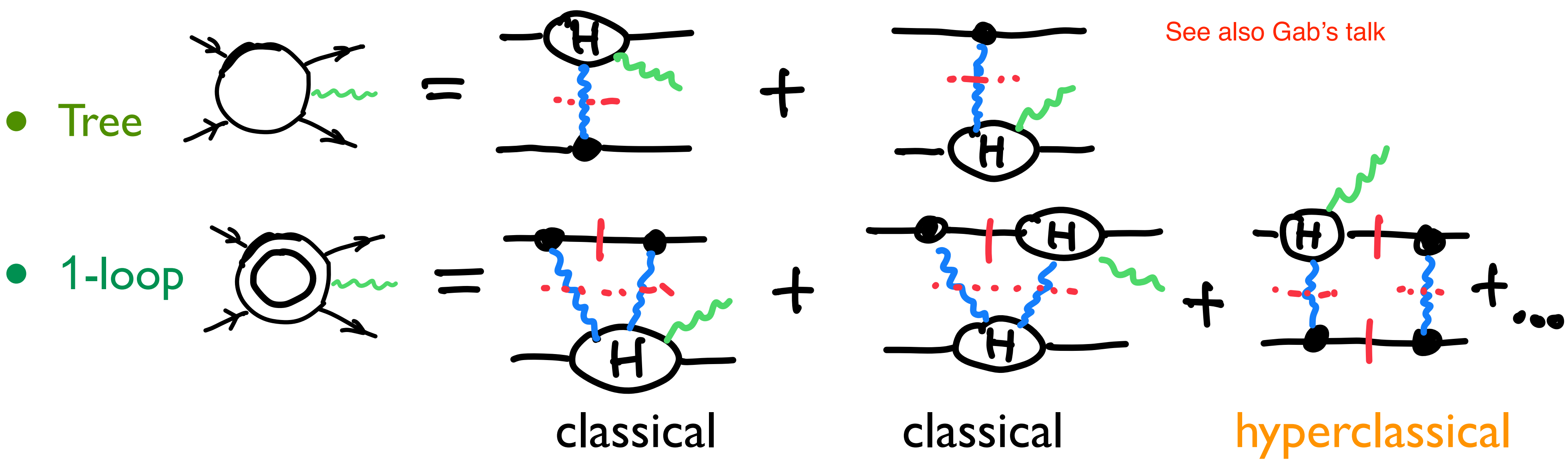


tree-level 1-loop KMOC "cut-term"

Radiation from Amplitudes

- Tree and 1-loop amplitudes using HEFT amplitudes

See also Gab's talk



- KMOC "cut-term" cancels hyperclassical diagram and gives a classical piece
- One-loop IR divergence agrees with Weinberg $\sim i\frac{G}{\epsilon}(p_1 \cdot k + p_2 \cdot k)M_5^{tree}$

  IR divergence from KMOC cut-term $\sim i\frac{G}{\epsilon}(p_1 \cdot k + p_2 \cdot k)\frac{y(y^2 - \frac{3}{2})}{(y^2-1)^{3/2}}M_5^{tree}$

- In time-domain absorb IR-divs in shift of retarded time (Shapiro time-delay)

Conclusions

- HEFT is an efficient tool to compute classical observables in GR
  - Scattering angle and waveform up to 3PM
  - All multiplicity, manifestly gauge invariant HEFT amplitudes
  - Quasi-shuffle Hopf kinematic algebra in HEFT and YM theory
    (see talk of Laurentiu)
- Some to-do's:
  - Analytic Waveforms at 1 loop
  - Higher PM orders
  - Spin, tidal effects, higher derivate interactions…