An unconventional approach to the Hierarchy problem

Emanuele Gendy Abd El Sayed
TUM

Based on ongoing work with N. Craig and J. Howard

Outline

The Hierarchy Problem (again?!) and supergroups

Super-scalars are super-finite

What to do with the dangerous modes?

Conclusions

The Hierarchy Problem

The masses of fundamental scalars appear to be UV sensitive, contradicting naturalness principles: this is the electroweak hierarchy problem

For the Higgs:

with cutoff regularization

$$\delta m_H^2 \sim -\frac{y_t^2}{16\pi^2}\Lambda_{UV}^2$$

in dim-reg after matching

$$\delta m_H^2 \sim \frac{\kappa}{16\pi^2} M^2 \left[\log\left(\frac{\mu_M^2}{M^2}\right) + 1\right]$$

A plethora of solutions

The problem has been approached from many angles

Cosmology

Relaxion

Anthropics

N-Naturalness

…

Symmetry

Supersymmetry

Higgs as a pNG

…

extra dimensions

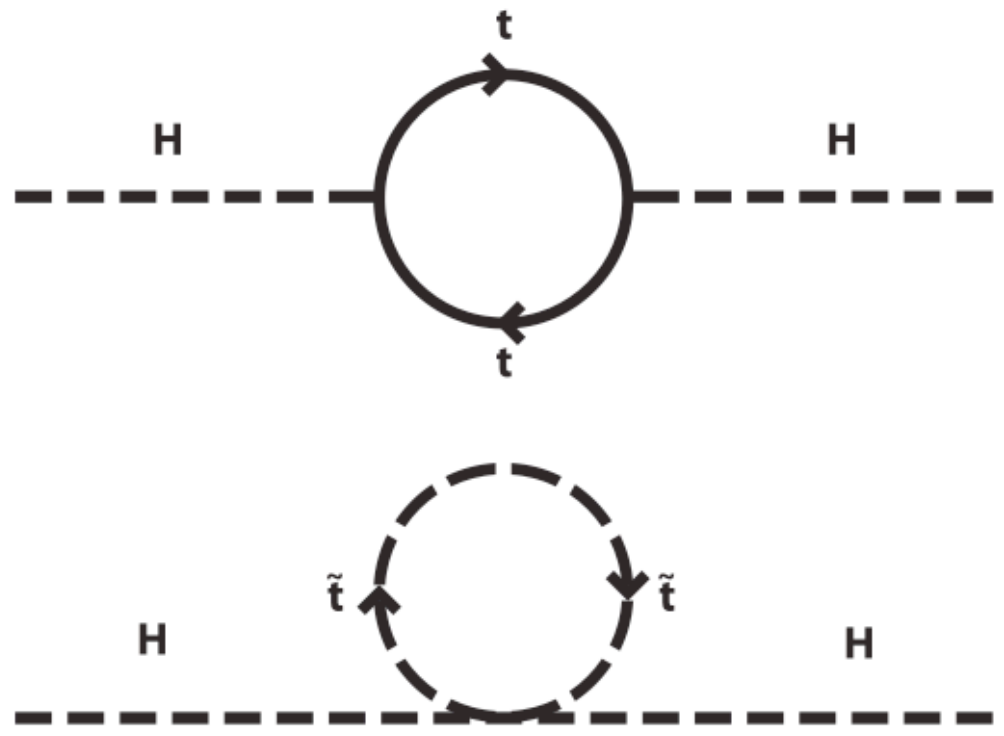
UV/IR mixing

…

Symmetry:
everyone likes a comeback story

Our idea walks on two legs:

1) the first comes from SUSY: loops of fermions and bosons have opposite signs and can be used to cancel each other



2) the second comes from Lee-Wick theories: fields with the wrong-sign kinetic term can be used to tame UV-divergences

$$\hat{D}(p) = \frac{i}{p^2 - p^4/M^2 - m^2}$$

Can we get both at the same time using symmetry?

