

Massive twistor worldline in electromagnetic fields

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Mainly based on: Joon-Hwi Kim, JWK, Sangmin Lee [2405.17056]

Others: Joon-Hwi Kim, JWK, Sangmin Lee [2102.0706]
Gang Chen, JWK, Tianheng Wang [2406.17658]

Massive twistor worldline on EM fields

[Kim, **JWK**, Lee]

- Toy model to study **spin resummation** in binary dynamics
 - Advantages: unitary (no unphysical DOF) & minimal constraints (**no SSC**)
 - EM coupling by (dynamical) Newman-Janis shift [Arkani-Hamed, Huang, Huang; Guevara, Ochirov, Vines; Chung, Huang, **JWK**, Lee; Arkani-Hamed, Huang, O'Connell]

$$S = \int \left[\lambda_{\alpha}^I \dot{\bar{\mu}}_I^{\alpha} + \bar{\lambda}_{I\dot{\alpha}} \dot{\mu}^{\dot{\alpha}I} - \kappa^0 \phi_0 - \kappa^1 \phi_1 \right] d\sigma + q \int \left[A_{\mu}^{+}(z) \dot{z}^{\mu} + A_{\mu}^{-}(\bar{z}) \dot{\bar{z}}^{\mu} \right] d\sigma$$

$$p^{\mu} = \frac{1}{2} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \lambda_{\alpha}^I \bar{\lambda}_{I\dot{\alpha}}$$

$$\Lambda_{(IJ)}^{\mu} = \frac{\bar{\sigma}^{\mu\dot{\alpha}\alpha} \lambda_{\alpha(I} \bar{\lambda}_{J)\dot{\alpha}}}{\sqrt{2}m}$$

$$\mu^{\dot{\alpha}I} = \frac{1}{2} z^{\dot{\alpha}\beta} \lambda_{\beta}^I$$

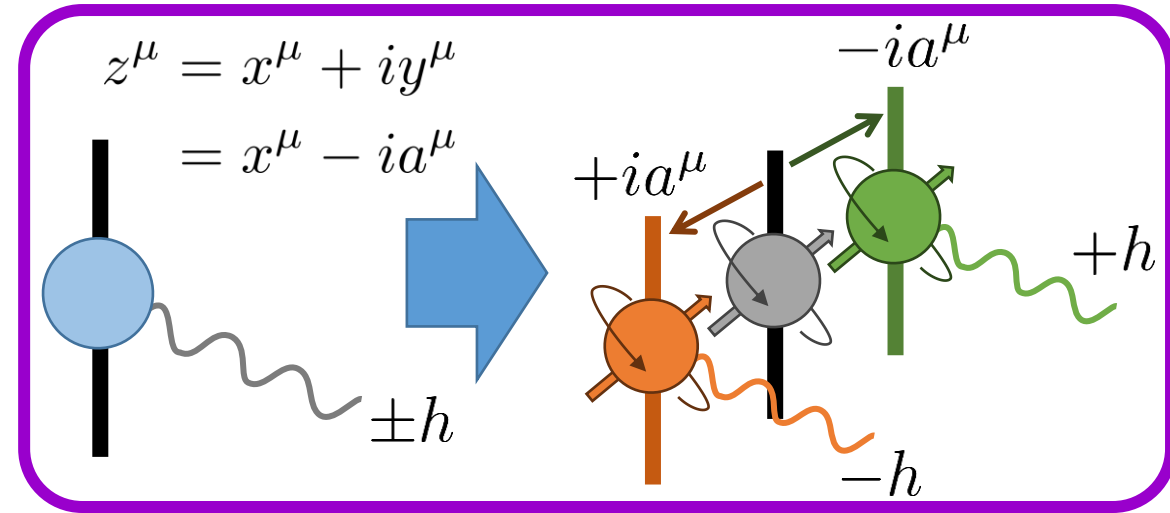
$$\bar{\mu}_I^{\alpha} = \frac{1}{2} \bar{\lambda}_{I\dot{\beta}} \bar{z}^{\dot{\beta}\alpha}$$

$$\phi_0 = \frac{1}{2} (m^2 - \Delta \bar{\Delta}) = \frac{1}{2} (p^2 + m^2)$$

$$\phi_1 = \frac{1}{2i} (\bar{\lambda}_{I\dot{\alpha}} \mu^{\dot{\alpha}I} - \bar{\mu}_I^{\alpha} \lambda_{\alpha}^I) = p \cdot y$$

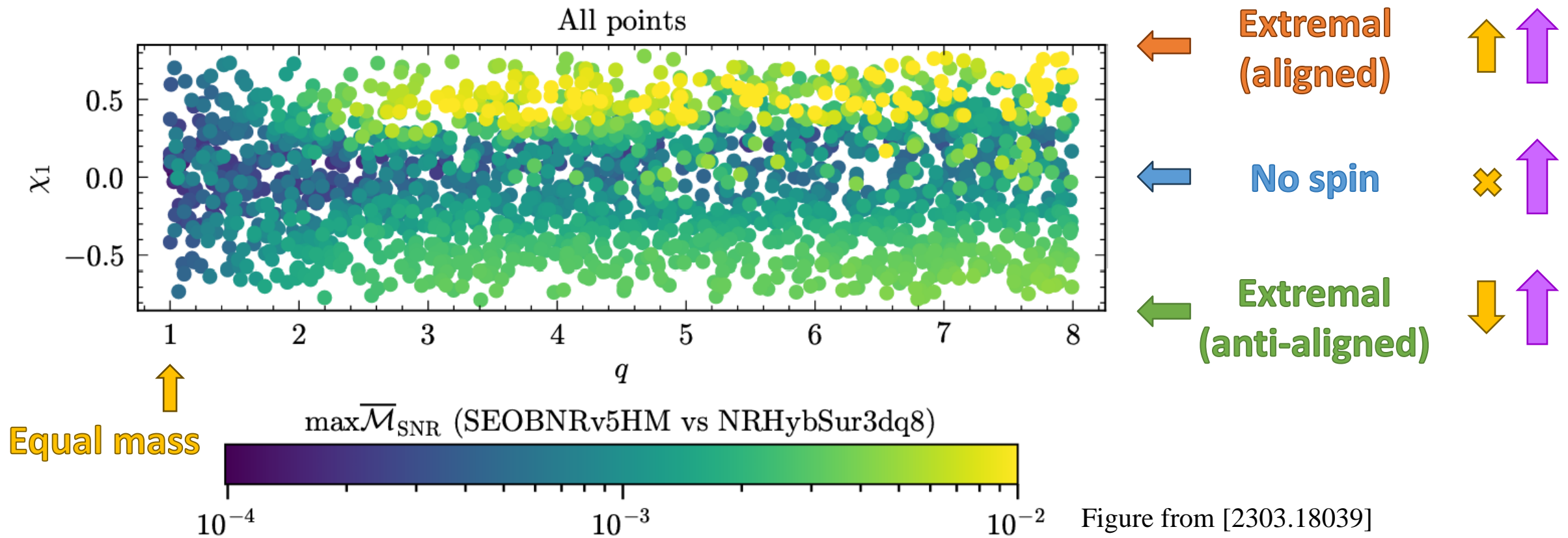
$$\Delta = \det(\lambda) = -\frac{1}{2} \epsilon^{\alpha\beta} \epsilon_{IJ} \lambda_{\alpha}^I \lambda_{\beta}^J$$

