Local renormalisation from Causal Loop-Tree Duality

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Abstract.

We report recent progress on the development of a **local renormalisation formalism based on Causal Loop-Tree Duality**. By performing an **expansion around the UV-propagator in an Euclidean space**, we manage to build counter-terms to cancel the non-integrable terms in the UV limit. This procedure is then **combined with the causal representation**, and the UV expansion is performed at the level of on-shell energies. The proposed **formalism is tested up to 3-loops**, with relevant families of topologies. In all the cases, we **successfully cancel the UV divergences and achieve a smooth numerical implementation**. These results constitute a first step towards a **new renormalisation program in four space-time dimensions** (by-passing DREG), perfectly suitable for fully numerical simulations.

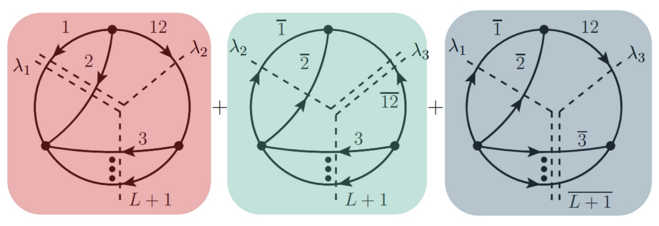
Loop-Tree Duality and Causality.

Basic idea behind LTD: **"open loops into trees"**

- 1st generation LTD: Dual representation (1 and 2-loops): Proposed in 2008, we cut once per loop. Replace cut with delta (remove energy component) and other propagators are promoted to "dual propagators" (modified prescription).
- 2nd generation LTD: Nested residues (multiloop): Iterated application of Cauchy residue theorem, removing one component of each loop momenta. Several cancellations take place; only a few physical residues remain.

[More details in Phys.Rev.Lett.124 (2020) 21, 211602; JHEP 01 (2021) 069 & JHEP 02 (2021) 112]

- 3rd generation LTD: Causal Loop-Tree Duality (multiloop): Generalization of nested residues; contributions reinterpreted in terms of **causal thresholds**.



$$\mathcal{A}^{(L)}_{\rm NMLT} = \int_{\vec\ell_1,\dots,\vec\ell_L} \frac{2}{x_{L+2}} \left(\frac{1}{\lambda_1\lambda_2} + \frac{1}{\lambda_2\lambda_3} + \frac{1}{\lambda_3\lambda_1} \right)$$

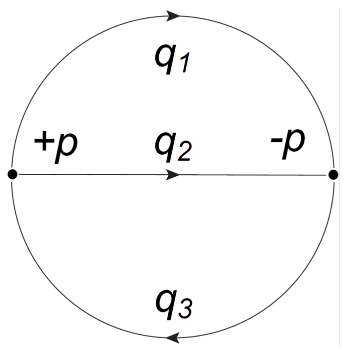
Causal representation example (scalar NMLT)

All-order generalization for arbitrary theories

$$\mathcal{A}^{(L)}_N = \sum_{\sigma\in\Sigma} \int_{\vec\ell_1,\cdots,\vec\ell_L} \frac{\mathcal{N}_\sigma(\{q^{(+)}_{r,0}\},\{p_{j,0}\})}{x_n} \times \prod_{i=1}^{k} \frac{1}{-\lambda_{\sigma(i)}} + (\sigma \leftrightarrow \bar\sigma)$$

with:

$x_n = \prod_{i\in 1\cup\ldots\cup n} 2q^{(+)}_{i,0}$ — PS-normalization; $k = V-1$ — Order of the diagram; $\lambda^\pm_j \equiv \sum_{i\in o_j} q^{(+)}_{i,0} \pm k_j$ — Causal threshold; Σ — Set of entangled causal thresholds