Lee-Wick theory

Lee-Wick theories are an unorthodox solution
(Lee, Wick 1969-70; Grinstein, O'Connell, Wise [0704.1845])

add higher derivative term to tame UV divergences

$$\mathcal{L}_{\rm hd} = \frac{1}{2}\partial_\mu \hat{\phi}\partial^\mu \hat{\phi} - \frac{1}{2M^2}(\partial^2 \hat{\phi})^2 - \frac{1}{2}m^2 \hat{\phi}^2 - \frac{1}{3!}g\hat{\phi}^3$$

faster propagator fall-off

$$\hat{D}(p) = \frac{i}{p^2 - p^4/M^2 - m^2}$$

$$\delta m^2 \sim$$

$$\sim \frac{g^2}{16\pi^2} M^2 \log\left(\frac{\Lambda^2}{M^2}\right)$$

equivalent to adding a field with negative kinetic term

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}\partial_\mu \tilde{\phi}\partial^\mu \tilde{\phi} + \frac{1}{2}M^2 \tilde{\phi}^2 - \frac{1}{2}m^2(\phi - \tilde{\phi})^2 - \frac{1}{3!}g(\phi - \tilde{\phi})^3$$

Lee-Wick theory

Wrong-sign kinetic terms are associated with apparent classical and quantum instabilities that could bring to violation of unitarity and/or causality

« Please read these lectures last ~~week~~ ». 10^{-30} s ago!

From S. Coleman, "Acausality"

But if wrong-sign particles decay fast enough and with modification to the Feynman $i\epsilon$ prescription → macroscopic causality violations avoided at every order in perturbation theory

At worst we have to deal with microscopic violations of unitarity

See e.g. [I. Bars, "Supergroups and Their Representations", 1984]

Supergroups: SU(N|M)

SU(N|M) has an algebra of the form

$$\mathcal{H} = \begin{pmatrix} H_N & \theta \\ \theta^\dagger & H_M \end{pmatrix}$$

- H_N: N x N complex Hermitian matrix
- θ: N x M complex Grassmann matrix
- H_M: M x M complex Hermitian matrix

Invariants are built with super-trace

$$\mathrm{str}(\mathcal{H}) \equiv \mathrm{tr}(H_N) - \mathrm{tr}(H_M)$$

Contains a bosonic subgroup $SU(N) \times SU(M) \times U(1) \subset SU(N|M)$

$$\mathrm{str}\left(\lambda_I \lambda_J\right) = \frac{1}{2} g_{IJ}$$

$$g_{IJ} = \left(\begin{array}{ccc|c} \mathbb{I}_N & & & 0 \\ & \mathrm{sgn}(N-M) & & \\ & & -\mathbb{I}_M & \\ \hline & 0 & & \ddots \end{array}\right)$$

- $\mathbb{I}_N$: $SU(N)$
- $\mathrm{sgn}(N-M)$: $U(1)$
- $-\mathbb{I}_M$: $SU(M)$
- $\ddots$: Fermionic part

Super-scalar

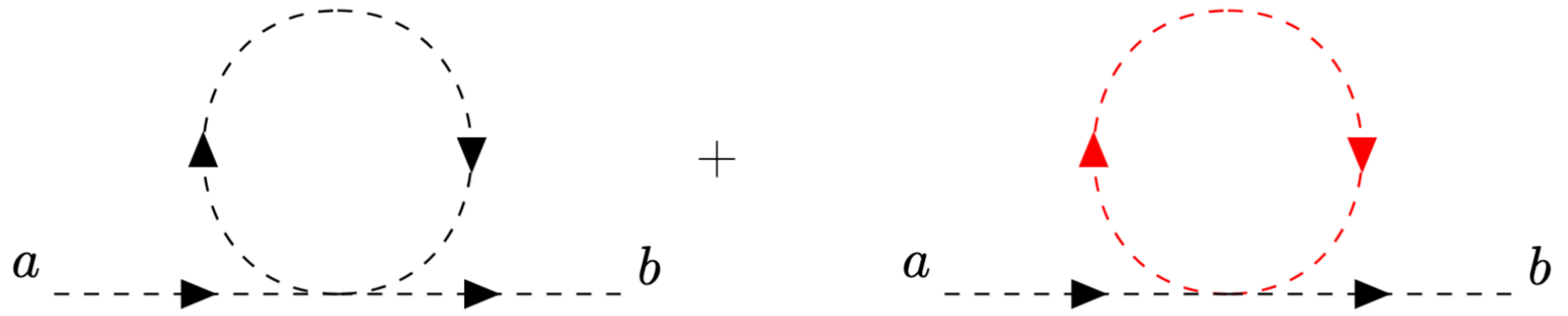
Introduce a scalar in the fundamental of SU(N|M)

