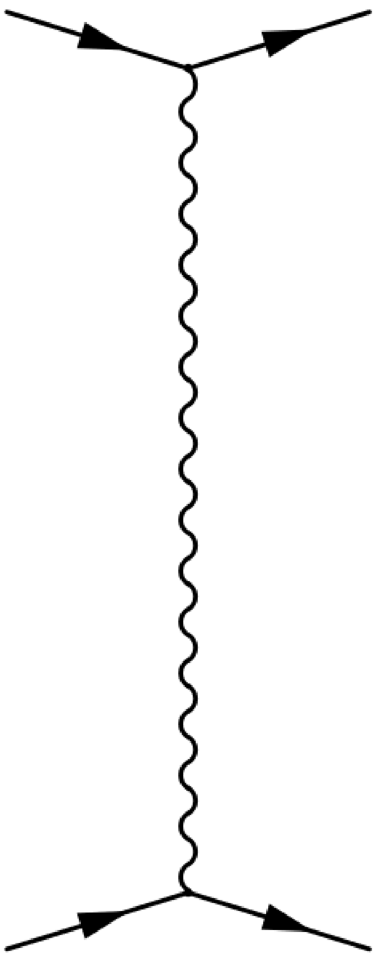
QCD corrections to Coulomb potential at vanishing transverse momenta

Martin Spousta

Charles University

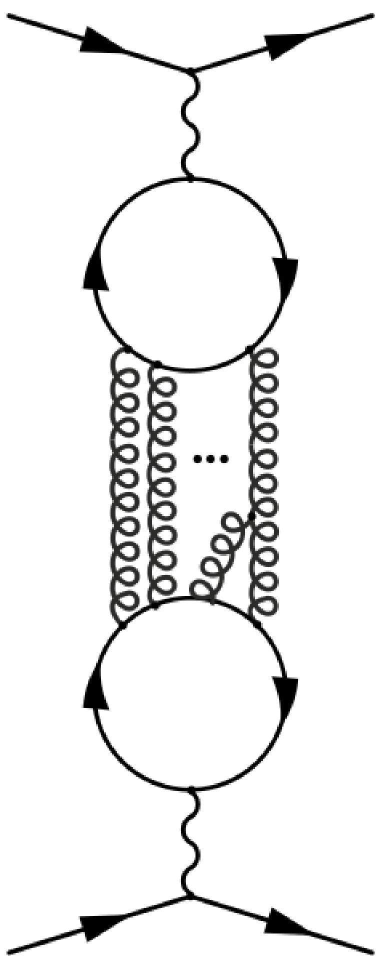
ICHEP 2024
Prague, 18-24 July 2024

Introduction



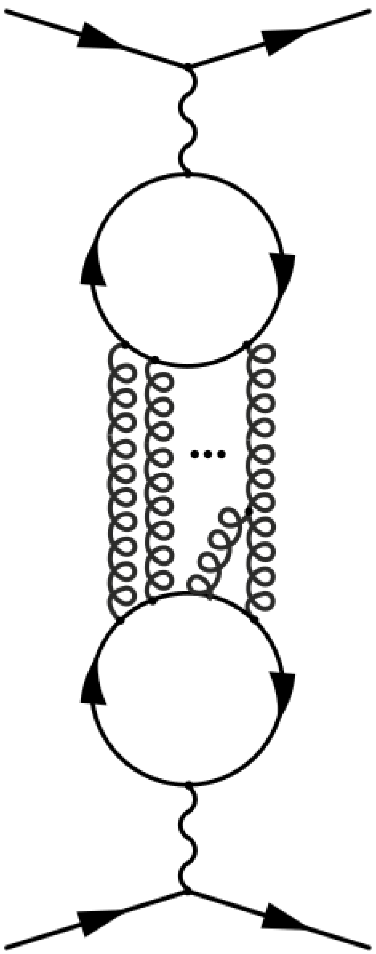
- Tree-level amplitude leading to Coulomb potential in $q^2 \to 0$ limit.

Introduction



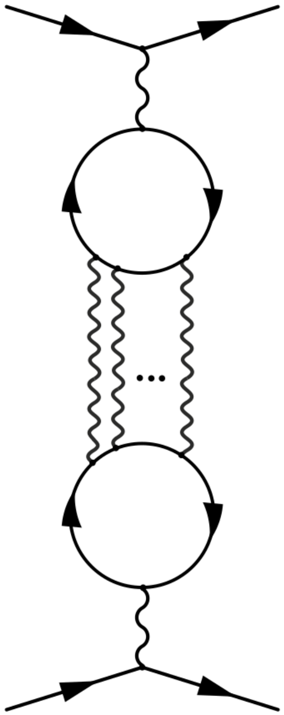
- Tree-level amplitude leading to Coulomb potential in $q^2 \to 0$ limit.
- How will the QCD corrections to the photon propagator affect the behavior in $q^2 \to 0$ limit?

