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Alexander von HUMBOLDT STIFTUNG

Renormalization Group Equations for the Dim-7 SMEFT Operators

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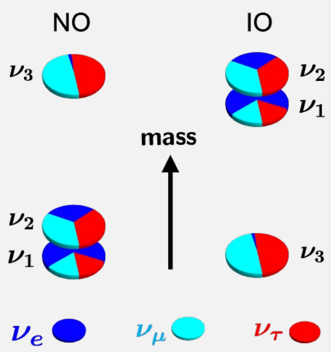
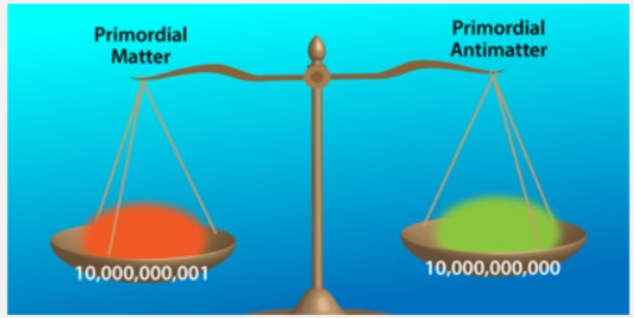
19 July 2024

The SMEFT and Operator Basis

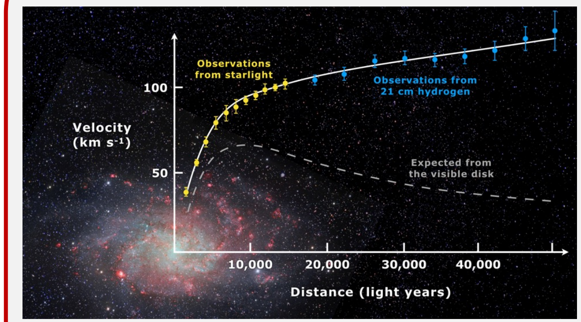
The Standard Model Effective Field Theory



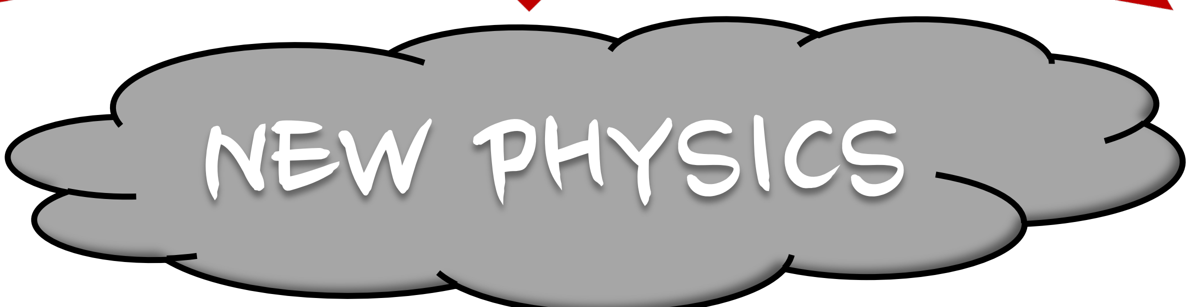
Neutrino Masses
Lepton Flavor Mixing



Matter-antimatter Asymmetry of the Universe



Dark Matter



- ☐ **Ultraviolet-complete Models**
- ☐ **Effective Field Theories**

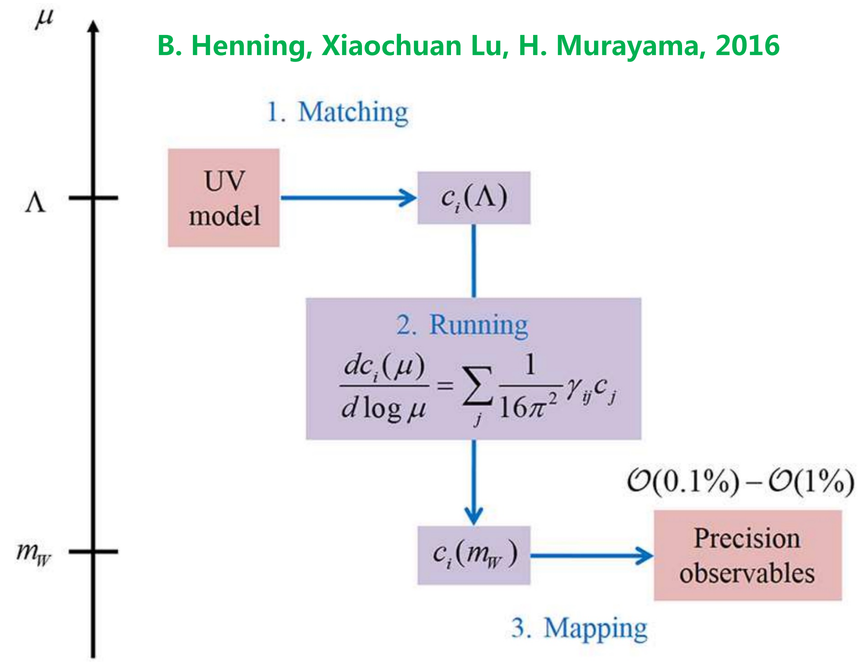
The Standard Model Effective Field Theory

The energy scale of new physics (NP) is much larger than the electroweak (EW) scale

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{d=5}^{\infty} \sum_{i=1}^{n_d} \left(\frac{1}{\Lambda^{d-4}} C_i^{(d)} \mathcal{O}_i^{(d)} + \mathrm{h.c.} \right)$$

W. Buchmuller, D. Wyler, 1986; B. Grzadkowski et al, 2010

Latest review: G. Isidori, F. Wilsch, D. Wyler, 2023



The SMEFT operators:

- consist of the **SM fields**
- satisfy the **SM gauge symmetries** and the Lorentz invariance
- capture all **indirect consequences** of NP

- **Indirectly** and **model-independently** investigate the low-energy consequences of NP
- Simply improve the quality of the **convergence of perturbation theory** with **multiple physical scales** via matching and RG running
- Reduce repeated calculations for different UV models: **running** and **mapping** only need to be done **once** in the SMEFT, though **matching** differs for **UV models**.

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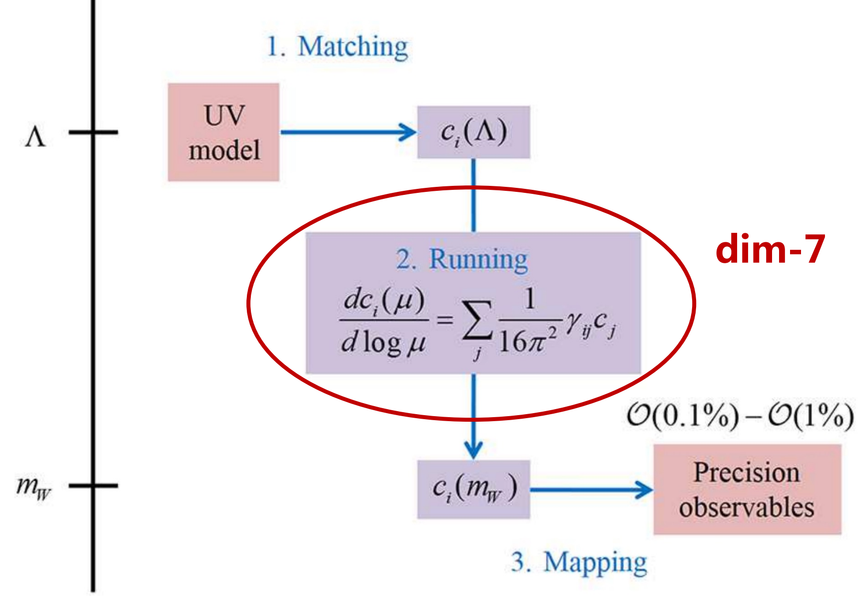
B. Henning, Xiaochuan Lu, H. Murayama, 2016



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The Physical Basis for Dim-7 Operators

Original "Basis" for dim-7 operators: **L. Lehman, 2014**

$\psi^2 H^4 + \text{h.c.}$		$\psi^2 H^3 D + \text{h.c.}$	
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^i C L^m) H^j H^n (H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon_{ij}\epsilon_{mn}(L^i C \gamma_\mu e) H^j H^m i D^\mu H^n$
$\psi^2 H^2 D^2 + \text{h.c.}$		$\psi^2 H^2 X + \text{h.c.}$	
$\mathcal{O}_{LHD1}$	$\epsilon_{ij}\epsilon_{mn}(L^i C D^\mu L^j) H^m (D_\mu H^n)$	$\mathcal{O}_{LHB}$	$\epsilon_{ij}\epsilon_{mn}(L^i C \sigma_{\mu\nu} L^m) H^j H^n B^{\mu\nu}$
$\mathcal{O}_{LHD2}$	$\epsilon_{im}\epsilon_{jn}(L^i C D^\mu L^j) H^m (D_\mu H^n)$	$\mathcal{O}_{LHW}$	$\epsilon_{ij}(\epsilon\tau^I)_{mn}(L^i C \sigma_{\mu\nu} L^m) H^j H^n W^{I\mu\nu}$
$\psi^4 D + \text{h.c.}$		$\psi^4 H + \text{h.c.}$	
$\mathcal{O}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C i D^\mu L^j)$	$\mathcal{O}_{\bar{e}LLLH}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^j C L^m) H^n$
$\mathcal{O}_{\bar{L}QddD}$	$(\bar{L}\gamma_\mu Q)(d C i D^\mu d)$	$\mathcal{O}_{\bar{d}LQLH1}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j C L^m) H^n$
$\mathcal{O}_{\bar{e}dddD}$	$(\bar{e}\gamma_\mu d)(d C i D^\mu d)$	$\mathcal{O}_{\bar{d}LQLH2}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j C L^m) H^n$
		$\mathcal{O}_{\bar{d}LueH}$	$\epsilon_{ij}(\bar{d}L^i)(u C e) H^j$
		$\mathcal{O}_{\bar{Q}uLLH}$	$\epsilon_{ij}(\bar{Q}u)(L C L^i) H^j$
		$\mathcal{O}_{\bar{L}dud\tilde{H}}$	$(\bar{L}d)(u C d)\tilde{H}$
		$\mathcal{O}_{\bar{L}dddH}$	$(\bar{L}d)(d C d) H$
		$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\epsilon_{ij}(\bar{e}Q^i)(d C d)\tilde{H}^j$
		$\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$\epsilon_{ij}(\bar{L}d)(Q C Q^i)\tilde{H}^j$
$\mathcal{O}^{(2)}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C \sigma^{\mu\nu} D_\nu L^j)$	$\mathcal{O}_{\bar{L}dQdD}$	$(\bar{L} i D^\mu d)(Q C \gamma_\mu d)$

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$\mathcal{O}_{\bar{L}QddD}$	$(\bar{L}\gamma_\mu Q)(d C iD^\mu d)$	$\mathcal{O}_{\bar{d}LQLH1}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j C L^m) H^n$
$\mathcal{O}_{\bar{e}dddD}$	$(\bar{e}\gamma_\mu d)(d C iD^\mu d)$	$\mathcal{O}_{\bar{d}LQLH2}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j C L^m) H^n$
		$\mathcal{O}_{\bar{d}LueH}$	$\epsilon_{ij}(\bar{d}L^i)(u C e) H^j$
		$\mathcal{O}_{\bar{Q}uLLH}$	$\epsilon_{ij}(\bar{Q}u)(L C L^i) H^j$
		$\mathcal{O}_{\bar{L}dud\tilde{H}}$	$(\bar{L}d)(u C d)\tilde{H}$
		$\mathcal{O}_{\bar{L}dddH}$	$(\bar{L}d)(d C d) H$
		$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\epsilon_{ij}(\bar{e}Q^i)(d C d)\tilde{H}^j$
		$\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$\epsilon_{ij}(\bar{L}d)(Q C Q^i)\tilde{H}^j$
$\mathcal{O}^{(2)}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C\sigma^{\mu\nu} D_\nu L^j)$	$\mathcal{O}_{\bar{L}dQdD}$	$(\bar{L} iD^\mu d)(Q C\gamma_\mu d)$

$$\psi_1 C \psi_2 \equiv \overline{\psi_1^c}\psi_2$$

Modified "Basis" : remove two **redundant** operators **Y. Liao, X. D. Ma, 2016**

$$\mathcal{O}^{(2)prst}_{\bar{d}uLLD} = 2(Y_e)_{tu}\mathcal{O}^{psru}_{\bar{d}LueH} - \mathcal{O}^{prst}_{\bar{d}uLLD} \qquad \mathcal{O}^{prst}_{\bar{L}QddD} = \left(Y_d^\dagger\right)_{tu}\mathcal{O}^{psru}_{\bar{L}dQQ\tilde{H}} + \mathcal{O}^{ptrs}_{\bar{L}dQdD}.$$

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Non-trivial flavor relations

$\psi^2 H^4 + \text{h.c.}$		$\psi^2 H^3 D + \text{h.c.}$	
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$\mathcal{O}_{\bar{L}QddD}$	$(\bar{L}\gamma_\mu Q)(d C i D^\mu d)$	$\mathcal{O}_{\bar{d}LQLH1}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j C L^m) H^n$
$\mathcal{O}_{\bar{e}dddD}$	$(\bar{e}\gamma_\mu d)(d C i D^\mu d)$	$\mathcal{O}_{\bar{d}LQLH2}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j C L^m) H^n$
		$\mathcal{O}_{\bar{d}LueH}$	$\epsilon_{ij}(\bar{d}L^i)(u C e) H^j$
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		$\mathcal{O}_{\bar{L}dud\tilde{H}}$	$(\bar{L}d)(u C d)\tilde{H}$
		$\mathcal{O}_{\bar{L}dddH}$	$(\bar{L}d)(d C d) H$
		$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\epsilon_{ij}(\bar{e}Q^i)(d C d)\tilde{H}^j$
		$\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$\epsilon_{ij}(\bar{L}d)(Q C Q^i)\tilde{H}^j$
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The story continues...

The Physical Basis for Dim-7 Operators

Non-trivial flavor relations: appear first at dimension seven

Y. Liao, X. D. Ma, 2019

Class	Operator	Flavor relations
$\psi^2 H^4$	$\mathcal{O}_{LH}$	$\mathcal{O}_{LH}^{pr} - p \leftrightarrow r = 0$
$\psi^2 H^3 D$	$\mathcal{O}_{LeHD}$	$\times$
$\psi^2 H^2 D^2$	$\mathcal{O}_{LHD1}$	$(\mathcal{O}_{LDH1}^{pr} + \mathcal{K}^{pr}) - p \leftrightarrow r = 0$
	$\mathcal{O}_{LHD2}$	$\left[4\mathcal{O}_{LHD2}^{pr} + 2(Y_e)_{rv}\mathcal{O}_{LeHD}^{pv} - \mathcal{O}_{LHW}^{pr} + 2\mathcal{K}^{pr}\right] - p \leftrightarrow r = \mathcal{O}_{LHB}^{pr}$
$\psi^2 H^2 X$	$\mathcal{O}_{LHB}$	$\mathcal{O}_{LHB}^{pr} + p \leftrightarrow r = 0$
	$\mathcal{O}_{LHW}$	$\times$
$\psi^4 H$	$\mathcal{O}_{\bar{e}LLLH}$	$(\mathcal{O}_{\bar{e}LLLH}^{prst} + r \leftrightarrow t) - r \leftrightarrow s = 0$
	$\mathcal{O}_{\bar{d}LQLH1}$	$\times$
	$\mathcal{O}_{\bar{d}LQLH2}$	$\times$
	$\mathcal{O}_{\bar{d}LueH}$	$\times$
	$\mathcal{O}_{\bar{Q}uLLH}$	$\times$
$\psi^4 D$	$\mathcal{O}_{\bar{d}uLLD}$	$\left[\mathcal{O}_{\bar{d}uLLD}^{prst} + (Y_d)_{vp}\mathcal{O}_{\bar{Q}uLLH}^{vrst} - (Y_u^\dagger)_{rv}\mathcal{O}_{\bar{d}LQLH2}^{psvt}\right] - s \leftrightarrow t = 0$
$\psi^4 H$	$\mathcal{O}_{\bar{L}dud\tilde{H}}$	$\times$
	$\mathcal{O}_{\bar{L}dddH}$	$\mathcal{O}_{\bar{L}dddH}^{prst} + s \leftrightarrow t = 0, \quad \mathcal{O}_{\bar{L}dddH}^{prst} + \mathcal{O}_{\bar{L}dddH}^{pstr} + \mathcal{O}_{\bar{L}dddH}^{ptrs} = 0$
	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{prst} + s \leftrightarrow t = 0$
	$\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$\times$
$\psi^4 D$	$\mathcal{O}_{\bar{L}QddD}$	$\left[\mathcal{O}_{\bar{L}QddD}^{prst} + (Y_u)_{rv}\mathcal{O}_{\bar{L}dud\tilde{H}}^{psvt}\right] - s \leftrightarrow t = -(Y_e^\dagger)_{vp}\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{vrst} - (Y_d)_{rv}\mathcal{O}_{\bar{L}dddH}^{pvst}$
	$\mathcal{O}_{\bar{e}dddD}$	$\mathcal{O}_{\bar{e}dddD}^{prst} - r \leftrightarrow s = (Y_d^\dagger)_{tv}\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{pvrs}$
		$(\mathcal{O}_{\bar{e}dddD}^{prst} + r \leftrightarrow t) - s \leftrightarrow t = (Y_e)_{vp}\mathcal{O}_{\bar{L}dddH}^{vrst}$