$$\bar{\Delta} = \det(\bar{\lambda}) = \frac{1}{2} \epsilon^{IJ} \epsilon^{\dot{\alpha}\dot{\beta}} \bar{\lambda}_{I\dot{\alpha}} \bar{\lambda}_{J\dot{\beta}}$$



Why study spin resummation?

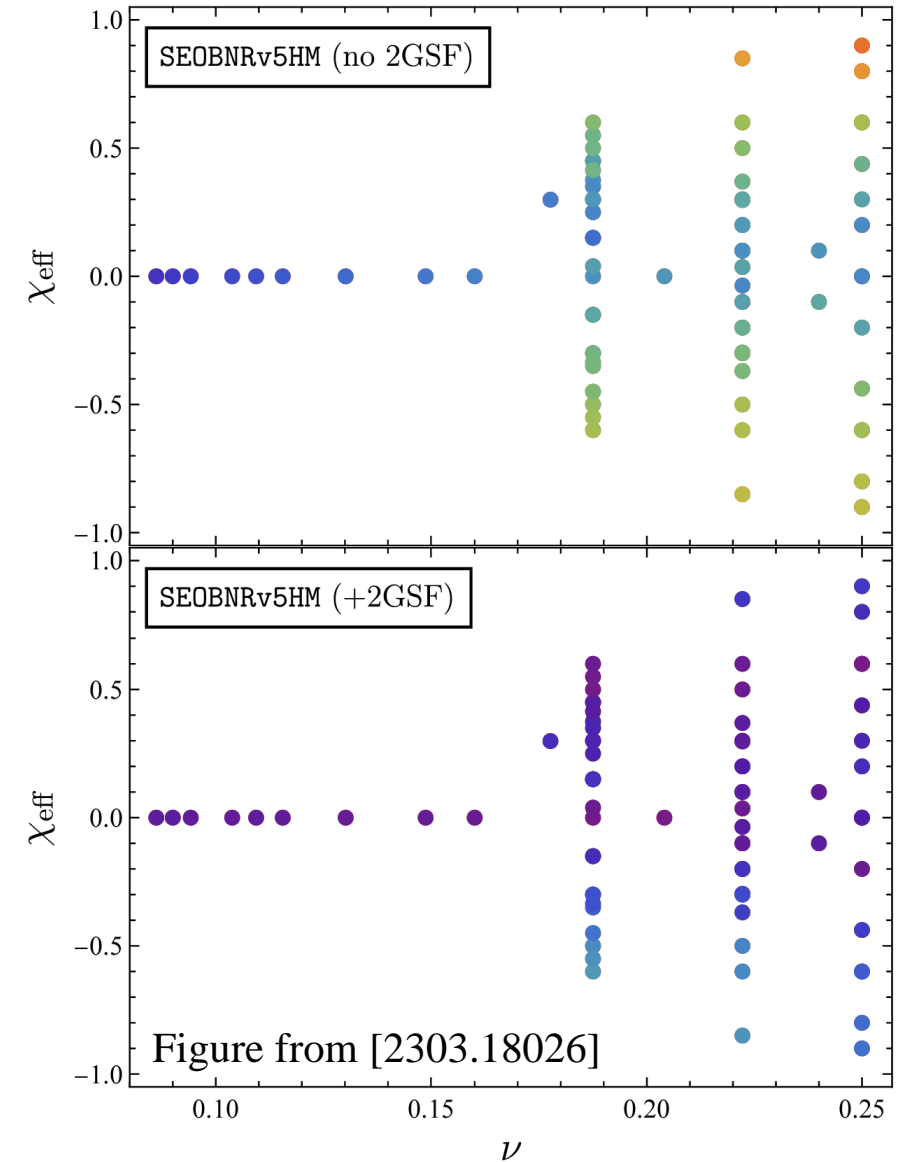
- Waveform models perform worse for high spins/mass ratios
 - Comparison with numerical relativity (NR) simulations



Why study spin resummation?

- Waveform models perform worse for high spins/mass ratios
- Limiting factor for (next-gen) GW physics
 - “we ascertain that current waveforms can accurately recover the distribution of masses in the LVK astrophysical population, **but not spins**”

[Dhani, Völkel, Buonanno, Estelles, Gair, Pfeiffer, Pompili, Toubiana]



Spin resummation in electrodynamics

[Kim, **JWK**, Lee]