Introduction



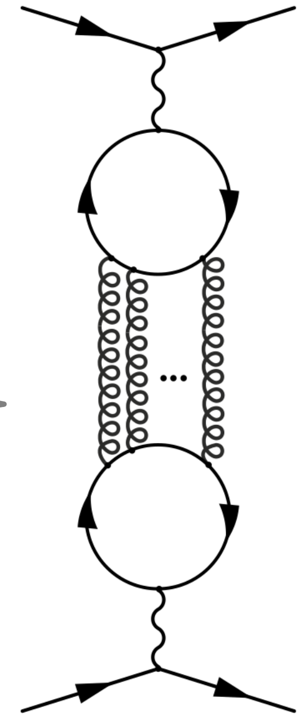
- Tree-level amplitude leading to Coulomb potential in $q^2 \to 0$ limit.
- How will the QCD corrections to the photon propagator affect the behavior in $q^2 \to 0$ limit?
- For $q^2 \gg 0$, these are local, diverging leading e.g. to running coupling.
- For $q^2 \to 0$, inter-loop gluon exchanges should be an important source of confining forces [arXiv:1604.08082].
- In many calculations [arXiv:0709.2877, arXiv:1408.5409, ...], $\alpha_S(q^2 \to 0) < 1$ => IR behavior can be investigated perturbatively.
- First, investigate amplitudes with 2 loops + multi-gluon exchanges in $q^2 \to 0$ limit in a systematic way.

General structure of amplitude



abelian case
(for reference)

non-abelian case



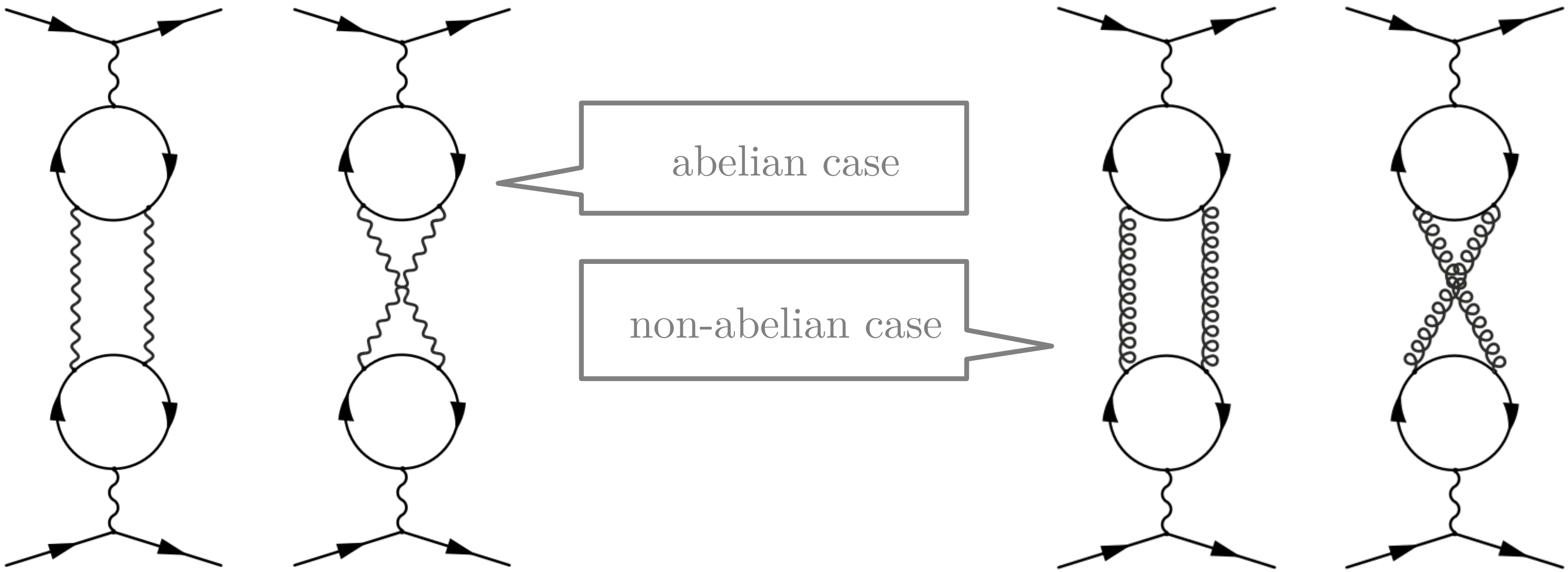
$$
\begin{aligned}
i\mathcal{M}^{\alpha\beta} &= (i\sqrt{a})^2 \frac{-ig^{\alpha\mu}}{q^2+i\epsilon} Y_{\mu\nu}(q) \frac{-ig^{\nu\beta}}{q^2+i\epsilon}, \\
Y_{\mu\nu}(q) &= \int \prod_i \mathrm{d}u_i \underbrace{A_{\mu,\mu_1\cdots\mu_{n-1}}(u_i,q)}_{\text{n-point quark loop}} \underbrace{B^{\mu_1\cdots\mu_{n-1},\nu_1\cdots\nu_{n'-1}}(u_i,q)}_{\text{bosonic exchanges}} \underbrace{A'_{\nu,\nu_1\cdots\nu_{n'-1}}(u_i,q)}_{\text{n'-point quark loop}}
\end{aligned}
$$

