Massive twistor worldline in electromagnetic fields

Jung-Wook Kim [MPI for Gravitational Physics (Albert Einstein Institute)]

Mainly based on: Joon-Hwi Kim, JWK, Sangmin Lee [2405.17056]

Others: Joon-Hwi Kim, JWK, Sangmin Lee [2102.0706]
Gang Chen, JWK, Tianheng Wang [2406.17658]

Massive twistor worldline on EM fields

[Kim, JWK, Lee]

- Toy model to study **spin resummation** in binary dynamics
  - Advantages: unitary (no unphysical DOF) & minimal constraints (**no SSC**)
  - EM coupling by (dynamical) Newman-Janis shift [Arkani-Hamed, Huang, Huang; Guevara, Ochirov, Vines; Chung, Huang, JWK, Lee; Arkani-Hamed, Huang, O'Connell]

$$S = \int \left[\lambda_\alpha{}^I \dot{\bar\mu}_I{}^\alpha + \bar\lambda_{I\dot\alpha}\dot\mu^{\dot\alpha I} - \kappa^0\phi_0 - \kappa^1\phi_1\right] d\sigma + q\int \left[A^+_\mu(z)\dot z^\mu + A^-_\mu(\bar z)\dot{\bar z}^\mu\right] d\sigma$$

$$p^\mu = \frac{1}{2}\bar\sigma^{\mu\dot\alpha\alpha}\lambda_\alpha{}^I\bar\lambda_{I\dot\alpha}$$

$$\Lambda^\mu_{(IJ)} = \frac{\bar\sigma^{\mu\dot\alpha\alpha}\lambda_{\alpha(I}\bar\lambda_{J)\dot\alpha}}{\sqrt{2}m}$$

$$\mu^{\dot\alpha I} = \frac{1}{2}z^{\dot\alpha\beta}\lambda_\beta{}^I$$

$$\bar\mu_I{}^\alpha = \frac{1}{2}\bar\lambda_{I\dot\beta}\bar z^{\dot\beta\alpha}$$

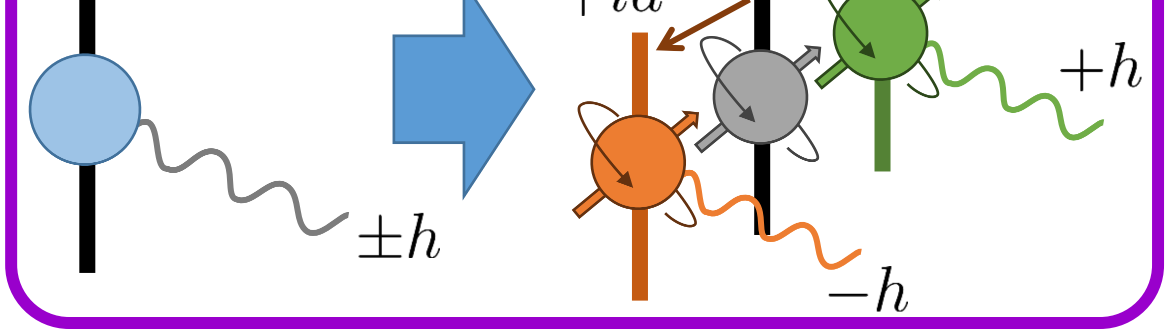
$$\phi_0 = \frac{1}{2}(m^2 - \Delta\bar\Delta) = \frac{1}{2}(p^2 + m^2)$$

$$\phi_1 = \frac{1}{2i}(\bar\lambda_{I\dot\alpha}\mu^{\dot\alpha I} - \bar\mu_I{}^\alpha\lambda_\alpha{}^I) = p\cdot y$$

$$\Delta = \det(\lambda) = -\frac{1}{2}\epsilon^{\alpha\beta}\epsilon_{IJ}\lambda_\alpha{}^I\lambda_\beta{}^J$$

$$\bar\Delta = \det(\bar\lambda) = \frac{1}{2}\epsilon^{IJ}\epsilon^{\dot\alpha\dot\beta}\bar\lambda_{I\dot\alpha}\bar\lambda_{J\dot\beta}$$

$$z^\mu = x^\mu + iy^\mu = x^\mu - ia^\mu$$



Why study spin resummation?

- Waveform models perform worse for high spins/mass ratios
  - Comparison with numerical relativity (NR) simulations