- Spin-resummed 1PL (post-Lorentzian) eikonal $\chi_{(1)}$

- Singularity structure similar to Kerr

[Vines, Steinhoff; Vines; Guevara, Ochirov, Vines; Chung, Huang, **JWK**, Lee]

$$\chi_{(1)} = \frac{q_1 q_2 \gamma}{4\pi \sqrt{\gamma^2 - 1}} \left[\frac{1}{\epsilon} + \Re \left(\log \frac{(b^\mu - i a_\perp^\mu)^2}{b_0^2} \right) \right. \\ \left. + \frac{\epsilon [b, v_1, v_2, a_\perp]}{2\gamma \sqrt{b^2 a_\perp^2 - (b \cdot a_\perp)^2}} \log \left(\frac{b^2 + a_\perp^2 + 2\sqrt{b^2 a_\perp^2 - (b \cdot a_\perp)^2}}{b^2 + a_\perp^2 - 2\sqrt{b^2 a_\perp^2 - (b \cdot a_\perp)^2}} \right) \right]$$

$$a^\mu = a_1^\mu + a_2^\mu, \quad a_1^\mu = \frac{S_1^\mu}{m_1}$$

- Eikonal = Scattering generator [Kim, **JWK**, Lee; Gonzo, Shi]

- Iterated brackets = Causality cuts [Kim, **JWK**, Lee; **JWK**, Kim, Lee (WIP)]

$$\mathcal{O}_f = e^{\{\chi, \bullet\}} [\mathcal{O}] = \mathcal{O} + \{\chi, \mathcal{O}\} + \frac{1}{2!} \{\chi, \{\chi, \mathcal{O}\}\} + \frac{1}{3!} \{\chi, \{\chi, \{\chi, \mathcal{O}\}\}\} + \dots$$

Scattering generator and causality cuts

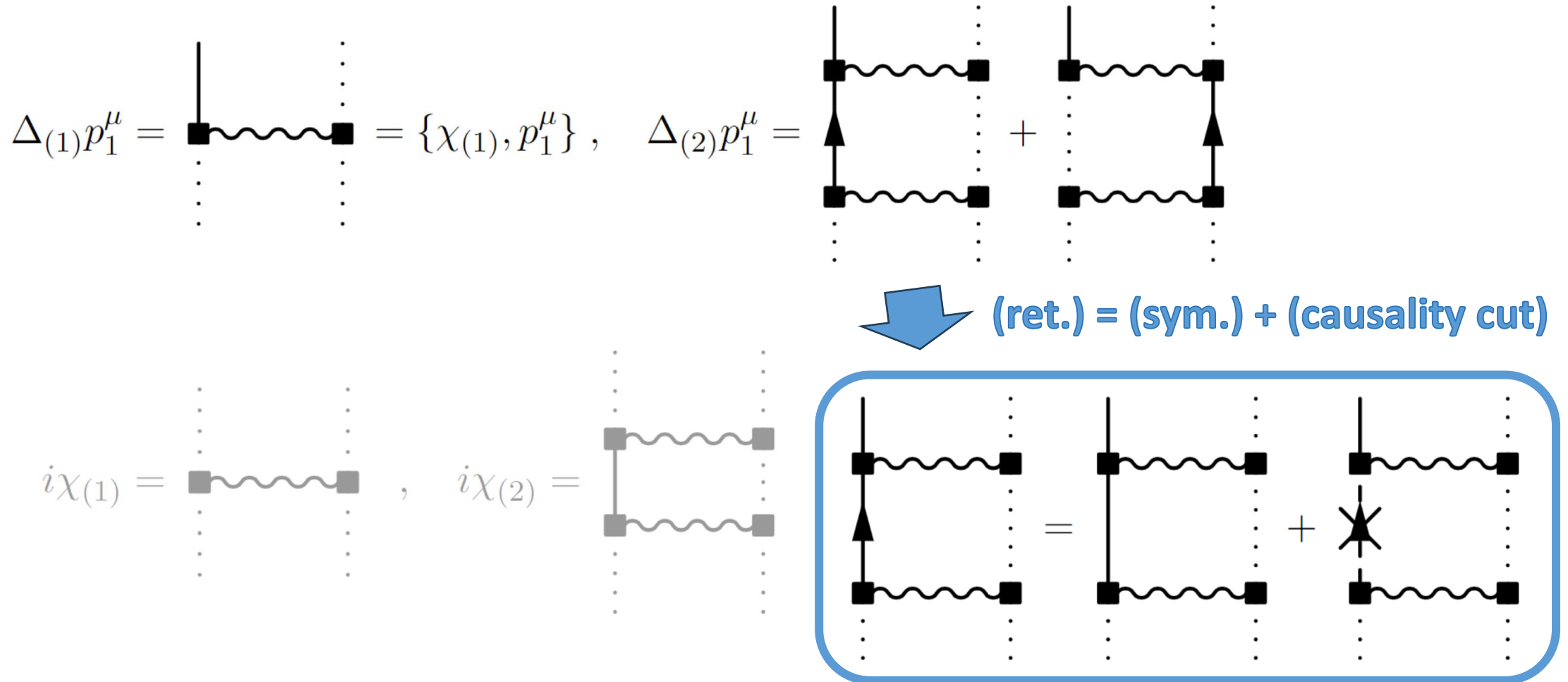
[Kim, **JWK**, Lee; **JWK**, Kim, Lee (WIP)]

$$\Delta_{(1)} p_1^\mu = \begin{array}{c} | \\ \blacksquare \\ \vdots \end{array} \text{---} \text{wavy} \text{---} \begin{array}{c} \vdots \\ \blacksquare \\ \vdots \end{array} = \{\chi_{(1)}, p_1^\mu\}, \quad \Delta_{(2)} p_1^\mu = \begin{array}{c} | \\ \blacksquare \\ \vdots \end{array} \text{---} \text{wavy} \text{---} \begin{array}{c} \vdots \\ \blacksquare \\ \vdots \end{array} + \begin{array}{c} | \\ \blacktriangle \\ \vdots \end{array} \text{---} \text{wavy} \text{---} \begin{array}{c} \vdots \\ \blacksquare \\ \vdots \end{array}$$

$$i\chi_{(1)} = \begin{array}{c} \vdots \\ \blacksquare \\ \vdots \end{array} \text{---} \text{wavy} \text{---} \begin{array}{c} \vdots \\ \blacksquare \\ \vdots \end{array}, \quad i\chi_{(2)} = \begin{array}{c} \vdots \\ \blacksquare \\ \vdots \end{array} \text{---} \text{wavy} \text{---} \begin{array}{c} \vdots \\ \blacksquare \\ \vdots \end{array} + (1 \leftrightarrow 2)$$