Table 2. Flavor relations for dim-7 operators. The symbol $\times$ indicates lack of such a relation.

$$\mathcal{K}^{pr} = (Y_u)_{vw}\mathcal{O}_{\bar{Q}uLLH}^{vwpr} - (Y_d^\dagger)_{vw}\mathcal{O}_{\bar{d}LQLH2}^{vpwr} - (Y_e^\dagger)_{vw}\mathcal{O}_{\bar{e}LLLH}^{vwpr}.$$

Operators with covariant derivative and repeated fermion fields

Induced by the EoMs

Not all degrees of freedom in operators are independent

The Physical Basis for Dim-7 Operators

A physical basis gets rid of all redundant degrees of freedom: Y. Liao, X. D. Ma, 2019

in the basis: for the subset $L=2,\ B=0$,

$$\frac{1}{2}(\mathcal{O}_{LH}^{pr}+\mathcal{O}_{LH}^{rp}),\quad \mathcal{O}_{LeHD}^{pr},\quad \frac{1}{2}(\mathcal{O}_{LHD1}^{pr}+\mathcal{O}_{LHD1}^{rp}),\quad \frac{1}{2}(\mathcal{O}_{LHD2}^{pr}+\mathcal{O}_{LHD2}^{rp}),\quad \frac{1}{2}(\mathcal{O}_{LHB}^{pr}-\mathcal{O}_{LHB}^{rp}),$$
$$\mathcal{O}_{LHW}^{pr},\quad \mathcal{O}_{\bar{d}LQLH1}^{prst},\quad \mathcal{O}_{\bar{d}LQLH2}^{prst},\quad \mathcal{O}_{\bar{d}LueH}^{prst},\quad \mathcal{O}_{\bar{Q}uLLH}^{prst},\quad \frac{1}{2}(\mathcal{O}_{\bar{d}uLLD}^{prst}+\mathcal{O}_{\bar{d}uLLD}^{prst}),$$
$$\frac{1}{4}(\mathcal{O}_{\bar{e}LLLH}^{prst}+\mathcal{O}_{\bar{e}LLLH}^{ptsr}+\mathcal{O}_{\bar{e}LLLH}^{psrt}+\mathcal{O}_{\bar{e}LLLH}^{ptrs})\ \text{(with at least two of } r,s,t \text{ being equal)},$$
$$\mathcal{O}_{\bar{e}LLLH}^{prst},\ \mathcal{O}_{\bar{e}LLLH}^{prts},\ \mathcal{O}_{\bar{e}LLLH}^{psrt},\ \mathcal{O}_{\bar{e}LLLH}^{pstr}\ (\text{for } r<s<t), \tag{2.3}$$

and for the subset $B=-L=1$,

$$\mathcal{O}_{\bar{L}dud\tilde{H}}^{prst},\quad \frac{1}{2}(\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{prst}-\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{prts}),\quad \mathcal{O}_{\bar{L}dQQ\tilde{H}}^{prst},\quad \frac{1}{2}(\mathcal{O}_{\bar{L}QddD}^{prst}+\mathcal{O}_{\bar{L}QddD}^{prts}),$$
$$\frac{1}{2}(\mathcal{O}_{\bar{L}dddH}^{prst}-\mathcal{O}_{\bar{L}dddH}^{prts})\ \text{(with at least two of } r,s,t \text{ being equal)},$$
$$\mathcal{O}_{\bar{L}dddH}^{prst},\ \mathcal{O}_{\bar{L}dddH}^{pstr}\ (\text{for } r<s<t),\ \frac{1}{6}(\mathcal{O}_{\bar{e}dddD}^{prst}+5 \text{ permutations of } (r,s,t)), \tag{2.4}$$

The Physical Basis for Dim-7 Operators

A physical basis gets rid of all redundant degrees of freedom: Y. Liao, X. D. Ma, 2019

in the basis: for the subset $L=2,\ B=0$,

$$\frac{1}{2}(\mathcal{O}_{LH}^{pr}+\mathcal{O}_{LH}^{rp}),\quad \mathcal{O}_{LeHD}^{pr},\quad \frac{1}{2}(\mathcal{O}_{LHD1}^{pr}+\mathcal{O}_{LHD1}^{rp}),\quad \frac{1}{2}(\mathcal{O}_{LHD2}^{pr}+\mathcal{O}_{LHD2}^{rp}),\quad \frac{1}{2}(\mathcal{O}_{LHB}^{pr}-\mathcal{O}_{LHB}^{rp}),$$
$$\mathcal{O}_{LHW}^{pr},\quad \mathcal{O}_{\bar{d}LQLH1}^{prst},\quad \mathcal{O}_{\bar{d}LQLH2}^{prst},\quad \mathcal{O}_{\bar{d}LueH}^{prst},\quad \mathcal{O}_{\bar{Q}uLLH}^{prst},\quad \frac{1}{2}(\mathcal{O}_{\bar{d}uLLD}^{prst}+\mathcal{O}_{\bar{d}uLLD}^{prst}),$$
$$\boxed{\begin{array}{l}\frac{1}{4}(\mathcal{O}_{\bar{e}LLLH}^{prst}+\mathcal{O}_{\bar{e}LLLH}^{ptsr}+\mathcal{O}_{\bar{e}LLLH}^{psrt}+\mathcal{O}_{\bar{e}LLLH}^{ptrs})\ \text{(with at least two of } r,s,t \text{ being equal)},\\ \mathcal{O}_{\bar{e}LLLH}^{prst},\ \mathcal{O}_{\bar{e}LLLH}^{prts},\ \mathcal{O}_{\bar{e}LLLH}^{psrt},\ \mathcal{O}_{\bar{e}LLLH}^{pstr}\ \text{(for } r<s<t),\end{array}} \tag{2.3}$$

and for the subset $B=-L=1$,

$$\mathcal{O}_{\bar{L}dud\tilde{H}}^{prst},\quad \frac{1}{2}(\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{prst}-\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{prts}),\quad \mathcal{O}_{\bar{L}dQQ\tilde{H}}^{prst},\quad \frac{1}{2}(\mathcal{O}_{\bar{L}QddD}^{prst}+\mathcal{O}_{\bar{L}QddD}^{prts}),$$
$$\boxed{\frac{1}{2}(\mathcal{O}_{\bar{L}dddH}^{prst}-\mathcal{O}_{\bar{L}dddH}^{prts})\ \text{(with at least two of } r,s,t \text{ being equal)},}$$
$$\boxed{\mathcal{O}_{\bar{L}dddH}^{prst},\ \mathcal{O}_{\bar{L}dddH}^{pstr}\ \text{(for } r<s<t),}\ \frac{1}{6}(\mathcal{O}_{\bar{e}dddD}^{prst}+5\ \text{permutations of }(r,s,t)), \tag{2.4}$$

- **Constraints on the flavor indices of operators**
- **Inconvenient for one-loop matching, derivation of RGEs, phenomenological studies, etc.**

Question: A more convenient physical basis for dim-7 operators?

The Physical Basis for Dim-7 Operators

	$\psi^2 H^4$	
$\mathcal{O}_{\ell H}^{(S)\alpha\beta} = \frac{1}{2}\left(\mathcal{O}_{\ell H}^{\alpha\beta} + \mathcal{O}_{\ell H}^{\beta\alpha}\right)$	$C_{\ell H}^{(S)\alpha\beta} = \frac{1}{2}\left(C_{\ell H}^{\alpha\beta} + C_{\ell H}^{\beta\alpha}\right)$	$\frac{1}{2}n(n+1)$
	$\psi^2 H^3 D$	
$\mathcal{O}_{\ell eHD}^{\alpha\beta}$	$C_{\ell eHD}^{\alpha\beta}$	n^2
	$\psi^2 H^2 D^2$	
$\mathcal{O}_{\ell HD1}^{(S)\alpha\beta} = \frac{1}{2}\left(\mathcal{O}_{\ell HD1}^{\alpha\beta} + \mathcal{O}_{\ell HD1}^{\beta\alpha}\right)$	$C_{\ell HD1}^{(S)\alpha\beta} = \frac{1}{2}\left(C_{\ell HD1}^{\alpha\beta} + C_{\ell HD1}^{\beta\alpha}\right)$	$\frac{1}{2}n(n+1)$
$\mathcal{O}_{\ell HD2}^{(S)\alpha\beta} = \frac{1}{2}\left(\mathcal{O}_{\ell HD2}^{\alpha\beta} + \mathcal{O}_{\ell HD2}^{\beta\alpha}\right)$	$C_{\ell HD2}^{(S)\alpha\beta} = \frac{1}{2}\left(C_{\ell HD2}^{\alpha\beta} + C_{\ell HD2}^{\beta\alpha}\right)$	$\frac{1}{2}n(n+1)$
	$\psi^2 H^2 X$	
$\mathcal{O}_{\ell HB}^{(A)\alpha\beta} = \frac{1}{2}\left(\mathcal{O}_{\ell HB}^{\alpha\beta} - \mathcal{O}_{\ell HB}^{\beta\alpha}\right)$	$C_{\ell HB}^{(A)\alpha\beta} = \frac{1}{2}\left(C_{\ell HB}^{\alpha\beta} - C_{\ell HB}^{\beta\alpha}\right)$	$\frac{1}{2}n(n-1)$
$\mathcal{O}_{\ell HW}^{\alpha\beta}$	$C_{\ell HW}^{\alpha\beta}$	n^2
	$\psi^4 H$	
$\mathcal{O}_{\bar{e}\ell\ell\ell H}^{(S)\alpha\beta\gamma\lambda} = \frac{1}{6}\left(\mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\beta\gamma} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\lambda\beta} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\lambda\gamma} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta}\right)$	$C_{\bar{e}\ell\ell\ell H}^{(S)\alpha\beta\gamma\lambda} = \frac{1}{6}\left(C_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + C_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\beta\gamma} + C_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\lambda\beta} + C_{\bar{e}\ell\ell\ell H}^{\alpha\beta\lambda\gamma} + C_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} + C_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta}\right)$	$\frac{1}{6}n^2(n+1)(n+2)$
$\mathcal{O}_{\bar{e}\ell\ell\ell H}^{(A)\alpha\beta\gamma\lambda} = \frac{1}{6}\left(\mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\beta\gamma} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\lambda\beta} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\lambda\gamma} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta}\right)$	$C_{\bar{e}\ell\ell\ell H}^{(A)\alpha\beta\gamma\lambda} = \frac{1}{6}\left(C_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + C_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\beta\gamma} + C_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\lambda\beta} - C_{\bar{e}\ell\ell\ell H}^{\alpha\beta\lambda\gamma} - C_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} - C_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta}\right)$	$\frac{1}{6}n^2(n-1)(n-2)$
$\mathcal{O}_{\bar{e}\ell\ell\ell H}^{(M)\alpha\beta\gamma\lambda} = \frac{1}{3}\left(\mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\lambda\beta}\right)$	$C_{\bar{e}\ell\ell\ell H}^{(M)\alpha\beta\gamma\lambda} = \frac{1}{6}\left(2C_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} - C_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\beta\gamma} - C_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\lambda\beta} + C_{\bar{e}\ell\ell\ell H}^{\alpha\beta\lambda\gamma} + C_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} - 2C_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta}\right)$	$\frac{1}{3}n^2(n-1)(n+1)$
$\mathcal{O}_{\bar{d}\ell q\ell H1}^{\alpha\beta\gamma\lambda}$	$C_{\bar{d}\ell q\ell H1}^{\alpha\beta\gamma\lambda}$	n^4
$\mathcal{O}_{\bar{d}\ell q\ell H2}^{\alpha\beta\gamma\lambda}$	$C_{\bar{d}\ell q\ell H2}^{\alpha\beta\gamma\lambda}$	n^4
$\mathcal{O}_{\bar{d}\ell ueH}^{\alpha\beta\gamma\lambda}$	$C_{\bar{d}\ell ueH}^{\alpha\beta\gamma\lambda}$	n^4
$\mathcal{O}_{\bar{q}u\ell\ell H}^{\alpha\beta\gamma\lambda}$	$C_{\bar{q}u\ell\ell H}^{\alpha\beta\gamma\lambda}$	n^4
$\mathcal{O}_{\bar{e}dud\tilde{H}}^{\alpha\beta\gamma\lambda}$	$C_{\bar{e}dud\tilde{H}}^{\alpha\beta\gamma\lambda}$	n^4
$\mathcal{O}_{\bar{\ell}dddH}^{(M)\alpha\beta\gamma\lambda} = \frac{1}{3}\left(\mathcal{O}_{\bar{\ell}dddH}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{\ell}dddH}^{\alpha\gamma\beta\lambda} - \mathcal{O}_{\bar{\ell}dddH}^{\alpha\beta\lambda\gamma} - \mathcal{O}_{\bar{\ell}dddH}^{\alpha\lambda\beta\gamma}\right)$	$C_{\bar{\ell}dddH}^{(M)\alpha\beta\gamma\lambda} = \frac{1}{6}\left(2C_{\bar{\ell}dddH}^{\alpha\beta\gamma\lambda} - C_{\bar{\ell}dddH}^{\alpha\lambda\beta\gamma} - C_{\bar{\ell}dddH}^{\alpha\gamma\lambda\beta} - 2C_{\bar{\ell}dddH}^{\alpha\beta\lambda\gamma} + C_{\bar{\ell}dddH}^{\alpha\gamma\beta\lambda} + C_{\bar{\ell}dddH}^{\alpha\lambda\gamma\beta}\right)$	$\frac{1}{3}n^2(n-1)(n+1)$
$\mathcal{O}_{\bar{e}qdd\tilde{H}}^{(A)\alpha\beta\gamma\lambda} = \frac{1}{2}\left(\mathcal{O}_{\bar{e}qdd\tilde{H}}^{\alpha\beta\gamma\lambda} - \mathcal{O}_{\bar{e}qdd\tilde{H}}^{\alpha\beta\lambda\gamma}\right)$	$C_{\bar{e}qdd\tilde{H}}^{(A)\alpha\beta\gamma\lambda} = \frac{1}{2}\left(C_{\bar{e}qdd\tilde{H}}^{\alpha\beta\gamma\lambda} - C_{\bar{e}qdd\tilde{H}}^{\alpha\beta\lambda\gamma}\right)$	$\frac{1}{2}n^3(n-1)$
$\mathcal{O}_{\bar{\ell}dqq\tilde{H}}^{\alpha\beta\gamma\lambda}$	$C_{\bar{\ell}dqq\tilde{H}}^{\alpha\beta\gamma\lambda}$	n^4
	$\psi^4 D$	
$\mathcal{O}_{\bar{e}dddD}^{(S)\alpha\beta\gamma\lambda} = \frac{1}{6}\left(\mathcal{O}_{\bar{e}dddD}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\beta\gamma} + \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\lambda\beta} + \mathcal{O}_{\bar{e}dddD}^{\alpha\beta\lambda\gamma} + \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\beta\lambda} + \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\gamma\beta}\right)$	$C_{\bar{e}dddD}^{(S)\alpha\beta\gamma\lambda} = \frac{1}{6}\left(C_{\bar{e}dddD}^{\alpha\beta\gamma\lambda} + C_{\bar{e}dddD}^{\alpha\lambda\beta\gamma} + C_{\bar{e}dddD}^{\alpha\gamma\lambda\beta} + C_{\bar{e}dddD}^{\alpha\beta\lambda\gamma} + C_{\bar{e}dddD}^{\alpha\gamma\beta\lambda} + C_{\bar{e}dddD}^{\alpha\lambda\gamma\beta}\right)$	$\frac{1}{6}n^2(n+1)(n+2)$
$\mathcal{O}_{\bar{d}u\ell\ell D}^{(S)\alpha\beta\gamma\lambda} = \frac{1}{2}\left(\mathcal{O}_{\bar{d}u\ell\ell D}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{d}u\ell\ell D}^{\alpha\beta\lambda\gamma}\right)$	$C_{\bar{d}u\ell\ell D}^{(S)\alpha\beta\gamma\lambda} = \frac{1}{2}\left(C_{\bar{d}u\ell\ell D}^{\alpha\beta\gamma\lambda} + C_{\bar{d}u\ell\ell D}^{\alpha\beta\lambda\gamma}\right)$	$\frac{1}{2}n^3(n+1)$
$\mathcal{O}_{\bar{\ell}qddD}^{(S)\alpha\beta\gamma\lambda} = \frac{1}{2}\left(\mathcal{O}_{\bar{\ell}qddD}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{\ell}qddD}^{\alpha\beta\lambda\gamma}\right)$	$C_{\bar{\ell}qddD}^{(S)\alpha\beta\gamma\lambda} = \frac{1}{2}\left(C_{\bar{\ell}qddD}^{\alpha\beta\gamma\lambda} + C_{\bar{\ell}qddD}^{\alpha\beta\lambda\gamma}\right)$	$\frac{1}{2}n^3(n+1)$