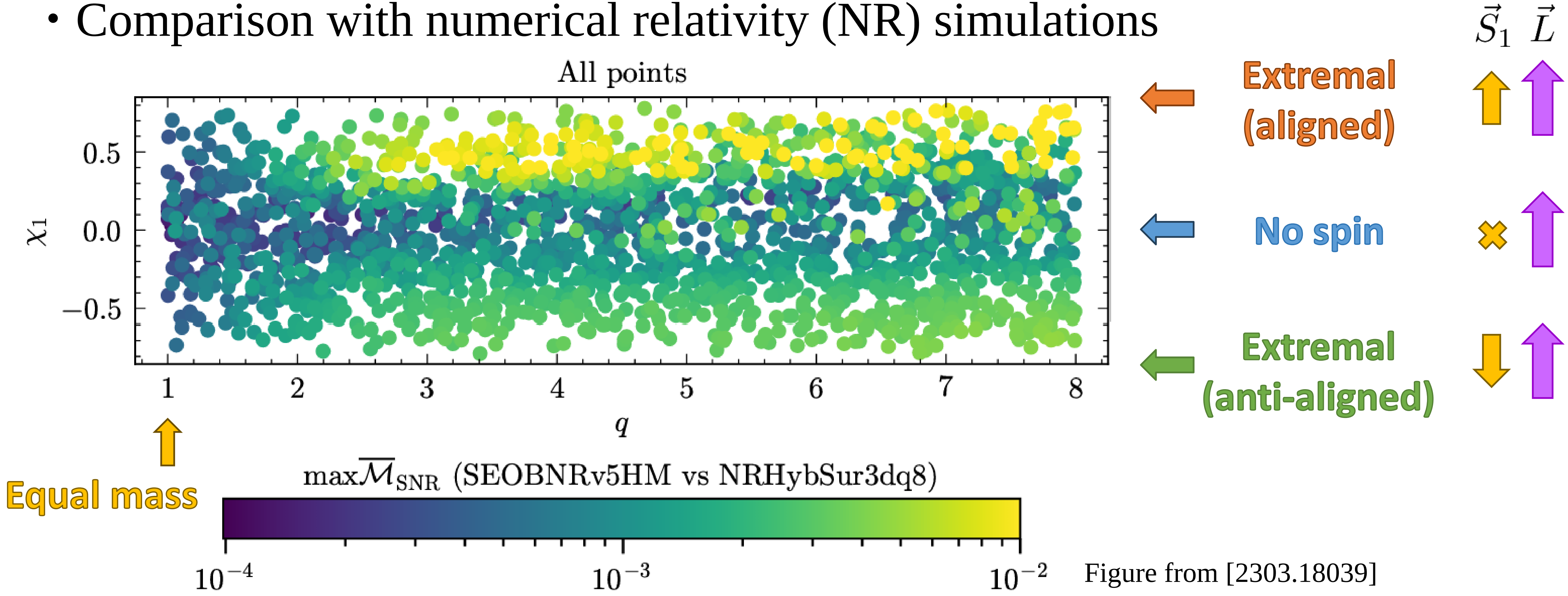


Figure from [2303.18039]

Why study spin resummation?

- Waveform models perform worse for high spins/mass ratios

- Limiting factor for (next-gen) GW physics
  - "we ascertain that current waveforms can accurately recover the distribution of masses in the LVK astrophysical population, **but not spins**"

    [Dhani, Völkel, Buonanno, Estelles, Gair, Pfeiffer, Pompili, Toubiana]

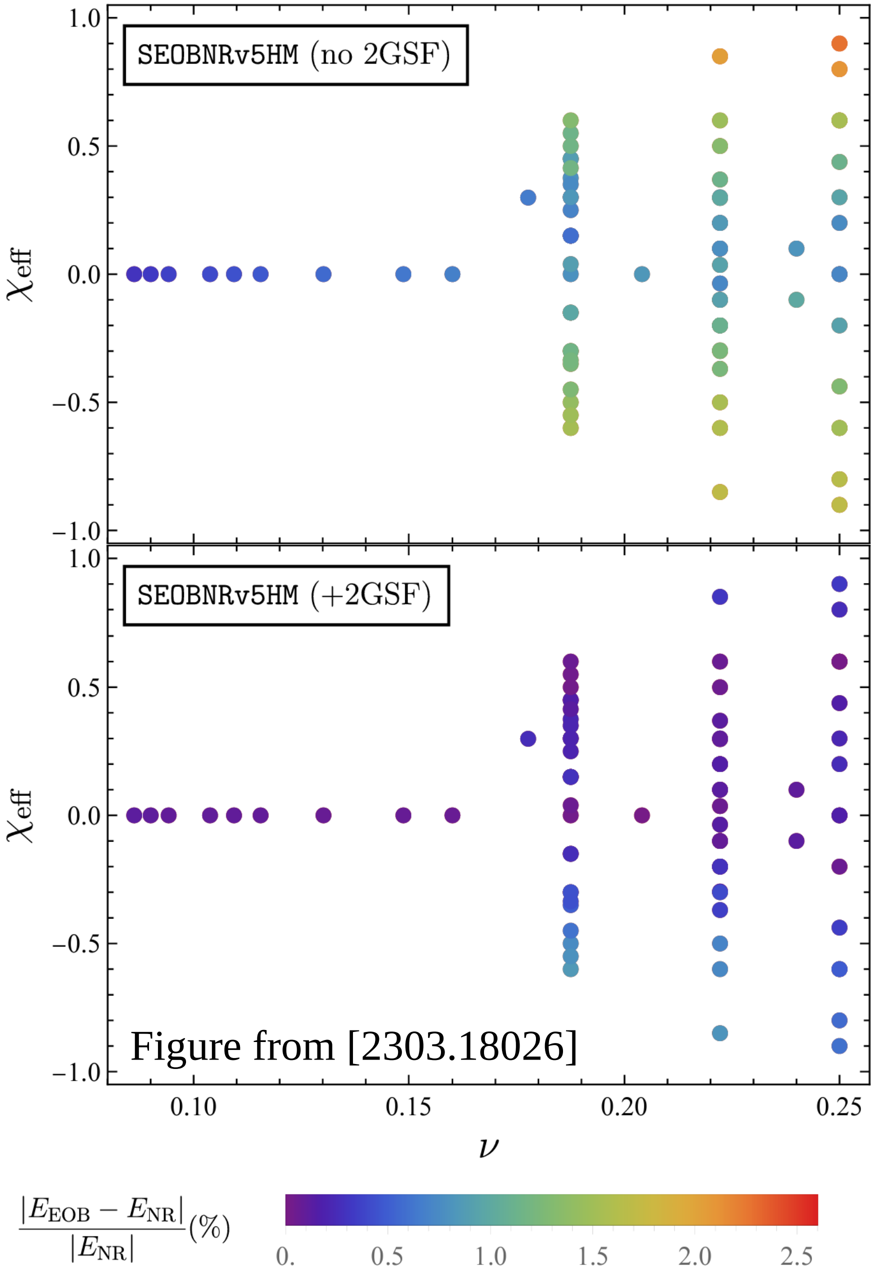


Figure from [2303.18026]

