From Feynman integrals to quantum algorithms: *the LTD connection*

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Junta de Castilla y León

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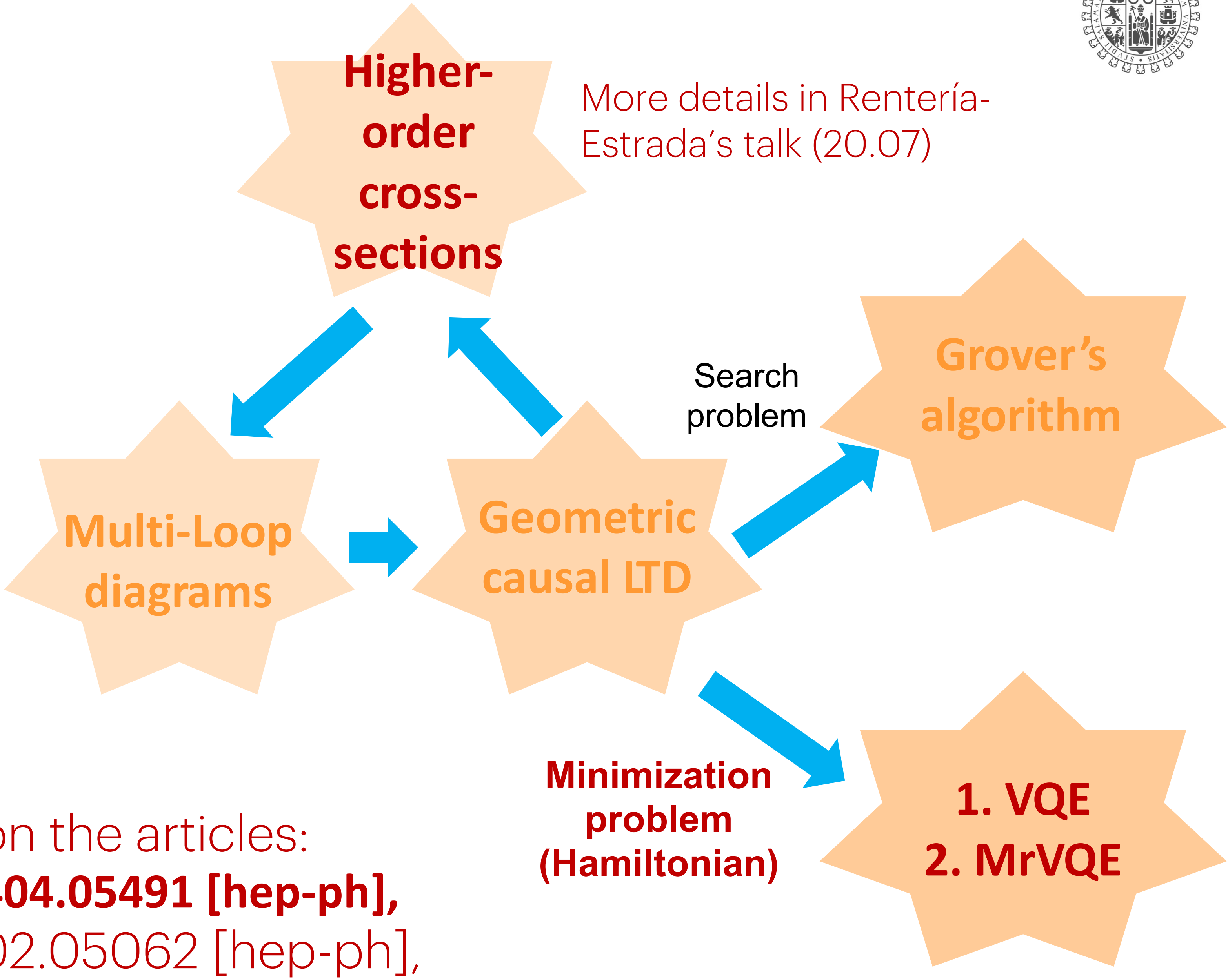
   - A. Geometrical causality
   - B. Cross-section from vacuum
2. Quantum algorithms for LTD
   - A. Grover search algorithm
   - B. Hamiltonian formulation
   - C. MrVQE
3. Conclusions

More details in Rentería-Estrada's talk (20.07)



This talk is based on the articles:
2404.05492 [hep-ph], 2404.05491 [hep-ph], 2210.13240 [hep-ph], 2102.05062 [hep-ph], 2001.03564 [hep-ph]

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PROBLEM

High-precision calculations require to deal with LOOPS and REAL-EMISSION. Current approaches calculate them separately.

QUESTION

Can we do something to combine them?

ANSWER

We can use…

PROBLEM

High-precision calculations require to deal with **LOOPS** and **REAL-EMISSION**. Current approaches calculate them separately.

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ANSWER

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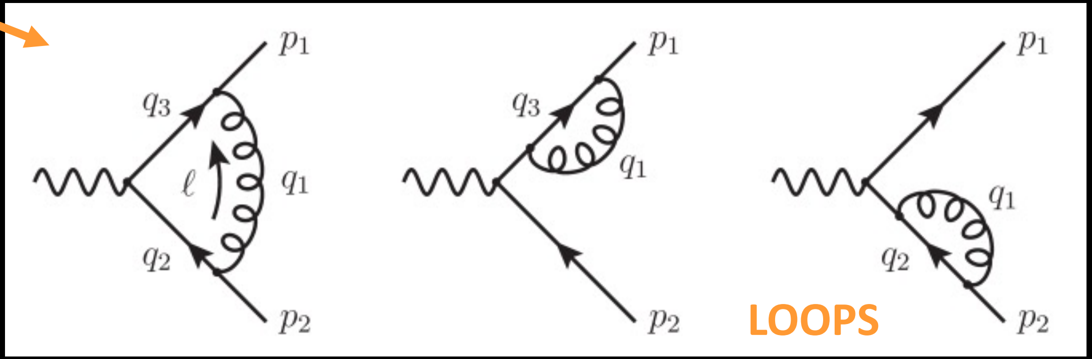
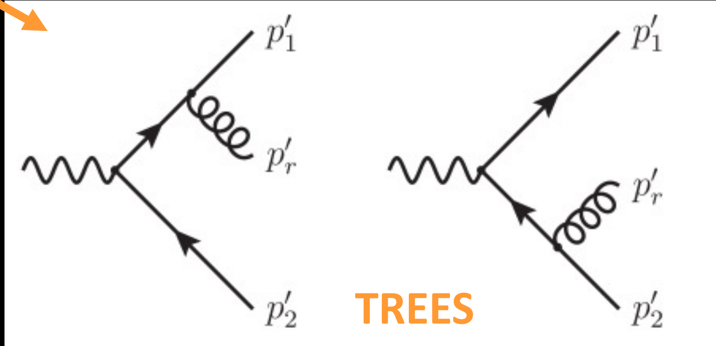
Loop-Tree Duality!