A more convenient physical basis:
DZ, 2023a

Operators with repeated fermions decomposed as SU(n) tensor

n is the number of flavors

- **No constraints on the flavor indices**
- **Each flavor index can run over all flavors**
- **Automatically get rid of redundant degrees of freedom in operators**

The Green's Basis for Dim-7 Operators

- The so-called **Green's basis** is directly related to **1PI Green's functions**, and usually needed for **off-shell scheme** M. Jiang et al, 2019; V. Gherardi, D. Marzocca, E. Venturini, 2020
- This basis is converted to the physical one via EoMs or field redefinitions

The Green's basis for dim-7 operators: DZ, 2023a (X. X. Li , Z. Ren , J. H. Yu, 2023)

	$\psi^2 H^4$			$\psi^2 H^3 D$	
$\mathcal{O}_{\ell H}$	$\epsilon^{ab}\epsilon^{de}\left(\ell^a_{\rm L} C \ell^d_{\rm L}\right) H^b H^e \left(H^\dagger H\right)$	$\frac{1}{2}n(n+1)$	$\mathcal{O}_{\ell eHD}$	$\epsilon^{ab}\epsilon^{de}\left(\ell^a_{\rm L} C \gamma_\mu E_{\rm R}\right) H^b H^d {\rm i}D^\mu H^e$	n^2
	$\psi^2 H^2 D^2$			$\psi^4 H$	
$\mathcal{O}_{\ell HD1}$	$\epsilon^{ab}\epsilon^{de}\left(\ell^a_{\rm L} C D^\mu \ell^b_{\rm L}\right) H^d D_\mu H^e$	n^2	$\mathcal{O}_{\bar{e}\ell\ell\ell H}$	$\epsilon^{ab}\epsilon^{de}\left(\overline{E_{\rm R}}\ell^a_{\rm L}\right)\left(\ell^b_{\rm L} C \ell^d_{\rm L}\right) H^e$	$\frac{1}{3}n^2\left(2n^2+1\right)$
$\mathcal{O}_{\ell HD2}$	$\epsilon^{ad}\epsilon^{be}\left(\ell^a_{\rm L} C D^\mu \ell^b_{\rm L}\right) H^d D_\mu H^e$	n^2	$\mathcal{O}_{\bar{d}\ell q\ell H1}$	$\epsilon^{ab}\epsilon^{de}\left(\overline{D_{\rm R}}\ell^a_{\rm L}\right)\left(Q^b_{\rm L} C \ell^d_{\rm L}\right) H^e$	n^4
$\mathcal{R}_{\ell HD3}$	$\epsilon^{ad}\epsilon^{be}\left(\ell^a_{\rm L} C \ell^b_{\rm L}\right) D^\mu H^d D_\mu H^e$	$\frac{1}{2}n(n+1)$	$\mathcal{O}_{\bar{d}\ell q\ell H2}$	$\epsilon^{ad}\epsilon^{be}\left(\overline{D_{\rm R}}\ell^a_{\rm L}\right)\left(Q^b_{\rm L} C \ell^d_{\rm L}\right) H^e$	n^4
$\mathcal{R}_{\ell HD4}$	$\epsilon^{ad}\epsilon^{be}\left(D^\mu \ell^a_{\rm L} C D_\mu \ell^b_{\rm L}\right) H^d H^e$	$\frac{1}{2}n(n+1)$	$\mathcal{O}_{\bar{d}\ell ueH}$	$\epsilon^{ab}\left(\overline{D_{\rm R}}\ell^a_{\rm L}\right)\left(U_{\rm R} C E_{\rm R}\right) H^b$	n^4
$\mathcal{R}_{\ell HD5}$	$\epsilon^{ab}\epsilon^{de}\left(\ell^a_{\rm L} C \sigma_{\mu\nu} D^\mu \ell^b_{\rm L}\right) H^d D^\nu H^e$	n^2	$\mathcal{O}_{\bar{q}u\ell\ell H}$	$\epsilon^{ab}\left(\overline{Q_{\rm L}}U_{\rm R}\right)\left(\ell_{\rm L} C \ell^a_{\rm L}\right) H^b$	n^4
$\mathcal{R}_{\ell HD6}$	$\epsilon^{ad}\epsilon^{be}\left(D^\mu \ell^a_{\rm L} C \sigma_{\mu\nu} D^\nu \ell^b_{\rm L}\right) H^d H^e$	$\frac{1}{2}n(n+1)$	$\mathcal{O}_{\bar{\ell}dud\tilde{H}}$	$\left(\overline{\ell_{\rm L}}D_{\rm R}\right)\left(U_{\rm R} C D_{\rm R}\right)\tilde{H}$	n^4
	$\psi^2 H^2 X$		$\mathcal{O}_{\bar{\ell}dddH}$	$\left(\overline{\ell_{\rm L}}D_{\rm R}\right)\left(D_{\rm R} C D_{\rm R}\right) H$	$\frac{1}{3}n^2\left(n^2-1\right)$
$\mathcal{O}_{\ell HB}$	$\epsilon^{ab}\epsilon^{de}\left(\ell^a_{\rm L} C \sigma_{\mu\nu} \ell^d_{\rm L}\right) H^b H^e B^{\mu\nu}$	$\frac{1}{2}n(n-1)$	$\mathcal{O}_{\bar{e}qdd\tilde{H}}$	$\epsilon^{ab}\left(\overline{E_{\rm R}}Q^a_{\rm L}\right)\left(D_{\rm R} C D_{\rm R}\right)\tilde{H}^b$	$\frac{1}{2}n^3(n-1)$
$\mathcal{O}_{\ell HW}$	$\epsilon^{ab}\left(\epsilon\sigma^I\right)^{de}\left(\ell^a_{\rm L} C \sigma_{\mu\nu} \ell^d_{\rm L}\right) H^b H^e W^{I\mu\nu}$	n^2	$\mathcal{O}_{\bar{\ell}dqq\tilde{H}}$	$\epsilon^{ab}\left(\overline{\ell_{\rm L}}D_{\rm R}\right)\left(Q_{\rm L} C Q^a_{\rm L}\right)\tilde{H}^b$	n^4
			$\psi^4 D$		
$\mathcal{O}_{\bar{e}dddD}$	$\left(\overline{E_{\rm R}}\gamma_\mu D_{\rm R}\right)\left(D_{\rm R} C {\rm i}D^\mu D_{\rm R}\right)$	n^4	$\mathcal{O}_{\bar{\ell}qddD}$	$\left(\overline{\ell_{\rm L}}\gamma_\mu Q_{\rm L}\right)\left(D_{\rm R} C {\rm i}D^\mu D_{\rm R}\right)$	n^4
$\mathcal{O}_{\bar{d}u\ell\ell D}$	$\epsilon^{ab}\left(\overline{D_{\rm R}}\gamma_\mu U_{\rm R}\right)\left(\ell^a_{\rm L} C {\rm i}D^\mu \ell^b_{\rm L}\right)$	n^4	$\mathcal{R}_{\bar{\ell}dDqd}$	$\left(\overline{\ell_{\rm L}}D_{\rm R}\right)\left({\rm i}D^\mu Q_{\rm L} C \gamma_\mu D_{\rm R}\right)$	n^4
$\mathcal{R}_{\bar{d}\ell\ell Du}$	$\epsilon^{ab}\left(\overline{D_{\rm R}}\ell^a_{\rm L}\right)\left(\ell^b_{\rm L} C \gamma_\mu {\rm i}D^\mu U_{\rm R}\right)$	n^4	$\mathcal{R}_{\bar{\ell}dqDd}$	$\left(\overline{\ell_{\rm L}}D_{\rm R}\right)\left(Q_{\rm L} C \gamma_\mu {\rm i}D^\mu D_{\rm R}\right)$	n^4
$\mathcal{R}_{\bar{d}D\ell\ell u}$	$\epsilon^{ab}\left(\overline{D_{\rm R}}{\rm i}D^\mu \ell^a_{\rm L}\right)\left(\ell^b_{\rm L} C \gamma_\mu U_{\rm R}\right)$	n^4			

Reduction Relations Between Two Bases

Apply EoMs to operators in the Green's basis: **DZ, 2023a**

$$\mathcal{R}^{\alpha\beta}_{\ell HD3} = \mathcal{O}^{\beta\alpha}_{\ell HD1} - \mathcal{O}^{\beta\alpha}_{\ell HD2} - \mathcal{O}^{\alpha\beta}_{\ell HD2} + m^2 \left(\mathcal{O}^{\dagger}_{5}\right)_{\alpha\beta} + 2\lambda \mathcal{O}^{\alpha\beta}_{\ell H} - \left(Y^{\dagger}_{l}\right)_{\gamma\lambda} \mathcal{O}^{\gamma\lambda\beta\alpha}_{\overline{e}\ell\ell\ell H} - \left(Y^{\dagger}_{\rm d}\right)_{\gamma\lambda} \mathcal{O}^{\gamma\beta\lambda\alpha}_{\overline{d}\ell q\ell H1}$$
$$- \left(Y^{\dagger}_{\rm d}\right)_{\gamma\lambda} \mathcal{O}^{\gamma\alpha\lambda\beta}_{\overline{d}\ell q\ell H1} + \left(Y^{\dagger}_{\rm d}\right)_{\gamma\lambda} \mathcal{O}^{\gamma\alpha\lambda\beta}_{\overline{d}\ell q\ell H2} + \left(Y_{\rm u}\right)_{\gamma\lambda} \mathcal{O}^{\gamma\lambda\beta\alpha}_{\overline{q}u\ell\ell H} ,$$

$$\mathcal{R}^{\alpha\beta}_{\ell HD4} = \mathcal{O}^{\alpha\beta}_{\ell HD1} - 2\mathcal{O}^{\alpha\beta}_{\ell HD2} - \left(Y_{l}\right)_{\beta\gamma} \mathcal{O}^{\alpha\gamma}_{\ell eHD} + \frac{1}{4} g_1 \mathcal{O}^{\alpha\beta}_{\ell HB} + \frac{1}{4} g_2 \mathcal{O}^{\alpha\beta}_{\ell HW} ,$$

$$\mathcal{R}^{\alpha\beta}_{\ell HD5} = {\rm i} \left[\mathcal{O}^{\alpha\beta}_{\ell HD1} + \left(Y_{l}\right)_{\beta\gamma} \mathcal{O}^{\alpha\gamma}_{\ell eHD}\right] ,$$

$$\mathcal{R}^{\alpha\beta}_{\ell HD6} = -{\rm i} \left[\mathcal{O}^{\alpha\beta}_{\ell HD1} - 2\mathcal{O}^{\alpha\beta}_{\ell HD2} - \left(Y_{l}\right)_{\beta\gamma} \mathcal{O}^{\alpha\gamma}_{\ell eHD} + \frac{1}{4} g_1 \mathcal{O}^{\alpha\beta}_{\ell HB} + \frac{1}{4} g_2 \mathcal{O}^{\alpha\beta}_{\ell HW}\right] ,$$

$$\mathcal{R}^{\alpha\beta\gamma\lambda}_{\overline{d}\ell\ell Du} = \left(Y^{\dagger}_{\rm u}\right)_{\lambda\rho} \mathcal{O}^{\alpha\beta\rho\gamma}_{\overline{d}\ell q\ell H2} ,$$

$$\mathcal{R}^{\alpha\beta\gamma\lambda}_{\overline{d}D\ell\ell u} = \mathcal{O}^{\alpha\lambda\gamma\beta}_{\overline{d}u\ell\ell D} - \left(Y_{l}\right)_{\beta\rho} \mathcal{O}^{\alpha\gamma\lambda\rho}_{\overline{d}\ell ueH} ,$$
$$\mathcal{R}^{\alpha\beta\gamma\lambda}_{\ell dDqd} = -\left(Y_{\rm u}\right)_{\gamma\rho} \mathcal{O}^{\alpha\beta\rho\lambda}_{\ell dud\widetilde{H}} + \left(Y_{\rm d}\right)_{\gamma\rho} \mathcal{O}^{\alpha\beta\lambda\rho}_{\ell dddH} ,$$
$$\mathcal{R}^{\alpha\beta\gamma\lambda}_{\ell dqDd} = -\left(Y^{\dagger}_{\rm d}\right)_{\lambda\rho} \mathcal{O}^{\alpha\beta\gamma\rho}_{\ell dqq\widetilde{H}} .$$

Reduction relations:

$$C^{(S)\alpha\beta}_{\ell H} = G^{(S)\alpha\beta}_{\ell H} + \left(G^{\dagger}_{5}\right)^{\alpha\beta} \left(\frac{1}{4} g^2_2 G_{2W} - g_2 G_{WDH} - 2\lambda G_{DH} - \frac{1}{2} G^{\prime}_{HD} - {\rm i} G^{\prime\prime}_{HD}\right)$$
$$+ \frac{1}{2} \left(G^{\dagger}_{5}\right)^{\alpha\gamma} \left(G^{\prime(3)\gamma\beta}_{H\ell} - {\rm i} G^{\prime\prime(3)\gamma\beta}_{H\ell} - G^{\prime(1)\gamma\beta}_{H\ell} + {\rm i} G^{\prime\prime(1)\gamma\beta}_{H\ell}\right) + \frac{1}{2} \left(G^{\dagger}_{5}\right)^{\beta\gamma}$$
$$\times \left(G^{\prime(3)\gamma\alpha}_{H\ell} - {\rm i} G^{\prime\prime(3)\gamma\alpha}_{H\ell} - G^{\prime(1)\gamma\alpha}_{H\ell} + {\rm i} G^{\prime\prime(1)\gamma\alpha}_{H\ell}\right) + 2\lambda G^{(S)\alpha\beta}_{\ell HD3} .$$

$$C^{\alpha\beta}_{\ell eHD} = G^{\alpha\beta}_{\ell eHD} + \frac{1}{2} \left(G^{\dagger}_{5}\right)^{\alpha\gamma} \left[G^{\gamma\beta}_{eHD2} - G^{\gamma\beta}_{eHD4} - 2 G^{\gamma\lambda}_{\ell D} \left(Y_{l}\right)_{\lambda\beta}\right]$$
$$+ \left({\rm i} G^{\alpha\gamma}_{\ell HD5} + {\rm i} G^{(S)\alpha\gamma}_{\ell HD6} - G^{(S)\alpha\gamma}_{\ell HD4}\right) \left(Y_{l}\right)_{\gamma\beta} - \underline{\frac{1}{4} \left(G^{\alpha\gamma}_{\ell HD2} - G^{\gamma\alpha}_{\ell HD2}\right) \left(Y_{l}\right)_{\gamma\beta}} .$$

⋮

$$C^{\alpha\beta}_{5} = G^{\alpha\beta}_{5} - 2m^2 G_{DH} G^{\alpha\beta}_{5} + 2m^2 \left(G^{(S)\dagger}_{\ell HD3}\right)^{\alpha\beta}$$

- **Terms in blue come from dim-6 operators in the Green's basis**

$$ {\rm i}\not{D} \ell^{a}_{\alpha {\rm L}} = \left(Y_{l}\right)_{\alpha\beta} H^{a} E_{\beta {\rm R}} - C^{\alpha\beta}_{5} \widetilde{H}^{a} \widetilde{H}^{\rm T} \ell^{\rm c}_{\beta {\rm L}}$$

$$\left(D^2 H\right)^{a} = -m^2 H^{a} - 2\lambda H^{a} \left(H^{\dagger} H\right)$$
$$- \left(\overline{E_{\rm R}} Y^{\dagger}_{l} \ell^{a}_{\rm L} + \overline{D_{\rm R}} Y^{\dagger}_{\rm d} Q^{a}_{\rm L} - \epsilon^{ab} \overline{Q^{b}_{\rm L}} Y_{\rm u} U_{\rm R}\right)$$
$$- \frac{1}{2} \epsilon^{ab} C^{\alpha\beta}_{5} \left(\overline{\ell^{b}_{\alpha {\rm L}}} \widetilde{H}^{\rm T} \ell^{\rm c}_{\beta {\rm L}} + \overline{\ell_{\alpha {\rm L}}} \widetilde{H} \ell^{bc}_{\beta {\rm L}}\right) ,$$