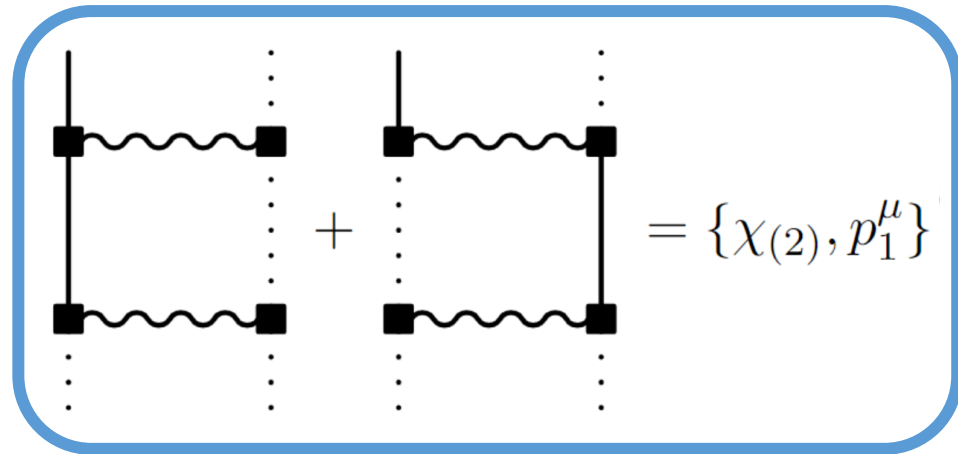
Scattering generator and causality cuts

[Kim, **JWK**, Lee; **JWK**, Kim, Lee (WIP)]

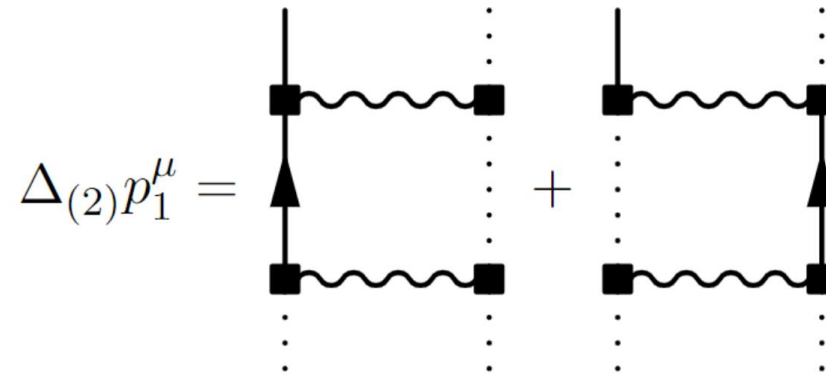


Scattering generator and causality cuts

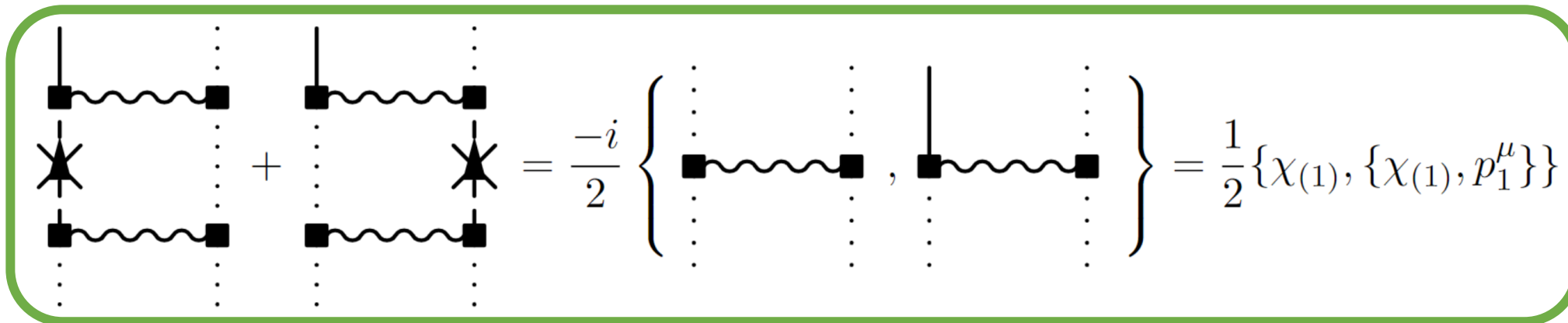
[Kim, **JWK**, Lee; **JWK**, Kim, Lee (WIP)]



(sym.) = (eikonal)

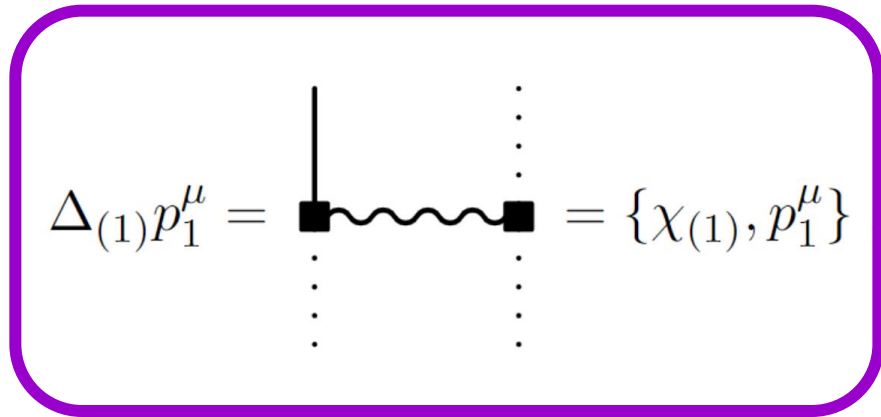


(c. cut) = (nested P.B.)