2-loop MLT with arbitrary masses and external momenta

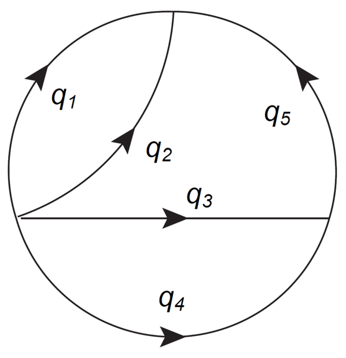
Causal representation:

$$\mathcal{A}^{(2)} = \int_{\vec\ell_1,\vec\ell_2} \frac{1}{x_3}\left(\frac{1}{\lambda_1^+} + \frac{1}{\lambda_1^-}\right)$$

that leads to the reduced causal representation:

$$\mathcal{A}^{(2)}_{RED} = \frac{1}{q^{(+)}_{1,0}+q^{(+)}_{2,0}+q^{(+)}_{3,0}+p_0} + \frac{1}{q^{(+)}_{1,0}+q^{(+)}_{2,0}+q^{(+)}_{3,0}-p_0}$$

λ_1^+ λ_1^-



3-loop NMLT vacuum diagram with arbitrary masses

Reduced causal representation:

$$\mathcal{A}^{(3)}_{RED} = \frac{1}{\lambda_1\lambda_2} + \frac{1}{\lambda_2\lambda_3} + \frac{1}{\lambda_3\lambda_1}$$

with the causal propagators:

$$\lambda_1 = q^{(+)}_{1,0}+q^{(+)}_{2,0}+q^{(+)}_{3,0}+q^{(+)}_{4,0}$$
$$\lambda_2 = q^{(+)}_{1,0}+q^{(+)}_{2,0}+q^{(+)}_{5,0}$$
$$\lambda_3 = q^{(+)}_{3,0}+q^{(+)}_{4,0}+q^{(+)}_{5,0}$$

Local renormalization.

- The aim is to **remove divergences from the causal LTD expression** with a method that consists of applying **Taylor Expansions of the loop-momenta in the UV limit**. The expansion is carried out up to logarithmic order and introduces a new energy scale: the renormalization scale.
- This method was **first developed in Minkowski space** and was modified to Euclidean space, to be applied on the LTD expression.
- The expansion is applied to every possible combination of loop momenta, so that all **simultaneous UV limits of internal lines are removed**. First, it is applied to the single UV limit. Then, single UV-limit counter-terms are subtracted from the amplitude to calculate multiple UV-limit counter-terms.

1st step: Select a loop momentum *j* and re-scale it inside on-shell energies, according to:

$$q^{(+)}_{j,0} = \sqrt{(\vec\ell_i+\vec k)^2 + m_j^2} \to \sqrt{\lambda^2\left(\vec\ell_i^{\,2}+\mu^2_{\rm UV}\right) + 2\lambda\,\vec\ell_i\cdot\vec k + \vec k^2 - \mu^2_{\rm UV} + m_j^2}$$

2nd step: Expand inside the on-shell energy and define replacement rules:

$$\mathcal{S}'_{{\rm UV},i} : q^{(+)}_{j,0} \to \lambda q^{(+)}_{i,0,{\rm UV}} + \frac{\vec\ell_i\cdot\vec k}{q^{(+)}_{i,0,{\rm UV}}} - \frac{(\vec\ell_i\cdot\vec k)^2}{2\lambda\,(q^{(+)}_{i,0,{\rm UV}})^3}$$

3rd step: Apply replacement rule and Taylor-expand, and remove non-integrable terms

$$\mathcal{A}^{(L)}_{RED,{\rm UV},i_1} = L_\lambda\left(\mathcal{A}^{(L)}_{RED}|_{\mathcal{S}'_{UV^1,i_1}}\right)$$

For 1-loop amplitudes: Consider all the single UV-limits and sum: you have the UV counter-term!

For multiloop amplitudes:

4th step: Remove the single UV-limit counter-terms

$$\mathcal{A}^{(L)}_{RED,1} = \mathcal{A}^{(L)}_{RED,0} - \sum_{i_1=1}^{L} \mathcal{A}^{(L)}_{RED,{\rm UV},i_1}$$

5th step: Expand inside the on-shell energy and define replacement rules:

$$\vec\ell_{j,\gamma} = \sum_{k\in\gamma\cap\delta_j} \vec\ell_k \qquad \vec v_j = \vec q_j - \vec\ell_{j,\gamma}$$

$$q^{(+)}_{j,0} \to \sqrt{\lambda^2\,(q^{(+)}_{\delta_j\cap\gamma,0,{\rm UV}})^2 + 2\lambda\,\vec\ell_{j,\gamma}\cdot\vec v_j + \vec v_j^{\,2} - \mu^2_{\rm UV} + m_j^2}$$