- **Terms in red come from dim-7 operators in the Green's basis**

Induced by non-trivial flavor relation

$$\left[4\mathcal{O}^{\alpha\beta}_{\ell HD2} + 2\mathcal{K}^{\alpha\beta} + 2 \left(Y_{l}\right)_{\beta\gamma} \mathcal{O}^{\alpha\gamma}_{\ell eHD} - \frac{1}{2} g_2 \mathcal{O}^{\alpha\beta}_{\ell HW}\right] - \alpha \leftrightarrow \beta = g_1 \mathcal{O}^{\alpha\beta}_{\ell HB}$$

Reduction Relations Between Two Bases

Apply EoMs to operators in the Green's basis: DZ, 2023a

$$\mathcal{R}^{\alpha\beta}_{\ell HD3} = \mathcal{O}^{\beta\alpha}_{\ell HD1} - \mathcal{O}^{\beta\alpha}_{\ell HD2} - \mathcal{O}^{\alpha\beta}_{\ell HD2} + m^2 \left(\mathcal{O}^{\dagger}_{5}\right)_{\alpha\beta} + 2\lambda \mathcal{O}^{\alpha\beta}_{\ell H} - \left(Y^{\dagger}_{l}\right)_{\gamma\lambda} \mathcal{O}^{\gamma\lambda\beta\alpha}_{\bar{e}\ell\ell\ell H} - \left(Y^{\dagger}_{\rm d}\right)_{\gamma\lambda} \mathcal{O}^{\gamma\beta\lambda\alpha}_{\bar{d}\ell q\ell H1}$$
$$- \left(Y^{\dagger}_{\rm d}\right)_{\gamma\lambda} \mathcal{O}^{\gamma\alpha\lambda\beta}_{\bar{d}\ell q\ell H1} + \left(Y^{\dagger}_{\rm d}\right)_{\gamma\lambda} \mathcal{O}^{\gamma\alpha\lambda\beta}_{\bar{d}\ell q\ell H2} + \left(Y_{\rm u}\right)_{\gamma\lambda} \mathcal{O}^{\gamma\lambda\beta\alpha}_{\bar{q}u\ell\ell H} \,,$$

$$\mathcal{R}^{\alpha\beta}_{\ell HD4} = \mathcal{O}^{\alpha\beta}_{\ell HD1} - 2\mathcal{O}^{\alpha\beta}_{\ell HD2} - \left(Y_{l}\right)_{\beta\gamma} \mathcal{O}^{\alpha\gamma}_{\ell eHD} + \frac{1}{4} g_1 \mathcal{O}^{\alpha\beta}_{\ell HB} + \frac{1}{4} g_2 \mathcal{O}^{\alpha\beta}_{\ell HW} \,,$$

$$\mathcal{R}^{\alpha\beta}_{\ell HD5} = {\rm i} \left[\mathcal{O}^{\alpha\beta}_{\ell HD1} + \left(Y_{l}\right)_{\beta\gamma} \mathcal{O}^{\alpha\gamma}_{\ell eHD}\right] \,,$$

$$\mathcal{R}^{\alpha\beta}_{\ell HD6} = -{\rm i} \left[\mathcal{O}^{\alpha\beta}_{\ell HD1} - 2\mathcal{O}^{\alpha\beta}_{\ell HD2} - \left(Y_{l}\right)_{\beta\gamma} \mathcal{O}^{\alpha\gamma}_{\ell eHD} + \frac{1}{4} g_1 \mathcal{O}^{\alpha\beta}_{\ell HB} + \frac{1}{4} g_2 \mathcal{O}^{\alpha\beta}_{\ell HW}\right] \,,$$

$$\mathcal{R}^{\alpha\beta\gamma\lambda}_{\bar{d}\ell\ell Du} = \left(Y^{\dagger}_{\rm u}\right)_{\lambda\rho} \mathcal{O}^{\alpha\beta\rho\gamma}_{\bar{d}\ell q\ell H2} \,,$$

$$\mathcal{R}^{\alpha\beta\gamma\lambda}_{\bar{d}D\ell\ell u} = \mathcal{O}^{\alpha\lambda\gamma\beta}_{\bar{d}u\ell\ell D} - \left(Y_{l}\right)_{\beta\rho} \mathcal{O}^{\alpha\gamma\lambda\rho}_{\bar{d}\ell ueH} \,,$$
$$\mathcal{R}^{\alpha\beta\gamma\lambda}_{\bar{\ell}dDqd} = -\left(Y_{\rm u}\right)_{\gamma\rho} \mathcal{O}^{\alpha\beta\rho\lambda}_{\bar{\ell}dud\tilde{H}} + \left(Y_{\rm d}\right)_{\gamma\rho} \mathcal{O}^{\alpha\beta\lambda\rho}_{\bar{\ell}dddH} \,,$$
$$\mathcal{R}^{\alpha\beta\gamma\lambda}_{\bar{\ell}dqDd} = -\left(Y^{\dagger}_{\rm d}\right)_{\lambda\rho} \mathcal{O}^{\alpha\beta\gamma\rho}_{\bar{\ell}dqq\tilde{H}} \,.$$

Reduction relations:

$$C^{(S)\alpha\beta}_{\ell H} = G^{(S)\alpha\beta}_{\ell H} + \left(G^{\dagger}_{5}\right)^{\alpha\beta} \left(\frac{1}{4} g_2^2 G_{2W} - g_2 G_{WDH} - 2\lambda G_{DH} - \frac{1}{2} G^{\prime}_{HD} - {\rm i} G^{\prime\prime}_{HD}\right)$$
$$+ \frac{1}{2} \left(G^{\dagger}_{5}\right)^{\alpha\gamma} \left(G^{\prime(3)\gamma\beta}_{H\ell} - {\rm i} G^{\prime\prime(3)\gamma\beta}_{H\ell} - G^{\prime(1)\gamma\beta}_{H\ell} + {\rm i} G^{\prime\prime(1)\gamma\beta}_{H\ell}\right) + \frac{1}{2} \left(G^{\dagger}_{5}\right)^{\beta\gamma}$$
$$\times \left(G^{\prime(3)\gamma\alpha}_{H\ell} - {\rm i} G^{\prime\prime(3)\gamma\alpha}_{H\ell} - G^{\prime(1)\gamma\alpha}_{H\ell} + {\rm i} G^{\prime\prime(1)\gamma\alpha}_{H\ell}\right) + 2\lambda G^{(S)\alpha\beta}_{\ell HD3} \,.$$

$$C^{\alpha\beta}_{\ell eHD} = G^{\alpha\beta}_{\ell eHD} + \frac{1}{2} \left(G^{\dagger}_{5}\right)^{\alpha\gamma} \left[G^{\gamma\beta}_{eHD2} - G^{\gamma\beta}_{eHD4} - 2 G^{\gamma\lambda}_{\ell D} \left(Y_{l}\right)_{\lambda\beta}\right]$$
$$+ \left({\rm i} G^{\alpha\gamma}_{\ell HD5} + {\rm i} G^{(S)\alpha\gamma}_{\ell HD6} - G^{(S)\alpha\gamma}_{\ell HD4}\right) \left(Y_{l}\right)_{\gamma\beta} - \frac{1}{4} \left(G^{\alpha\gamma}_{\ell HD2} - G^{\gamma\alpha}_{\ell HD2}\right) \left(Y_{l}\right)_{\gamma\beta} \,.$$

⋮

$$C^{\alpha\beta}_{5} = G^{\alpha\beta}_{5} - 2m^2 G_{DH} G^{\alpha\beta}_{5} + 2m^2 \left(G^{(S)\dagger}_{\ell HD3}\right)^{\alpha\beta}$$

- ✓ The physical basis for dim-7 operators
- ✓ The Green's basis for dim-7 operators
- ✓ The reduction relations between two bases

The RGEs of the LNV SMEFT Operators

Status of RGEs in the SMEFT

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			✓ [51]						✓ [27]		✓ [28]
$d_{\leq 4}$ (fermionic)			✓ [51]						X		X
d_5	✓ [52–54]				✓ [8]	✓ [8]					
d_6 (bosonic)		✓ [44]	✓ [51, 55, 56]					This work	✓ [27]	This work	✓ [28]
d_6 (fermionic)		✓ [44]	✓ [51, 55–57]					X		X	X
d_7				✓ [8]	✓ [8]	✓ [42, 58]					
d_8 (bosonic)							This work	This work	✓ [27]	This work	✓ [28]
d_8 (fermionic)							X	X	X	X	✓ [26]

Table 3: *State of the art of the SMEFT renormalisation (adapted from Refs. [27, 28]). The rows show the renormalised operators (categorized by dimensions and statistics). The columns show the operators contributing to RG running. Blank entries vanish, ✓ denotes that the complete contribution is available, ✓ implies that only (but substantial) partial results is present, and, X indicates that nothing, or very little, is known. The contribution made in this paper is marked by This work.* **S. Das Bakshi, A. Diaz-Carmona, 2023**

[8] M. Chala, A. Titov, 2021

[26] M. A. Huber, S. De Angelis, 2021

[27] M. Chala et al, 2021

[28] S. Das Bakshi et al, 2022

[42] Y. Liao, X. D. Ma, 2016

[44] S. Davidson, M. Gorbahn, M. Leak, 2018;
Y. L. Wang, D. Zhang, S. Zhou, 2023

[51] E. E. Jenkins, A. V. Manohar, M. Trott, 2013

[52] P. H. Chankowski, Z. Pluciennik, 1993

[53] K. S. Babu, C. N. Leung, J. T. Pantaleone, 1993

[54] S. Antusch et al, 2001

[55] E. E. Jenkins, A. V. Manohar, M. Trott, 2014

[56] R. Alonso et al, 2014a

[57] R. Alonso et al, 2014b

[58] Y. Liao, X. D. Ma, 2019

Status of RGEs in the SMEFT

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			✓ [51]						✓ [27]		✓ [28]
$d_{\leq 4}$ (fermionic)			✓ [51]						X		X
d_5	✓ [52–54]				✓ [8]	✓ [8]					
d_6 (bosonic)		✓ [44]	✓ [51, 55, 56]					This work	✓ [27]	This work	✓ [28]
d_6 (fermionic)		✓ [44]	✓ [51, 55–57]					X		X	X
d_7				✓ [8]	✓ [8]	✓ [42, 58]					
d_8 (bosonic)							This work	This work	✓ [27]	This work	✓ [28]
d_8 (fermionic)							X	X	X	X	✓ [26]

Table 3: *State of the art of the SMEFT renormalisation (adapted from Refs. [27, 28]). The rows show the renormalised operators (categorized by dimensions and statistics). The columns show the operators contributing to RG running. Blank entries vanish, ✓ denotes that the complete contribution is available, ✓ implies that only (but substantial) partial results is present, and, X indicates that nothing, or very little, is known. The contribution made in this paper is marked by This work.* **S. Das Bakshi, A. Diaz-Carmona, 2023**

[8] M. Chala, A. Titov, 2021 — **Only one dim-7 operator, namely $\mathcal{O}_{\ell H}$, is considered and $Y_{d,l}$ are ignored**

[42] Y. Liao, X. D. Ma, 2016 — **The non-trivial flavor relations are not considered**

[58] Y. Liao, X. D. Ma, 2019 — **No final results in a physical basis, only counterterms in the flavor-blind basis**

- **Working out the complete results in a more convenient physical basis**
- **As a crosscheck (Indeed, typos/mistakes in [8] and [58] are found and later confirmed by the authors)**

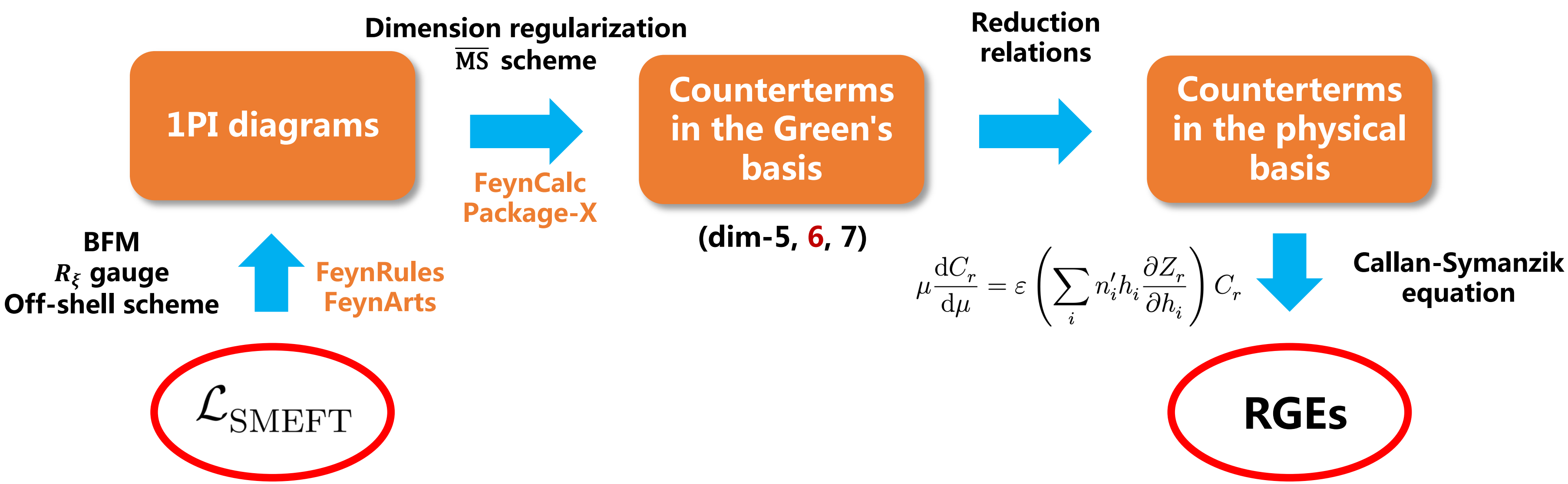
RGEs of LNV Operators up to Dimension Seven

General structures of RGEs up to $\mathcal{O}(\Lambda^{-3})$:

$$16\pi^2\mu\frac{\mathrm{d}C_5}{\mathrm{d}\mu} = \gamma^{(5,5)}C_5 + \hat{\gamma}^{(5,5)}C_5C_5C_5 + \gamma_i^{(5,6)}C_5C_6^i + \gamma_i^{(5,7)}C_7^i \ ,$$

$$16\pi^2\mu\frac{\mathrm{d}C_7^i}{\mathrm{d}\mu} = \gamma_{ij}^{(7,7)}C_7^j + \gamma_i^{(7,5)}C_5C_5C_5 + \gamma_{ij}^{(7,6)}C_5C_6^j \ ,$$

Procedure for calculations:



- **A large amount of diagrams and also lots of calculations; FeynRules/Arts/Calc and Package-X**
- **FeynCalc can not properly deal with four-fermion vertices, thus diagrams involving four-fermion vertices are calculated by hand**
- **Crosscheck can be done by modifying the package Matchmakereft** A. Carmona, et al., 2021