n-point
quark loop

bosonic exchanges

n'-point
quark loop

n=n'=3 contribution



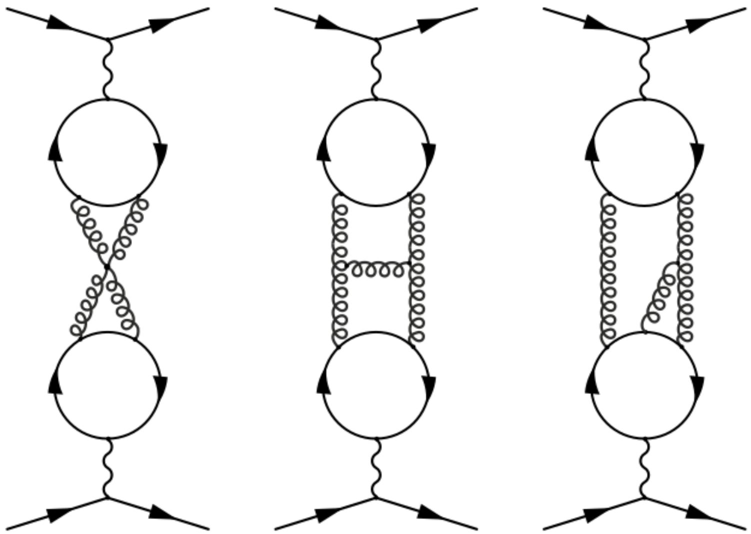
$$Y_{\mu\nu} = \alpha_{(\mathrm{s})}^2 \int \mathrm{d}^4 u_1 \int \mathrm{d}^4 p \, \mathrm{Tr}\left\{ \gamma_\mu \frac{i}{\not{p} + \not{q} - m} \gamma_{\mu_1} \frac{i}{\not{p} + \not{u}_1 + \not{q} - m} \gamma_{\mu_2} \frac{i}{\not{p} - m} \right\} \frac{-i g_{\mu_1 \nu_1}}{(u_1 + q)^2 - M^2} \frac{-i g_{\mu_2 \nu_2}}{u_1^2 - M^2}$$
$$\int \mathrm{d}^4 p' \, \mathrm{Tr}\left\{ \gamma_\nu \frac{i}{\not{p}' - m} \gamma_{\nu_2} \frac{i}{\not{p}' + \not{u}_1 + \not{q} - m} \gamma_{\nu_1} \frac{i}{\not{p}' + \not{q} - m} \right\},$$

$= 0$ → Furry's theorem

$= 0$ → If $u_{\mathrm{i}}^2 = M^2 = 0$ (abelian case)

$\neq 0$ → If $u_{\mathrm{i}}^2 = M^2 \neq 0$ (non-abelian case, IR gluon mass [arXiv:2201.09747])

n=3, n'=3,4 contribution



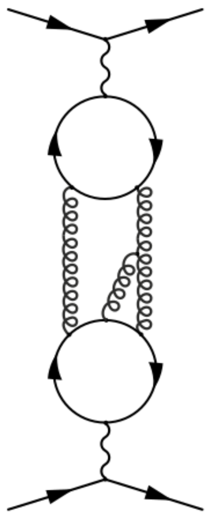
non-abelian only

- Calculations performed using Package-X + tailored Python interface
- Same limiting results as for n=n'=3

=> discuss the structure of solution for one particular amplitude

n=3, n'=4 contribution

$$Y_{\mu\nu}(q) \;=\; \int \prod_i \mathrm{d}u_i A_{\mu,\mu_1\cdots\mu_{n-1}}(u_i,q) B^{\mu_1\cdots\mu_{n-1},\nu_1\cdots\nu_{n'-1}}(u_i,q) A'_{\nu,\nu_1\cdots\nu_{n'-1}}(u_i,q)$$



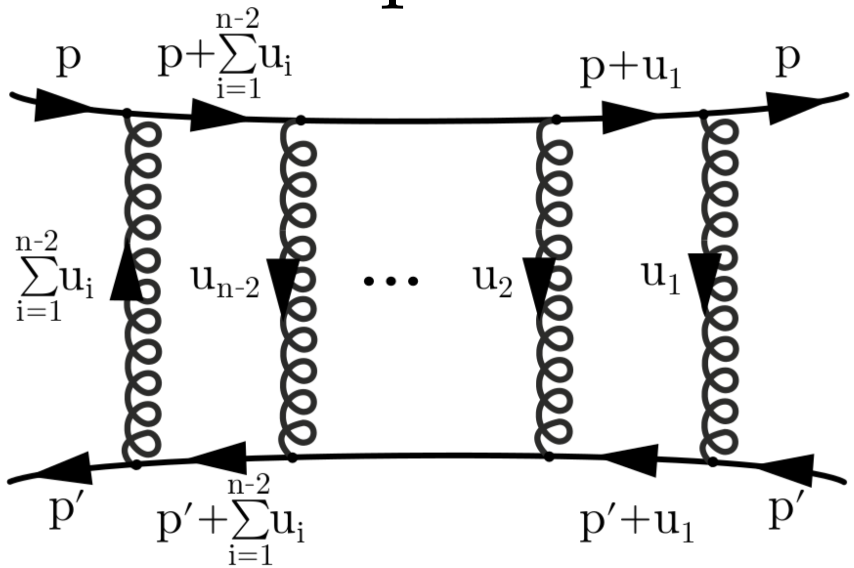
- After performing the contraction due to bosonic fields – several thousands terms.
- Some terms contain scalar product $u_\mathrm{i} \cdot u_\mathrm{j}$ inside PV coefficients.
- Currently possible to calculate only part of the amplitude with no scalar products:

$$Y^{\mathrm{part}}_{\mu\nu} = g_{\mu\nu}\, \alpha_\mathrm{s}^{\frac{5}{2}} \left[\sum_{j=0}^{4} c_{0j} \log^j \frac{\mu^2}{M^2} + \frac{1}{\epsilon} \sum_{j=0}^{3} c_{1j} \log^j \frac{\mu^2}{M^2} + \frac{1}{\epsilon^2} \sum_{j=0}^{2} c_{2j} \log^j \frac{\mu^2}{M^2} + \mathcal{O}(\epsilon) \right]$$