$$\Phi_i = \begin{pmatrix} \phi_a \\ \psi_\alpha \end{pmatrix}$$

- ϕ_a ← N-dimensional complex scalar
- ψ_α ← M-dimensional complex scalar with fermionic statistics $\mathrm{f}(\psi_a) = 1$

How to make sense of this wrong-statistics field? Push the question to later and compute

$$\mathcal{L}_\Phi = \partial_\mu \Phi^{\dagger i} \partial^\mu \Phi_i - m^2 \Phi^{\dagger i} \Phi_i + \lambda (\Phi^{\dagger i} \Phi_i)^2$$



$$-2(N+1) \times \lambda \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - m^2} \qquad +M \times (2\lambda) \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - m^2}$$

Cancellation for M=N+1!

Super-vectors

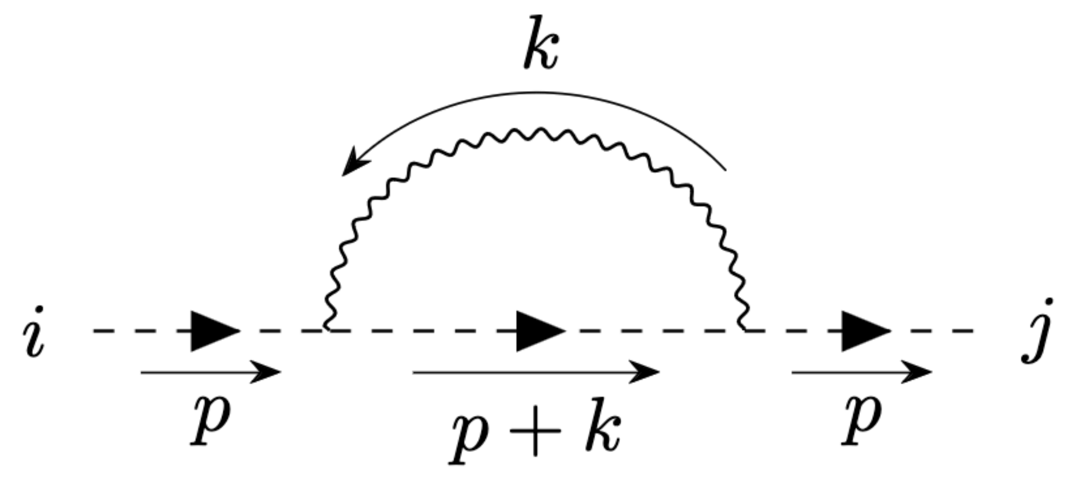
Let's add gauge interactions by gauging SU(N|M)