Generalization of Step 1: then expand and define the replacement rules as in Step 2

6th step: Apply the replacement rules induced by Step 5, and Taylor expand in the simultaneous UV-limit (analogous to Step 3). Go to Step 4, and iterate the procedure L times.

$$\mathcal{A}^{(L)}_{RED,{\rm UV}} = \mathcal{A}^{(L)}_{RED,L-1} - L_\lambda\left(\mathcal{A}^{(L)}_{RED,L-1}|_{\mathcal{S}'_{UV^L,\{1,\ldots,L\}}}\right)$$

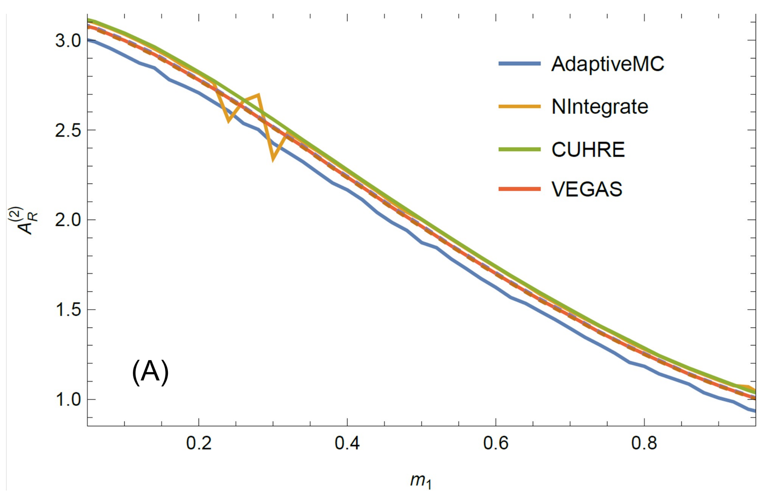
- The **loop momenta dependence** of the integrand is **hidden inside the on-shell energies** of the internal lines.
- It is shown that this method is **equivalent to BPHZ renormalization** when the renormalization scale is set to the mass of the internal lines.

2-loop example.

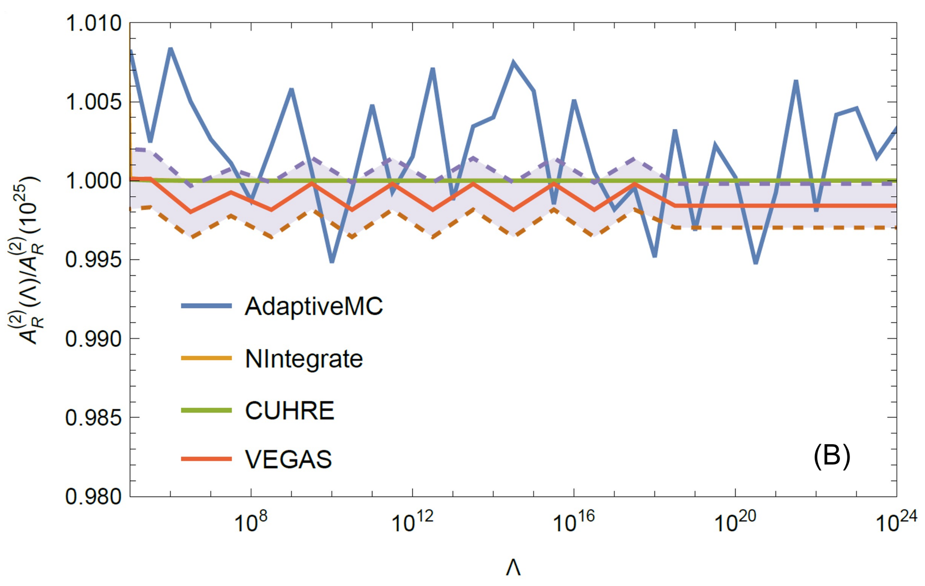
- We first test our formalism with a **2-loop sunrise diagram**. The local UV counter-term is:

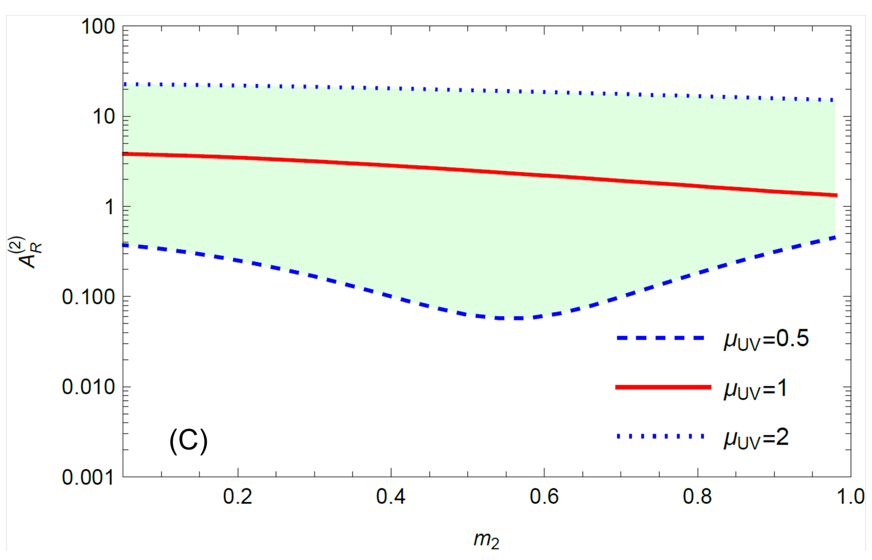
$$\mathcal{A}^{(2)}_{\rm UV} = \int_{\vec\ell_1,\vec\ell_2} \frac{1}{x_3}\left\{\frac{2}{Q_{\rm UV}} + \frac{1}{Q^2_{\rm UV}}\left[2\,\vec\ell_{12}\cdot\vec p + \frac{m_1^2}{q^{(+)}_{1,0,{\rm UV}}} + \frac{m_2^2}{q^{(+)}_{2,0,{\rm UV}}} - \mu^2_{\rm UV}\left(\frac{1}{q^{(+)}_{1,0,{\rm UV}}} + \frac{1}{q^{(+)}_{2,0,{\rm UV}}}\right)\right] + \frac{2p_0^2}{Q^3_{\rm UV}} + \frac{1}{Q^2_{\rm UV}(q^{(+)}_{12,0,{\rm UV}})^3}\left[\vec\ell^{\,2}_{12}\,(m_3^2+\vec p^{\,2}) - \left(1+\frac{2\,q^{(+)}_{12,0,{\rm UV}}}{Q_{\rm UV}}\right)(\vec\ell_{12}\cdot\vec p)^2 - \mu^4_{\rm UV} + \mu^2_{\rm UV}(m_3^2+\vec p^{\,2}-\vec\ell^{\,2}_{12})\right]\right\}$$

with $Q_{\rm UV} = q^{(+)}_{1,0,{\rm UV}} + q^{(+)}_{2,0,{\rm UV}} + q^{(+)}_{12,0,{\rm UV}}$



- We study the **stability of the numerical integration**, changing the UV cut-off (B) and modifying the masses (A) and the UV scale (C). Different integration strategies are used, showing a **smooth convergence in the UV region**.

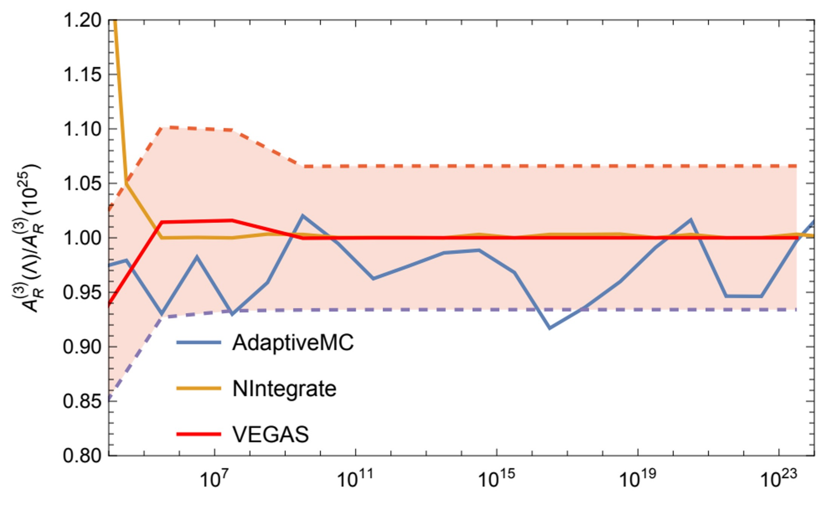




- Changing the UV scale, the renormalized amplitude varies drastically: this indicates a **very divergent UV limit, and a good local cancelation**.