Spin resummation in electrodynamics

[Kim, JWK, Lee]

- Spin-resummed 1PL (post-Lorentzian) eikonal $\chi_{(1)}$
  - Singularity structure similar to Kerr
    [Vines, Steinhoff; Vines; Guevara, Ochirov, Vines; Chung, Huang, JWK, Lee]

$$\chi_{(1)} = \frac{q_1 q_2 \gamma}{4\pi\sqrt{\gamma^2-1}} \left[\frac{1}{\epsilon} + \Re\left(\log \frac{(b^\mu - i a_\perp^\mu)^2}{b_0^2} \right) + \frac{\epsilon[b, v_1, v_2, a_\perp]}{2\gamma\sqrt{b^2 a_\perp^2 - (b\cdot a_\perp)^2}} \log\left(\frac{b^2 + a_\perp^2 + 2\sqrt{b^2 a_\perp^2 - (b\cdot a_\perp)^2}}{b^2 + a_\perp^2 - 2\sqrt{b^2 a_\perp^2 - (b\cdot a_\perp)^2}} \right) \right]$$

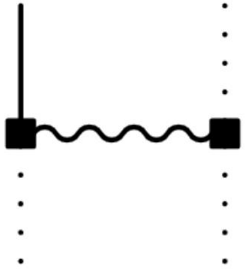
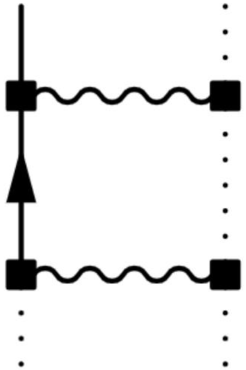
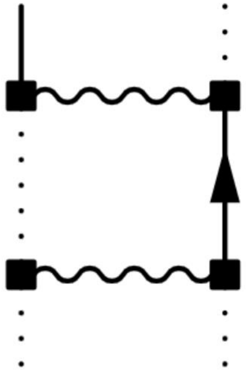
$$a^\mu = a_1^\mu + a_2^\mu \,,\; a_1^\mu = \frac{S_1^\mu}{m_1}$$

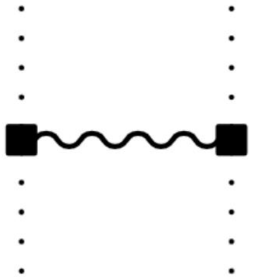
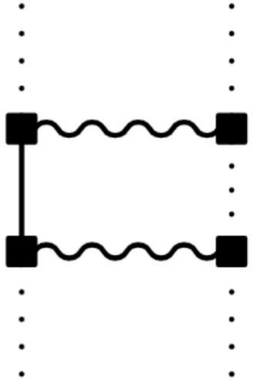
- Eikonal = Scattering generator [Kim, JWK, Lee; Gonzo, Shi]
  - Iterated brackets = *Causality cuts* [Kim, JWK, Lee; JWK, Kim, Lee (WIP)]

$$\mathcal{O}_f = e^{\{\chi,\bullet\}}[\mathcal{O}] = \mathcal{O} + \{\chi,\mathcal{O}\} + \frac{1}{2!}\{\chi,\{\chi,\mathcal{O}\}\} + \frac{1}{3!}\{\chi,\{\chi,\{\chi,\mathcal{O}\}\}\} + \cdots$$