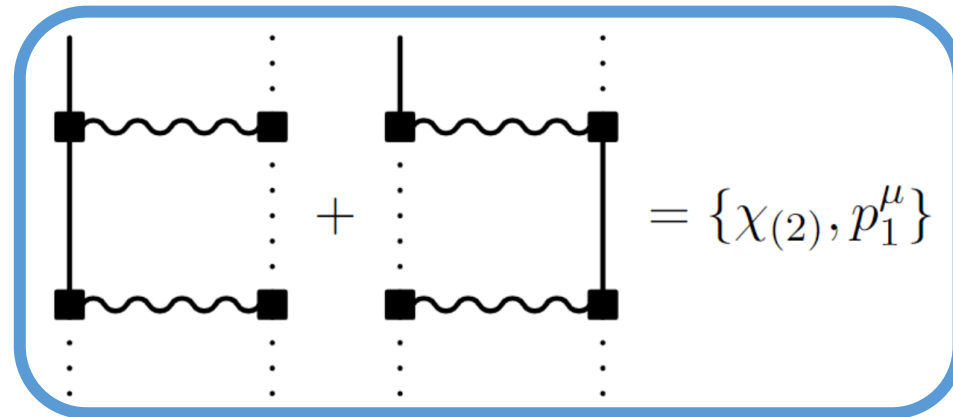


Scattering generator and causality cuts

[Kim, **JWK**, Lee; **JWK**, Kim, Lee (WIP)]



(1PL eikonal)

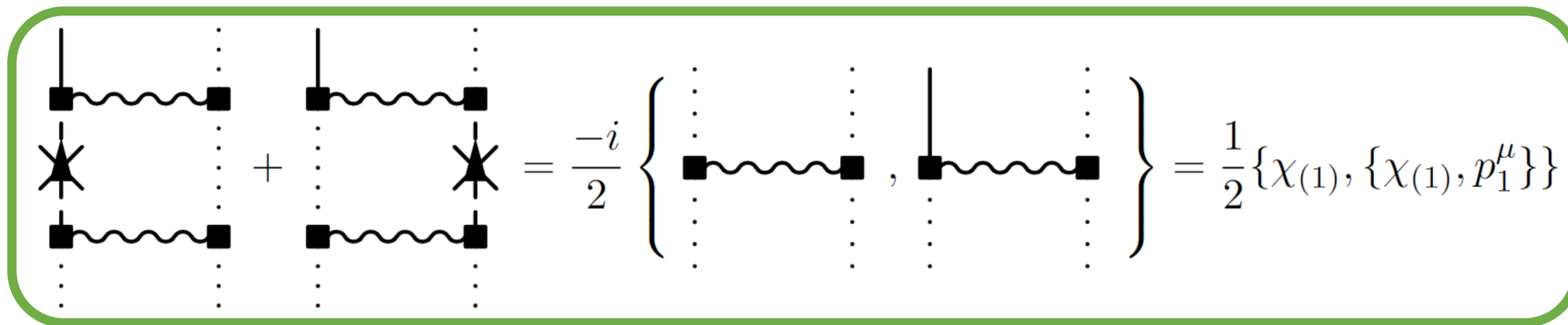


(2PL eikonal)

$$\mathcal{O}_f = \mathcal{O} + \{\chi, \mathcal{O}\} + \frac{1}{2!} \{\chi, \{\chi, \mathcal{O}\}\} + \dots$$

(c. cut) = (iterated 1PL eikonal)

= (long. imp.)



Spin resummation in electrodynamics

[Kim, **JWK**, Lee]

- Spin-resummed 2PL twistor worldline eikonal
 - Key master integrals

$$\int \frac{\mu^{2\epsilon} d^D \ell_E e^{2\ell_E \cdot a} \leftarrow}{(\ell_E^2)^{\lambda_1} [(k - \ell_E)^2]^{\lambda_2} (2v \cdot \ell_E - i0^+)^{\lambda_3}} = \frac{i^{\lambda_3} \pi^{D/2} \mu^{2\epsilon}}{2\Gamma(\lambda_1)\Gamma(\lambda_2)\Gamma(\lambda_3)(k^2)^{\lambda_1+\lambda_2+\frac{\lambda_3}{2}-\frac{D}{2}}(v^2)^{\frac{\lambda_3}{2}}}$$

$$\times \sum_{l,m,n=0}^{\infty} \frac{(2k \cdot a)^l \left(-2i(v \cdot a)\sqrt{\frac{k^2}{v^2}}\right)^m (k^2 a^2)^n}{l!m!n!} \frac{\Gamma(\lambda_1 + \lambda_2 + \frac{\lambda_3}{2} - \frac{m}{2} - n - \frac{D}{2})\Gamma(\frac{\lambda_3+m}{2})}{\Gamma(D - \lambda_1 - \lambda_2 - \lambda_3 + l + m + 2n)}$$

$$\times \Gamma\left(\frac{D}{2} - \lambda_2 - \frac{\lambda_3}{2} + \frac{m}{2} + n\right) \Gamma\left(\frac{D}{2} - \lambda_1 - \frac{\lambda_3}{2} + l + \frac{m}{2} + n\right)$$

$$\int \frac{d^D k_E}{(2\pi)^D} \frac{(k_E \cdot a)^l e^{i(k_E \cdot b)}}{[k_E^2]^\lambda} = \frac{l!\Gamma(\frac{D}{2} - \lambda)}{2^{2\lambda} \pi^{\frac{D}{2}} \Gamma(\lambda)} \frac{i^l C_l^{(\frac{D}{2}-\lambda)} \left(\frac{(a \cdot b)}{(a^2 b^2)^{1/2}}\right) (a^2)^{l/2}}{(b^2)^{\frac{D}{2} - \lambda + \frac{l}{2}}}$$

Spin resummation in electrodynamics

[Kim, **JWK**, Lee]