$$c_{ij} = c_{ij}(m^2, M^2, \log(m^2/M^2), \mathrm{PV}(m,M))$$

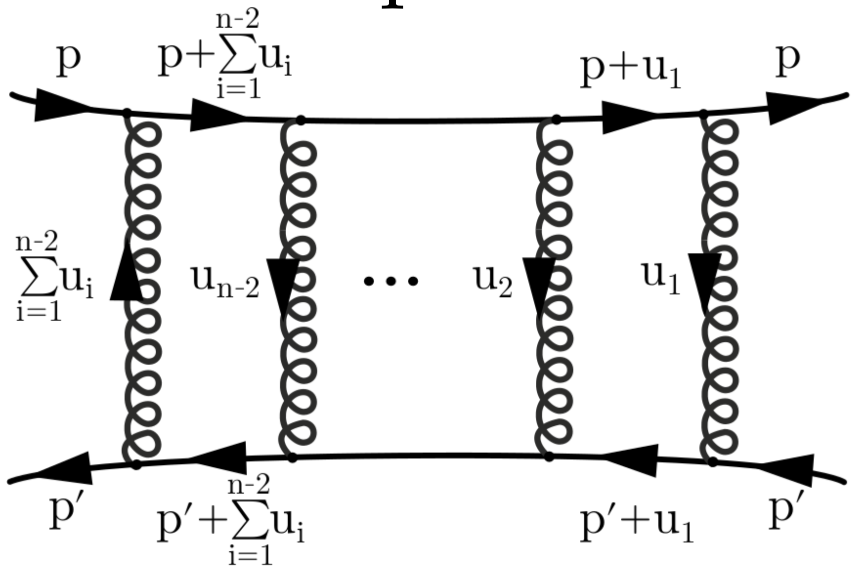
- Remarkably $Y^{\mathrm{part}}_{\mu\nu} \neq 0$ for $M^2 \neq 0$. This unexpected **IR finite behavior** can be generalized ...

Generalized partial solution



... for any n-even

Generalized partial solution



... for any n-even

$$Y^{\mathrm{part}}_{\mu\nu} \sim g_{\mu\nu}\,\alpha_s^{n-1}\left[M^2(1+\log M^2+\log^2 M^2)+\mathcal{O}(M^4)\right] \equiv g_{\mu\nu}\,f_M$$

... that is $\boldsymbol{Y^{\mathrm{part}}_{\mu\nu} \neq 0}$ **for** $\boldsymbol{M^2 \neq 0}$. Full solution for any n-even is then:

$$Y_{\mu\nu} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left(f_M(\alpha_\mathrm{s}, M) + f_{\mathbb{A}}(\alpha_\mathrm{s}, m, M, q) + f_{\mathbb{N}}(\alpha_\mathrm{s}, m, M, q)\right)$$

finite

analytical function of q

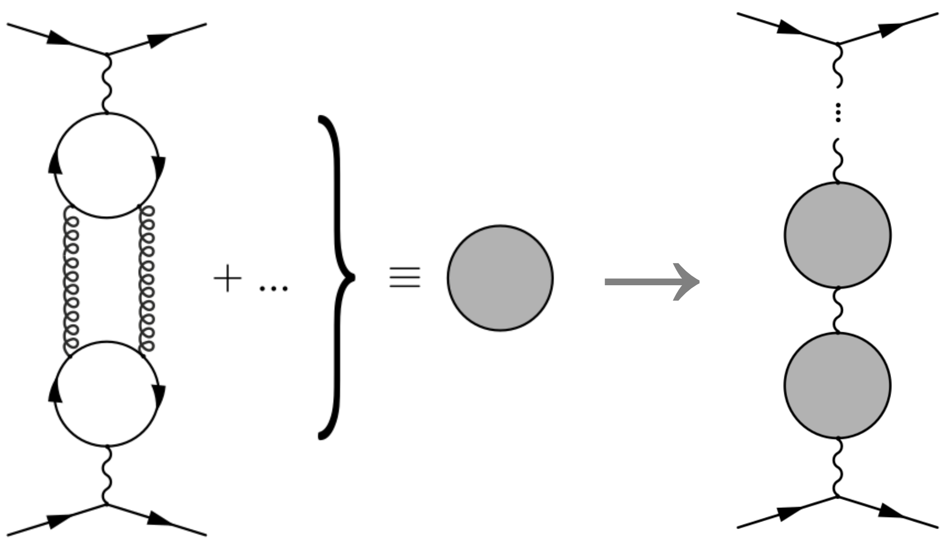
non-analytical function of q

Full solution: resummation

- Need to **resum** these contributions to get full solution.
- We empirically know that these loop interactions give rise to confining forces in the $q^2 \to 0$ limit which are different than Coulomb interaction => resum them **separately** from free photon propagator in $q^2 \to 0$ limit.
- Presence of **finite solution** allows to perform this resummation.

Full solution: resummation

- Need to **resum** these contributions to get full solution.
- We empirically know that these loop interactions give rise to confining forces in the $q^2 \to 0$ limit which are different than Coulomb interaction => resum them **separately** from free photon propagator in $q^2 \to 0$ limit.
- Presence of **finite solution** allows to perform this resummation.