NLO computations in a single slide



Loops contain a virtual particle, not observable
(IR/UV singularities)

\+



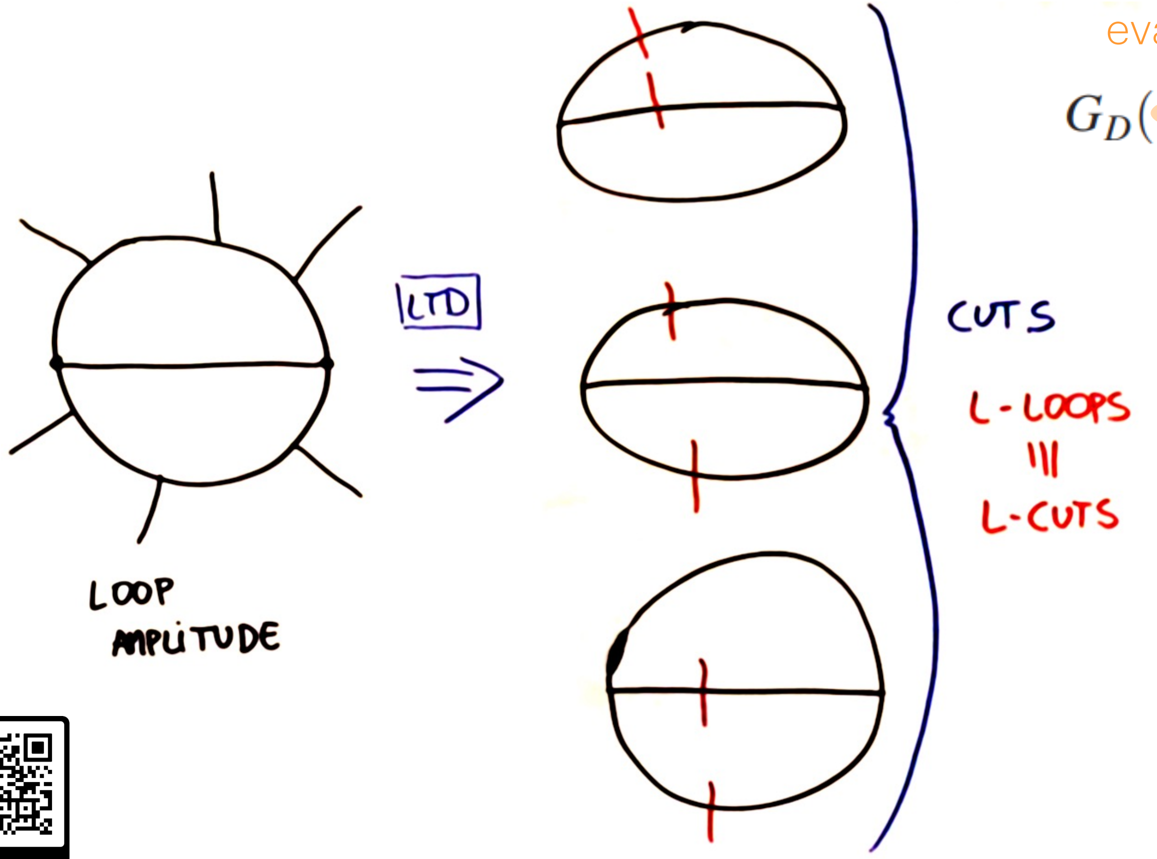
This is the real-radiation, because one extra particle is produced
(IR singularities)

\+ **UV counter-terms** = **Finite result!**

Tackling the loops: Loop-Tree Duality

- **<u>Main strategy:</u>** iterate Cauchy's theorem to *open loops into trees*
- Energy component is removed by using **<u>Cauchy's residue theorem</u>**
- **Multiloop require to iterate ("nest") the procedure (remove all the energy components)**

Rodrigo & collaborators, PRL 124 (2020) 21, 211602; JHEP 12 (2019) 163; JHEP 01 (2021) 069; JHEP 02 (2021) 112; JHEP 04 (2021) 129



r^{th} residue evaluation

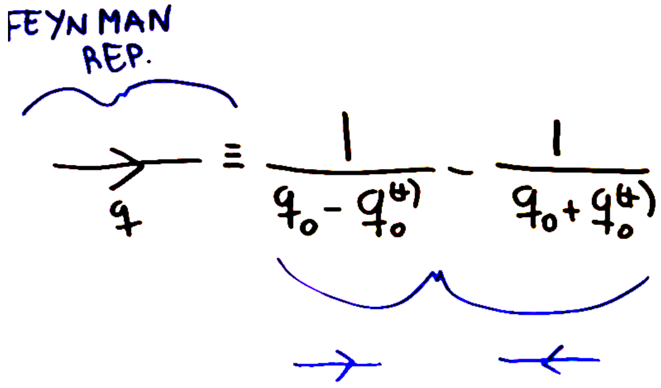
Remaining sets (no residue evaluation)

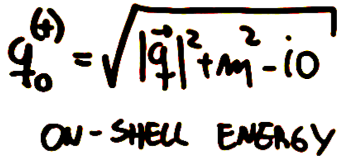
$$G_D(1,\ldots,r;n) = -2\pi\imath \sum_{i_r \in r} \text{Res}(G_D(1,\ldots,r-1;r,n), \text{Im}(\eta \cdot q_{i_r}) < 0)$$

Sum over all the elements of the r^{th} set

$(r-1)^{th}$ dual function

Depends on integration variables (q_i)





Master formula $$\mathcal{A}_N^{(L)}(1,\dots,L+k) = \sum_{\sigma\in\Sigma} \int_{\vec{\ell}_1,\cdots,\vec{\ell}_L} \frac{\mathcal{N}_\sigma(\{q_{r,0}^{(+)}\},\{p_{j,0}\})}{x_{L+k}} \times \prod_{i=1}^{k} \frac{1}{-\lambda_{\sigma(i)}} + (\sigma \leftrightarrow \bar{\sigma})$$

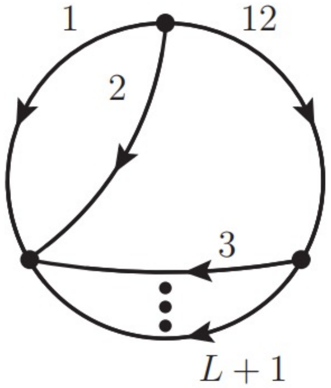
Set of entangled thresholds

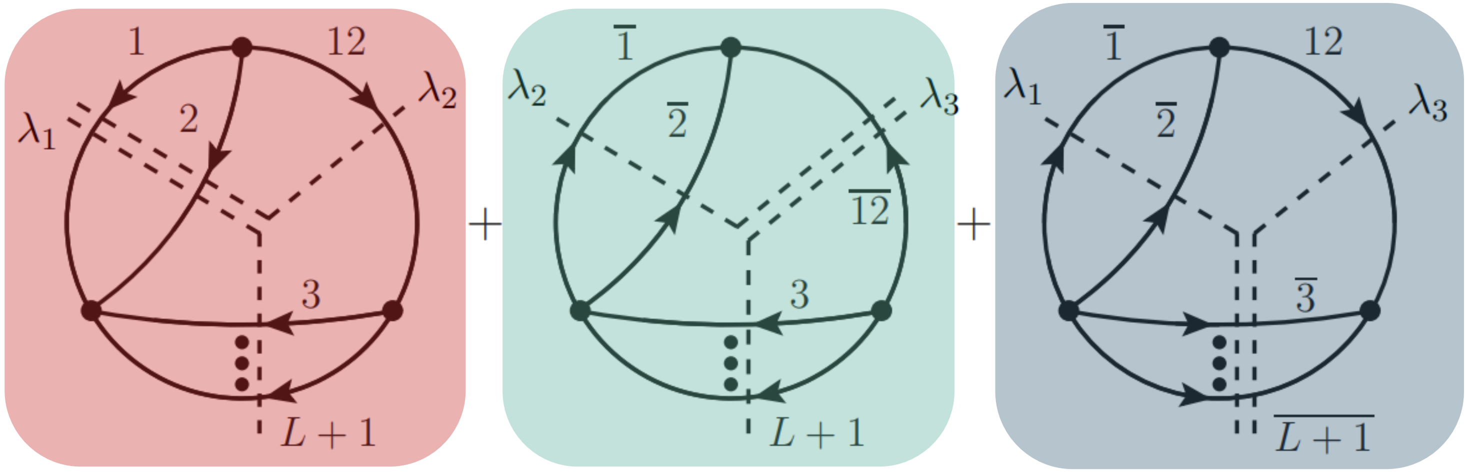
Products of *k* causal propagators

- **Graphical interpretation in terms of entangled thresholds**
  1. **Each causal** propagator represents a **threshold** of the diagram
  2. **Each diagram** contains **several thresholds**
  3. **The causal representation involves products of (*compatible*) thresholds**

More details in: JHEP 01 (2021) 069; JHEP 04 (2021) 129; JHEP 04 (2021) 183; Eur.Phys.J.C 81 (2021) 6, 514