- Spin-resummed 2PL twistor worldline eikonal
 - Eikonal is a double infinite sum of 2F1, but simplifies in special kinematics

$$\begin{aligned}
 & \left(\pi^{3/2} \text{dot}[b, b]^{\frac{5}{2}-1-m} (-\text{dot}[y, y] - \text{dot}[y1, v2]^2)^m \left(-(1+2l) \text{dot}[b, ymb] \text{dot}[b, ypb] + 2(1+2m) \text{dot}[b, b] (\text{dot}[y1, v2]^2 + \text{dot}[y1, y1] + \text{dot}[y1, y2]) + (1+2l) \text{dot}[b, b] \text{dot}[ypb, ymb] \right) \right. \\
 & \quad \left. (-\text{dot}[ypb, ypb])^1 \text{Gamma}\left[\frac{1}{2}+1\right] \text{Gamma}\left[\frac{1}{2}+m\right] \text{GegenbauerC}\left[2l, \frac{3}{2}+m, \frac{\text{dot}[b, ypb]}{\sqrt{\text{dot}[b, b] \text{dot}[ypb, ypb]}}\right] \right) / \left(2 \text{Gamma}\left[-\frac{1}{2}-m\right] \text{Gamma}[1+m] \text{Gamma}[2+1+m] \right) + \\
 & \left((-1)^1 \pi^{3/2} \text{dot}[b, b]^{-2-1-m} \text{dot}[b, ymb] (-\text{dot}[y, y] - \text{dot}[y1, v2]^2)^m \text{dot}[ypb, ypb]^{\frac{1}{2}+1} \text{Gamma}\left[\frac{3}{2}+1\right] \text{Gamma}\left[\frac{1}{2}+m\right] \text{GegenbauerC}\left[1+2l, \frac{1}{2}+m, \frac{\text{dot}[b, ypb]}{\sqrt{\text{dot}[b, b] \text{dot}[ypb, ypb]}}\right] \right) / \\
 & \left(\text{Gamma}\left[\frac{1}{2}-m\right] \text{Gamma}[1+m] \text{Gamma}[1+1+m] \right) + \left(\pi^{3/2} \text{dot}[b, b]^{\frac{1}{2}-1-m} (-\text{dot}[y, y] - \text{dot}[y1, v2]^2)^m (-\text{dot}[ypb, ypb])^1 \right. \\
 & \quad \left. \text{Gamma}\left[\frac{1}{2}+1\right] \text{Gamma}\left[\frac{1}{2}+m\right] \text{GegenbauerC}\left[2l, \frac{1}{2}+m, \frac{\text{dot}[b, ypb]}{\sqrt{\text{dot}[b, b] \text{dot}[ypb, ypb]}}\right] \left((-1+\text{gamma}^2) (\text{gamma} \text{dot}[y1, v2] - \text{dot}[y2, v1]) + \right. \right. \\
 & \quad \left. \left. 2(1+m) \left(2 \text{gamma} (-1+\text{gamma}^2) \text{dot}[y1, v2] + (\text{dot}[y2, v1] + \text{gamma} (\text{dot}[y1, v2] - 2 \text{gamma} \text{dot}[y2, v1])) \text{Hypergeometric2F1}\left[1, -m, \frac{1}{2}, \frac{(-\text{gamma} \text{dot}[y1, v2] + \text{dot}[y2, v1])^2}{(-1+\text{gamma}^2) (\text{dot}[y, y] + \text{dot}[y1, v2]^2)}\right] \right) \right) \right) / \\
 & \left((-1+\text{gamma}^2) (\text{gamma} \text{dot}[y1, v2] - \text{dot}[y2, v1]) \text{Gamma}\left[\frac{1}{2}-m\right] \text{Gamma}[1+m] \text{Gamma}[1+1+m] \right) + \left(\pi^2 \text{dot}[b, b]^{\frac{3}{2}-1-m} (-\text{dot}[y, y] - \text{dot}[y1, v2]^2)^m (-\text{dot}[ypb, ypb])^1 \right. \\
 & \quad \left. \text{dot}[ypb, ypb] \text{Gamma}\left[\frac{3}{2}+1+m\right] \text{Hypergeometric2F1Regularized}\left[-2(1+l), 3+2l+2m, 1+m, \frac{1}{2} - \frac{\text{dot}[b, ypb]}{2\sqrt{\text{dot}[b, b] \text{dot}[ypb, ypb]}}\right] \right) / \left(\text{Gamma}[1+l] \text{Gamma}\left[\frac{1}{2}-m\right] \text{Gamma}[1+m] \right)
 \end{aligned}$$

Spin resummation in electrodynamics

[Kim, **JWK**, Lee]

- Spin-resummed 2PL twistor worldline eikonal
 - Eikonal is a double infinite sum of 2F1, but simplifies in special kinematics