RGEs of LNV Operators up to Dimension Seven

Examples for the final results ($\mathcal{O}_5^{\alpha\beta} = \overline{\ell_{\alpha\mathrm{L}}}\widetilde{H}\widetilde{H}^{\mathrm{T}}\ell^{\mathrm{c}}_{\beta\mathrm{L}}$, $\mathcal{O}_{\ell H}^{\alpha\beta} = \overline{\ell^{\mathrm{c}}_{\alpha\mathrm{L}}}\widetilde{H}^*\widetilde{H}^\dagger\ell_{\beta\mathrm{L}}\left(H^\dagger H\right)$ **):** **DZ, 2023a;2023b**

$$
\begin{aligned}
\dot{C}_5^{\alpha\beta} = {} & \frac{1}{2}\left(-3g_2^2+4\lambda+2T\right)C_5^{\alpha\beta} - \frac{3}{2}\left(Y_l Y_l^\dagger C_5\right)^{\alpha\beta} + m^2\left(8C_{H\Box}-C_{HD}\right)C_5^{\alpha\beta} \\
& +m^2\Big\{8C_{\ell H}^{(S)*\alpha\beta} + \frac{3}{2}g_2^2\left(2C_{\ell HD1}^{(S)*\alpha\beta}+C_{\ell HD2}^{(S)*\alpha\beta}\right) + \left(Y_lY_l^\dagger C_{\ell HD1}^{(S)\dagger}\right)^{\alpha\beta} - \frac{1}{2}\left(Y_lY_l^\dagger C_{\ell HD2}^{(S)\dagger}\right)^{\alpha\beta} + 2\left(Y_l C_{\ell eHD}^\dagger\right)^{\alpha\beta} \\
& -\left(Y_l^\dagger\right)_{\gamma\lambda}\left(3C_{\bar{e}\ell\ell\ell H}^{(S)*\gamma\lambda\alpha\beta}+2C_{\bar{e}\ell\ell\ell H}^{(M)*\gamma\lambda\alpha\beta}\right) - 3\left(Y_{\mathrm{d}}^\dagger\right)_{\gamma\lambda}C_{\bar{d}\ell q\ell H1}^{*\gamma\alpha\lambda\beta} + 6\left(Y_{\mathrm{u}}\right)_{\lambda\gamma}C_{\bar{q}u\ell\ell H}^{*\lambda\gamma\alpha\beta}\Big\} + \alpha\leftrightarrow\beta
\end{aligned}
$$

$$
\begin{aligned}
\dot{C}_{\ell H}^{(S)\alpha\beta} = {} & \frac{1}{2}\mathrm{Tr}\left(C_5C_5^\dagger\right)C_5^{*\alpha\beta} + \frac{5}{4}\left(C_5^\dagger C_5 C_5^\dagger\right)^{\alpha\beta} + C_5^{*\alpha\beta}\Big\{-3C_H - \frac{3}{4}\left(g_1^2-g_2^2+4\lambda\right)C_{HD} + \left(16\lambda-\frac{5}{3}g_2^2\right)C_{H\Box} \\
& -3g_2^2C_{HW} + \frac{3}{2}\mathrm{i}\left(g_1^2C_{H\widetilde{B}}+3g_2^2C_{H\widetilde{W}}+g_1g_2C_{H\widetilde{W}B}\right) - \mathrm{Tr}\Big[2g_2^2\left(C_{Hq}^{(3)}+\frac{1}{3}C_{H\ell}^{(3)}\right) + C_{eH}Y_l^\dagger + 3C_{dH}Y_{\mathrm{d}}^\dagger \\
& +3Y_{\mathrm{u}}C_{uH}^\dagger - 2\left(Y_l^\dagger C_{H\ell}^{(3)}Y_l + 3Y_{\mathrm{d}}^\dagger C_{Hq}^{(3)}Y_{\mathrm{d}} + 3Y_{\mathrm{u}}^\dagger C_{Hq}^{(3)}Y_{\mathrm{u}}\right) + 3\left(Y_{\mathrm{u}}C_{Hud}Y_{\mathrm{d}}^\dagger + Y_{\mathrm{d}}C_{Hud}^\dagger Y_{\mathrm{u}}^\dagger\right)\Big]\Big\} - 3g_2\left(C_5^\dagger Y_l C_{eW}^\dagger\right)^{\alpha\beta} \\
& +\frac{3}{2}\left(g_1^2+g_2^2\right)\left[\left(C_5^\dagger C_{H\ell}^{(3)}\right)^{\alpha\beta} - \left(C_5^\dagger C_{H\ell}^{(1)}\right)^{\alpha\beta}\right] + \frac{1}{2}\left(C_5^\dagger Y_l C_{eH}^\dagger\right)^{\alpha\beta} + \left(C_5^\dagger C_{eH}Y_l^\dagger\right)^{\alpha\beta} - 3\left(C_5^\dagger Y_lY_l^\dagger C_{H\ell}^{(3)}\right)^{\alpha\beta} \\
& -\frac{1}{4}\left(3g_1^2+15g_2^2-80\lambda-8T\right)C_{\ell H}^{(S)\alpha\beta} - \frac{3}{2}\left(C_{\ell H}^{(S)}Y_lY_l^\dagger\right)^{\alpha\beta} + \left(2\lambda-\frac{3}{2}g_2^2\right)\left(C_{\ell eHD}Y_l^\dagger\right)^{\alpha\beta} + \left(C_{\ell eHD}Y_l^\dagger Y_lY_l^\dagger\right)^{\alpha\beta} \\
& -\frac{3}{4}g_2^2\left(g_2^2-4\lambda\right)C_{\ell HD1}^{(S)\alpha\beta} + \lambda\left(C_{\ell HD1}^{(S)}Y_lY_l^\dagger\right)^{\alpha\beta} - \left(C_{\ell HD1}^{(S)}Y_lY_l^\dagger Y_lY_l^\dagger\right)^{\alpha\beta} - \frac{3}{8}\left(g_1^4+2g_1^2g_2^2+3g_2^4-4g_2^2\lambda\right)C_{\ell HD2}^{(S)\alpha\beta} \\
& -\frac{1}{2}\lambda\left(C_{\ell HD2}^{(S)}Y_lY_l^\dagger\right)^{\alpha\beta} - \left(C_{\ell HD2}^{(S)}Y_lY_l^\dagger Y_lY_l^\dagger\right)^{\alpha\beta} - 3g_2^3C_{\ell HW}^{\alpha\beta} - 6g_2\left(C_{\ell HW}Y_lY_l^\dagger\right)^{\alpha\beta} - 3C_{\bar{e}\ell\ell\ell H}^{(S)\gamma\lambda\alpha\beta}\Big[\lambda\left(Y_l\right)_{\lambda\gamma} \\
& -\left(Y_lY_l^\dagger Y_l\right)_{\lambda\gamma}\Big] - 2C_{\bar{e}\ell\ell\ell H}^{(M)\gamma\lambda\alpha\beta}\left[\lambda\left(Y_l\right)_{\lambda\gamma} - \left(Y_lY_l^\dagger Y_l\right)_{\lambda\gamma}\right] - 3C_{\bar{d}\ell q\ell H1}^{\gamma\alpha\lambda\beta}\left[\lambda\left(Y_{\mathrm{d}}\right)_{\lambda\gamma} - \left(Y_{\mathrm{d}}Y_{\mathrm{d}}^\dagger Y_{\mathrm{d}}\right)_{\lambda\gamma}\right] \\
& +6C_{\bar{q}u\ell\ell H}^{\gamma\lambda\alpha\beta}\left[\lambda\left(Y_{\mathrm{u}}^\dagger\right)_{\lambda\gamma} - \left(Y_{\mathrm{u}}^\dagger Y_{\mathrm{u}}Y_{\mathrm{u}}^\dagger\right)_{\lambda\gamma}\right] + \alpha\leftrightarrow\beta
\end{aligned}
$$

RGEs of LNV Operators up to Dimension Seven

Examples for the final results ($\mathcal{O}_5^{\alpha\beta} = \overline{\ell_{\alpha\mathrm{L}}} \widetilde{H} \widetilde{H}^{\mathrm{T}} \ell^{\mathrm{c}}_{\beta\mathrm{L}}$, $\mathcal{O}_{\ell H}^{\alpha\beta} = \overline{\ell^{\mathrm{c}}_{\alpha\mathrm{L}}} \widetilde{H}^{*} \widetilde{H}^{\dagger} \ell_{\beta\mathrm{L}} \left(H^\dagger H\right)$ **):** **DZ, 2023a;2023b**

$$\gamma^{(5,5)} C_5 + \cancel{\hat{\gamma}^{(5,5)} C_5 C_5 C_5} + \gamma_i^{(5,6)} C_5 C_6^i + \gamma_i^{(5,7)} C_7^i$$

$$\begin{aligned}
\dot{C}_5^{\alpha\beta} = {} & \frac{1}{2}\left(-3g_2^2 + 4\lambda + 2T\right) C_5^{\alpha\beta} - \frac{3}{2}\left(Y_l Y_l^\dagger C_5\right)^{\alpha\beta} + m^2 \left(8C_{H\Box} - C_{HD}\right) C_5^{\alpha\beta} \\
& + m^2 \left\{ 8C_{\ell H}^{(S)*\alpha\beta} + \frac{3}{2} g_2^2 \left(2C_{\ell HD1}^{(S)*\alpha\beta} + C_{\ell HD2}^{(S)*\alpha\beta}\right) + \left(Y_l Y_l^\dagger C_{\ell HD1}^{(S)\dagger}\right)^{\alpha\beta} - \frac{1}{2}\left(Y_l Y_l^\dagger C_{\ell HD2}^{(S)\dagger}\right)^{\alpha\beta} + 2\left(Y_l C_{\ell eHD}^\dagger\right)^{\alpha\beta} \right. \\
& \left. - \left(Y_l^\dagger\right)_{\gamma\lambda} \left(3C_{\overline{e}\ell\ell\ell H}^{(S)*\gamma\lambda\alpha\beta} + 2C_{\overline{e}\ell\ell\ell H}^{(M)*\gamma\lambda\alpha\beta}\right) - 3\left(Y_{\mathrm{d}}^\dagger\right)_{\gamma\lambda} C_{\overline{d}\ell q\ell H1}^{*\gamma\alpha\lambda\beta} + 6\left(Y_{\mathrm{u}}\right)_{\lambda\gamma} C_{\overline{q}u\ell\ell H}^{*\lambda\gamma\alpha\beta} \right\} + \alpha \leftrightarrow \beta
\end{aligned}$$

$$\begin{aligned}
\dot{C}_{\ell H}^{(S)\alpha\beta} = {} & \frac{1}{2}\mathrm{Tr}\left(C_5 C_5^\dagger\right) C_5^{*\alpha\beta} + \frac{5}{4}\left(C_5^\dagger C_5 C_5^\dagger\right)^{\alpha\beta} + C_5^{*\alpha\beta} \left\{ -3C_H - \frac{3}{4}\left(g_1^2 - g_2^2 + 4\lambda\right) C_{HD} + \left(16\lambda - \frac{5}{3}g_2^2\right) C_{H\Box} \right. \\
& - 3g_2^2 C_{HW} + \frac{3}{2}\mathrm{i}\left(g_1^2 C_{H\widetilde{B}} + 3g_2^2 C_{H\widetilde{W}} + g_1 g_2 C_{H\widetilde{W}B}\right) - \mathrm{Tr}\left[2g_2^2\left(C_{Hq}^{(3)} + \frac{1}{3}C_{H\ell}^{(3)}\right) + C_{eH} Y_l^\dagger + 3C_{dH} Y_{\mathrm{d}}^\dagger \right. \\
& \left.\left. + 3Y_{\mathrm{u}} C_{uH}^\dagger - 2\left(Y_l^\dagger C_{H\ell}^{(3)} Y_l + 3Y_{\mathrm{d}}^\dagger C_{Hq}^{(3)} Y_{\mathrm{d}} + 3Y_{\mathrm{u}}^\dagger C_{Hq}^{(3)} Y_{\mathrm{u}}\right) + 3\left(Y_{\mathrm{u}} C_{Hud} Y_{\mathrm{d}}^\dagger + Y_{\mathrm{d}} C_{Hud}^\dagger Y_{\mathrm{u}}^\dagger\right)\right]\right\} - 3g_2\left(C_5^\dagger Y_l C_{eW}^\dagger\right)^{\alpha\beta} \\
& + \frac{3}{2}\left(g_1^2 + g_2^2\right)\left[\left(C_5^\dagger C_{H\ell}^{(3)}\right)^{\alpha\beta} - \left(C_5^\dagger C_{H\ell}^{(1)}\right)^{\alpha\beta}\right] + \frac{1}{2}\left(C_5^\dagger Y_l C_{eH}^\dagger\right)^{\alpha\beta} + \left(C_5^\dagger C_{eH} Y_l^\dagger\right)^{\alpha\beta} - 3\left(C_5^\dagger Y_l Y_l^\dagger C_{H\ell}^{(3)}\right)^{\alpha\beta} \\
& - \frac{1}{4}\left(3g_1^2 + 15g_2^2 - 80\lambda - 8T\right) C_{\ell H}^{(S)\alpha\beta} - \frac{3}{2}\left(C_{\ell H}^{(S)} Y_l Y_l^\dagger\right)^{\alpha\beta} + \left(2\lambda - \frac{3}{2}g_2^2\right)\left(C_{\ell eHD} Y_l^\dagger\right)^{\alpha\beta} + \left(C_{\ell eHD} Y_l^\dagger Y_l Y_l^\dagger\right)^{\alpha\beta} \\
& - \frac{3}{4}g_2^2\left(g_2^2 - 4\lambda\right) C_{\ell HD1}^{(S)\alpha\beta} + \lambda\left(C_{\ell HD1}^{(S)} Y_l Y_l^\dagger\right)^{\alpha\beta} - \left(C_{\ell HD1}^{(S)} Y_l Y_l^\dagger Y_l Y_l^\dagger\right)^{\alpha\beta} - \frac{3}{8}\left(g_1^4 + 2g_1^2 g_2^2 + 3g_2^4 - 4g_2^2\lambda\right) C_{\ell HD2}^{(S)\alpha\beta} \\
& - \frac{1}{2}\lambda\left(C_{\ell HD2}^{(S)} Y_l Y_l^\dagger\right)^{\alpha\beta} - \left(C_{\ell HD2}^{(S)} Y_l Y_l^\dagger Y_l Y_l^\dagger\right)^{\alpha\beta} - 3g_2^3 C_{\ell HW}^{\alpha\beta} - 6g_2\left(C_{\ell HW} Y_l Y_l^\dagger\right)^{\alpha\beta} - 3C_{\overline{e}\ell\ell\ell H}^{(S)\gamma\lambda\alpha\beta}\left[\lambda\left(Y_l\right)_{\lambda\gamma} \right. \\
& \left. - \left(Y_l Y_l^\dagger Y_l\right)_{\lambda\gamma}\right] - 2C_{\overline{e}\ell\ell\ell H}^{(M)\gamma\lambda\alpha\beta}\left[\lambda\left(Y_l\right)_{\lambda\gamma} - \left(Y_l Y_l^\dagger Y_l\right)_{\lambda\gamma}\right] - 3C_{\overline{d}\ell q\ell H1}^{\gamma\alpha\lambda\beta}\left[\lambda\left(Y_{\mathrm{d}}\right)_{\lambda\gamma} - \left(Y_{\mathrm{d}} Y_{\mathrm{d}}^\dagger Y_{\mathrm{d}}\right)_{\lambda\gamma}\right] \\
& + 6C_{\overline{q}u\ell\ell H}^{\gamma\lambda\alpha\beta}\left[\lambda\left(Y_{\mathrm{u}}^\dagger\right)_{\lambda\gamma} - \left(Y_{\mathrm{u}}^\dagger Y_{\mathrm{u}} Y_{\mathrm{u}}^\dagger\right)_{\lambda\gamma}\right] + \alpha \leftrightarrow \beta
\end{aligned}$$

$$\gamma_{ij}^{(7,7)} C_7^j + \gamma_i^{(7,5)} C_5 C_5 C_5 + \gamma_{ij}^{(7,6)} C_5 C_6^j$$

Used for a complete and consistent one-loop analysis of the SMEFT up to dimension seven

Applications

Non-renormalization Theorem

- Every operator $\mathcal{O}$ has the so-called holomorphic weight ω and antiholomophic weight $\overline{\omega}$
- For the same mass dimensional operators $\mathcal{O}_i$ and $\mathcal{O}_j$, contributions from $\mathcal{O}_j$ to the one-loop anomalous dimension matrix γ_{ij} of $\mathcal{O}_i$ satisfies

C. Cheung and C. H. Shen, 2015

$$\boxed{\gamma_{ij} = 0 \ \text{ if } \omega_i < \omega_j \text{ or } \overline{\omega}_i < \overline{\omega}_j}$$

$$(4\pi)^2 \frac{dc_i}{d\log\mu} = \sum_j \gamma_{ij} c_j$$

- But an exception in the SM(EFT), i.e., Yukawa couplings of nonholomorphic form, like $Y_{\rm u} Y_{{\rm d},l}$

$$16\pi^2 \mu \frac{\mathrm{d}C_7^i}{\mathrm{d}\mu} = \gamma_{ij}^{(7,7)} C_7^j + \gamma_i^{(7,5)} C_5 C_5 C_5 + \gamma_{ij}^{(7,6)} C_5 C_6^j$$

The structure of the one-loop anomalous dimension matrix γ_{ij} for dim-7 baryon-number-violating operators

DZ, 2023b

γ_{ij} ; C_j $(w_j, \overline{w}_j)$; C_i $(w_i, \overline{w}_i)$	$C^{(S)}_{\bar{e}dddD}$ $(5,3)$	$C^{(S)}_{\bar{\ell}qddD}$ $(5,3)$	$C^{(A)}_{\bar{e}qdd\tilde{H}}$ $(5,5)$	$C_{\bar{\ell}dqq\tilde{H}}$ $(5,5)$	$C_{\bar{\ell}dud\tilde{H}}$ $(7,3)$	$C^{(M)}_{\bar{\ell}dddH}$ $(7,3)$
$C^{(S)}_{\bar{e}dddD}$ $(5,3)$	g^2, y^2	y^2	0	0	0	0
$C^{(S)}_{\bar{\ell}qddD}$ $(5,3)$	y^2	g^2, y^2	0	0	0	0
$C^{(A)}_{\bar{e}qdd\tilde{H}}$ $(5,5)$	$g^2 y, y^3$	y^3	g^2, y^2	y^2	$\overline{y}^2$	0
$C_{\bar{\ell}dqq\tilde{H}}$ $(5,5)$	y^3	$g^2 y, y^3$	y^2	g^2, y^2	$\overline{y}^2$	0
$C_{\bar{\ell}dud\tilde{H}}$ $(7,3)$	y^3	$g^2 y, y^3$	y^2	y^2	g^2, y^2	y^2
$C^{(M)}_{\bar{\ell}dddH}$ $(7,3)$	y^3	$g^2 y, y^3$	0	0	y^2	g^2, y^2

Fully consistent with the above non-renormalization theorem!!!

