YUSHAN
FELLOW PROGRAM
MINISTRY OF EDUCATION, TAIWAN

Fermionic Geometry for Effective Field Theory

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Based on 2307.03187 (JHEP) & work in progress
w/ Benoit Assi, Andreas Helset, Aneesh Manohar, Julie Pagès

the Standard Model EFT: $\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{n,i} \frac{1}{\Lambda^{n-4}} \mathcal{O}_{n,i}$

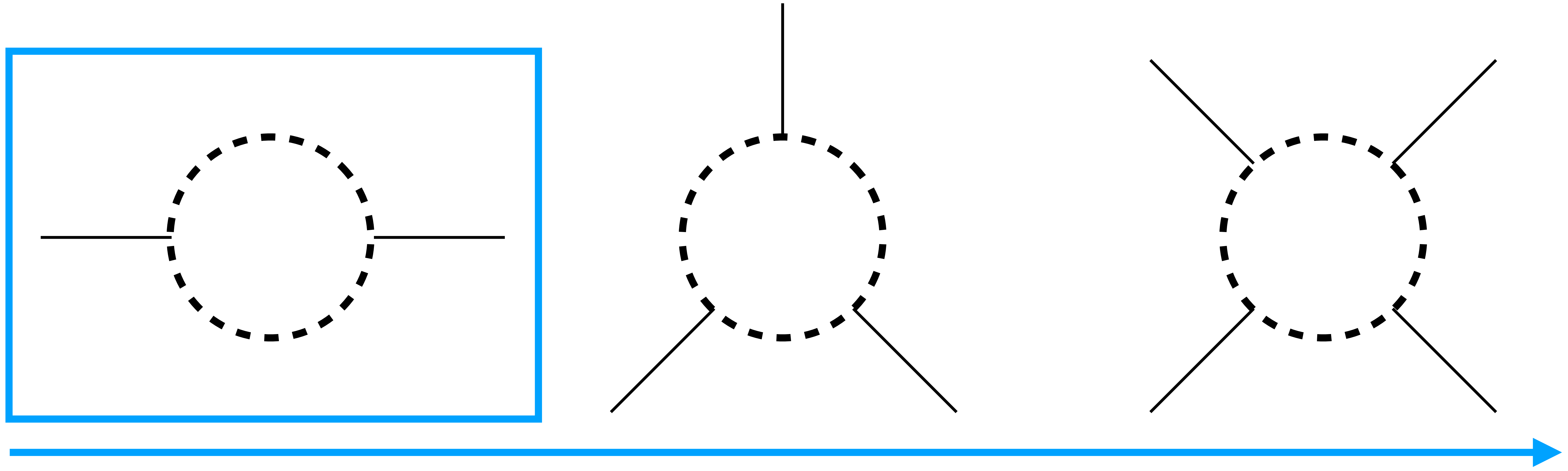
Order	dim 4	dim 5	dim 6	dim 7	dim 8
# of interactions	Standard Model	*2*	*84*	*30*	*993*

Important to flesh out all structures as EFT gets complicated

→ ***Use field-redefinition invariance***

What can we learn from field-redefinition invariance?

- Lesson from QCD:



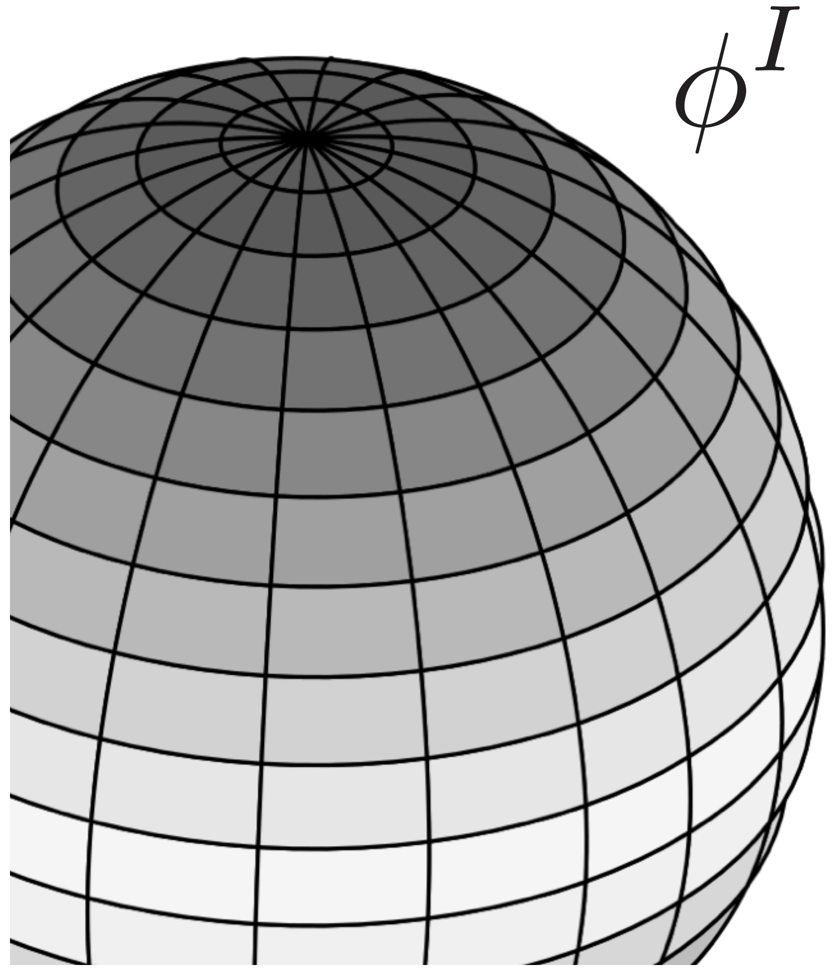
All these terms are related by (background field) gauge invariance

- Why off-shell? All one-loop running of operators are given by simple masters

Geometry in the Scalar Sector

[Meetz, Honerkamp, …]
[Alonso, Manohar, Jenkins]

The field redefinition w/o derivatives is identical to diffeomorphism



$$\begin{tikzcd}\phi^I \to \phi'^I(\phi) \arrow[d, leftrightarrow] \\ x^\mu \to x'^\mu(x)\end{tikzcd}$$

$$\mathcal{L} = \frac{1}{2}\underline{h_{IJ}(\phi)}\partial_\mu\phi^I\partial^\mu\phi^J - V(\phi)$$

target space metric
(resuming an infinite towers of operators)

$$h_{IJ}(\phi) \to h'_{IJ}(\phi') = h_{AB}(\phi)\frac{\partial\phi^A}{\partial\phi'^I}\frac{\partial\phi^B}{\partial\phi'^J}$$

Diffeo-invariant quantities ↔ ***field-basis invariant quantities***

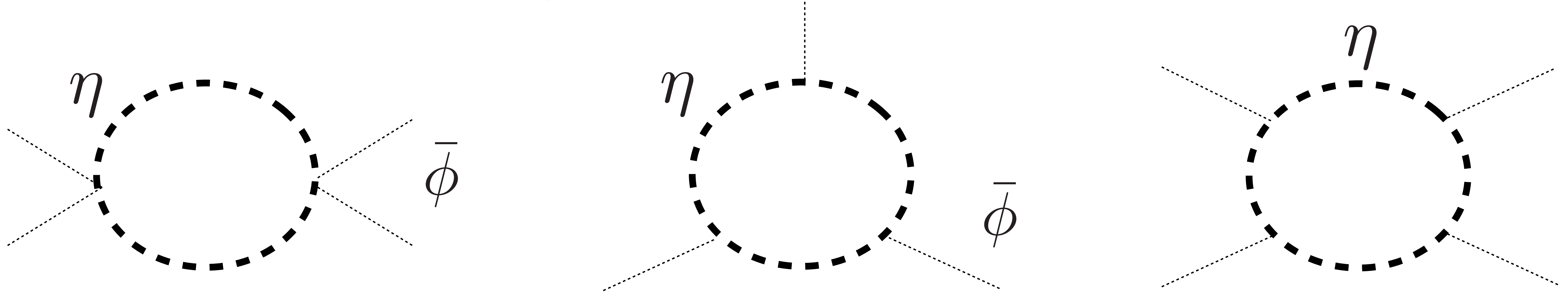
curvatures, covariant derivatives, …

amplitudes, RG equations, …

Applications of Geometry

- Manifest basis-invariance in amplitudes, can be applied to soft theorems, double copy, identifying hidden symmetries, etc.
  [Alonso, Manohar, Jenkins; Cohen, Craig, Lu, Sutherland; Cheung, Helset, Parra-Martinez; Helset, …]
- **Loop-level application: simplifying RG**