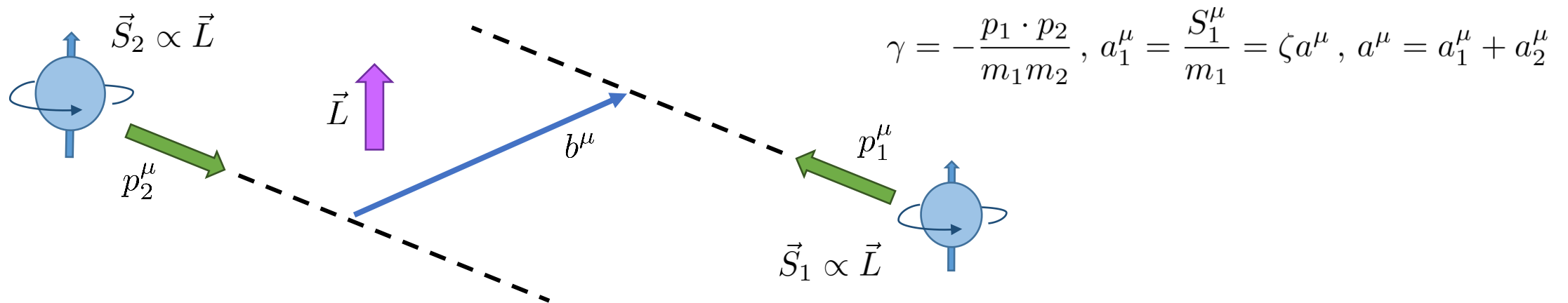
$$\begin{aligned}
 & \left(\pi^{3/2} \dot{[b, b]}^{\frac{5}{2}-1-m} (-\dot{[y, y]} \right. \\
 & \quad \left. (-\dot{[y, y]} - \dot{[y_1, v_2]^2})^{-1-n} \dot{[y, y]}^{-\frac{1}{2}-1} e^{[b, v_1, y_1, y_2]} \Gamma\left[\frac{1}{2} + 1\right] \Gamma\left[\frac{1}{2} + m\right] \right. \\
 & \quad \left. \left((1+21+2m) \dot{[b, b]} \sqrt{\dot{[y, y]}} \text{GegenbauerC}\left[21, \frac{1}{2} + m, \frac{\dot{[b, y, ypb]}}{\sqrt{\dot{[b, b]} \dot{[y, y]}}}\right] + (1+2m) \sqrt{\dot{[b, b]}} \dot{[b, y, ypb]} \text{GegenbauerC}\left[-1+21, \frac{3}{2} + m, \frac{\dot{[b, y, ypb]}}{\sqrt{\dot{[b, b]} \dot{[y, y]}}}\right] \right) \\
 & \quad \left(\dot{[y, y]} - \text{gamma}^2 \dot{[y, y]} + \dot{[y_1, v_2]^2} - 2 \text{gamma} \dot{[y_1, v_2]} \dot{[y_2, v_1]} + \dot{[y_2, v_1]^2} \right) \text{Hypergeometric2F1}\left[1, 1-m, -\frac{1}{2}, \frac{(-\text{gamma} \dot{[y_1, v_2]} + \dot{[y_2, v_1]^2})}{(-1+\text{gamma}^2) (\dot{[y, y]} + \dot{[y_1, v_2]^2})}\right] + \\
 & \quad \left((-1+\text{gamma}^2) \dot{[y, y]} + (-1+\text{gamma}^2 (-3+2m)) \dot{[y_1, v_2]^2} - 4 \text{gamma} (-2+m) \dot{[y_1, v_2]} \dot{[y_2, v_1]} + 2 (-2+m) \dot{[y_2, v_1]^2} \right) \text{Hypergeometric2F1}\left[1, 1-m, \frac{1}{2}, \frac{(-\text{gamma} \dot{[y_1, v_2]} + \dot{[y_2, v_1]^2})}{(-1+\text{gamma}^2) (\dot{[y, y]} + \dot{[y_1, v_2]^2})}\right] \Big) / \\
 & \quad \left((-1+\text{gamma}^2) (\text{gamma} \dot{[y_1, v_2]} - \dot{[y_2, v_1]}) \Gamma\left[\frac{3}{2} - m\right] \Gamma[m] \Gamma[1+m] \right) + \left((-1)^4 4^{1-n} \text{gamma} \pi^{3/2} \dot{[b, b]}^{-1-1-n} (-\dot{[y, y]} - \dot{[y_1, v_2]^2})^n \dot{[y, y]}^{-\frac{1}{2}-1} e^{[v_1, v_2, y_1, y_2]} \right. \\
 & \quad \left. \Gamma\left[\frac{1}{2} + m\right] \Gamma\left[\frac{3}{2} + 1 + m\right] \text{Hypergeometric2F1Regularized}\left[1-21, 2(1+m), 2+m, \frac{1}{2} \left(1 - \frac{\dot{[b, y, ypb]}}{\sqrt{\dot{[b, b]} \dot{[y, y]}}}\right)\right] \right) / \\
 & \quad \left((-1+\text{gamma}^2) \Gamma[1] \Gamma\left[-\frac{1}{2} - m\right] \Gamma[2+2m] \right) + \left(2 (-1)^4 \pi^2 \dot{[b, b]}^{-2-1-n} (-\dot{[y, y]} - \dot{[y_1, v_2]^2})^n \dot{[y, y]}^{-\frac{1}{2}-1} e^{[b, v_1, v_2, y_2]} \Gamma\left[\frac{3}{2} + 1 + m\right] \right. \\
 & \quad \left. \left((-1+\text{gamma}^2) \dot{[y_1, v_2]} + (\dot{[y_1, v_2]} + \text{gamma}^2 \dot{[y_1, v_2]} - 2 \text{gamma} \dot{[y_2, v_1]}) \text{Hypergeometric2F1}\left[1, -m, \frac{1}{2}, \frac{(-\text{gamma} \dot{[y_1, v_2]} + \dot{[y_2, v_1]^2})}{(-1+\text{gamma}^2) (\dot{[y, y]} + \dot{[y_1, v_2]^2})}\right] \right) \right) / \\
 & \quad \left(1 \dot{[b, y, ypb]} \text{Hypergeometric2F1Regularized}\left[1-21, 2(1+m), 2+m, \frac{1}{2} \left(1 - \frac{\dot{[b, y, ypb]}}{\sqrt{\dot{[b, b]} \dot{[y, y]}}}\right)\right] + \sqrt{\dot{[b, b]} \dot{[y, y]}} \text{Hypergeometric2F1Regularized}\left[-21, 1+21+2m, 1+m, \frac{1}{2} \left(1 - \frac{\dot{[b, y, ypb]}}{\sqrt{\dot{[b, b]} \dot{[y, y]}}}\right)\right] \right) / \\
 & \quad \left((-1+\text{gamma}^2) (\text{gamma} \dot{[y_1, v_2]} - \dot{[y_2, v_1]}) \Gamma[1+1] \Gamma\left[\frac{1}{2} - m\right] \Gamma[1+m] \right) + \left(2 \pi^2 \dot{[b, b]}^{-2-1-n} (-\dot{[y, y]} - \dot{[y_1, v_2]^2})^n (-\dot{[y, y]} - \dot{[y_1, v_2]^2})^1 e^{[b, v_1, v_2, y_1]} \Gamma\left[\frac{3}{2} + 1 + m\right] \right. \\
 & \quad \left. \left((-1+\text{gamma}^2) (\dot{[y_1, v_2]} - \text{gamma} \dot{[y_2, v_1]}) + (\dot{[y_1, v_2]} + \text{gamma} (-2+\text{gamma}^2) \dot{[y_2, v_1]}) \text{Hypergeometric2F1}\left[1, -m, \frac{1}{2}, \frac{(-\text{gamma} \dot{[y_1, v_2]} + \dot{[y_2, v_1]^2})}{(-1+\text{gamma}^2) (\dot{[y, y]} + \dot{[y_1, v_2]^2})}\right] \right) \right) / \\
 & \quad \left(1 \dot{[b, y, ypb]} \text{Hypergeometric2F1Regularized}\left[1-21, 2(1+m), 2+m, \frac{1}{2} - \frac{\dot{[b, y, ypb]}}{2 \sqrt{\dot{[b, b]} \dot{[y, y]}}}\right] + \sqrt{\dot{[b, b]} \dot{[y, y]}} \text{Hypergeometric2F1Regularized}\left[-21, 1+21+2m, 1+m, \frac{1}{2} - \frac{\dot{[b, y, ypb]}}{2 \sqrt{\dot{[b, b]} \dot{[y, y]}}}\right] \right) / \\
 & \quad \left((-1+\text{gamma}^2) (\text{gamma} \dot{[y_1, v_2]} - \dot{[y_2, v_1]}) \sqrt{\dot{[y, y]}} \Gamma[1+1] \Gamma\left[\frac{1}{2} - m\right] \Gamma[1+m] \right)
 \end{aligned}$$