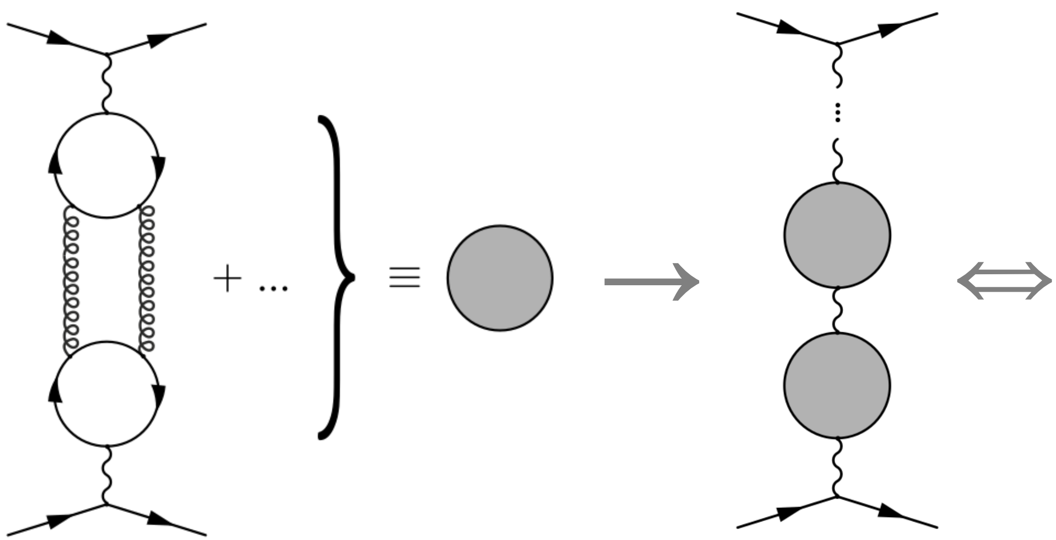


$$\Longleftrightarrow \quad \frac{1}{X} Y \frac{1}{X} + \frac{1}{X} Y \frac{1}{X} Y \frac{1}{X} + \ldots = $$
$$= \frac{1}{X} Y \left[\frac{1}{X - Y} \right]$$

Full solution: resummation

$$i\mathcal{M}^{\alpha\beta} = -g^{\alpha\beta}\left(\frac{ia}{q^2} - \frac{1}{f_M(\alpha_s, M) + f_{\mathbb{A},0}(\alpha_s, m, M) + f_{\mathbb{N}}(\alpha_s, m, M, q)} + \mathcal{O}(q)\right)$$

$q^2 \to 0$

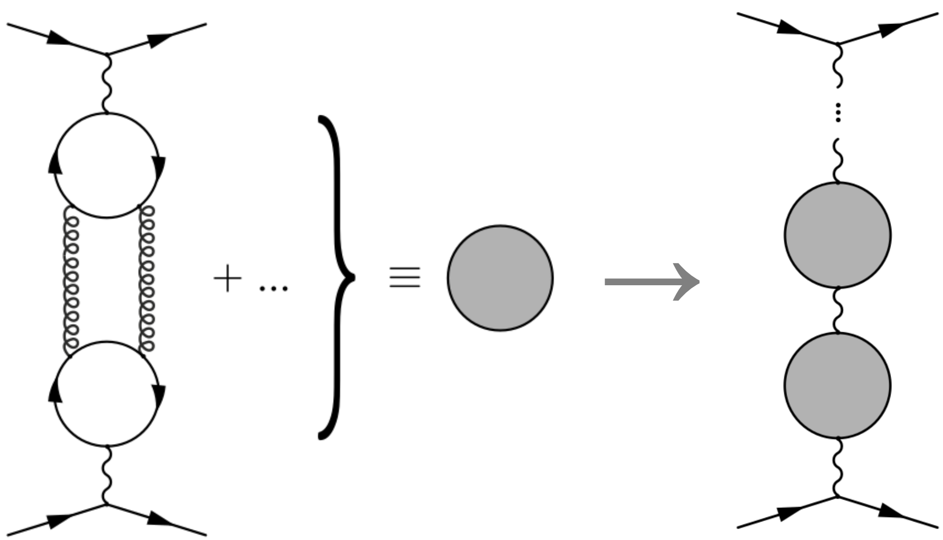


$$\frac{1}{X}Y\frac{1}{X} + \frac{1}{X}Y\frac{1}{X}Y\frac{1}{X} + \ldots =$$
$$= \frac{1}{X}Y\left[\frac{1}{X-Y}\right]$$

Full solution: resummation

$$i\mathcal{M}^{\alpha\beta} = -g^{\alpha\beta}\left(\frac{ia}{q^2} - \frac{1}{f_M(\alpha_s, M) + f_{\mathbb{A},0}(\alpha_s, m, M) + f_{\mathbb{N}}(\alpha_s, m, M, q)} + \mathcal{O}(q)\right)$$

$q^2 \to 0$



$$\Longleftrightarrow \frac{1}{X}Y\frac{1}{X} + \frac{1}{X}Y\frac{1}{X}Y\frac{1}{X} + \ldots =$$

$$= \frac{1}{X}Y\left[\frac{1}{X-Y}\right]$$

Full solution: resummation

$$i\mathcal{M}^{\alpha\beta} = -g^{\alpha\beta}\left(\frac{ia}{q^2} - \frac{1}{f_M(\alpha_s, M) + f_{\mathbb{A},0}(\alpha_s, m, M) + f_{\mathbb{N}}(\alpha_s, m, M, q)} + \mathcal{O}(q)\right)$$

Change in the **sign** wrt Coulomb interaction

Exact $\boldsymbol{1/r}$ behavior of resulting potential:

- due to the presence of $\boldsymbol{f_M}$, $\boldsymbol{f_{\mathbb{N}}}$ can have any q^2 behavior and any μ^2 dependence
- no $1/r^{\mathrm{k}}$ corrections to $1/r$

Can be generalized (backup), but sufficient to see for one class of amplitudes.