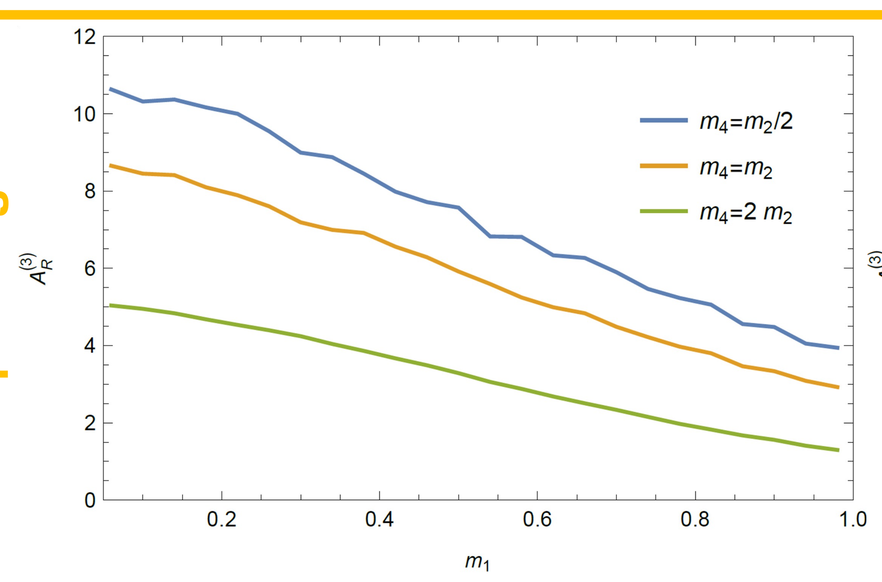
3-loop examples.

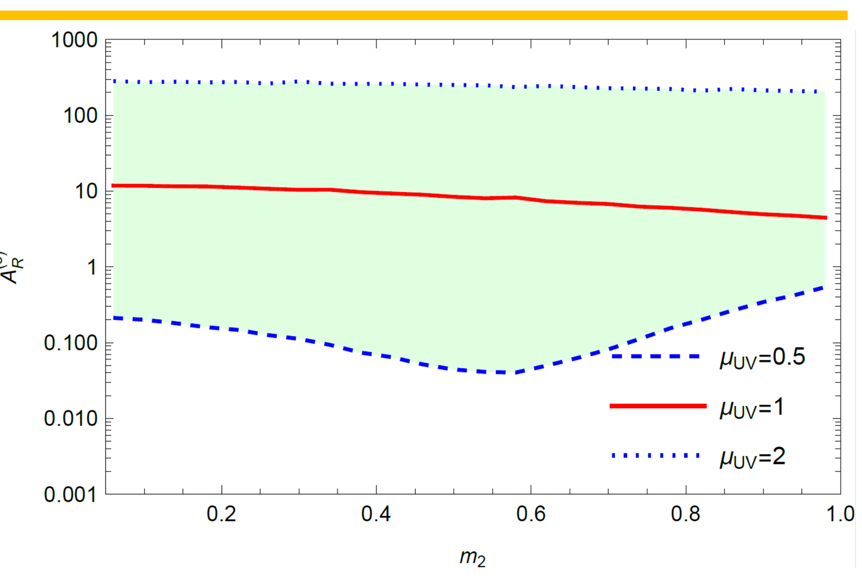
- Generic **3-loop MLT and 3-loop NMLT** vacuum diagrams are considered.
- The UV counter-terms **successfully cancel** the divergent behavior.
- Compared to the 2-loop case, more iterations and points required in Vegas integrators to achieve decent errors.
- The **numerical convergence is good enough**, even if **wild UV divergences** are present (specially in the 3-loop MLT).