$$\phi^I = \underset{\substack{\uparrow \\ \text{background}}}{\bar{\phi}^I} + \underset{\substack{\uparrow \\ \text{quantum fluctuation}}}{\eta^I}$$



many seemingly unrelated terms

Applications of Geometry

- **Loop-level application: simplifying RG**

$$\phi^I = \underset{\uparrow}{\bar{\phi}^I} + \underset{\uparrow}{\eta^I} - \frac{1}{2}\Gamma^I_{JK}\underset{\uparrow}{\eta^J}\eta^K + \dots \qquad \eta'^I = \frac{\delta \phi'^I}{\delta \phi^J}\eta^J$$

background covariant quantum fluctuation

$$\delta^2 S = \frac{1}{2}\int d^4x \left[g_{IJ}\underline{(\mathcal{D}_\mu \eta)^I}(\mathcal{D}_\mu \eta)^J - \underline{R_{IJKL}}\eta^I (D_\mu \phi)^J \eta^K (D_\mu \phi)^L + \underline{V_{;IJ}}\eta^I \eta^J\right]$$

$$\swarrow \qquad\qquad\qquad\qquad \downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$\mathcal{D}_\mu \eta^I = D_\mu \eta^I + \Gamma^I_{JK} D_\mu \phi^K \eta^J \qquad R^A{}_{BIJ} = [\mathcal{D}_I, \mathcal{D}_J]^A{}_B \qquad \mathcal{D}_J \mathcal{D}_I V$$

Applications of Geometry

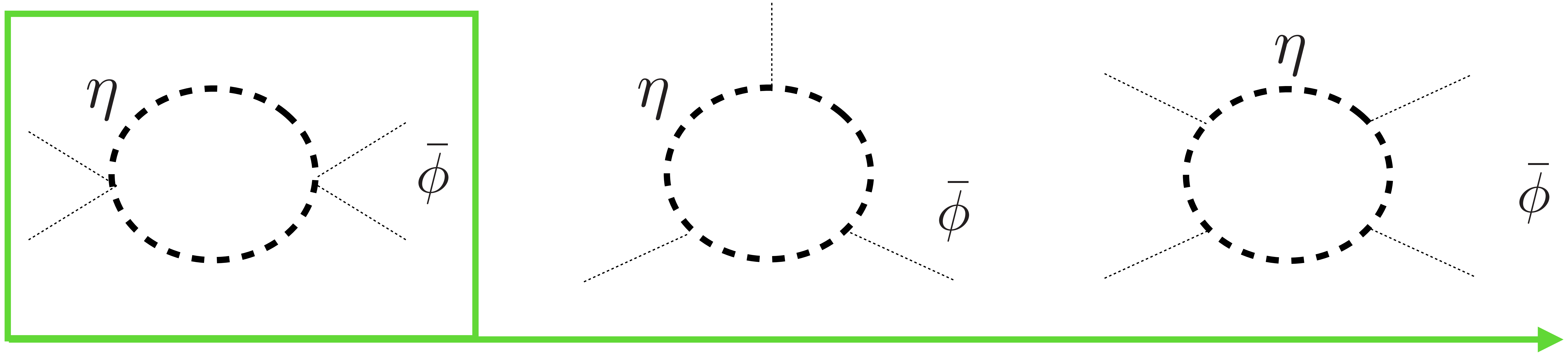
- **Loop-level application: simplifying RG**