Causal denominators (λ) are associated to *cut lines* in the diagrams: momenta flow must be adjusted to be compatible





$$\mathcal{A}_{\rm NMLT}^{(L)}(1,2,\dots,L+2) = \int_{\vec{\ell}_1,\dots,\vec{\ell}_L} \frac{2}{x_{L+2}} \left(\frac{1}{\lambda_1\lambda_2} + \frac{1}{\lambda_2\lambda_3} + \frac{1}{\lambda_3\lambda_1} \right)$$

Aguilera-Verdugo et al, JHEP 01 (2021) 069
Ramírez Uribe et al, JHEP 04 (2021) 129

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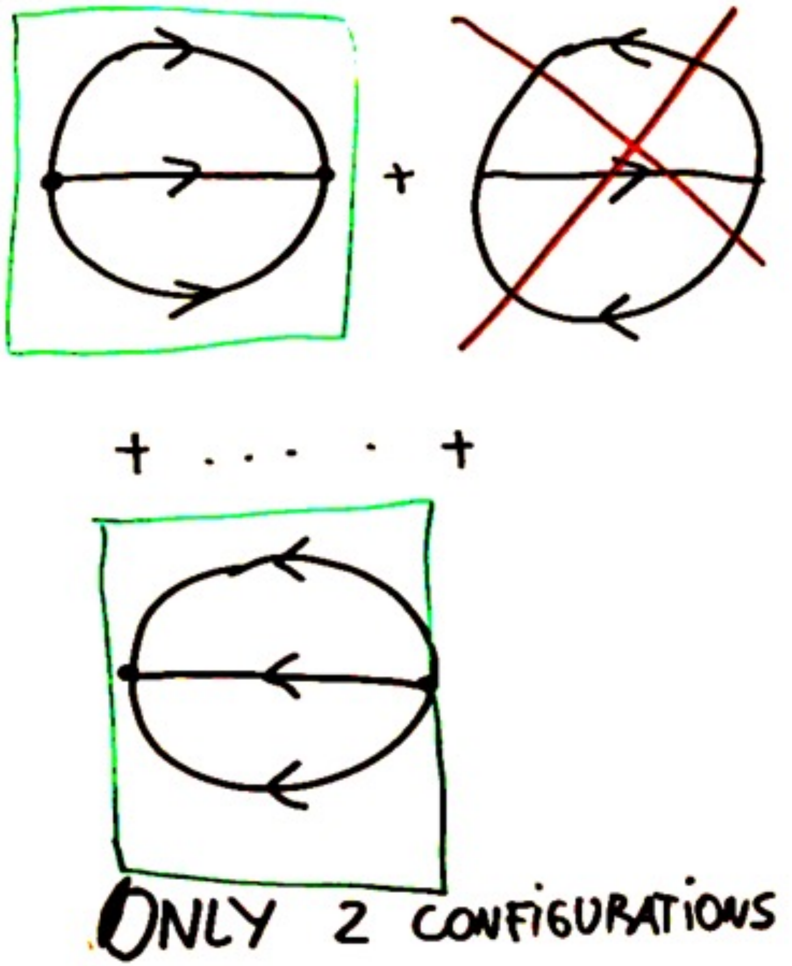
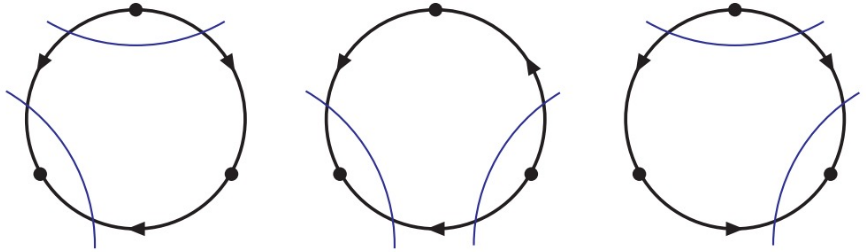


- *Geometrical rules look into several properties of the graph*

  "If the graph can be cut in tree-level pieces (thresholds) with consistent momentum flow (entangled thresholds), then we have a causal representation"

- **IMPORTANT:** **Causal configurations MUST FULLFIL the causal-compatible flux condition → Bootstrapping!!**



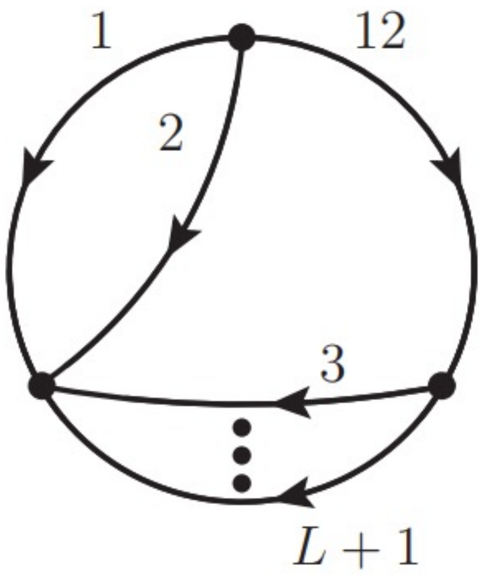
- **From graph theory:** *"Directed oriented graphs that can be split into binary partitions fulfilling (*) have no closed directed cycles"*

Non-cyclical configurations = Causal flux

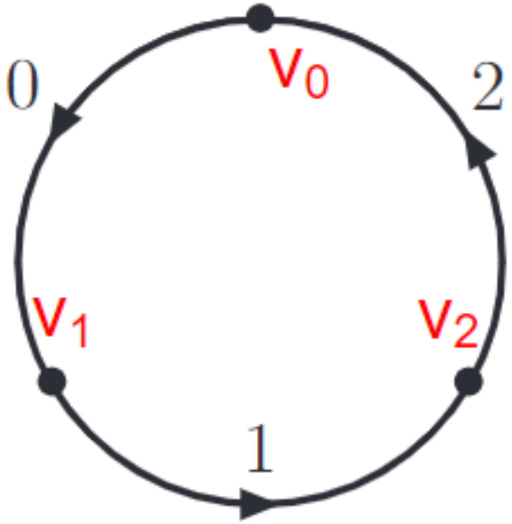
More detailed explanation arXiv:2102.05062 [hep-ph] & arXiv:2105.08703 [hep-ph]

Aguilera-Verdugo et al, JHEP 01 (2021) 069
Ramírez Uribe et al, JHEP 04 (2021) 129

Causal Reconstruction from graphs



Topologically equivalent diagrams



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- *Geometrical rules look into several properties of the graph*

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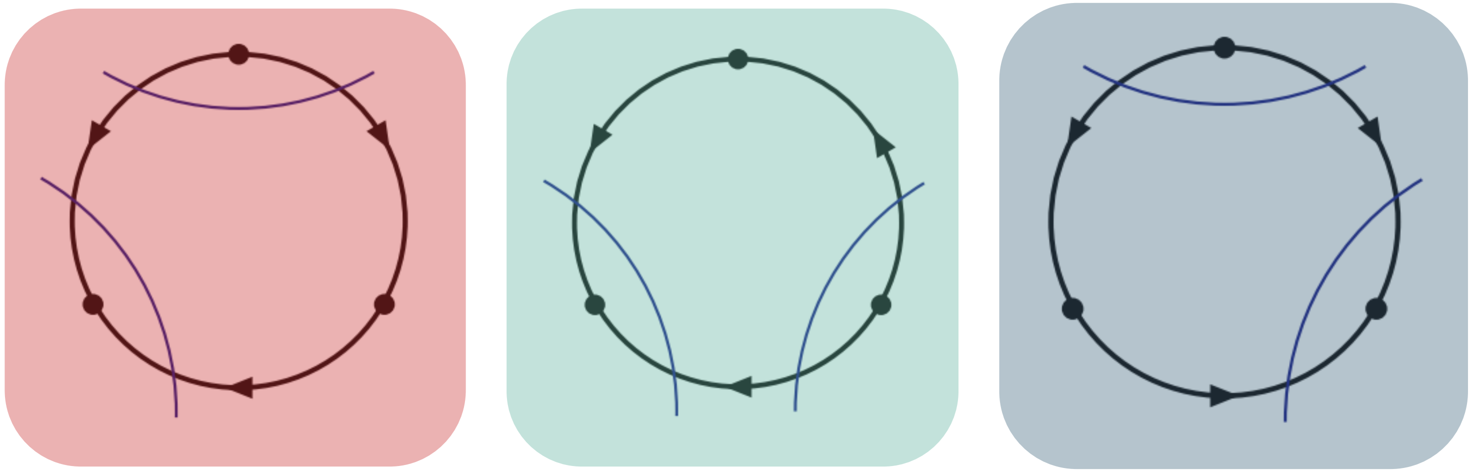

$$= \int_{\vec{\ell}_1} \frac{2}{x_{L+2}} \left(\frac{1}{\lambda_1 \lambda_2} + \frac{1}{\lambda_2 \lambda_3} + \frac{1}{\lambda_3 \lambda_1} \right)$$