Sign of the interaction

	Yukawa		electromagnetic			
	$f\,f$	$f\,\bar{f}$	$f\,f$	$f\,\bar{f}$	$f_1^-\,f_2^+$	$f_1^-\,\bar{f}_2^-$
fermion current	$+1$	-1	$+1$	$+1$	$+1$	$+1$
intermediate particle	$+1$	$+1$	-1	-1	-1	-1
normal product	-1	$+1$	-1	$+1$	-1	-1
charge	$+1$	$+1$	$+1$	$+1$	-1	$+1$
$(-1)\cdot$ total	$+1$	$+1$	-1	$+1$	$+1$	-1

- Sign of Coulomb interaction is **complex** (!)

Sign of the interaction

	Yukawa		"residual"			
	$f\,f$	$f\,\bar{f}$	$f\,f$	$f\,\bar{f}$	$f_1^-\,f_2^+$	$f_1^-\,\bar{f}_2^-$
fermion current	+1	−1	+1	+1	+1	+1
intermediate particle	+1	+1	−1	−1	−1	−1
normal product	−1	+1	−1	+1	−1	−1
charge	+1	+1	+1	+1	+1	+1
summation			−1	−1	−1	−1
$(-1)\cdot$ total	+1	+1	+1	−1	+1	+1

- Sign of Coulomb interaction is **complex** (!)
- Probing **strongly interacting degrees of freedom** in fermion's wave function => electromagnetic charge is irrelevant.

Sign of the interaction

	Yukawa		"residual"			
	ff	$f\bar{f}$	ff	$f\bar{f}$	$f_1^- f_2^+$	$f_1^- \bar{f}_2^-$
fermion current	$+1$	-1	$+1$	$+1$	$+1$	$+1$
intermediate particle	$+1$	$+1$	-1	-1	-1	-1
normal product	-1	$+1$	-1	$+1$	-1	-1
charge	$+1$	$+1$	$+1$	$+1$	$+1$	$+1$
summation			-1	-1	-1	-1
$(-1)\cdot$ total	$+1$	$+1$	$+1$	-1	$+1$	$+1$

- Sign of Coulomb interaction is **complex** (!)
- Probing **strongly interacting degrees of freedom** in fermion's wave function => electromagnetic charge is irrelevant.
- Total sign of the "QCD residual interaction": **attractive except for fermion-anti-fermion** which is repulsive.

Strength of the interaction

- Strength given by the hadronic part of elmg. coupling:

$$\alpha(q^2) = \frac{\alpha_0}{1 - \Delta\alpha(q^2)} = \alpha_0[1 + \Delta\alpha(q^2)_{\rm had} + \\ + \Delta\alpha(q^2)_{\rm lep} + \Delta\alpha(q^2)_{\rm top} + \mathcal{O}(\Delta\alpha^2)]$$

$$\alpha_0 \Delta\alpha(q^2)_{\rm had} \equiv a(q^2)$$

- Magnitude of a can be estimated using **alphaQED** package [arXiv:1905.05078].
- The ratio of a to fine structure constant α_0 is:

$$a / \alpha_0 = 10^{-20} \quad \text{... for eV scale}$$

$$a / \alpha_0 = 10^{-26} \quad \text{... for meV scale}$$

(i.e. not far from the ratio of gravity to elmg. interaction let's say for e^{-})

Where it can act

- Terrestrial experiments have no sensitivity to measure it (20 orders of magnitude smaller than Coulomb force).
- It can play a role in **large-scale astrophysical objects** due a presence of QED plasma in thermal equilibrium, e.g.
  - Intracluster medium (**ICM**) in clusters of galaxies
  - Warm ionized medium (**WIM**) in interstellar space
- Magnitude of residual force in a Debye volume of plasma:

$$\Phi_{\mathrm{deb}}(r) = \frac{1}{4\pi\epsilon_0}\frac{e}{r}\exp\left(-\frac{\sqrt{2}r}{\lambda_{\mathrm{D}}}\right) \rightarrow F_{\mathrm{res}}(\lambda_{\mathrm{D}}) = 2n_i e \int^{\lambda_{\mathrm{D}}} \frac{\partial\Phi_{\mathrm{deb}}(r)}{\partial r}\mathrm{d}^3x\Big|_{\alpha_0\to a(q^2)}$$

- **Comparison to gravity** within Debye volume:

$F_{\mathrm{res}} \sim 10^{-40}$ GeV/fm $\quad F_{\mathrm{G}} \sim 10^{-61}$ GeV/fm ... for ICM

$F_{\mathrm{res}} \sim 10^{-46}$ GeV/fm $\quad F_{\mathrm{G}} \sim 10^{-62}$ GeV/fm ... for WIM