[Alonso, Manohar, Jenkins]

$$\Delta S^{1-\mathrm{loop}} = \frac{1}{32\pi^2\epsilon}\int d^4x \left(\frac{1}{12}\mathrm{Tr}[Y_{\mu\nu}Y^{\mu\nu}] + \frac{1}{2}\mathrm{Tr}[X^2]\right)$$

$$[Y_{\mu\nu}]^I{}_J = [\mathcal{D}_\mu, \mathcal{D}_\nu]^I{}_J$$

$$[X]^I{}_J = -R^I{}_{JKL}(D_\mu\phi)^K(D^\mu\phi)^L + g^{IK}V_{;JK}$$



Field-Redefinition invariance

Fermionic Sector

[Assi, Helset, Manohar, CHS, Pages]

$$\mathcal{L} = \underbrace{\frac{1}{2} i \underline{k_{\bar{p}r}(\phi)} \left(\bar{\psi}^{\bar{p}} \gamma^{\mu} \overleftrightarrow{D}_{\mu} \psi^{r}\right)}_{\textit{kinetic term}} + \underbrace{i \underline{\omega_{\bar{p}rI}(\phi)} (D_{\mu}\phi)^{I} \bar{\psi}^{\bar{p}} \gamma^{\mu} \psi^{r}}_{\textit{spin connection}} - \underbrace{\bar{\psi}^{\bar{p}} \underline{\mathcal{M}_{\bar{p}r}(\phi)} \psi^{r}}_{\textit{Yukawa}} + \underbrace{\bar{\psi}^{\bar{p}} \sigma_{\mu\nu} \underline{\mathcal{T}^{\mu\nu}_{\bar{p}r}(\phi, F)} \psi^{r}}_{\textit{Dipole}}$$

	Dim-6 Examples		
trivial in Warsaw basis	$(\bar{\ell}_p \gamma^{\mu} \ell_r)\,(H^{\dagger} i \overleftrightarrow{D}_{\mu} H)$	$(H^{\dagger} H)(\bar{\ell}_p e_r H)$	$(\bar{\ell}_p \sigma^{\mu\nu} e_r H) B_{\mu\nu}$

Sufficient for all dim-6 fermonic operators* and most of dim 8.

Fermionic Sector

$$\mathcal{L} = \underbrace{\frac{1}{2} i \underline{k_{\bar p r}(\phi)} \left(\bar\psi^{\bar p}\gamma^\mu \overset{\leftrightarrow}{D}_\mu \psi^r\right)}_{\textbf{kinetic term}} + \underbrace{i\underline{\omega_{\bar p r I}(\phi)}(D_\mu\phi)^I \bar\psi^{\bar p}\gamma^\mu\psi^r}_{\textbf{spin connection}} - \underbrace{\bar\psi^{\bar p}\underline{\mathcal{M}_{\bar p r}(\phi)}\psi^r}_{\textbf{Yukawa}} + \underbrace{\bar\psi^{\bar p}\sigma_{\mu\nu}\underline{\mathcal{T}^{\mu\nu}_{\bar p r}(\phi, F)}\psi^r}_{\textbf{Dipole}}$$

Field Redefinition

$$\psi^p \to R^p{}_r(\phi)\psi^r$$

$$k \to (R^\dagger)^{-1} k R^{-1}\,,$$

$$\omega_I \to (R^\dagger)^{-1}\omega_I R^{-1} + \frac{1}{2}(R^\dagger)^{-1}k(\partial_I R^{-1}) - \frac{1}{2}(\partial_I (R^\dagger)^{-1})kR^{-1}\,,$$

$$\mathcal{M} \to (R^\dagger)^{-1}\mathcal{M}R^{-1}\,,$$

$$\mathcal{T}_{\mu\nu} \to (R^\dagger)^{-1}\mathcal{T}_{\mu\nu}R^{-1}$$

Geometry in the Fermionic Sector

$$\psi^p \to R^p{}_r(\phi)\psi^r$$

$$\Phi^a = \begin{pmatrix} \phi^I \\ \psi^p \\ \bar{\psi}^{\bar{p}} \end{pmatrix} . \qquad \frac{\delta \Phi'^a}{\delta \Phi^b} = \begin{pmatrix} \frac{\delta {\phi'}^I}{\delta \phi^J} & 0 & 0 \\ (\partial_I R)\ \psi & R & 0 \\ \bar{\psi}\left(\partial_I R^\dagger\right) & 0 & R^\dagger \end{pmatrix} .$$

scalars and fermions mix

$$\bar{g}_{ab}(\phi,\psi) = \begin{pmatrix} h_{IJ} & -\left(\frac{1}{2}k_{\bar{s}r,I} - \omega_{\bar{s}rI}\right)\bar{\psi}^{\bar{s}} & \left(\frac{1}{2}k_{\bar{r}s,I} + \omega_{\bar{r}sI}\right)\psi^s \\ \left(\frac{1}{2}k_{\bar{s}p,J} - \omega_{\bar{s}pJ}\right)\bar{\psi}^{\bar{s}} & 0 & k_{\bar{r}p} \\ -\left(\frac{1}{2}k_{\bar{p}s,J} + \omega_{\bar{p}sJ}\right)\psi^s & -k_{\bar{p}r} & 0 \end{pmatrix} .$$