Non-renormalization Theorem

Results for dim-7 baryon-number-conserving operators DZ, 2023b

γ_{ij} \ C_j $(w_j, \overline{w}_j)$ / C_i $(w_i, \overline{w}_i)$	$C^{(S)}_{\ell HD1}$ $(3,5)$	$C^{(S)}_{\ell HD2}$ $(3,5)$	$C^{(S)}_{\bar{d}u\ell\ell D}$ $(3,5)$	$C^{(A)}_{\ell HB}$ $(3,7)$	$C_{\ell HW}$ $(3,7)$	$C^{(S,A,M)}_{\bar{e}\ell\ell\ell H}$ $(3,7)$	$C_{\bar{d}\ell q\ell H1}$ $(3,7)$	$C_{\bar{d}\ell q\ell H2}$ $(3,7)$	$C_{\ell eHD}$ $(5,5)$	$C_{\bar{d}\ell ueH}$ $(5,5)$	$C_{\bar{q}u\ell\ell H}$ $(5,5)$	$C^{(S)}_{\ell H}$ $(5,7)$
$C^{(S)}_{\ell HD1}$ $(3,5)$	g^2, y^2	g^2, λ	y^2	0	0	0	0	0	0	0	0	0
$C^{(S)}_{\ell HD2}$ $(3,5)$	g^2, y^2	g^2, y^2, λ	0	0	0	0	0	0	0	0	0	0
$C^{(S)}_{\bar{d}u\ell\ell D}$ $(3,5)$	y^2	y^2	g^2, y^2	0	0	0	0	0	0	0	0	0
$C^{(A)}_{\ell HB}$ $(3,7)$	gy^2	gy^2	0	g^2, y^2, λ	g^2	gy	gy	0	0	0	0	0
$C_{\ell HW}$ $(3,7)$	g^3, gy^2	g^3, gy^2	0	g^2	g^2, y^2, λ	gy	gy	gy	0	0	0	0
$C^{(S,A,M)}_{\bar{e}\ell\ell\ell H}$ $(3,7)$	g^2y, y^3	g^2y, y^3	0	gy	gy	g^2, y^2	y^2	y^2	0	0	$\overline{y}^2$	0
$C_{\bar{d}\ell q\ell H1}$ $(3,7)$	g^2y, y^3	g^2y, y^3	g^2y, y^3	gy	gy	y^2	g^2, y^2	g^2, y^2	0	$\overline{y}^2$	$\overline{y}^2$	0
$C_{\bar{d}\ell q\ell H2}$ $(3,7)$	g^2y, y^3	g^2y, y^3	g^2y, y^3	gy	gy	y^2	g^2, y^2	g^2, y^2	0	$\overline{y}^2$	$\overline{y}^2$	0
$C_{\ell eHD}$ $(5,5)$	g^2y, y^3	$g^2y, \lambda y, y^3$	0	0	0	0	0	0	g^2, y^2, λ	y^2	0	0
$C_{\bar{d}\ell ueH}$ $(5,5)$	y^3	y^3	g^2y, y^3	0	0	0	y^2	y^2	y^2	g^2, y^2	y^2	0
$C_{\bar{q}u\ell\ell H}$ $(5,5)$	g^2y, y^3	g^2y, y^3	g^2y, y^3	0	0	y^2	y^2	y^2	0	y^2	g^2, y^2	0
$C^{(S)}_{\ell H}$ $(5,7)$	$g^4, \lambda g^2, \lambda y^2, y^4$	$g^4, \lambda g^2, \lambda y^2, y^4$	0	0	g^3, gy^2	$\lambda y, y^3$	$\lambda y, y^3$	0	$\lambda y, g^2y, y^3$	0	$\lambda y, y^3$	g^2, y^2, λ

Radiative Corrections to Neutrino Masses

Neutrino masses in the SMEFT:

$$M_\nu\left(\Lambda_{\rm EW}\right) = -\frac{v^2}{2}\left[C_5\left(\Lambda_{\rm EW}\right) + v^2 C_{\ell H}^{(S)\dagger}\left(\Lambda_{\rm EW}\right)\right]$$

The leading-logarithmic approximation:

Comes from two redundant dim-6 operators, $R'^{(1)}_{H\ell}$ and $R'^{(3)}_{H\ell}$

$$\begin{aligned}
\delta M_\nu^{\alpha\beta} = -\frac{v^2}{2}\frac{1}{16\pi^2}\ln\left(\frac{v}{\Lambda}\right)\Big\{ & C_5^{\alpha\beta}\left[-3g_2^2 + 4\lambda + 6{\rm Tr}\left(Y_{\rm u}Y_{\rm u}^\dagger\right)\right] \\
& + 2v^2 C_5^{\alpha\beta}\left[-3C_H + \left(8\lambda - \frac{5}{3}g_2^2\right)C_{H\Box} - \left(2\lambda + \frac{3}{4}g_1^2 - \frac{3}{4}g_2^2\right)C_{HD} - 3g_2^2 C_{HW}\right. \\
& - \frac{3}{2}{\rm i}\left(g_1^2 C_{H\widetilde{B}} + 3g_2^2 C_{H\widetilde{W}} + g_1 g_2 C_{H\widetilde{W}B}\right) + \frac{1}{2}{\rm Tr}\left(C_5 C_5^\dagger\right) - \frac{2}{3}g_2^2{\rm Tr}\left(3C_{Hq}^{(3)\dagger} + C_{H\ell}^{(3)\dagger}\right) \\
& \left. - 3{\rm Tr}\left(C_{uH}Y_{\rm u}^\dagger\right) + 6{\rm Tr}\left(Y_{\rm u}^\dagger C_{Hq}^{(3)\dagger}Y_{\rm u}\right)\right] + \boxed{\frac{5}{2}v^2\left(C_5 C_5^\dagger C_5\right)^{\alpha\beta}} + \frac{3}{2}\left(g_1^2 + g_2^2\right)v^2 \\
& \times\left[\left(C_{H\ell}^{(3)\dagger}C_5\right)^{\alpha\beta} + \left(C_{H\ell}^{(3)\dagger}C_5\right)^{\beta\alpha} - \left(C_{H\ell}^{(1)\dagger}C_5\right)^{\alpha\beta} - \left(C_{H\ell}^{(1)\dagger}C_5\right)^{\beta\alpha}\right]\Big\}
\end{aligned}$$

- **The small down-type quark and lepton Yukawa couplings, and contributions from dim-7 operators are ignored**
- **Apart from the C_5 cubic term in red, the above result is consistent with that obtained in [M. Chala, A. Titov, 2021]**

Radiative Corrections to Neutrino Masses

Numerical analysis in [M. Chala, A. Titov, 2021]: one generation

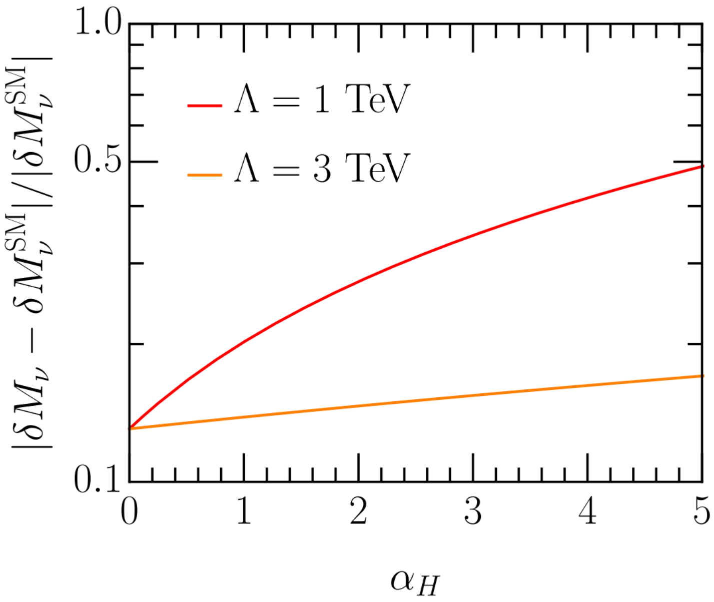


FIG. 2. *Impact of dimension-six interactions on the size of the leading-logarithmic correction to M_ν as a function of α_H for different values of the new physics scale Λ. The Wilson coefficients of other dimension-six operators have been set to their best-fit values from [44]. Dimension-seven operators have been assumed to vanish. See the text for further details.*

Corrections from insertion of dim-5 + dim-6 operators can reach 50% of those purely from the dim-5 operator for Λ = 1 TeV, depending on the Wilson coefficient of $\mathcal{O}_H = (H^\dagger H)^3$

Summary

Summary

- A Green's basis and a new physical basis for the SMEFT dim-7 operators are proposed
- The reduction relations between the above two bases have been achieved, where some redundant dim-6 operators are involved
- With the above bases, the complete one-loop RGEs for the dim-5 and dim-7 operators up to $\mathcal{O}(\Lambda^{-3})$ have been derived for the first time
- These results can be used for a complete and consistent one-loop analysis of the SMEFT up to dimension seven

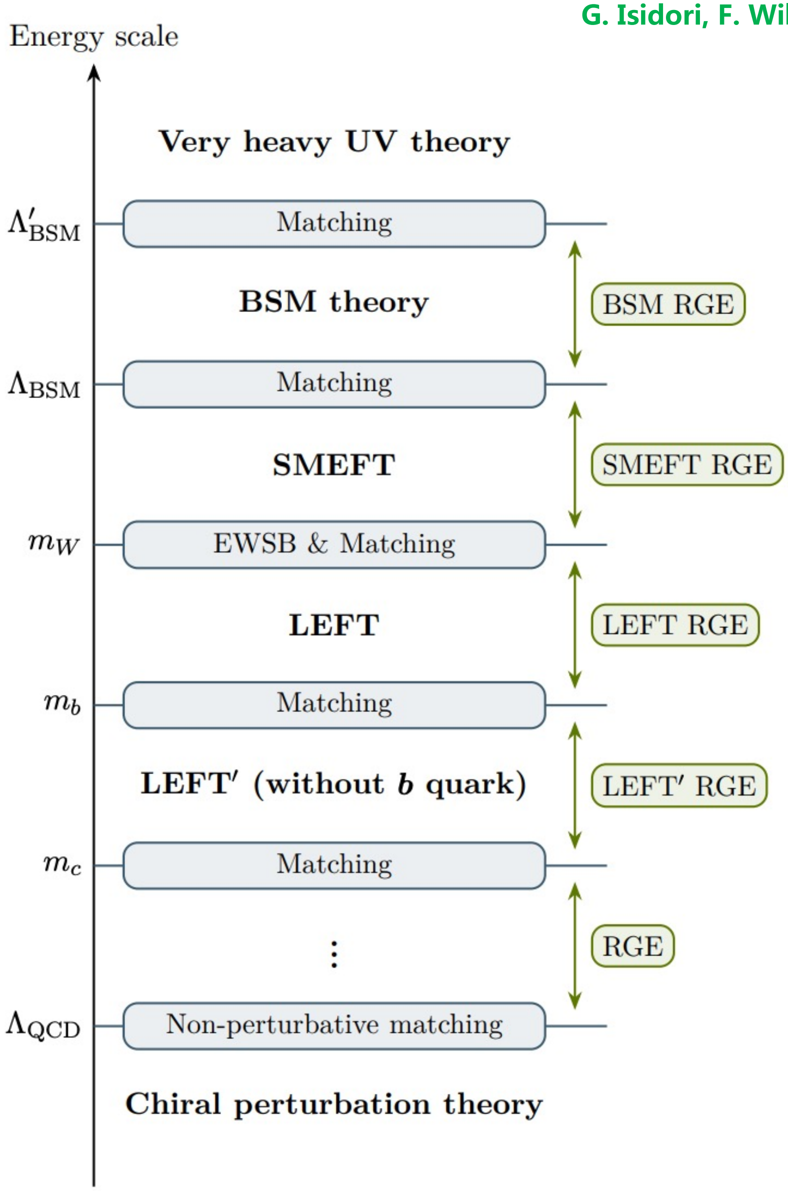
THANKS FOR YOUR ATTENTION

Backup

Backup

The usage of EFTs

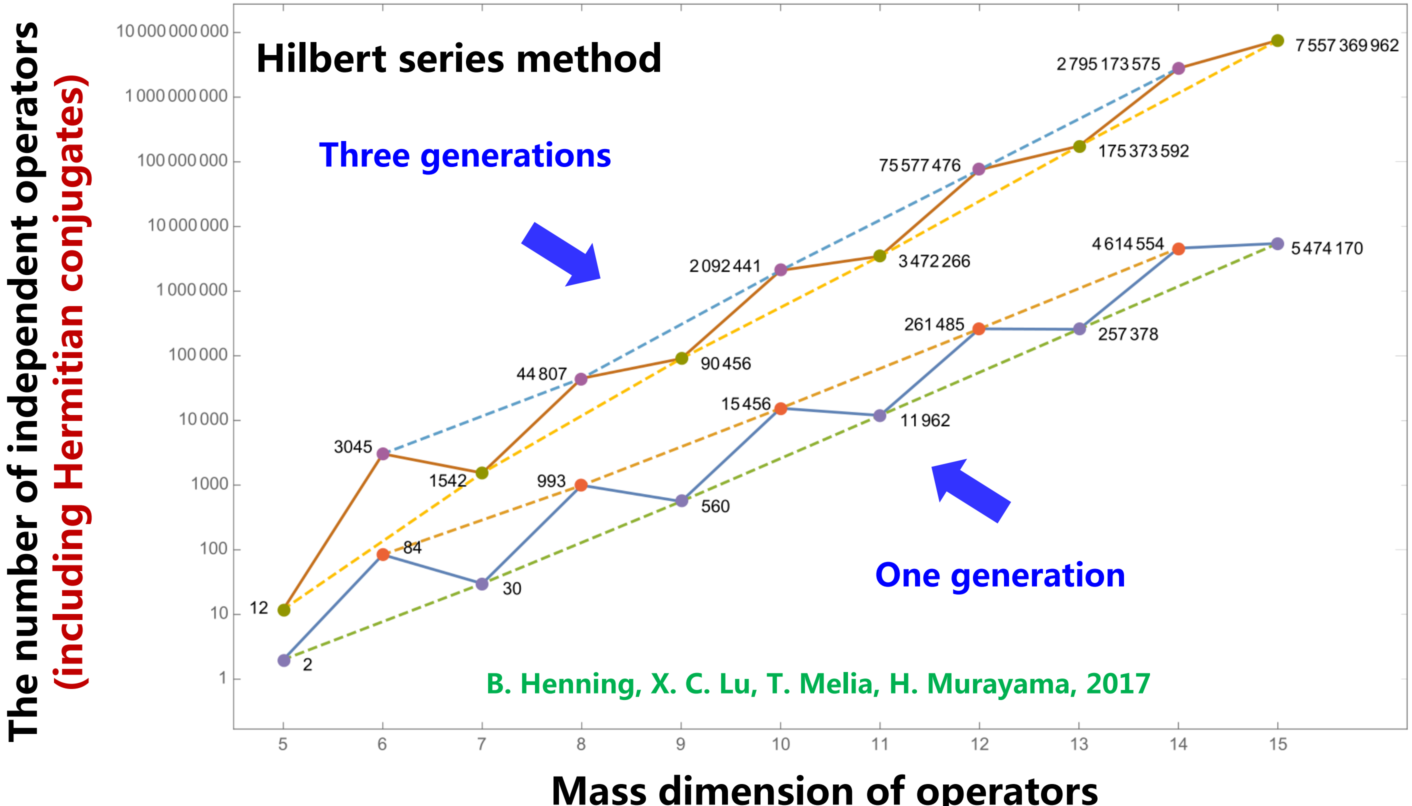
G. Isidori, F. Wilsch, D. Wyler, 2023



Backup

The number of **independent** operators in the SMEFT:

IBP, **Fierz transformations**, **Algebraic relations**, **Field Redefinitions** (or equations of motion)



Growing roughly exponentially with the mass dimension

- ☐ **Hilbert series method**
- ☐ Operator-counting packages: **Basisgen** (J. C. Criado, 2019) and **Sym2Int** (R. M. Fonseca, 2019)

The number of operators ✓

Field ingredients in operators ✓

Lorentz and gauge invariant structure of operators ✗

Backup

How to construct a complete operator basis:

- **Brute force (enumeration) method**

dim-5 S. Weinberg, 1979 Unique

dim-6 W. Buchmuller, D. Wyler, 1986; B. Grzadkowski et al, 2010 24 years!!!