Scattering generator and causality cuts

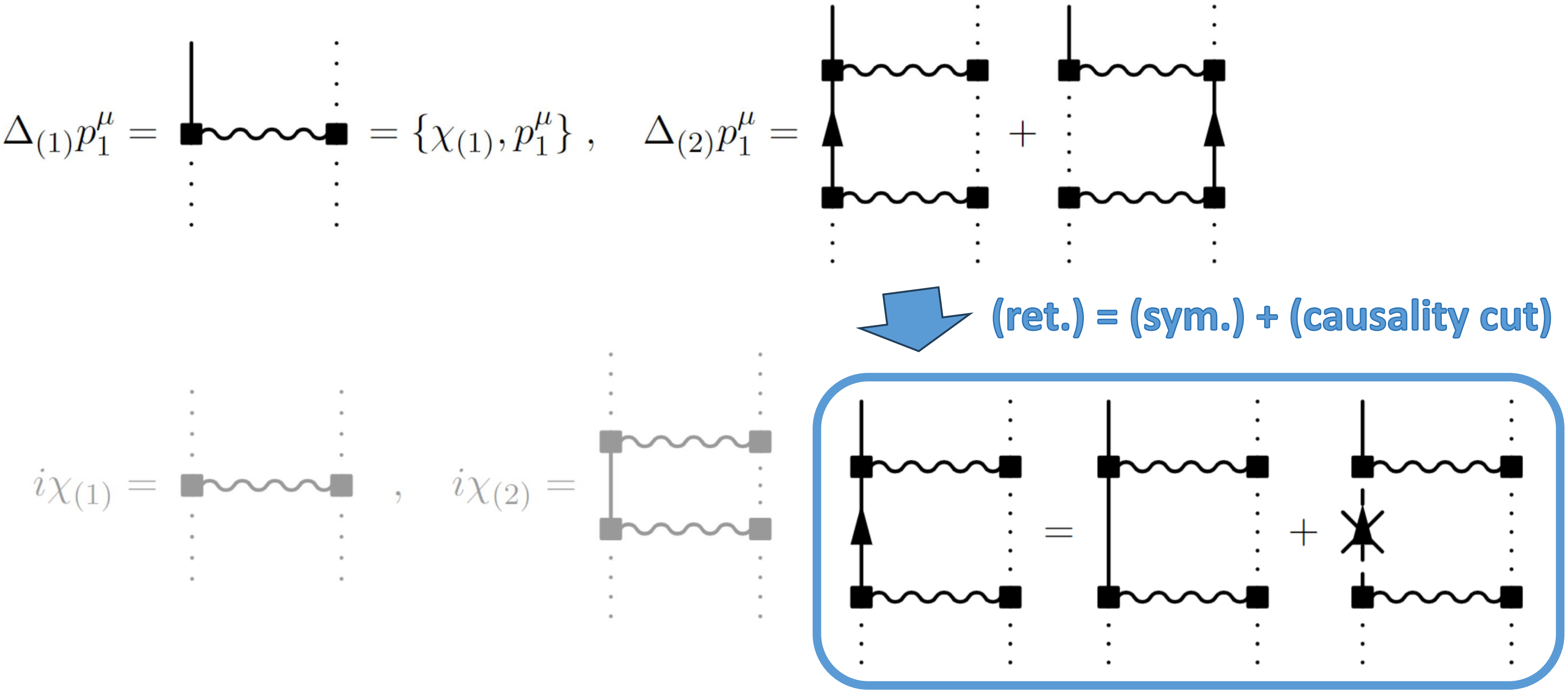
[Kim, JWK, Lee; JWK, Kim, Lee (WIP)]

$\Delta_{(1)} p_1^\mu =$  $= \{\chi_{(1)}, p_1^\mu\}$, $\Delta_{(2)} p_1^\mu =$  $+$ 

$i\chi_{(1)} =$  , $i\chi_{(2)} =$  $+ (1 \leftrightarrow 2)$

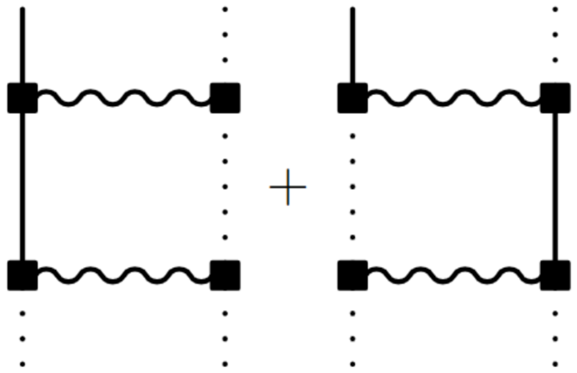
Scattering generator and causality cuts

[Kim, JWK, Lee; JWK, Kim, Lee (WIP)]



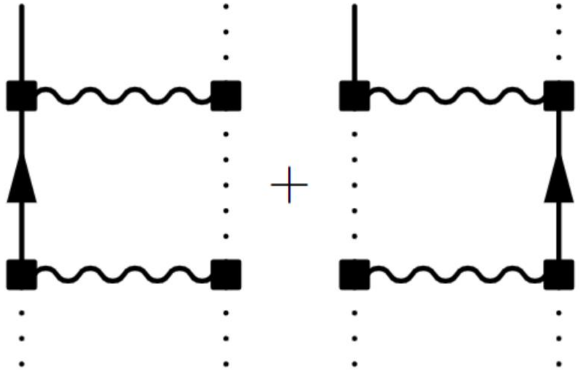
Scattering generator and causality cuts

[Kim, JWK, Lee; JWK, Kim, Lee (WIP)]



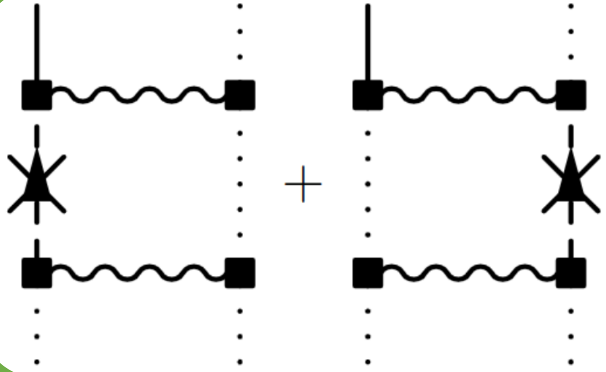
$$= \{\chi_{(2)}, p_1^\mu\}$$

$$\Delta_{(2)} p_1^\mu =$$

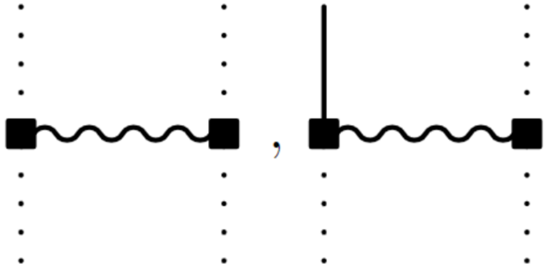


(sym.) = (eikonal)