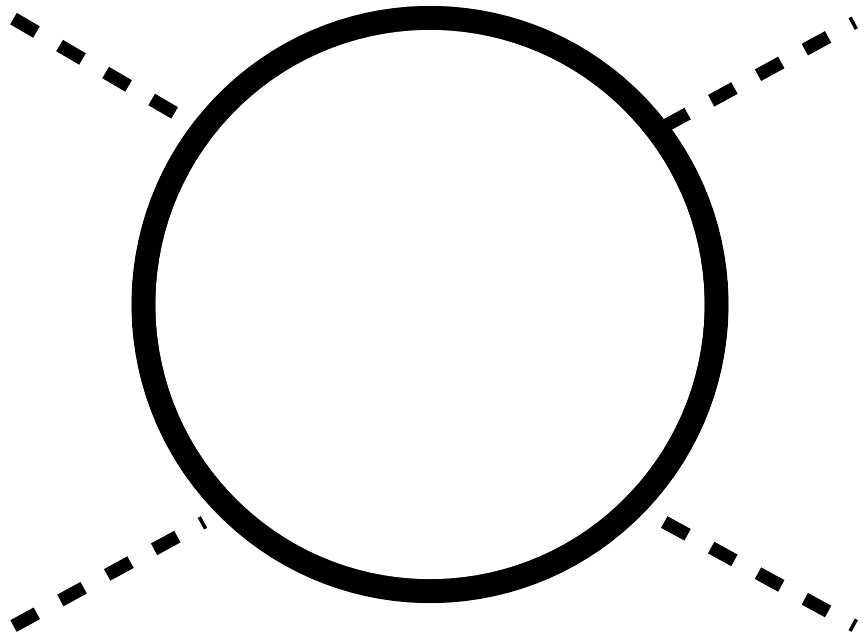
$$\bar{R}_{\bar{p}rIJ} = \omega_{\bar{p}rJ,I} - \left(\frac{1}{2}k_{\bar{p}s,I} - \omega_{\bar{p}sI}\right)k^{s\bar{t}}\left(\frac{1}{2}k_{\bar{t}r,J} + \omega_{\bar{t}rJ}\right) - (I \leftrightarrow J)$$

nontrivial combinations

RG with Fermionic Loops

[Assi, Helset, Manohar, CHS, Pages]

$$\begin{aligned}\Delta S =&\frac{1}{32\pi^2\epsilon}\int \mathrm{d}^4x\,\Bigg\{\frac{1}{3}\mathrm{Tr}\,\underline{[\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu}]}+\mathrm{Tr}\,\left[\underline{(\mathcal{D}_\mu\mathcal{M})(\mathcal{D}^\mu\mathcal{M})}-(\mathcal{M}\mathcal{M})^2\right]\\&-\frac{16}{3}\mathrm{Tr}[\underline{(\mathcal{D}_\mu\mathcal{T}^{\mu\alpha})(\mathcal{D}_\nu\mathcal{T}^{\nu\alpha})}-(\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^2]\\&-4i\mathrm{Tr}\underline{[\mathcal{Y}_{\mu\nu}(\mathcal{M}\mathcal{T}^{\mu\nu}+\mathcal{T}^{\mu\nu}\mathcal{M})]}-8\mathrm{Tr}(\mathcal{M}\mathcal{T}^{\mu\nu})^2\Bigg\}\,,\end{aligned}$$