Spin resummation in electrodynamics

[Kim, **JWK**, Lee]

- Spin-resummed 2PL twistor worldline eikonal
 - Eikonal is a double infinite sum of 2F1, but simplifies in special kinematics
 - Aligned-spin configuration [Aoude, Haddad, Helset; Damgaard, Hoogeveen, Luna, Vines]

$$\chi_{(2,\text{aligned})} = \frac{(q_1 q_2)^2 \left(b^2 + \frac{(\zeta - 2)\gamma}{(\gamma^2 - 1)} \epsilon[b, v_1, v_2, a] + \frac{\gamma^2(1 - \zeta) + \zeta}{\gamma^2 - 1} a^2 \right)}{32\pi m_1 \sqrt{\gamma^2 - 1} (b^2 - a^2)^{3/2}} + (1 \leftrightarrow 2)$$

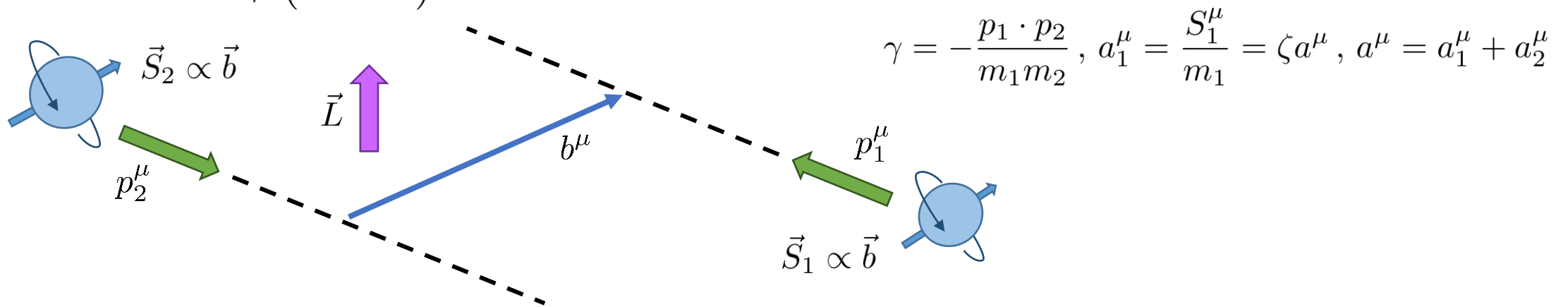


Spin resummation in electrodynamics

[Kim, **JWK**, Lee]

- Spin-resummed 2PL twistor worldline eikonal
 - Eikonal is a double infinite sum of 2F1, but simplifies in special kinematics
 - Axial scattering configuration

$$\chi_{(2,\text{axial})} = \frac{(q_1 q_2)^2 \sqrt{b^2}}{16\pi^2 m_1 (\gamma^2 - 1)^{3/2}} \left[\frac{\gamma^2 (\zeta - 1) - \zeta}{b^2} K\left(-\frac{a^2}{b^2}\right) - \frac{\gamma^2 (\zeta - 2) - (\zeta - 1)}{b^2 + a^2} E\left(-\frac{a^2}{b^2}\right) \right] + (1 \leftrightarrow 2)$$



Challenges for the future

- Spin resummation in gravity / NLO in mass-ratio
 - Coupling massive twistor worldline to gravity
 - (Full analytic) 2PM from HEFT/HPET [Chen, **JWK**, Wang (WIP)]
 - 3PL/PM: first beyond-probe-limit (NLO in mass-ratio)
- Eikonal as the scattering generator (all-order proof) [**JWK**, Kim, Lee (WIP)]
 - Including radiation / continuation to bound [Kälin, Porto; Cho, Kälin, Porto]
- Better spin-resummation schemes for waveform models
 - PM-based waveform: SEOBNR-PM [Buonanno, Mogull, Patil, Pompili]

Other future directions

- Tensor Integral Generating Functions [Feng]
 - Feynman integrals deformed by an exponential factor
 - Generally encountered in spin-resummed PM dynamics ($e^{(ia)\cdot\nabla} \Leftrightarrow e^{a\cdot k}$)
waveform calculations

$$\mathcal{I}_{\lambda_k}[\alpha_i^\mu] = \int \prod_{j=1}^L d^D \ell_j \frac{\exp(\sum_{j=1}^L \alpha_j \cdot \ell_j)}{\mathcal{D}_1^{\lambda_1} \cdots \mathcal{D}_n^{\lambda_n}}$$

$$\mathcal{D}_j = (\ell + q_j)^2 - m_j^2$$

- Provides alternative methods for tensor reduction: $\frac{\partial}{\partial \alpha_j^\mu} \Leftrightarrow \ell_j^\mu$
- Can be computed by conventional multiloop techniques [Chen, **JWK**, Wang]
- Can we reduce irreducible numerators more efficiently?

A wide-angle photograph of the Seoul National University campus. In the foreground, a large, light-colored paved plaza features a large, stylized 'SNU' logo. A road with several lanes and a few cars runs alongside the plaza. In the background, there are several large, green mountains under a clear blue sky. Some university buildings are visible on the slopes of the mountains.

Amplitudes 2025

16-20 June

Conference

Seoul National University