Summary + speculation

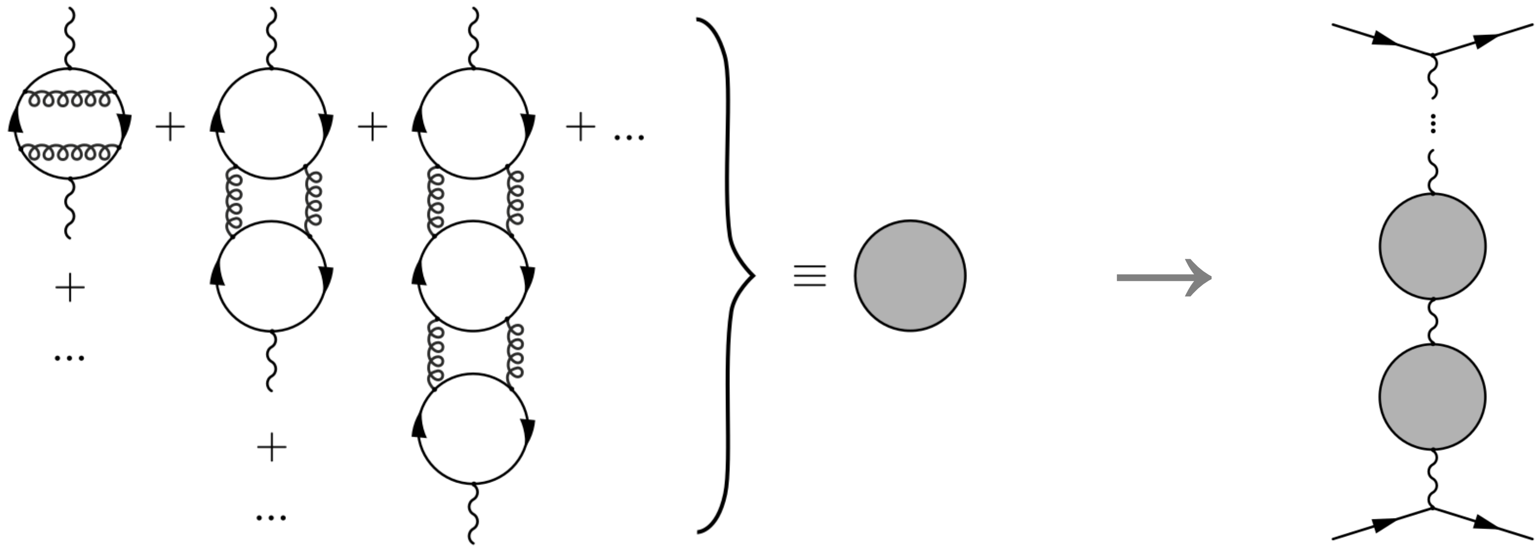
- Basic properties of residual interaction:
  - Potential with exact **1/r** behavior.
  - **Attractive** for all fermions except for fermion-anti-fermion.
  - **Universal** – each particle has a hadronic part in its wave function (including e.g. neutrina).
  - **Not localized** to a single point but has an intrinsic spacial structure for $q^2 \nrightarrow 0$ (c.f. polarization tensor).
  - **Small interaction strength** (not completely far from that of gravity).
  - Linear **dependence on mass** (cf. normalization of bispinors – backup).
- Speculation: Can this be a new candidate for an **emergent gravity**?
  - Only if virtual photons are universally present unaffected by multipole structure of charged particles.
  - If not, still important for large scale astrophysical objects: ICM, WIM, etc. and possibly a part of DM puzzle.

More information

- For details see EPJP 139 (2024) 5, 374 + publication in preparation
- Little more in the backup

Backup slides

Generalization



$$\frac{1}{X}Y\frac{1}{X}+\frac{1}{X}Y\frac{1}{X}Y\frac{1}{X}+\ldots=\frac{1}{X}Y\left[\frac{1}{X-Y}\right]$$

$$\frac{y}{q^2(q^2-y)}=-\frac{1}{q^2}+\frac{1}{q^2-y}=-\frac{1}{q^2}+\frac{1}{q^2-y}=-\frac{1}{q^2}-\frac{1}{y}\left(1+\mathcal{O}(q^2)\right)$$

- Why **not to** resum them to the photon propagator? Because we empirically know that they (or their subclass) give rise to confining forces in the $q^2 \to 0$ limit (which are different than Coulomb force).

$\alpha_s(0)$ I.