(c. cut) = (nested P.B.)



$$= \frac{-i}{2} \{ \quad , \quad \}$$

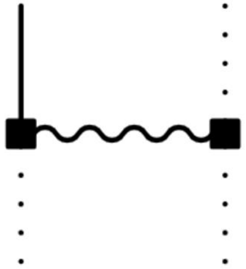


$$= \frac{1}{2}\{\chi_{(1)}, \{\chi_{(1)}, p_1^\mu\}\}$$

Scattering generator and causality cuts

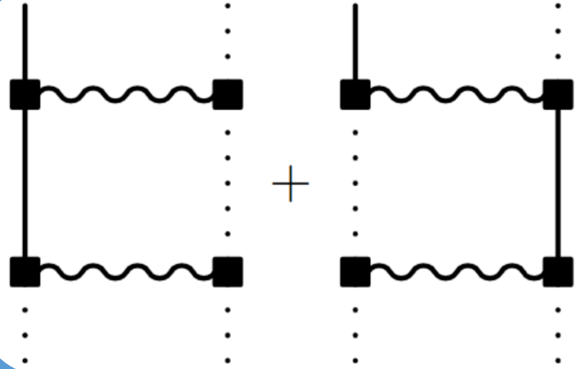
[Kim, JWK, Lee; JWK, Kim, Lee (WIP)]

$\Delta_{(1)} p_1^\mu =$



$= \{\chi_{(1)}, p_1^\mu\}$

(1PL eikonal)

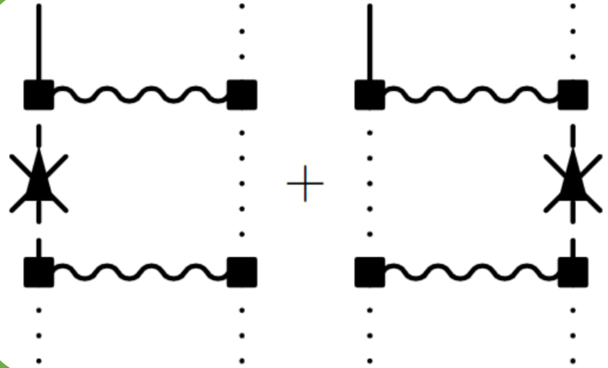


$= \{\chi_{(2)}, p_1^\mu\}$

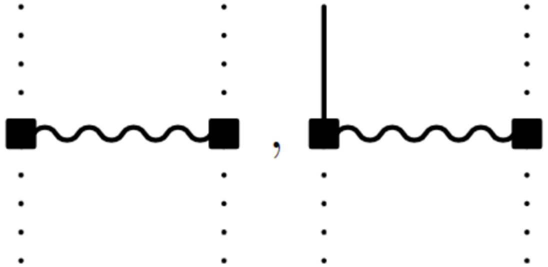
(2PL eikonal)

$$\mathcal{O}_f = \mathcal{O} + \{\chi, \mathcal{O}\} + \frac{1}{2!}\{\chi, \{\chi, \mathcal{O}\}\} + \cdots$$

(c. cut) = (iterated 1PL eikonal)
= (long. imp.)



$= \frac{-i}{2} \Big\{$



$\Big\} = \frac{1}{2}\{\chi_{(1)}, \{\chi_{(1)}, p_1^\mu\}\}$

Spin resummation in electrodynamics

[Kim, JWK, Lee]

- Spin-resummed 2PL twistor worldline eikonal
  - Key master integrals

$$\int \frac{\mu^{2\epsilon} d^D\ell_E \; e^{2\ell_E\cdot a}}{(\ell_E^2)^{\lambda_1}[(k-\ell_E)^2]^{\lambda_2}(2v\cdot\ell_E - i0^+)^{\lambda_3}} = \frac{i^{\lambda_3}\pi^{D/2}\mu^{2\epsilon}}{2\Gamma(\lambda_1)\Gamma(\lambda_2)\Gamma(\lambda_3)(k^2)^{\lambda_1+\lambda_2+\frac{\lambda_3}{2}-\frac{D}{2}}(v^2)^{\frac{\lambda_3}{2}}}$$

$$\times \sum_{l,m,n=0}^{\infty} \frac{(2k\cdot a)^l\left(-2i(v\cdot a)\sqrt{\frac{k^2}{v^2}}\right)^m (k^2a^2)^n}{l!m!n!} \frac{\Gamma(\lambda_1+\lambda_2+\frac{\lambda_3}{2}-\frac{m}{2}-n-\frac{D}{2})\Gamma(\frac{\lambda_3+m}{2})}{\Gamma(D-\lambda_1-\lambda_2-\lambda_3+l+m+2n)}$$

$$\times\,\Gamma\left(\frac{D}{2}-\lambda_2-\frac{\lambda_3}{2}+\frac{m}{2}+n\right)\Gamma\left(\frac{D}{2}-\lambda_1-\frac{\lambda_3}{2}+l+\frac{m}{2}+n\right)$$

$$\int \frac{d^Dk_E}{(2\pi)^D}\frac{(k_E\cdot a)^l e^{i(k_E\cdot b)}}{[k_E^2]^\lambda} = \frac{l!\Gamma(\frac{D}{2}-\lambda)}{2^{2\lambda}\pi^{\frac{D}{2}}\Gamma(\lambda)} \frac{i^l C_l^{(\frac{D}{2}-\lambda)}\left(\frac{(a\cdot b)}{(a^2b^2)^{1/2}}\right)(a^2)^{l/2}}{(b^2)^{\frac{D}{2}-\lambda+\frac{l}{2}}}$$