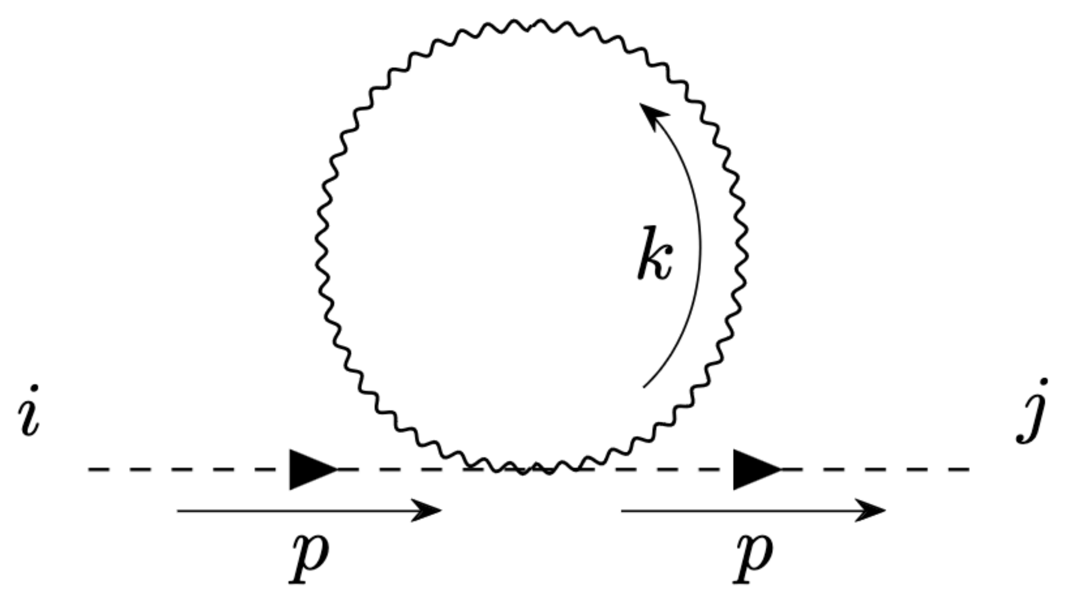
$$\mathcal{A}_\mu = \begin{pmatrix} A^{1a}_\mu t^a_N & B^i_\mu (s_1 + \tilde{s}_i) \\ (B^\dagger_\mu)^i (s_1^\dagger + \tilde{s}_i^\dagger) & A^{2b}_\mu t^b_M \end{pmatrix} + A^\lambda_\mu \lambda$$

Wrong statistics

Wrong-sign kinetic term (Lee-Wick)

Compute scalar one-loop mass correction





$$\propto (\lambda_I)_{ik} g^{IJ} (\lambda_J)_{kj} = \frac{1}{2}\delta_{ij}\left((N-M) - \frac{1}{N-M}\right)$$

Again, cancellation for M=N+1!

The same happens if coupling to a spinor in the fundamental ξ_i and one in the adjoint Θ^I of SU(N|M)

Recap

Let us pause to recap:

A scalar Φ_i in the fundamental of SU(N|M) coupled to:

- Itself, via a quartic coupling $\lambda(\Phi^{\dagger i}\Phi_i)^2$
- Gauge bosons, with minimal coupling $ig\partial_\mu\Phi^{\dagger i}(\mathcal{A}^\mu)_i^j\Phi_j - ig\Phi^{\dagger i}(\mathcal{A}^\mu)_i^j\partial_\mu\Phi_j + g^2\Phi^{\dagger i}(\mathcal{A}_\mu\mathcal{A}^\mu)_i^j\Phi_j$
- Fermions, one in the adjoint and one in the fundamental of SU(N|M), with coupling $-y\Phi_i\bar{\xi}_j(\lambda_I)_{ji}\Theta^I + \text{ h.c.}$

does not receive correction to its mass at one loop, provided M=N+1

This is nice enough to keep going!

Breaking SU(N|M)

Can we spontaneously break SU(N|N+1) and give a mass to all (or at least some) of the unwanted d.o.f.s?

Add a scalar transforming as a direct product of fundamental × antifundamental, with potential

$$V[\Sigma] = -\frac{1}{2}\mu^2 \Sigma^{\tilde{I}} g_{\tilde{I}\tilde{J}} \Sigma^{\tilde{J}} + \frac{1}{4}\lambda_1 \left(\Sigma^{\tilde{I}} g_{\tilde{I}\tilde{J}} \Sigma^{\tilde{J}}\right)^2 + \frac{1}{4}\lambda_2 \Sigma^{\tilde{I}}\Sigma^{\tilde{J}}\Sigma^{\tilde{K}}\Sigma^{\tilde{L}} T_{\tilde{I}\tilde{J}\tilde{K}\tilde{L}}$$