	$\alpha_s(0)$	IR behavior	Definition	RS	Gauge	Quarks	Framework	References
	3.2	freezes	gh.-gl. vertex	MOM	Landau	Yes	Lattice	[331]
	4.45	freezes	gh.-gl. vertex	MOM	Landau	No	Lattice	[329]
●	0.5	freezes	pQCD form	MOM	Indep.	No	Lattice	[329]
●	0.29 ± 0.03	freezes	Q-Q pot.	V	Indep.	No	Lattice	[212]
●	0.40	freezes	Q-Q pot.	V	Indep.	Yes	Lattice	[212]
	3.3 ± 0.7	freezes	gh.-gl. vertex	(τ)	Landau	No	Lattice $\Lambda = 1.1$	[237]
●	0	vanishes	gh.-gl. vertex	MOM	Landau	No	Lattice	see caption
●	0	vanishes	gh.-gl. vertex	MOM	Landau	Yes	Lattice	[264,276]
	2.5 ± 0.5	freezes	gh.-gl., q.-gl. v.	MOM	Land. Coul.	Yes	Lattice	[258]
●	0.52	freezes	Q-Q pot.	V	Indep.	Yes	Lat. pert. stoch. th.	[338]
	diverges	$e^{m/Q}$	Schrod. func.	$\overline{MS}$	–	Yes	Lattice	[341]
	2.97	freezes	gh.-gl. vertex	–	indep.	Yes	Stoch. Quant.	[244,348]
	∞	$1/Q^2$	pQCD form	V	Coulomb	Yes	Variational ap.	[163]
●	0.71	freezes	pQCD form	$\overline{MS}$	Indep.	Yes	Pheno.	[222]
	∞	$1/Q^2$	pQCD form	V	Indep.	Yes	Pheno.	[294]
●	0.58	freezes	pQCD form	V	Indep.	Yes	Pheno.	[221],
	∞	$1/Q^2$	pQCD form	$\overline{MS}$	Indep.	Yes	Pheno.	[226]
●	0.42 ± 0.03	(Av. IR val.)	pQCD form	–	Indep.	Yes	Pheno.	[206]
●	$\simeq 0.6$	freezes	pQCD form	–	Indep.	Yes	Pheno.	[206]

A. Deur et al., Prog. Part. Nucl. Phys. 90 (2016) 1–74 [arXiv:1604.08082]

23 out of 39 papers in the review suggest $\mathbf{0 \leq \alpha_s(0) < 1}$.

$\alpha_s(0)$ II.

$\alpha_s(0)$	IR behavior	Definition	RS	Gauge	Quarks	Framework	References
● 0.78	mono. incr.	pQCD form	$\overline{MS}$	Indep.	Yes	Pheno.	[374]
● 0.60	freezes	pQCD form	($\overline{MS}$)	Indep.	Yes	Pheno. $\Lambda = 0.2$	[219]
● 0.82	freezes	pQCD form	($\overline{MS}$)	Indep.	Yes	Pheno.	[292]
diverges	$1/Q^2$	pQCD form	V	Indep.	Yes	Pheno.	[224]
● 0.83	freezes	pQCD form	–	Indep.	Yes	Pheno.	[220]
● 0.83	freezes	α_R	Indep.	Indep.	Yes	Opt. Pert. Theo.	[388]
1.77	freezes	gh.-gl. vertex	$\overline{MS}$, MOM	Landau	Yes	Gribov–Zwanziger	[351,214]
7.7 ± 2	freezes	–	–	Landau	No	FRG	[343]
2.97	freezes	gh.-gl. vertex	–	Landau	No	FRG	[257]
~ 4	freezes	–	Indep.	Indep.	Yes	Analytic Ap. $\Lambda = 0.56$	[166]
1.25	mono. decr.	–	($\overline{MS}$)	Indep.	Yes	Analytic Ap. $\Lambda = 0.32$	[362,359]
diverges	$1/Q^2$	–	Indep.	Indep.	Yes	Analytic Approach	[368,377]
● 0.47	freezes	–	Indep.	Indep.	Yes	Analytic Approach	[378]
● 0.37–0.60	freezes	pQCD form	$\overline{MS}$	Indep.	No	BPT	[218,386]
● 0.81	freezes	pQCD form	V	Indep.	Yes	BPT	[45]
● 0.97 ± 0.44	freezes	pQCD form	V	Indep.	Yes	BPT	[45,387]
● 0.50 ± 0.08	freezes	pQCD form	$\overline{MS}$	indep.	Yes	duality.	[389]
● 0	vanishes	gh.-gl. vertex	–	–	No	ϕ^4-YM mapping	[390]
● 0	vanishes	pQCD form	$\overline{MS}$	indep.	No	Bogoliubov comp.	[170]
● 0	vanishes	pQCD form	$\overline{MS}$	Landau	Yes	Curci–Ferrari Model	[392]

A. Deur et al., Prog. Part. Nucl. Phys. 90 (2016) 1–74 [arXiv:1604.08082]

23 out of 39 papers in the review suggest $\mathbf{0 \leq \alpha_s(0) < 1}$.

Normalization

- **Coulomb case**. Full amplitude, including also initial and final state description

$$i\mathcal{M} = \frac{-ie^2}{4\pi\epsilon_0\hbar c}\,\frac{1}{|\mathbf{q}|^2}\,(2m\xi'^{\dagger}\xi)_p(2m\xi'^{\dagger}\xi)_k$$

- Non-relativistic Born approximation:

$$\langle p'|iT|p\rangle = -i\tilde{\mathcal{V}}(\mathbf{q})\,(2\pi)\,\delta(E_{p'} - E_p)$$

- **Mass factors** (and $\hbar c$) in relativistic amplitude **dropped manually** to match non-relativistic conventions.

Open questions

- Can implementing such attractive force into **astrophysical simulations** help with interpretation of the data? Could it be a part of the dark matter puzzle?
- Can we prove that this force is not an **emergent gravity**?
- Can we improve on the **magnitude of hadronic part** of α_0 at (m)eV scales to have better estimates strength of the force (e.g. using lattice QCD)?

Open questions

- Can implementing such attractive force into **astrophysical simulations** help with interpretation of the data? Could it be a part of the dark matter puzzle?
- Can we prove that this force is not an **emergent gravity**?
- Can we improve on the **magnitude of hadronic part** of α_0 at (m)eV scales to have better estimates strength of the force (e.g. using lattice QCD)?