Spin resummation in electrodynamics

[Kim, JWK, Lee]

- Spin-resummed 2PL twistor worldline eikonal
  - Eikonal is a double infinite sum of 2F1, but simplifies in special kinematics

$$\Big(\pi^{3/2}\,\mathrm{dot}[b,b]^{-\frac{5}{2}-l-m}\left(-\mathrm{dot}[y,y]-\mathrm{dot}[y1,v2]^2\right)^m\left(-(1+2l)\,\mathrm{dot}[b,ymb]\,\mathrm{dot}[b,ypb]+2(1+2m)\,\mathrm{dot}[b,b]\left(\mathrm{dot}[y1,v2]^2+\mathrm{dot}[y1,y1]+\mathrm{dot}[y1,y2]\right)+(1+2l)\,\mathrm{dot}[b,b]\,\mathrm{dot}[ypb,ymb]\right)$$

$$(-\mathrm{dot}[ypb,ypb])^l\,\mathrm{Gamma}\left[\frac{1}{2}+l\right]\mathrm{Gamma}\left[\frac{1}{2}+m\right]\mathrm{GegenbauerC}\left[2l,\frac{3}{2}+m,\frac{\mathrm{dot}[b,ypb]}{\sqrt{\mathrm{dot}[b,b]\,\mathrm{dot}[ypb,ypb]}}\right]\Big)\Big/\left(2\,\mathrm{Gamma}\left[-\frac{1}{2}-m\right]\mathrm{Gamma}[1+m]\,\mathrm{Gamma}[2+l+m]\right)+$$

$$\left((-1)^l\,\pi^{3/2}\,\mathrm{dot}[b,b]^{-2-l-m}\,\mathrm{dot}[b,ymb]\left(-\mathrm{dot}[y,y]-\mathrm{dot}[y1,v2]^2\right)^m\mathrm{dot}[ypb,ypb]^{\frac{1}{2}+l}\,\mathrm{Gamma}\left[\frac{3}{2}+l\right]\mathrm{Gamma}\left[\frac{1}{2}+m\right]\mathrm{GegenbauerC}\left[1+2l,\frac{1}{2}+m,\frac{\mathrm{dot}[b,ypb]}{\sqrt{\mathrm{dot}[b,b]\,\mathrm{dot}[ypb,ypb]}}\right]\right)\Big/$$

$$\left(\mathrm{Gamma}\left[\frac{1}{2}-m\right]\mathrm{Gamma}[1+m]\,\mathrm{Gamma}[1+l+m]\right)+\Big(\pi^{3/2}\,\mathrm{dot}[b,b]^{-\frac{1}{2}-l-m}\left(-\mathrm{dot}[y,y]-\mathrm{dot}[y1,v2]^2\right)^m(-\mathrm{dot}[ypb,ypb])^l$$

$$\mathrm{Gamma}\left[\frac{1}{2}+l\right]\mathrm{Gamma}\left[\frac{1}{2}+m\right]\mathrm{GegenbauerC}\left[2l,\frac{1}{2}+m,\frac{\mathrm{dot}[b,ypb]}{\sqrt{\mathrm{dot}[b,b]\,\mathrm{dot}[ypb,ypb]}}\right]\Big(\left(-1+\mathrm{gamma}^2\right)(\mathrm{gamma}\,\mathrm{dot}[y1,v2]-\mathrm{dot}[y2,v1])+$$

$$2(l+m)\Big(2\,\mathrm{gamma}\left(-1+\mathrm{gamma}^2\right)\mathrm{dot}[y1,v2]+(\mathrm{dot}[y2,v1]+\mathrm{gamma}\,(\mathrm{dot}[y1,v2]-2\,\mathrm{gamma}\,\mathrm{dot}[y2,v1]))\,\mathrm{Hypergeometric2F1}\left[1,-m,\frac{1}{2},\frac{(-\mathrm{gamma}\,\mathrm{dot}[y1,v2]+\mathrm{dot}[y2,v1])^2}{\left(-1+\mathrm{gamma}^2\right)\left(\mathrm{dot}[y,y]+\mathrm{dot}[y1,v2]^2\right)}\right]\Big)\Big)\Big)\Big/$$