Aguilera-Verdugo et al, JHEP 01 (2021) 069
Ramírez Uribe et al, JHEP 04 (2021) 129

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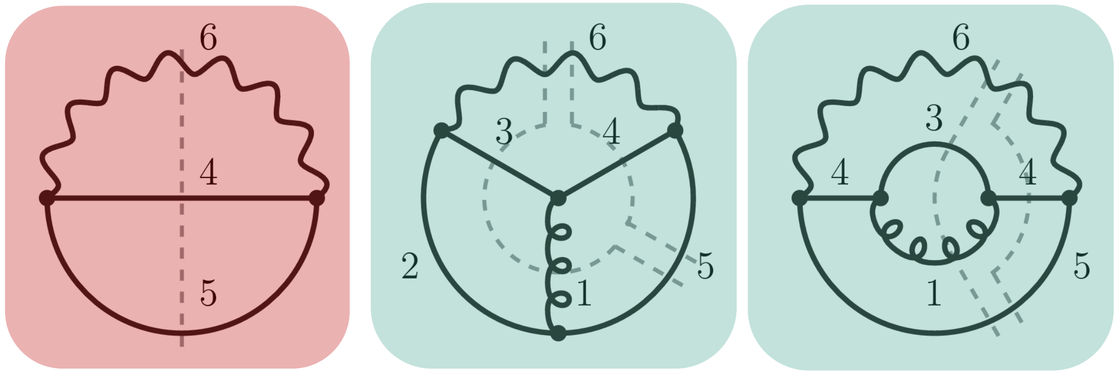
Cross-sections from multiloop vacuum diagrams

Developed by LTD Collaboration

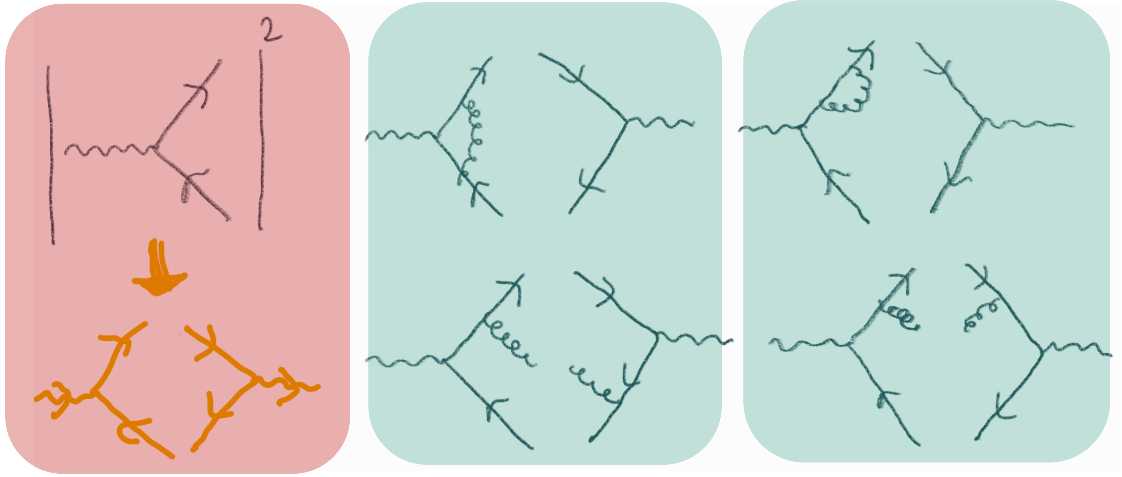
- **Novel technology to compute cross-sections: vacuum diagrams!**
- Applying residue theorem to the causal LTD representation, real, virtual and IR counter-terms are simultaneously generated! ➡ **Fully local cancellation of physical singularities!**

$\gamma^* \to q\bar{q}(g)$



Tree-level (eq. 2-loop)

Next-to-leading order (eq. 3-loop)



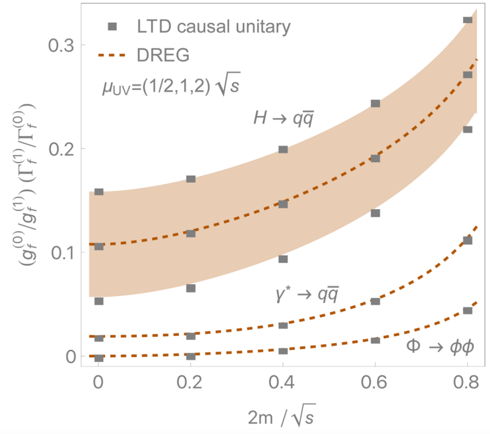
Master formula

$$d\Gamma_a^{(k)} = \frac{d\Lambda}{2m_a} \sum_{(i_1\cdots i_n a)\in\Sigma} \mathcal{A}_{\rm D}^{(\Lambda,{\rm R})}(i_1\cdots i_n a)\, \tilde{\Delta}_{i_1\cdots i_n\bar{a}}$$

with $\mathcal{A}_{\rm D}^{(\Lambda)}(i_1\cdots i_n a) = {\rm Res}\left(\frac{x_a}{2}\mathcal{A}_{\rm D}^{(\Lambda)}, \lambda_{i_1\cdots i_n a}\right)$

the residue of the dual amplitude and the momentum conservation delta:

$\tilde{\Delta}_{i_1\cdots i_n\bar{a}} = 2\pi\,\delta(\lambda_{i_1\cdots i_n\bar{a}})$ where $\lambda_{i_1\cdots i_n\bar{a}} = \sum_{s=1}^{n} q_{i_s,0}^{(+)} - p_{a,0}^{(+)}$



More details in Rentería-Estrada's talk (20.07)

Rodrigo et al, arXiv:2404.05491 & 2404.05492 [hep-ph]

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PROBLEM

Complex topologies have many causal configurations; it takes "a lot of time" to test all the possibilities.

QUESTION

Can we use other techniques to identify the causal terms?