dim-7 L. Lehman, 2014; Y. Liao, X. D. Ma, 2016; 2019 Not so satisfactory

dim-8 C. W. Murphy, 2020

dim-9 Y. Liao, X. D. Ma, 2020

- **Amplitude operator correspondence and group theoretic techniques**

dim-8 H. L. Li et al, 2021a

dim-9 H. L. Li et al, 2021b

Mathematica package: ABC4EFT (Amplitude Basis Construction for Effective Field Theories)
H. L. Li et al, 2022

up to dim-12 R. V. Harlander, T. Kempkens, M. C. Schaaf, 2023
Package: AutoEFT (re-implementation of the algorithm of ABC4EFT)

- **On-shell amplitude method**

up to dim- 6 T. Ma, J. Shu, M. L. Xiao, 2019; R. Aoudea, C. S. Machado, 2019; Zi-Yu Dong et al, 2022

up to dim- 8 G. Durieux, C. S. Machado, 2019; M. A. Hubera, S. De Angelis, 2021

Mathematica package: MassiveGraphs S. De Angelis, 2022

Backup

$$\mathcal{O}^{\alpha\beta\gamma\lambda}_{\bar{e}\ell\ell\ell H} + \mathcal{O}^{\alpha\lambda\gamma\beta}_{\bar{e}\ell\ell\ell H} - \mathcal{O}^{\alpha\lambda\beta\gamma}_{\bar{e}\ell\ell\ell H} - \mathcal{O}^{\alpha\gamma\beta\lambda}_{\bar{e}\ell\ell\ell H} = 0 \,,$$

$$\mathcal{O}^{\alpha\beta\gamma\lambda}_{\bar{\ell}dddH} + \mathcal{O}^{\alpha\beta\lambda\gamma}_{\bar{\ell}dddH} = 0 \,, \quad \mathcal{O}^{\alpha\beta\gamma\lambda}_{\bar{\ell}dddH} + \mathcal{O}^{\alpha\gamma\lambda\beta}_{\bar{\ell}dddH} + \mathcal{O}^{\alpha\lambda\beta\gamma}_{\bar{\ell}dddH} = 0 \,,$$

Mixed symmetries among flavor indices of repeated fermion fields

DZ, 2023a

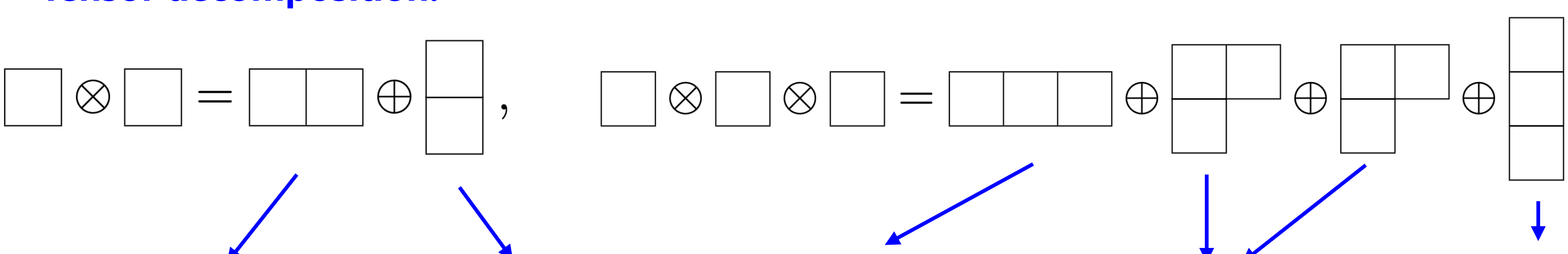
Decomposing operators to those with explicit (mixed) symmetries in the flavor space

- **Fermion fields can be regarded as the fundamental representation of SU(n) or U(n) flavor symmetry with n being the number of generations (flavors)**
- **Operators with several identical fermion fields denote the tensor of SU(n), and can be decomposed to direct sums of irreducible representations of SU(n)**

➢ **Repeated fermion fields transform under SU(n) as** **Young tableau**

$\square \otimes \square$ **for 2 identical fermion fields** $\square \otimes \square \otimes \square$ **for 3 identical fermion fields**

➢ **Tensor decomposition:**



symmetric $\frac{n(n+1)}{2}$

antisymmetric $\frac{n(n-1)}{2}$

totally symmetric $\frac{n(n+1)(n+2)}{6}$

mixed-symmetric $\frac{n(n+1)(n-1)}{6}$

totally anti-symmetric $\frac{n(n-1)(n-2)}{6}$

Backup

Examples: **DZ, 2023a**

■ Operator with 2 identical fermion fields

$$\mathcal{O}_{\ell H}^{\alpha\beta} = \mathcal{O}_{\ell H}^{(S)\alpha\beta} + \mathcal{O}_{\ell H}^{(A)\alpha\beta}$$

$$\boxed{\mathcal{O}_{\ell H}^{(S)\alpha\beta} = \frac{1}{2}\left(\mathcal{O}_{\ell H}^{\alpha\beta} + \mathcal{O}_{\ell H}^{\beta\alpha}\right)}\,, \quad \cancel{\mathcal{O}_{\ell H}^{(A)\alpha\beta} = \frac{1}{2}\left(\mathcal{O}_{\ell H}^{\alpha\beta} - \mathcal{O}_{\ell H}^{\beta\alpha}\right)}\,,$$

$$\mathcal{O}_{\ell H}^{\alpha\beta} - \mathcal{O}_{\ell H}^{\beta\alpha} = 0$$

■ Operator with 3 identical fermion fields

$$\mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} = \mathcal{O}_{\bar{e}\ell\ell\ell H}^{(S)\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{(A)\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{(M)\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{(M')\alpha\beta\gamma\lambda}$$

$$\begin{aligned}
\mathcal{O}_{\bar{e}\ell\ell\ell H}^{(S)\alpha\beta\gamma\lambda} &= \frac{1}{6}\left(\mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\beta\gamma} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\lambda\beta} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\lambda\gamma} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta}\right) \\
\mathcal{O}_{\bar{e}\ell\ell\ell H}^{(A)\alpha\beta\gamma\lambda} &= \frac{1}{6}\left(\mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\beta\gamma} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\lambda\beta} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\lambda\gamma} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta}\right) \\
\mathcal{O}_{\bar{e}\ell\ell\ell H}^{(M)\alpha\beta\gamma\lambda} &= \frac{1}{3}\left(\mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\lambda\beta}\right)\,,
\end{aligned}$$

$$\cancel{\mathcal{O}_{\bar{e}\ell\ell\ell H}^{(M')\alpha\beta\gamma\lambda} = \frac{1}{3}\left(\mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\beta\gamma} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda}\right)}\,,$$

$$\mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\beta\gamma} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} = 0$$

■ Operator with non-trivial flavor relations

$$\mathcal{O}_{\bar{e}dddD}^{\alpha\beta\gamma\lambda} = \mathcal{O}_{\bar{e}dddD}^{(S)\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}dddD}^{(A)\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}dddD}^{(M)\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}dddD}^{(M')\alpha\beta\gamma\lambda}$$

$$\boxed{\mathcal{O}_{\bar{e}dddD}^{(S)\alpha\beta\gamma\lambda} = \frac{1}{6}\left(\mathcal{O}_{\bar{e}dddD}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\beta\gamma} + \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\lambda\beta} + \mathcal{O}_{\bar{e}dddD}^{\alpha\beta\lambda\gamma} + \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\beta\lambda} + \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\gamma\beta}\right)}$$

$$\begin{aligned}
\mathcal{O}_{\bar{e}dddD}^{(A)\alpha\beta\gamma\lambda} &= \frac{1}{6}\left(\mathcal{O}_{\bar{e}dddD}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\beta\gamma} + \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\lambda\beta} - \mathcal{O}_{\bar{e}dddD}^{\alpha\beta\lambda\gamma} - \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\beta\lambda} - \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\gamma\beta}\right) \\
&= \frac{1}{6}\left[\left(Y_{\rm d}^\dagger\right)_{\lambda\rho}\mathcal{O}_{\bar{e}qdd\widetilde{H}}^{\alpha\rho\beta\gamma} + \left(Y_{\rm d}^\dagger\right)_{\gamma\rho}\mathcal{O}_{\bar{e}qdd\widetilde{H}}^{\alpha\rho\lambda\beta} + \left(Y_{\rm d}^\dagger\right)_{\beta\rho}\mathcal{O}_{\bar{e}qdd\widetilde{H}}^{\alpha\rho\gamma\lambda}\right]\,, \\
\mathcal{O}_{\bar{e}dddD}^{(M)\alpha\beta\gamma\lambda} &= \frac{1}{3}\left(\mathcal{O}_{\bar{e}dddD}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\gamma\beta} - \mathcal{O}_{\bar{e}dddD}^{\alpha\beta\lambda\gamma} - \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\lambda\beta}\right) \\
\mathcal{O}_{\bar{e}dddD}^{(M')\alpha\beta\gamma\lambda} &= \frac{1}{3}\left(\mathcal{O}_{\bar{e}dddD}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}dddD}^{\alpha\beta\lambda\gamma} - \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\gamma\beta} - \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\beta\gamma}\right) \\
&= \frac{1}{3}\left[\left(\mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\beta\gamma} + \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\beta\lambda} - \mathcal{O}_{\bar{e}dddD}^{\alpha\beta\lambda\gamma} - \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\lambda\beta}\right) + \left(\mathcal{O}_{\bar{e}dddD}^{\alpha\beta\gamma\lambda} - \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\beta\lambda}\right)\right. \\
&\quad \left. + \left(\mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\lambda\beta} - \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\gamma\beta}\right) - 2\left(\mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\beta\gamma} - \mathcal{O}_{\bar{e}dddD}^{\alpha\beta\lambda\gamma}\right)\right] \\
&= \frac{1}{3}\left[\left(Y_l\right)_{\rho\alpha}\mathcal{O}_{\bar{\ell}dddH}^{\rho\gamma\beta\lambda} + \left(Y_{\rm d}^\dagger\right)_{\lambda\rho}\mathcal{O}_{\bar{e}qdd\widetilde{H}}^{\alpha\rho\beta\gamma} + \left(Y_{\rm d}^\dagger\right)_{\beta\rho}\mathcal{O}_{\bar{e}qdd\widetilde{H}}^{\alpha\rho\gamma\lambda} - 2\left(Y_{\rm d}^\dagger\right)_{\gamma\rho}\mathcal{O}_{\bar{e}qdd\widetilde{H}}^{\alpha\rho\lambda\beta}\right]
\end{aligned}$$

$$\mathcal{O}_{\bar{e}dddD}^{\alpha\beta\gamma\lambda} - \mathcal{O}_{\bar{e}dddD}^{\alpha\gamma\beta\lambda} = \left(Y_{\rm d}^\dagger\right)_{\lambda\rho}\mathcal{O}_{\bar{e}qdd\widetilde{H}}^{\alpha\rho\beta\gamma}\,,$$

$$\mathcal{O}_{\bar{e}dddD}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}dddD}^{\alpha\lambda\gamma\beta} - \gamma \leftrightarrow \lambda = \left(Y_l\right)_{\rho\alpha}\mathcal{O}_{\bar{\ell}dddH}^{\rho\beta\gamma\lambda}$$

The remaining combinations are free from (linearly independent of) all (non-trivial) flavor relations

Backup

- For one-loop matchings and calculations of RGEs with **Feynman diagrammatic approach**, usually **off-shell scheme** is used and hence only **1PI diagrams** are involved
- In such a case, the so-called **Green's basis** is needed, which is directly related to **1PI Green's functions**
  M. Jiang et al, 2019; V. Gherardi, D. Marzocca, E. Venturini, 2020
- Operators in the Green's basis are independent in the sense of **IBP**, **Fierz transformations**, **Algebraic relations**, **but NOT EoMs**
- Operators in the Green's basis can be converted to those in the physical basis with the help of EoMs

Matching Mode $\mathcal{L}_{\rm UV}$ → **1PI diagrams** ← $\mathcal{L}_{\rm EFT}$ **RGEs Mode**

↓ Off-shell scheme

Results in the Green's basis

↓ Reduction relations

Results in the physical basis

The Green's basis for **dim-6** operators (including reduction relations):

V. Gherardi, D. Marzocca, E. Venturini, 2020

The Green's basis for **dim-8** operators:

M. Chala, A. Diaz-Carmona, G. Guedes, 2022

Z. Ren , J. H. Yu, 2022

The strategy adopted by **Matchmakereft**

A. Carmona, A. Lazopoulos, P. Olgoso, J. Santiago, 2021

Backup

$\psi^2 H^4$			$\psi^2 H^3 D$		
$\mathcal{O}_{\ell H}$	$\epsilon^{ab}\epsilon^{de}\left(\ell_{\rm L}^a C \ell_{\rm L}^d\right) H^b H^e \left(H^\dagger H\right)$	$\frac{1}{2}n(n+1)$	$\mathcal{O}_{\ell e H D}$	$\epsilon^{ab}\epsilon^{de}\left(\ell_{\rm L}^a C \gamma_\mu E_{\rm R}\right) H^b H^d {\rm i} D^\mu H^e$	n^2
$\psi^2 H^2 D^2$			$\psi^4 H$		
$\mathcal{O}_{\ell H D1}$	$\epsilon^{ab}\epsilon^{de}\left(\ell_{\rm L}^a C D^\mu \ell_{\rm L}^b\right) H^d D_\mu H^e$	n^2	$\mathcal{O}_{\bar{e}\ell\ell\ell H}$	$\epsilon^{ab}\epsilon^{de}\left(\overline{E_{\rm R}}\ell_{\rm L}^a\right)\left(\ell_{\rm L}^b C \ell_{\rm L}^d\right) H^e$	$\frac{1}{3}n^2\left(2n^2+1\right)$
$\mathcal{O}_{\ell H D2}$	$\epsilon^{ad}\epsilon^{be}\left(\ell_{\rm L}^a C D^\mu \ell_{\rm L}^b\right) H^d D_\mu H^e$	n^2	$\mathcal{O}_{\bar{d}\ell q\ell H1}$	$\epsilon^{ab}\epsilon^{de}\left(\overline{D_{\rm R}}\ell_{\rm L}^a\right)\left(Q_{\rm L}^b C \ell_{\rm L}^d\right) H^e$	n^4
$\mathcal{R}_{\ell H D3}$	$\epsilon^{ad}\epsilon^{be}\left(\ell_{\rm L}^a C \ell_{\rm L}^b\right) D^\mu H^d D_\mu H^e$	$\frac{1}{2}n(n+1)$	$\mathcal{O}_{\bar{d}\ell q\ell H2}$	$\epsilon^{ad}\epsilon^{be}\left(\overline{D_{\rm R}}\ell_{\rm L}^a\right)\left(Q_{\rm L}^b C \ell_{\rm L}^d\right) H^e$	n^4
$\mathcal{R}_{\ell H D4}$	$\epsilon^{ad}\epsilon^{be}\left(D^\mu \ell_{\rm L}^a C D_\mu \ell_{\rm L}^b\right) H^d H^e$	$\frac{1}{2}n(n+1)$	$\mathcal{O}_{\bar{d}\ell u e H}$	$\epsilon^{ab}\left(\overline{D_{\rm R}}\ell_{\rm L}^a\right)\left(U_{\rm R} C E_{\rm R}\right) H^b$	n^4
$\mathcal{R}_{\ell H D5}$	$\epsilon^{ab}\epsilon^{de}\left(\ell_{\rm L}^a C \sigma_{\mu\nu} D^\mu \ell_{\rm L}^b\right) H^d D^\nu H^e$	n^2	$\mathcal{O}_{\bar{q}u\ell\ell H}$	$\epsilon^{ab}\left(\overline{Q_{\rm L}} U_{\rm R}\right)\left(\ell_{\rm L} C \ell_{\rm L}^a\right) H^b$	n^4
$\mathcal{R}_{\ell H D6}$	$\epsilon^{ad}\epsilon^{be}\left(D^\mu \ell_{\rm L}^a C \sigma_{\mu\nu} D^\nu \ell_{\rm L}^b\right) H^d H^e$	$\frac{1}{2}n(n+1)$	$\mathcal{O}_{\bar{\ell}dud\tilde{H}}$	$\left(\overline{\ell_{\rm L}} D_{\rm R}\right)\left(U_{\rm R} C D_{\rm R}\right)\widetilde{H}$	n^4
$\psi^2 H^2 X$			$\mathcal{O}_{\bar{\ell}dddH}$	$\left(\overline{\ell_{\rm L}} D_{\rm R}\right)\left(D_{\rm R} C D_{\rm R}\right) H$	$\frac{1}{3}n^2\left(n^2-1\right)$
$\mathcal{O}_{\ell HB}$	$\epsilon^{ab}\epsilon^{de}\left(\ell_{\rm L}^a C \sigma_{\mu\nu} \ell_{\rm L}^d\right) H^b H^e B^{\mu\nu}$	$\frac{1}{2}n(n-1)$	$\mathcal{O}_{\bar{e}qdd\tilde{H}}$	$\epsilon^{ab}\left(\overline{E_{\rm R}} Q_{\rm L}^a\right)\left(D_{\rm R} C D_{\rm R}\right)\widetilde{H}^b$	$\frac{1}{2}n^3(n-1)$
$\mathcal{O}_{\ell HW}$	$\epsilon^{ab}\left(\epsilon\sigma^I\right)^{de}\left(\ell_{\rm L}^a C \sigma_{\mu\nu} \ell_{\rm L}^d\right) H^b H^e W^{I\mu\nu}$	n^2	$\mathcal{O}_{\bar{\ell}dqq\tilde{H}}$	$\epsilon^{ab}\left(\overline{\ell_{\rm L}} D_{\rm R}\right)\left(Q_{\rm L} C Q_{\rm L}^a\right)\widetilde{H}^b$	n^4
$\psi^4 D$					
$\mathcal{O}_{\bar{e}dddD}$	$\left(\overline{E_{\rm R}}\gamma_\mu D_{\rm R}\right)\left(D_{\rm R} C {\rm i} D^\mu D_{\rm R}\right)$	n^4	$\mathcal{O}_{\bar{\ell}qddD}$	$\left(\overline{\ell_{\rm L}}\gamma_\mu Q_{\rm L}\right)\left(D_{\rm R} C {\rm i} D^\mu D_{\rm R}\right)$	n^4
$\mathcal{O}_{\bar{d}u\ell\ell D}$	$\epsilon^{ab}\left(\overline{D_{\rm R}}\gamma_\mu U_{\rm R}\right)\left(\ell_{\rm L}^a C {\rm i} D^\mu \ell_{\rm L}^b\right)$	n^4	$\mathcal{R}_{\bar{\ell}dDqd}$	$\left(\overline{\ell_{\rm L}} D_{\rm R}\right)\left({\rm i} D^\mu Q_{\rm L} C \gamma_\mu D_{\rm R}\right)$	n^4
$\mathcal{R}_{\bar{d}\ell\ell Du}$	$\epsilon^{ab}\left(\overline{D_{\rm R}}\ell_{\rm L}^a\right)\left(\ell_{\rm L}^b C \gamma_\mu {\rm i} D^\mu U_{\rm R}\right)$	n^4	$\mathcal{R}_{\bar{\ell}dqDd}$	$\left(\overline{\ell_{\rm L}} D_{\rm R}\right)\left(Q_{\rm L} C \gamma_\mu {\rm i} D^\mu D_{\rm R}\right)$	n^4
$\mathcal{R}_{\bar{d}D\ell\ell u}$	$\epsilon^{ab}\left(\overline{D_{\rm R}}{\rm i} D^\mu \ell_{\rm L}^a\right)\left(\ell_{\rm L}^b C \gamma_\mu U_{\rm R}\right)$	n^4			