$$\begin{aligned}[\mathcal{Y}_{\mu\nu}]^p{}_r &=[\mathcal{D}_\mu,\mathcal{D}_\nu]^p{}_r=\bar{R}^p{}_{rIJ}(D_\mu\phi)^I(D_\nu\phi)^J+\left(\bar{\nabla}_r t^p_A\right)F^A_{\mu\nu}\,,\\(\mathcal{D}_\mu\mathcal{M})^p{}_r &=k^{p\bar{t}}(\mathcal{D}_\mu\mathcal{M}_{\bar{t}r})=k^{p\bar{t}}\left[D_\mu\mathcal{M}_{\bar{t}r}-\bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I\mathcal{M}_{\bar{s}r}-\bar{\Gamma}^s_{Ir}(D_\mu\phi)^I\mathcal{M}_{\bar{t}s}\right]\,,\\(\mathcal{M}\mathcal{M})^p{}_r &=k^{p\bar{t}}\mathcal{M}_{\bar{t}q}k^{q\bar{s}}\mathcal{M}_{\bar{s}r}\,,\\(\mathcal{D}_\mu\mathcal{T}^{\alpha\beta})^p{}_r &=k^{p\bar{t}}(\mathcal{D}_\mu\mathcal{T}^{\alpha\beta}_{\bar{t}r})=k^{p\bar{t}}\left[D_\mu\mathcal{T}^{\alpha\beta}_{\bar{t}r}-\bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I\mathcal{T}^{\alpha\beta}_{\bar{s}r}-\bar{\Gamma}^s_{Ir}(D_\mu\phi)^I\mathcal{T}^{\alpha\beta}_{\bar{t}s}\right]\,,\\(\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^p{}_r &=k^{p\bar{t}}\mathcal{T}^{\mu\nu}_{\bar{t}q}k^{q\bar{s}}\mathcal{T}^{\alpha\beta}_{\bar{s}r}\,.\end{aligned}$$

same geometric structure

Simple structures in RG from Geometry

- New RG equations for dim-8 operators in the SMEFT

[Agree w/ Chala et al.; Das Bakshi et al.; Accettulli et al. in overlap regions]

$$\begin{aligned} {}^6\dot{C}_{H^4\Box} =& \frac{2}{3}g_1^2\kappa_1 + 2g_2^2\kappa_2 - 2\kappa_9 - 6\kappa_{10} - 2\kappa_{11}, \\ {}^6\dot{C}_{H^4D^2} =& \frac{8}{3}g_1^2\kappa_1 - 8\kappa_9 + 4\kappa_{11}\,. \end{aligned}$$

Geometry combines terms together

$$\begin{aligned} \kappa_1 =& \left[y_e\ {}^6C_{e^2\underset{tt}{H}^2D} + 2y_\ell\ {}^6C^{(1)}_{\ell^2\underset{tt}{H}^2D} + N_cy_u\ {}^6C_{u^2\underset{tt}{H}^2D} + N_cy_d\ {}^6C_{d^2\underset{tt}{H}^2D} + 2N_cy_q\ {}^6C^{(1)}_{q^2\underset{tt}{H}^2D}\right] \\ \kappa_9 =& \mathrm{Tr}\left[-Y_eY_e^{\dagger}{}^6C_{e^2H^2D} + Y_e^{\dagger}Y_e{}^6C^{(1)}_{\ell^2H^2D} - N_cY_dY_d^{\dagger}{}^6C_{d^2H^2D} + N_cY_d^{\dagger}Y_d{}^6C^{(1)}_{q^2H^2D}\right. \\ & \left. +N_cY_uY_u^{\dagger}{}^6C_{u^2H^2D} - N_cY_u^{\dagger}Y_u{}^6C^{(1)}_{q^2H^2D}\right]\,, \\ \kappa_{11} =& \mathrm{Tr}\left[-N_cY_dY_u^{\dagger}{}^6C_{udH^2D} - N_cY_uY_d^{\dagger}{}^6C^{\dagger}_{udH^2D}\right] \end{aligned}$$

Summary & Outlook

- Geometry structures in EFT emerge from field-redefinition invariance
  - Organize physical quantities (amplitudes/RGE/…) into simple building blocks
- We extend this to the fermionic sector and obtain new RG equations
- Extension to mixed type of loops [work in progress w/ Assi, Helset, Pages]
- Understand the relation to SUSY? New seeds for double copy?
- Still off-shell so far. Merging with on-shell calculation?

QCD Meets Gravity in Taiwan

- Dec. 2-6: Cosmological Correlators in Taiwan
- Dec. 9-13: QCD Meets Gravity