ANSWER

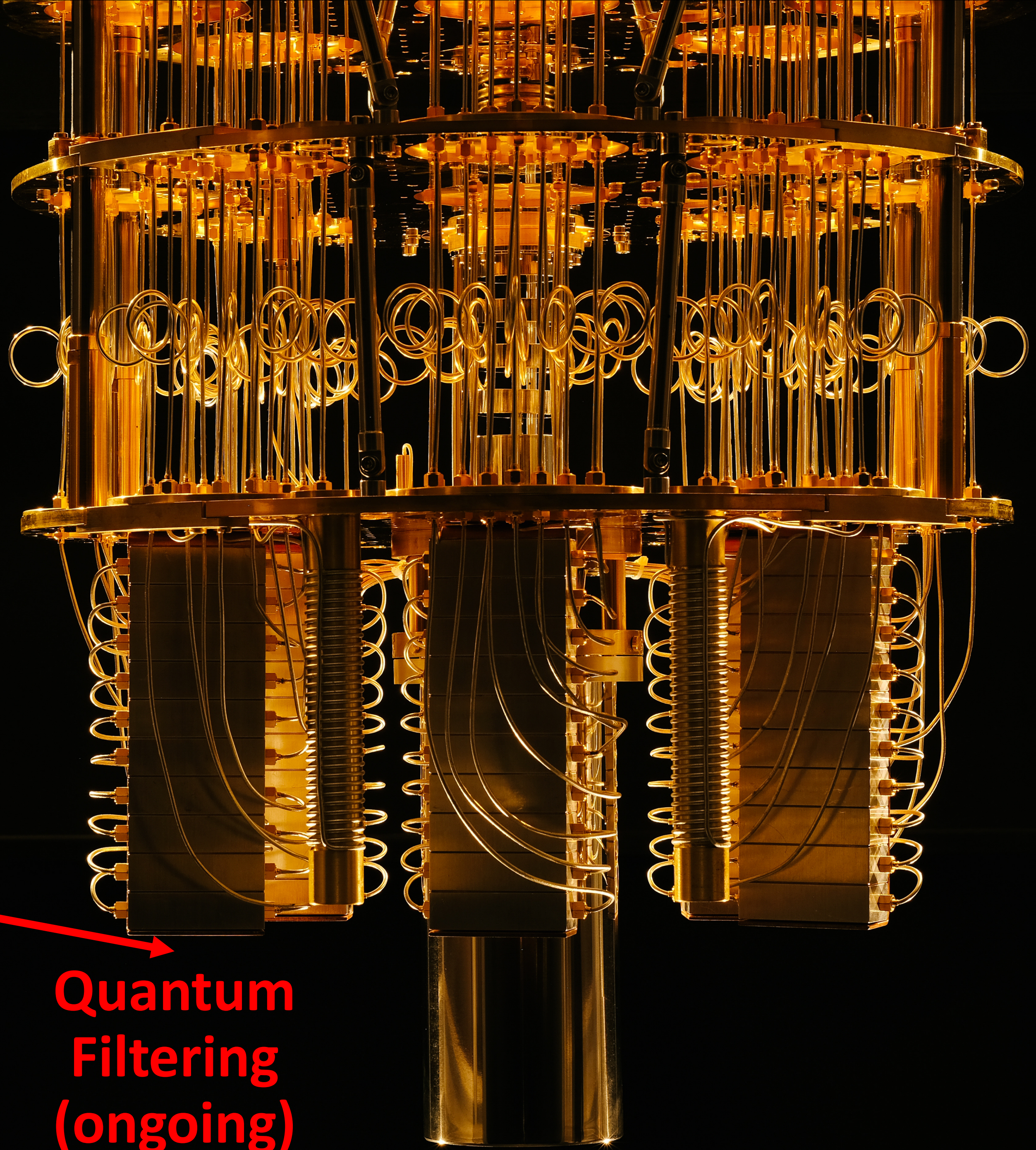
We can explore …

Quantum Algorithms!

Quantum Search (Grover)

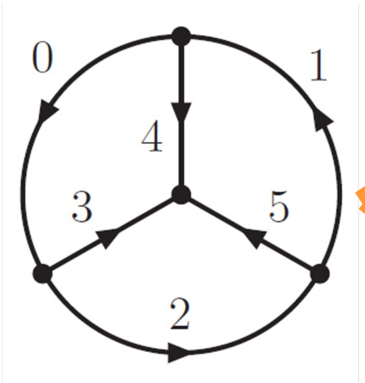
Quantum Minimization (VQE)

Quantum Filtering (ongoing)



Hamiltonian construction for minimization

- Geometrical information is codified in the **adjacency matrix**

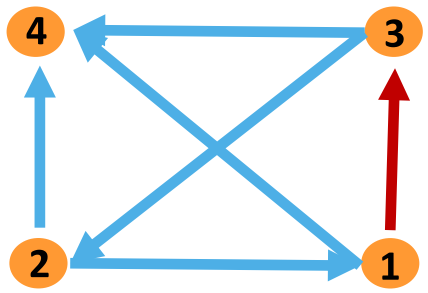


N2MLT topology:
Mercedes-Benz diagram

```
(*Configuracion momentos*)
NumeroVertices = 4; Orden = NumeroVertices - 1;
Eq[1] = {q[6] + q[4] - q[1] + p[1]};
Eq[2] = {q[1] + q[5] - q[2] + p[2]};
Eq[3]  = {q[3] - q[6] + q[2] + p[3]};
Eq[4] = {-q[3] - q[4] - q[5] - (p[1] + p[2] + p[3])};
```

$$\left(\begin{array}{ccccc|cccc} -1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & 0 & -1 & -1 & -1 \end{array}\right)$$

Vertex matrix
(from the conservation equations)



$$A = \begin{pmatrix} 0 & a_{12} & a_{13} & \cdots \\ \neg a_{12} & 0 & a_{23} & \cdots \\ \neg a_{13} & \neg a_{23} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Adjacency matrix
(how are vertices connected)

Clemente et al, PRD 108 (2023) 9, 096035
Sborlini, PRD 104 (2021) 3, 036014

- <u>**Useful properties:**</u>

$$f_3(A) \equiv \Sigma_{i_1,i_2,i_3}^{i_1 \neq i_2 \neq i_3} a_{i_1 i_2} \times a_{i_2 i_3} \times a_{i_3 i_1} \neq 0$$ ⟺ There are triangles

$$f_4(A) \equiv \Sigma_{i_1,i_2,i_3,i_4}^{i_1 \neq i_2 \neq i_3 \neq i_4} a_{i_1 i_2} \times a_{i_2 i_3} \times a_{i_3 i_4} \times a_{i_4 i_1} \neq 0$$ ⟺ There are boxes

In general, there are N-cycles **iff** $f_N(A)$ is non zero

A graph with associated adjacency matrix A is acyclic if:

$$\sum_{n=1}^{V} \mathrm{tr}(A^n) = 0$$

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Hamiltonian construction for minimization

- Exploit the **adjacency matrix** to build a **Hamiltonian** ➡ **Ground state = Acyclic graph**
- **1st approach:** Penalize oriented cycles using projectors to build the **loop Hamiltonian**

$$H_G = \sum_{\gamma \in \Gamma_{G_0}} \prod_{e \in \gamma} \pi_e^{s(e;G_0)}$$

Orientation (0 or 1)

Cycles (within graph)

Edges (within cycle)

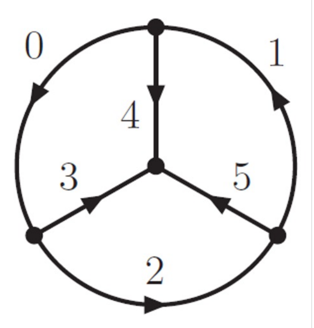
Projector

with

$$\pi_e^0 \equiv |0\rangle\langle 0|_e = \frac{1}{2}(I + Z)_e$$

$$\pi_e^1 \equiv |1\rangle\langle 1|_e = \frac{1}{2}(I - Z)_e$$

0 original direction
1 reversed direction



Need to fix a initial orientation, but results are independent of this choice!!