$$\mathcal{O}_{\ell H}^{\alpha\beta} - \mathcal{O}_{\ell H}^{\beta\alpha} = 0\,,$$
$$\mathcal{O}_{\ell HB}^{\alpha\beta} + \mathcal{O}_{\ell HB}^{\beta\alpha} = 0\,,$$
$$\mathcal{O}_{\bar{e}qdd\tilde{H}}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}qdd\tilde{H}}^{\alpha\beta\lambda\gamma} = 0\,,$$
$$\mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\gamma\beta} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\lambda\beta\gamma} - \mathcal{O}_{\bar{e}\ell\ell\ell H}^{\alpha\gamma\beta\lambda} = 0\,,$$
$$\mathcal{O}_{\bar{\ell}dddH}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{\ell}dddH}^{\alpha\beta\lambda\gamma} = 0\,, \quad \mathcal{O}_{\bar{\ell}dddH}^{\alpha\beta\gamma\lambda} + \mathcal{O}_{\bar{\ell}dddH}^{\alpha\gamma\lambda\beta} + \mathcal{O}_{\bar{\ell}dddH}^{\alpha\lambda\beta\gamma} = 0\,,$$

$$\mathcal{R}_{\ell HD3}^{\alpha\beta} - \mathcal{R}_{\ell HD3}^{\beta\alpha} = 0\,,$$
$$\mathcal{R}_{\ell HD4}^{\alpha\beta} - \mathcal{R}_{\ell HD4}^{\beta\alpha} = 0\,,$$
$$\mathcal{R}_{\ell HD6}^{\alpha\beta} - \mathcal{R}_{\ell HD6}^{\beta\alpha} = 0\,.$$

DZ, 2023a

X. X. Li , Z. Ren , J. H. Yu, 2023

No non-trivial flavor relations

Also decomposed as SU(3) tensor

Backup

A list of dim-6 operator in the Greens basis involved in reduction relations for dim-7 operators

$\mathcal{R}_{2W}$	$-\frac{1}{2}\left(D_\mu W^{I\mu\nu}\right)\left(D^\rho W^I_{\rho\nu}\right)$	$\mathcal{R}_{WDH}$	$D_\nu W^{I\mu\nu}\left(H^\dagger \mathrm{i}\overleftrightarrow{D}^I_\mu H\right)$	$\mathcal{R}_{DH}$	$\left(D_\mu D^\mu H\right)^\dagger\left(D_\nu D^\nu H\right)$
$\mathcal{R}'_{HD}$	$\left(H^\dagger H\right)\left(D_\nu H\right)^\dagger\left(D^\mu H\right)$	$\mathcal{R}''_{HD}$	$\left(H^\dagger H\right) D_\mu\left(H^\dagger \mathrm{i}\overleftrightarrow{D}_\mu H\right)$	$\mathcal{R}^{\alpha\beta}_{\ell D}$	$\frac{\mathrm{i}}{2}\overline{\ell_{\alpha\mathrm{L}}}\left\{D_\mu D^\mu, \not{D}\right\}\ell_{\beta\mathrm{L}}$
$\mathcal{R}^{\alpha\beta}_{uHD1}$	$\left(\overline{Q_{\alpha\mathrm{L}}}U_{\beta\mathrm{R}}\right) D_\mu D^\mu \widetilde{H}$	$\mathcal{R}^{\alpha\beta}_{uHD2}$	$\left(\overline{Q_{\alpha\mathrm{L}}}\mathrm{i}\sigma_{\mu\nu}D^\mu U_{\beta\mathrm{R}}\right) D^\nu \widetilde{H}$	$\mathcal{R}^{\alpha\beta}_{uHD4}$	$\left(\overline{Q_{\alpha\mathrm{L}}}D^\mu U_{\beta\mathrm{R}}\right) D^\mu \widetilde{H}$
$\mathcal{R}^{\alpha\beta}_{dHD1}$	$\left(\overline{Q_{\alpha\mathrm{L}}}D_{\beta\mathrm{R}}\right) D_\mu D^\mu H$	$\mathcal{R}^{\alpha\beta}_{dHD2}$	$\left(\overline{Q_{\alpha\mathrm{L}}}\mathrm{i}\sigma_{\mu\nu}D^\mu D_{\beta\mathrm{R}}\right) D^\nu H$	$\mathcal{R}^{\alpha\beta}_{dHD4}$	$\left(\overline{Q_{\alpha\mathrm{L}}}D_\mu D_{\beta\mathrm{R}}\right) D^\mu H$
$\mathcal{R}^{\alpha\beta}_{eHD1}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}E_{\beta\mathrm{R}}\right) D_\mu D^\mu H$	$\mathcal{R}^{\alpha\beta}_{eHD2}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\mathrm{i}\sigma_{\mu\nu}D^\mu E_{\beta\mathrm{R}}\right) D^\nu H$	$\mathcal{R}^{\alpha\beta}_{eHD4}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}D_\mu E_{\beta\mathrm{R}}\right) D^\mu H$
$\mathcal{R}'^{\alpha\beta}_{W\ell}$	$\frac{1}{2}\left(\overline{\ell_{\alpha\mathrm{L}}}\sigma^I\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu \ell_{\beta\mathrm{L}}\right) W^I_{\mu\nu}$	$\mathcal{R}'^{\alpha\beta}_{\widetilde{W}\ell}$	$\frac{1}{2}\left(\overline{\ell_{\alpha\mathrm{L}}}\sigma^I\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu \ell_{\beta\mathrm{L}}\right) \widetilde{W}^I_{\mu\nu}$	$\mathcal{R}'^{\alpha\beta}_{B\ell}$	$\frac{1}{2}\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu \ell_{\beta\mathrm{L}}\right) B_{\mu\nu}$
$\mathcal{R}'^{\alpha\beta}_{\widetilde{B}\ell}$	$\frac{1}{2}\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu \ell_{\beta\mathrm{L}}\right) \widetilde{B}_{\mu\nu}$	$\mathcal{R}'^{(1)\alpha\beta}_{H\ell}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\mathrm{i}\overleftrightarrow{\not{D}}\ell_{\beta\mathrm{L}}\right)\left(H^\dagger H\right)$	$\mathcal{R}''^{(1)\alpha\beta}_{H\ell}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu \ell_{\beta\mathrm{L}}\right)\partial_\mu\left(H^\dagger H\right)$
$\mathcal{R}'^{(3)\alpha\beta}_{H\ell}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\mathrm{i}\overleftrightarrow{\not{D}}^I\ell_{\beta\mathrm{L}}\right)\left(H^\dagger\sigma^I H\right)$	$\mathcal{R}''^{(3)\alpha\beta}_{H\ell}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\sigma^I\gamma^\mu \ell_{\beta\mathrm{L}}\right) D_\mu\left(H^\dagger\sigma^I H\right)$		

Table 3: Dim-6 operators in the Green's basis converted to physical operators with the help of EOMs of the lepton and Higgs doublets. They may give contributions to the RGEs of the dim-5 and dim-7 operators. The dual tensors are defined by $\widetilde{X}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}$ with $\epsilon_{0123} = +1$ and X denoting W^I and B.

Backup

Non-renormalization Theorem without SUSY

C. Cheung and C. H. Shen, 2015
Z. Bern, J. Parra-Martinez and E. Sawyer, 2019

Each operator $\mathcal{O}$ (or on-shell amplitude) has the so-called holomorphic weight ω and antiholomophic weight $\bar{\omega}$, defined as

$$w(\mathcal{O}) = n(\mathcal{O}) - h(\mathcal{O}), \qquad \bar{w}(\mathcal{O}) = n(\mathcal{O}) + h(\mathcal{O})$$

$n(\mathcal{O})$ is the number of particles created by $\mathcal{O}$ and $h(\mathcal{O})$ is their total helicity

$\mathcal{O}$	$F_{\alpha\beta}$	ψ_α	ϕ	$\bar{\psi}_{\dot{\alpha}}$	$\bar{F}_{\dot{\alpha}\dot{\beta}}$
h	$+1$	$+1/2$	0	$-1/2$	-1
$(w, \bar{w})$	$(0,\ 2)$	$(1/2, 3/2)$	$(1,\ 1)$	$(3/2, 1/2)$	$(2,\ 0)$

An operator $\mathcal{O}_i$ can only be renormalized by an operator $\mathcal{O}_j$ at one loop if the corresponding weights $(\omega_i, \bar{\omega}_i)$, and $(\omega_j, \bar{\omega}_j)$ satisfy the inequalities

$$\omega_i \geq \omega_j \text{ and } \bar{\omega}_i \geq \bar{\omega}_j$$

and all Yukawa couplings are of a "holomorphic" form consistent with a superpotential.

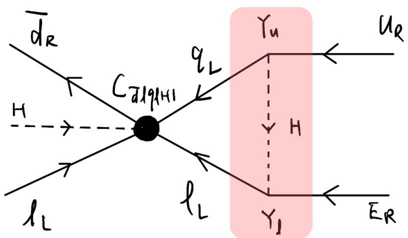
Non-renormalization Theorem: $\boxed{\gamma_{ij} = 0 \text{ if } \omega_i < \omega_j \text{ or } \bar{\omega}_i < \bar{\omega}_j}$

$$(4\pi)^2 \frac{dc_i}{d\log\mu} = \sum_j \gamma_{ij} c_j$$

The exceptional amplitudes: $A(\psi^+\psi^+\psi^+\psi^+), \quad A(F^+\phi\phi\phi), \quad A(\psi^+\psi^+\phi\phi)$

Only the first one is involved in the SM(EFT) and proportional to the product of up-type and down-type Yukawa couplings, caused by Higgs doublet exchange, such as

$\mathcal{O}_{\bar{d}lqlH1} \to \mathcal{O}_{\bar{d}lueH}$



$\propto C_{\bar{d}lqlH1}^{\alpha\beta\rho\sigma} (Y_u)_{\rho r} (Y_l)_{\sigma\lambda}$

Radiative Corrections to Neutrino Masses

Numerical analysis in [M. Chala, A. Titov, 2021]: one generation

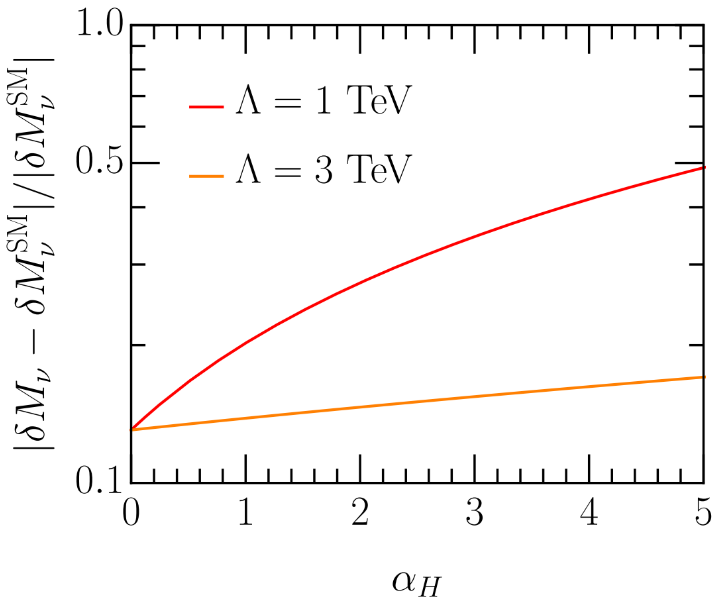


FIG. 2. *Impact of dimension-six interactions on the size of the leading-logarithmic correction to M_ν as a function of α_H for different values of the new physics scale Λ. The Wilson coefficients of other dimension-six operators have been set to their best-fit values from [44]. Dimension-seven operators have been assumed to vanish. See the text for further details.*

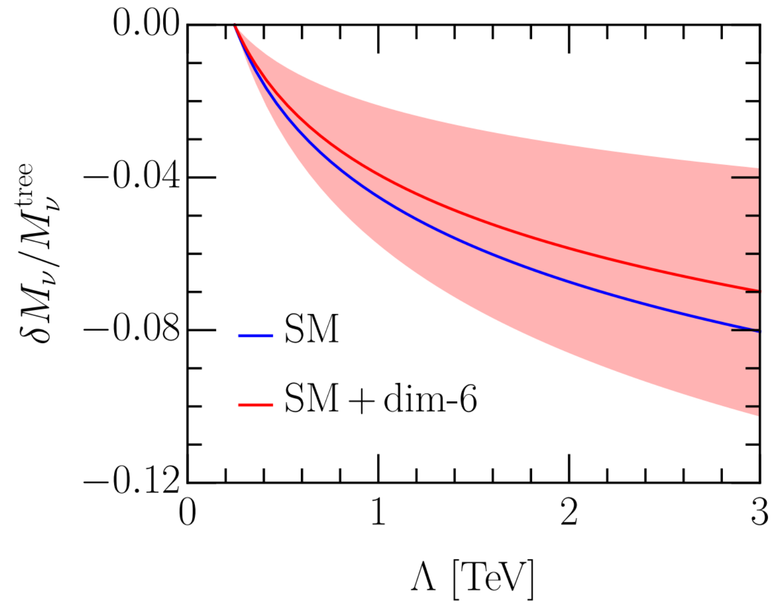


FIG. 3. *Relative size of the leading-logarithmic correction to M_ν as a function of Λ, with the Wilson coefficients of dimension-six operators set to zero (blue line) and to their best-fit values from [44] (red line). The band represents the variation of the Wilson coefficients of dimension-six operators across the 95 % confidence level ranges derived in [44]. Dimension-seven operators have been assumed to vanish. See the text for further details.*

- **Corrections from insertions of dim-5 + dim-6 operators can reach 50% of those from single insertions of the dim-5 operator for Λ = 1 TeV, depending on the Wilson coefficient of $\mathcal{O}_H = (H^\dagger H)^3$**
- **The relative size of the total correction is about 4%-8% compared to the tree-level neutrino mass for Λ $\in$ [1 TeV, 3 TeV]**