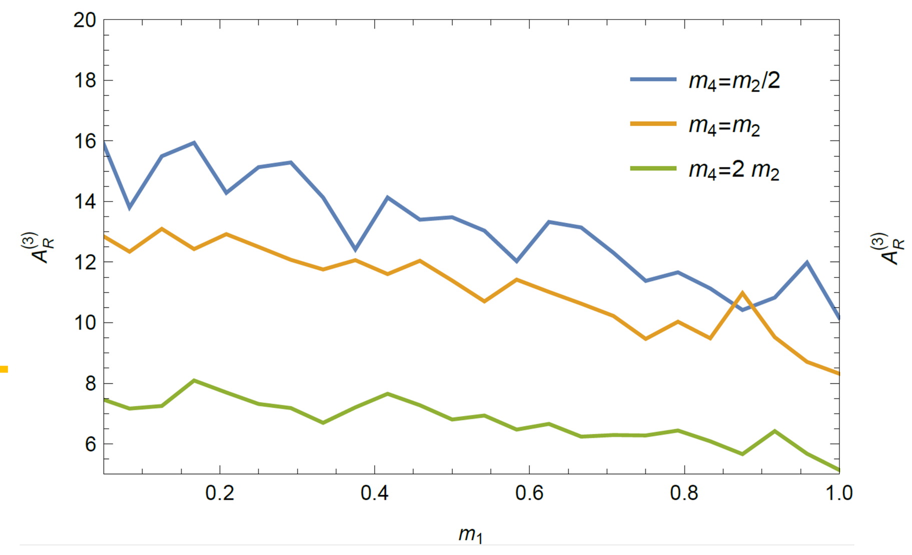
Convergence of 3-loop NMLT (different integrators)

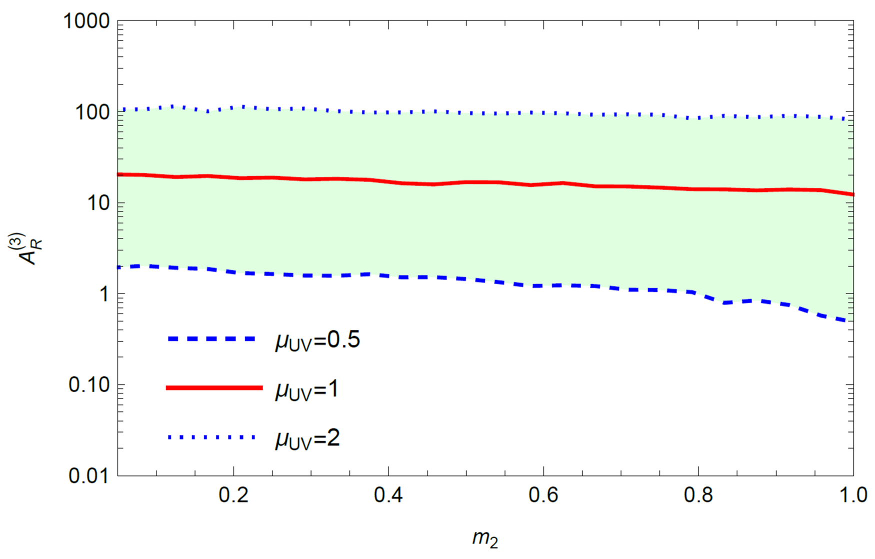
3-loop MLT generic





3-loop NMLT vacuum





Mass dependence of the renormalized amplitude

Renormalization scale and mass dependence

Conclusions.

- We explored techniques for **integrand-level renormalization of multiloop scattering amplitudes**, based on Causal Loop-Tree Duality [3].
- We extended the method of expansions around the UV-propagator in Minkowski space [2], through **Taylor-like expansions in Euclidean space**.
- Our approach is compatible with BPHZ renormalization, and it is a **crucial ingredient** to extend **Four-Dimensional Unsubtraction (FDU)** and **Causal Unitarity** to higher perturbative orders, local methods based on LTD.

References

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[3] J.J. Aguilera-Verdugo et al, ***Open Loop Amplitudes and Causality at All Orders and Powers from the Loop-Tree Duality***, Phys.Rev.Lett 124 (2020) 21, 211602. arXiv:2001.03564 [hep-ph].

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