- **2nd approach (BETTER):** Promote adjacency matrix to operator, and *trace over all possible cycles*

$$A \equiv \sum_{e \equiv (v_0, v_1) \in \bar{E}} \left[\sigma_{v_0}^- \pi_e^0 \sigma_{v_1}^+ + \sigma_{v_1}^- \pi_e^1 \sigma_{v_0}^+ \right]$$

Edge = (origin, end)

Pauli +/- operators

$$H_G = \sum_{n=1}^{M_G} \mathrm{Tr}_V [A^n]$$

Trace over vertex space

n = cycle length

Clemente et al, PRD 108 (2023) 9, 096035
Sborlini, PRD 104 (2021) 3, 036014

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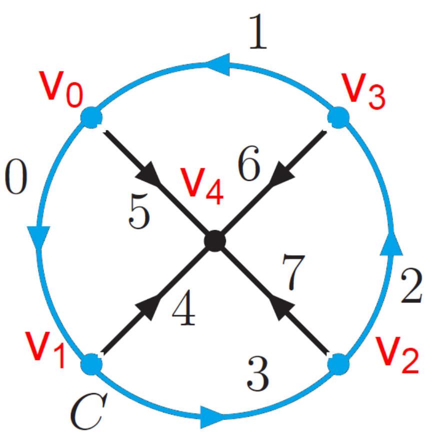
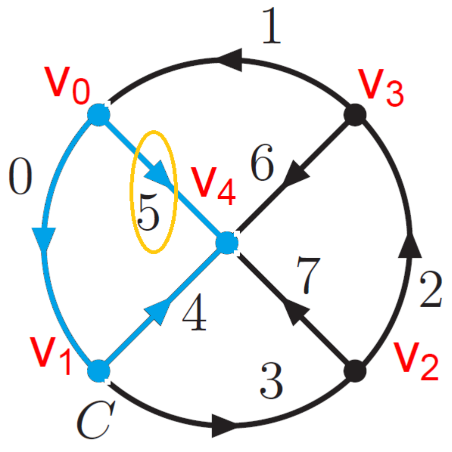
More details arXiv:2210.13240 [hep-ph]

Minimizing H, we find the acyclic graphs (0 energy)

Hamiltonian construction for minimization

- Exploit the **adjacency matrix** to build a **Hamiltonian** ➡ **Ground state = Acyclic graph**





A few comments:

- **For each cycle**, we have **1 penalization term**
- **0 (1) corresponds to the original (reversed)** direction w.r.t. the starting oriented graph
- Scaling depends on the connectivity of the graph: **lower bound (1) independent on the number of vertices V**

$$
\begin{aligned}
H_{\rm Top.C} = {} & 4\pi_0^0\,\pi_1^0\,\pi_2^0\,\pi_3^0 + 4\pi_0^1\,\pi_1^1\,\pi_2^1\,\pi_3^1 + 3\pi_0^1\,\pi_4^1\,\pi_5^0 + 5\pi_1^0\,\pi_2^0\,\pi_3^0\,\pi_4^1\,\pi_5^0 + 3\pi_0^0\,\pi_4^0\,\pi_5^1 \\
& + 5\pi_1^1\,\pi_2^1\,\pi_3^1\,\pi_4^0\,\pi_5^1 + 4\pi_0^1\,\pi_1^1\,\pi_4^1\,\pi_6^0 + 4\pi_2^0\,\pi_3^0\,\pi_4^1\,\pi_6^0 + 3\pi_1^1\,\pi_5^1\,\pi_6^0 + 5\pi_0^0\,\pi_2^0\,\pi_3^0\,\pi_5^1\,\pi_6^0 \\
& + 4\pi_0^0\,\pi_1^0\,\pi_4^0\,\pi_6^1 + 4\pi_2^1\,\pi_3^1\,\pi_4^0\,\pi_6^1 + 3\pi_1^0\,\pi_5^0\,\pi_6^1 + 5\pi_0^1\,\pi_2^1\,\pi_3^1\,\pi_5^0\,\pi_6^1 + 5\pi_0^1\,\pi_1^1\,\pi_2^1\,\pi_4^1\,\pi_7^0 \\
& + 3\pi_3^0\,\pi_4^1\,\pi_7^0 + 4\pi_1^1\,\pi_2^1\,\pi_5^1\,\pi_7^0 + 4\pi_0^0\,\pi_3^0\,\pi_5^1\,\pi_7^0 + 3\pi_2^1\,\pi_6^1\,\pi_7^0 + 5\pi_0^0\,\pi_1^0\,\pi_3^0\,\pi_6^1\,\pi_7^0 \\
& + 5\pi_0^0\,\pi_1^0\,\pi_2^0\,\pi_4^0\,\pi_7^1 + 3\pi_3^1\,\pi_4^0\,\pi_7^1 + 4\pi_1^0\,\pi_2^0\,\pi_5^0\,\pi_7^1 + 4\pi_0^1\,\pi_3^1\,\pi_5^0\,\pi_7^1 + 3\pi_2^0\,\pi_6^0\,\pi_7^1 \\
& + 5\pi_0^1\,\pi_1^1\,\pi_3^1\,\pi_6^0\,\pi_7^1
\end{aligned}
$$

Clemente et al, PRD 108 (2023) 9, 096035
Sborlini, PRD 104 (2021) 3, 036014

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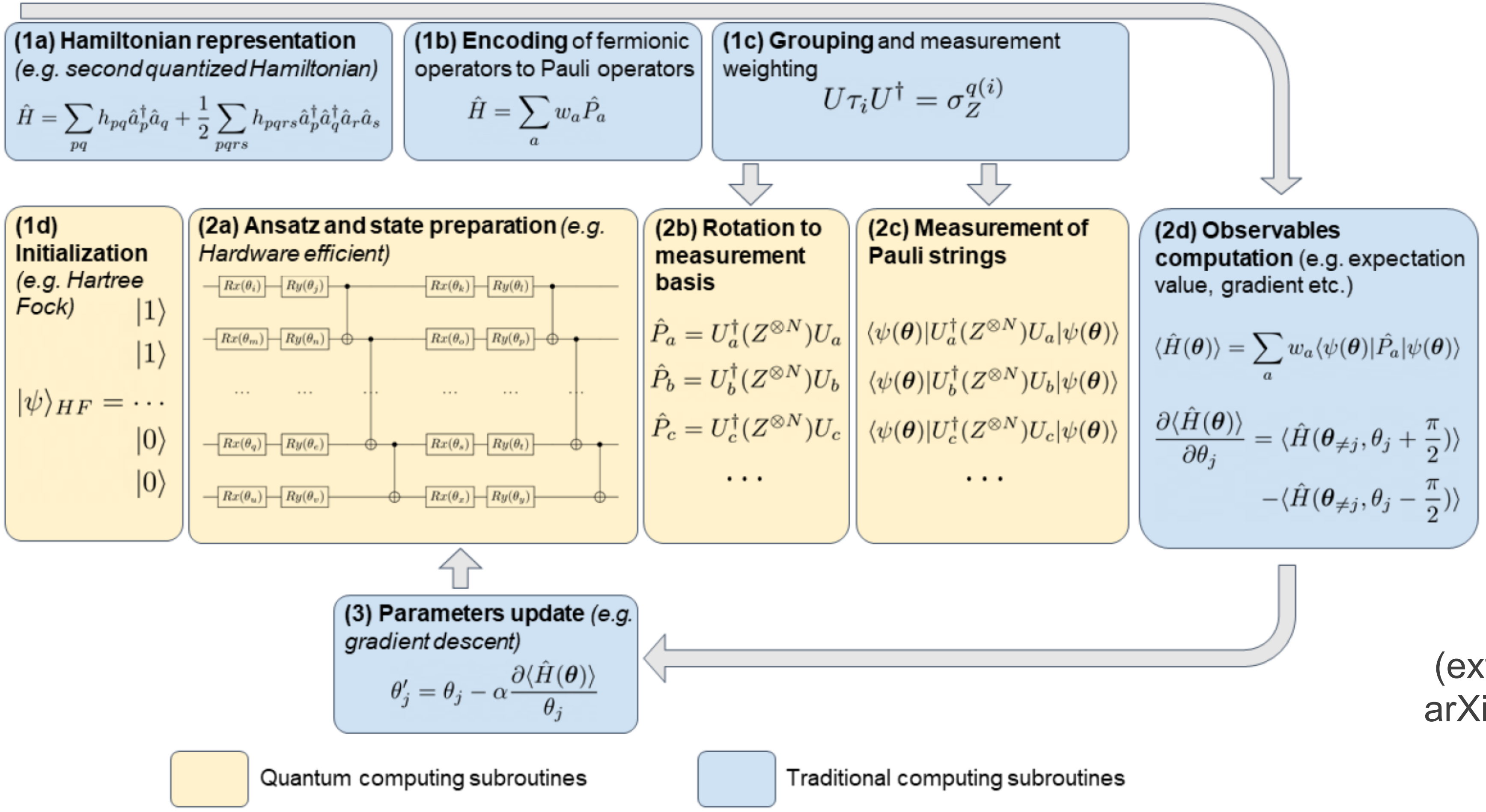
More details arXiv:2210.13240 [hep-ph]