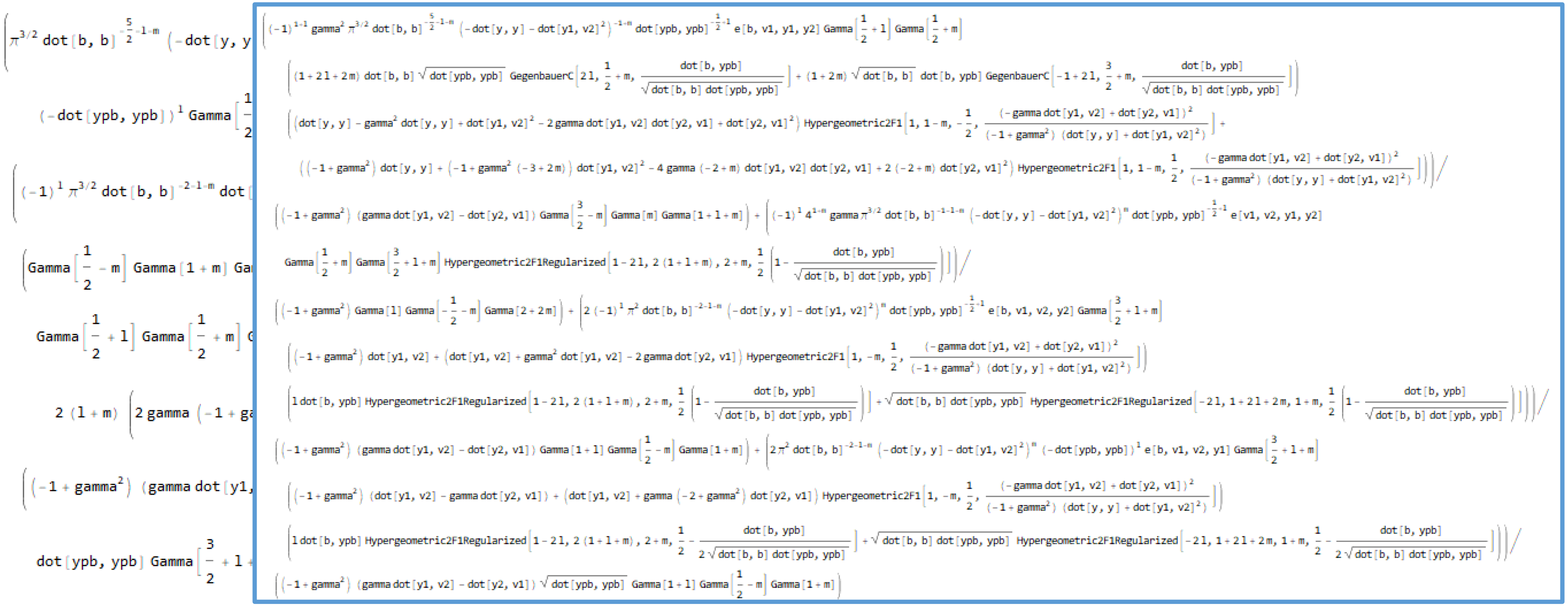
$$\left(\left(-1+\mathrm{gamma}^2\right)(\mathrm{gamma}\,\mathrm{dot}[y1,v2]-\mathrm{dot}[y2,v1])\,\mathrm{Gamma}\left[\frac{1}{2}-m\right]\mathrm{Gamma}[1+m]\,\mathrm{Gamma}[1+l+m]\right)+\Big(\pi^2\,\mathrm{dot}[b,b]^{-\frac{3}{2}-l-m}\left(-\mathrm{dot}[y,y]-\mathrm{dot}[y1,v2]^2\right)^m(-\mathrm{dot}[ypb,ypb])^l$$

$$\mathrm{dot}[ypb,ypb]\,\mathrm{Gamma}\left[\frac{3}{2}+l+m\right]\mathrm{Hypergeometric2F1Regularized}\left[-2(1+l),3+2l+2m,1+m,\frac{1}{2}-\frac{\mathrm{dot}[b,ypb]}{2\sqrt{\mathrm{dot}[b,b]\,\mathrm{dot}[ypb,ypb]}}\right]\Big)\Big/\left(\mathrm{Gamma}[1+l]\,\mathrm{Gamma}\left[\frac{1}{2}-m\right]\mathrm{Gamma}[1+m]\right)$$

Spin resummation in electrodynamics

[Kim, JWK, Lee]

- Spin-resummed 2PL twistor worldline eikonal
  - Eikonal is a double infinite sum of 2F1, but simplifies in special kinematics



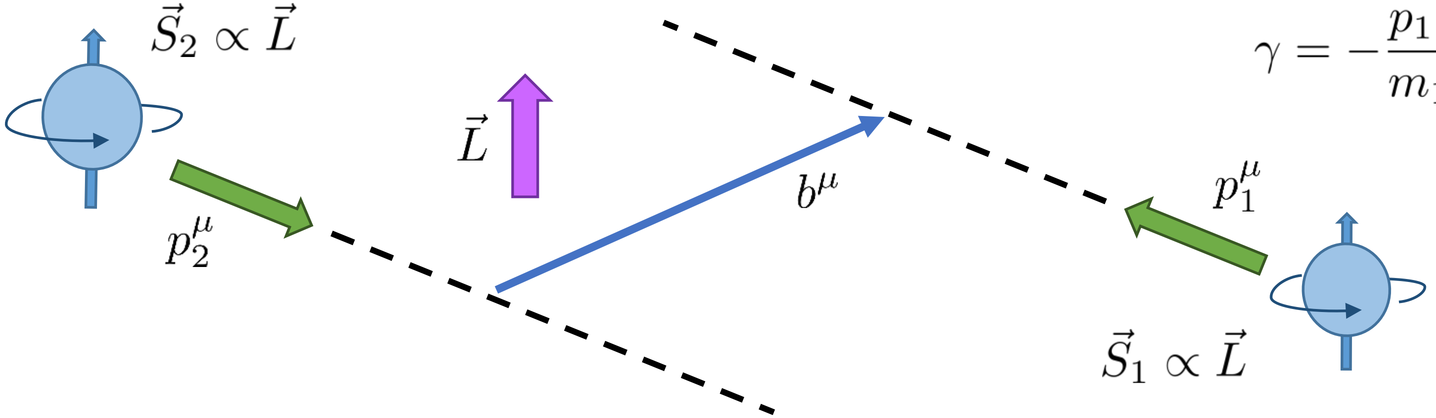
Spin resummation in electrodynamics

[Kim, JWK, Lee]