Stable local minimum $\langle \Sigma \rangle = \rho\sigma_3$ that breaks $SU(N|M) \to SU(N) \times SU(M) \times U(1)$ and realizes a Higgs-like mechanism

$$\mathcal{A}_\mu = \begin{pmatrix} A_\mu^{1a} t_N^a & B_\mu^i (s_1 + \tilde{s}_i) \\ (B_\mu^\dagger)^i (s_1^\dagger + \tilde{s}_i^\dagger) & A_\mu^{2b} t_M^b \end{pmatrix} + A_\mu^\lambda \lambda$$

They get a mass $m_B = g\rho$

They stay massless

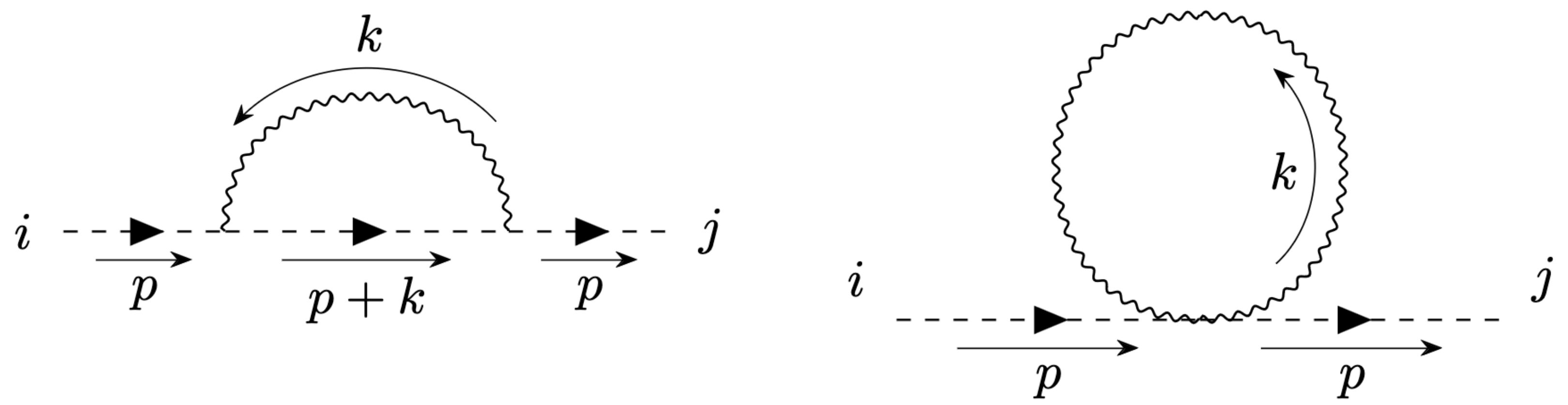
Breaking SU(N|M): Higgs mechanism

We could be tempted to take the limit $\rho \to \infty$: this way the wrong-statistics d.o.f.s in $\mathscr{A}_\mu$ and Σ are lifted

It would also decouple the wrong-sign d.o.f.s in $\mathscr{A}_\mu$ since the first coupling between A^1_μ and A^2_μ is irrelevant

$$\sim \frac{1}{\rho^4}(F^{(1)}_{\mu\nu})^2(F^{(2)}_{\alpha\beta})^2$$

However, we cannot choose ρ to be too large, since it gives a mass correction to Φ (similar to a soft-mass term)



$$\to \delta m^2_\Phi \approx m^2_B \frac{g^2}{16\pi^2}\left(\frac{N+1}{2}\right)\left(1 + 2\log\left(\frac{\mu^2}{m^2}\right)\right)$$

Cannot solve naturalness and have decoupling at the same time, but can be a good start!

Conclusions

Adding fermionic generators to gauge groups could be a way to address the EW hierarchy problem, creating a frameworks that takes ideas from Lee-Wick-like theories and SUSY, in a way completely described by symmetry

Why don't we just keep SUSY? While SUSY is either there or not, supergroups are more flexible: you can e.g. add supergroup structure to just a subgroup of the SM gauge-group

Dangers and issues are to be taken seriously, but the advantages can be worth the effort!