Using a Variational Quantum Eigensolver

- *VQE is a hybrid quantum-classical algorithm, optimized for minimization problems*
- **<u>QUANTUM PART</u>: Evaluation of the Hamiltonian applied to an ansatz (parametrized quantum circuit)**
- **<u>CLASSICAL PART</u>: Modification of the parameters, through minimization algorithms**



VQE pipeline

(extracted from J. Tilly et al, arXiv:2111.05176 [quant-ph])

J. Tilly et al (2022), Phys. Rep. 986, 1-128

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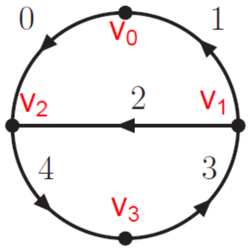
1. Our implementation with Qiskit: **Real Amplitudes** (ansatz) **+ COBYLA** (optimizer)
2. Improved results with **multi-run VQE (MrVQE)**: set a selection **threshold**, collect **solutions** and modify the Hamiltonian with **penalization terms**



Two eloop 5-point

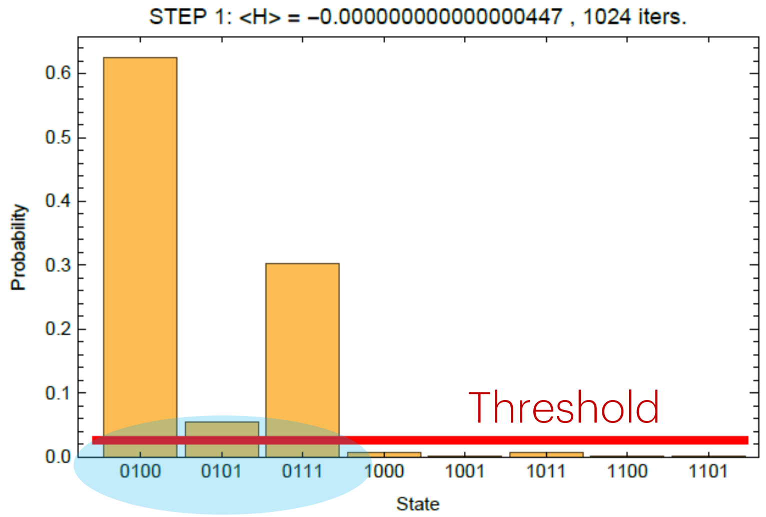
$$
\begin{aligned}
H_{E/\{e_0\}} = &\ 4\, I_4 \otimes I_3 \otimes I_2 \otimes I_1 + 2\, I_4 \otimes I_3 \otimes I_2 \otimes Z_1 - I_4 \otimes I_3 \otimes Z_2 \otimes I_1 \\
&- I_4 \otimes I_3 \otimes Z_2 \otimes Z_1 + I_4 \otimes Z_3 \otimes I_2 \otimes I_1 + I_4 \otimes Z_3 \otimes I_2 \otimes Z_1 \\
&+ 2\, I_4 \otimes Z_3 \otimes Z_2 \otimes I_1 + Z_4 \otimes I_3 \otimes I_2 \otimes I_1 + Z_4 \otimes I_3 \otimes I_2 \otimes Z_1 \\
&+ 2\, Z_4 \otimes I_3 \otimes Z_2 \otimes I_1 + 3\, Z_4 \otimes Z_3 \otimes I_2 \otimes I_1 + Z_4 \otimes Z_3 \otimes I_2 \otimes Z_1
\end{aligned}
$$

Reduced Hamiltonian

Classical methods

Complete set of solutions

$$\{|0011\rangle, |0100\rangle, |0101\rangle, |0111\rangle, |1000\rangle, |1001\rangle, |1011\rangle, |1100\rangle, |1101\rangle\}$$



$$\mathcal{S}_1 = \{|0100\rangle, |0101\rangle, |0111\rangle\}$$

Selected solutions (1st run)

Clemente et al, PRD 108 (2023) 9, 096035
Sborlini, PRD 104 (2021) 3, 036014

$$\Pi^{(1)} = \sum_{|\phi_l\rangle \in \mathcal{S}_1} b_l^{(1)} |\phi_l\rangle\langle\phi_l|$$

Penalization term ...

$$H^{(1)} = H^{(0)} + \Pi^{(1)}$$

... to be added to the Hamiltonian for next run!!

$$|\psi^{(1)}\rangle = \sum_j c_j^{(1)} |\phi_j\rangle$$

Approximated ground state found by VQE

$$\mathcal{S}_1 = \{|\phi_j\rangle \,|\, |c_j^{(1)}|^2 > \lambda\}$$

and subset of terms above the selection threshold

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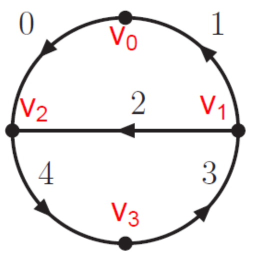
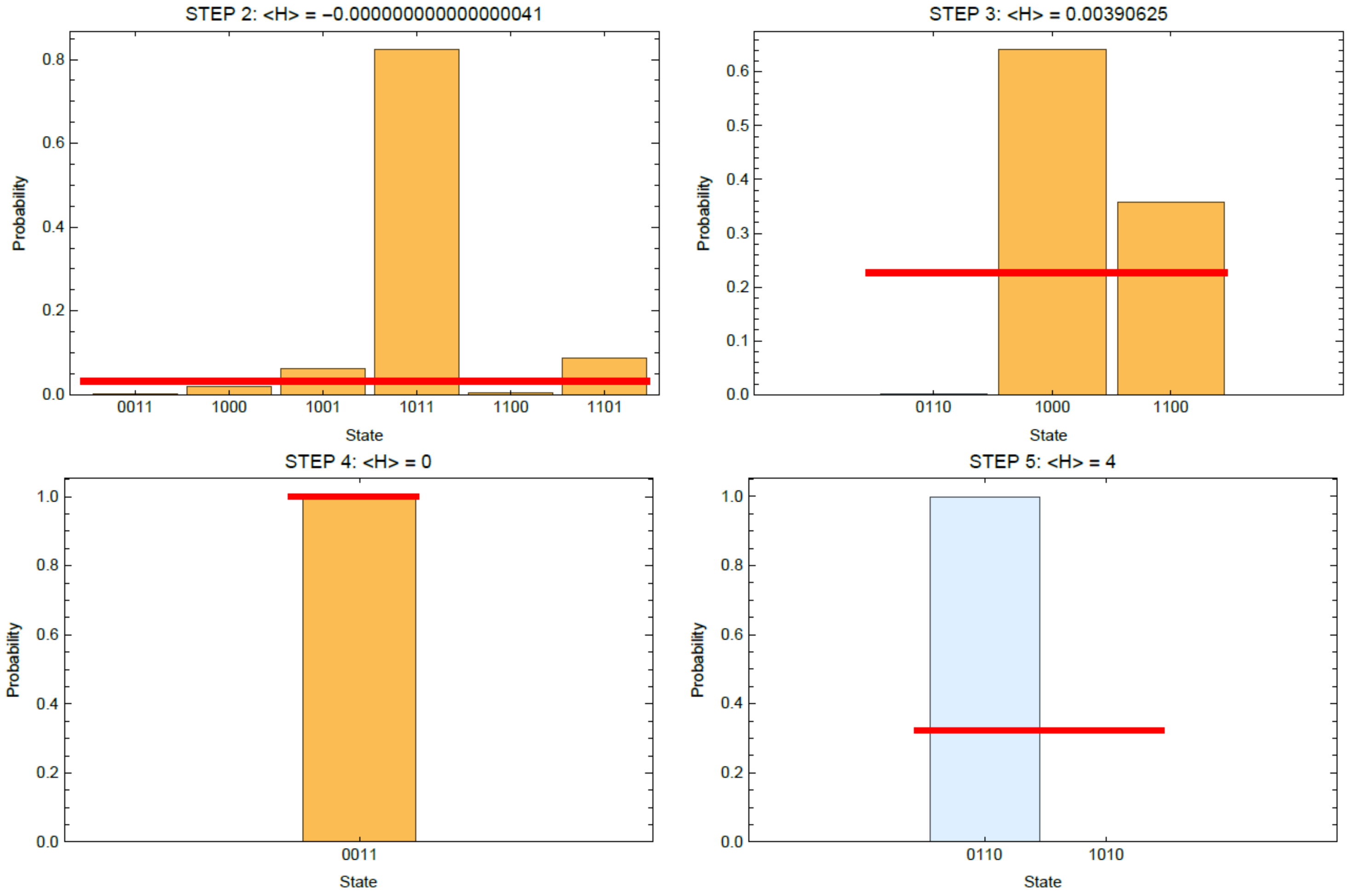
1. Our implementation with Qiskit: **Real Amplitudes** (ansatz) + **COBYLA** (optimizer)
2. Improved results with **multi-run VQE (MrVQE)**: set a selection **threshold**, collect **solutions** and modify the Hamiltonian with **penalization terms**