- Spin-resummed 2PL twistor worldline eikonal
  - Eikonal is a double infinite sum of 2F1, but simplifies in special kinematics
  - Aligned-spin configuration [Aoude, Haddad, Helset; Damgaard, Hoogeveen, Luna, Vines]

$$\chi_{(2,\text{aligned})} = \frac{(q_1 q_2)^2 \left(b^2 + \frac{(\zeta-2)\gamma}{(\gamma^2-1)} \epsilon[b, v_1, v_2, a] + \frac{\gamma^2(1-\zeta)+\zeta}{\gamma^2-1} a^2 \right)}{32\pi m_1 \sqrt{\gamma^2-1}\,(b^2-a^2)^{3/2}} + (1 \leftrightarrow 2)$$

$$\gamma = -\frac{p_1 \cdot p_2}{m_1 m_2}\,,\ a_1^\mu = \frac{S_1^\mu}{m_1} = \zeta a^\mu\,,\ a^\mu = a_1^\mu + a_2^\mu$$



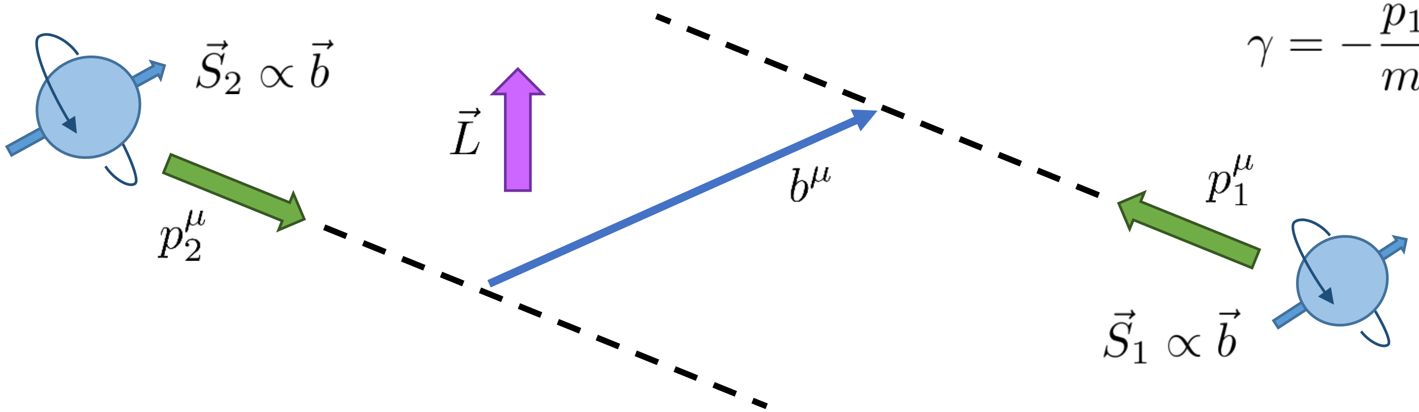
Spin resummation in electrodynamics

[Kim, JWK, Lee]

- Spin-resummed 2PL twistor worldline eikonal
  - Eikonal is a double infinite sum of 2F1, but simplifies in special kinematics
  - Axial scattering configuration

$$\chi_{(2,\text{axial})} = \frac{(q_1 q_2)^2 \sqrt{b^2}}{16\pi^2 m_1 (\gamma^2-1)^{3/2}} \left[\frac{\gamma^2(\zeta-1)-\zeta}{b^2} K\left(-\frac{a^2}{b^2}\right) - \frac{\gamma^2(\zeta-2)-(\zeta-1)}{b^2+a^2} E\left(-\frac{a^2}{b^2}\right) \right] + (1 \leftrightarrow 2)$$

$$\gamma = -\frac{p_1 \cdot p_2}{m_1 m_2}\,,\; a_1^\mu = \frac{S_1^\mu}{m_1} = \zeta a^\mu\,,\; a^\mu = a_1^\mu + a_2^\mu$$



Challenges for the future

- Spin resummation in gravity / NLO in mass-ratio
  - Coupling massive twistor worldline to gravity
  - (Full analytic) 2PM from HEFT/HPET [Chen, JWK, Wang (WIP)]
  - 3PL/PM: first beyond-probe-limit (NLO in mass-ratio)

- Eikonal as the scattering generator (all-order proof) [JWK, Kim, Lee (WIP)]
  - Including radiation / continuation to bound [Kälin, Porto; Cho, Kälin, Porto]

- Better spin-resummation schemes for waveform models
  - PM-based waveform: SEOBNR-PM [Buonanno, Mogull, Patil, Pompili]

Other future directions

- Tensor Integral Generating Functions [Feng]
  - Feynman integrals deformed by an exponential factor
    - Generally encountered in spin-resummed PM dynamics ($e^{(ia)\cdot\nabla} \Leftrightarrow e^{a\cdot k}$) waveform calculations

$$\mathcal{I}_{\lambda_k}[\alpha_i^\mu] = \int \prod_{j=1}^{L} d^D\ell_j \frac{\exp(\sum_{j=1}^{L} \alpha_j \cdot \ell_j)}{\mathcal{D}_1^{\lambda_1} \cdots \mathcal{D}_n^{\lambda_n}}$$

$$\mathcal{D}_j = (\ell + q_j)^2 - m_j^2$$

  - Provides alternative methods for tensor reduction: $\frac{\partial}{\partial \alpha_j^\mu} \Leftrightarrow \ell_j^\mu$
  - Can be computed by *conventional* multiloop techniques [Chen, JWK, Wang]
  - Can we reduce irreducible numerators more efficiently?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