Two eloop 5-point



- We collect solutions step by step, till the algorithm converges (if <H> >1)
- *Problem:* it is not guaranteed that all the solutions are collected ~~(work in progress!!)~~ **IMPROVED!!!**

Clemente et al, PRD 108 (2023) 9, 096035
Sborlini, PRD 104 (2021) 3, 036014

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Conclusions

- **Nested residues leads to manifestly causal representations** in QFT
- **Geometrical rules** select **entangled thresholds. Complete reconstruction** of multiloop amplitudes!
- **LTD + causality** ➡ **New strategy to compute cross-sections from multi-loop vacuum diagrams** ➡ **Several loops required (3 loops = 2->2 tree-level process)**

More details in Rentería-Estrada's talk (20.07)

- **Quantum algorithms** to speed-up **causal flux selection**.
- **VQE** is a promising candidate to **calculate Feynman integrals/amplitudes from minimization problems!**
- **Optimized MrVQE can tackle complicated (several) multiloop diagrams**

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THANKS!
SCAN ME!

BACKUP

- **Causal representation** obtained directly after **summing over all the nested residues**

Master formula

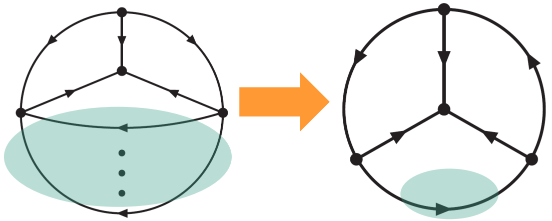
$$\mathcal{A}_N^{(L)}(1,\ldots,L+k) = \sum_{\sigma\in\Sigma} \int_{\vec{\ell}_1,\cdots,\vec{\ell}_L} \frac{\mathcal{N}_\sigma(\{q_{r,0}^{(+)}\},\{p_{j,0}\})}{x_{L+k}} \times \prod_{i=1}^{k} \frac{1}{-\lambda_{\sigma(i)}} + (\sigma \leftrightarrow \bar{\sigma})$$

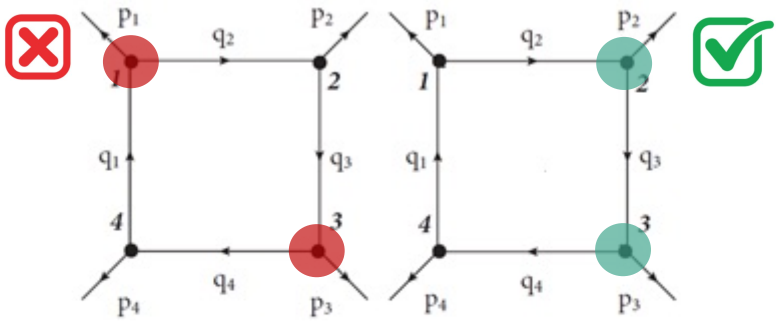
Set of entangled thresholds

Products of k causal propagators

- *Is it possible to do it in other way?* →
  - **Geometrical reconstruction** Sborlini '21
  - **Algebraic reconstruction (Lotty)** Torres Bobadilla '21

- **Important GEOMETRICAL concepts:**
  1. **Diagrams** are made of **vertices** and **multi-edges** *(bunches of propagators, connecting two given vertices)*
  2. **Multi-edges** define a **basis of momenta**, that lead to the **"vertex" and adjacency matrices** → **Defines the casual structure!**
  3. **Binary partitions** are given by **subsets of vertices** that **splits in two** the original diagram → **Connected partitions!**

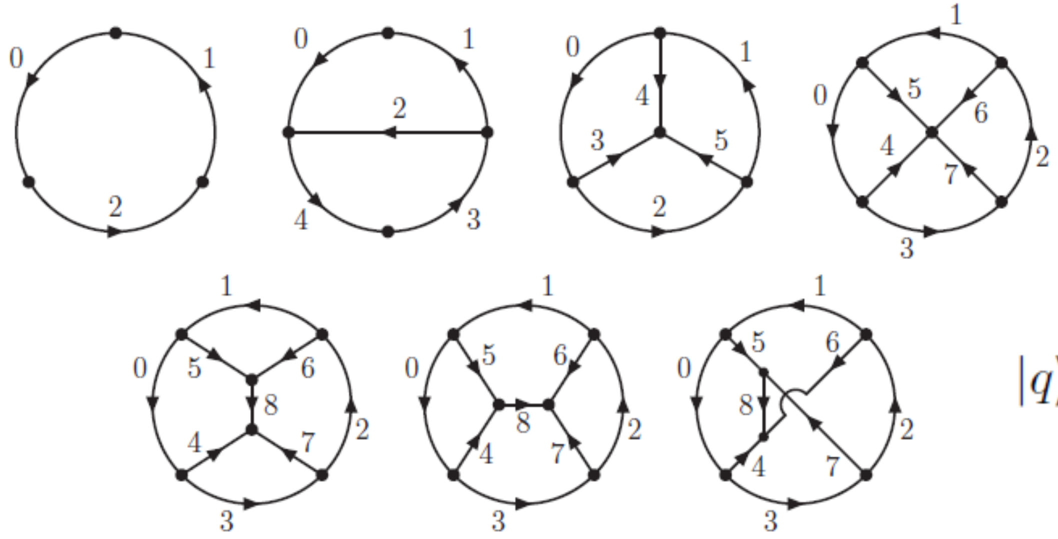




Sborlini, Phys.Rev.D 104 (2021) 3, 036014

Grover's algorithm for Causal Reconstruction

- **Identify momentum-orderings compatible with causality using Grover's search algorithm.**
- We assign **1 qubit to each edge**, and impose logical conditions to select configurations without closed directed cycles → **Non-cyclical configurations = Causal flux**
- **Important:** **"loop"** refers to **integration variables; "eloop"** to loops in the **graph**



Total number of orderings (n = nº of edges)

$$N = 2^n \rightarrow |q\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

Quantum superposition of N flux configurations

$$|q\rangle = \cos\theta\, |q_\perp\rangle + \sin\theta\, |w\rangle$$

$$|w\rangle = \frac{1}{\sqrt{r}} \sum_{x\in w} |x\rangle$$

States with causal flow = "Winning states"

$$|q_\perp\rangle = \frac{1}{\sqrt{N-r}} \sum_{x\notin w} |x\rangle$$

States with non-causal flow = "Orthogonal states"

Rodrigo, Ramírez-Uribe et al, JHEP 05 (2022) 100

- We use Grover's algorithm to **enhances** the probability of the **causal states**:

$$U_w = \boldsymbol{I} - 2|w\rangle\langle w|$$

Oracle operator (changes sign of causal states)

$$U_q = 2|q\rangle\langle q| - \boldsymbol{I}$$

Diffusion operator (reflects with respect to initial state)

$$\rightarrow (U_q U_w)^t |q\rangle = \cos\theta_t\, |q_\perp\rangle + \sin\theta_t\, |w\rangle$$

with $\sin^2\theta_t \sim 1$