Quasielastic Lepton-Nucleus Scattering and the Correlated Fermi Gas Model

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- nuclear physics via a nuclear model

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- Precision needs separate control of *form factors* and *nuclear effects*
- Achieved for the first time in arXiv: 2405.05342

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- Quasi-Elastic Lepton Nucleus Scattering and the Correlated Fermi Gas Model [Bhattacharya, Carey, Cohen, GP, arXiv:2405.05342]

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- Nuclear cross section is

$$\sigma_{\rm nuclear} = n_i(\boldsymbol{p}) \otimes \sigma_{\rm nucleon}(\boldsymbol{p} \to \boldsymbol{p}') \otimes [1 - n_f(\boldsymbol{p}')]$$

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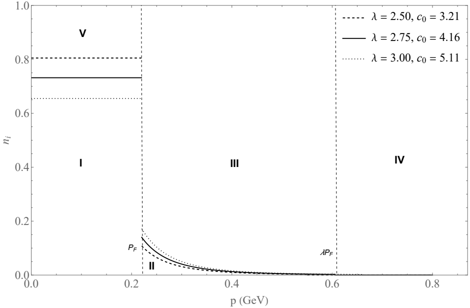

Highlights of CFG implementation

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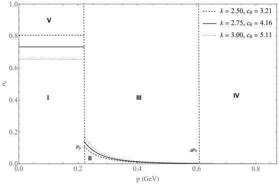

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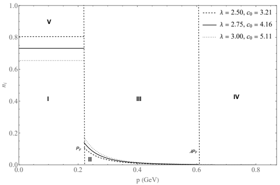

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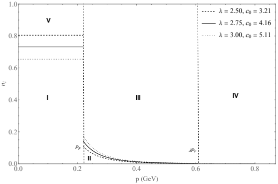

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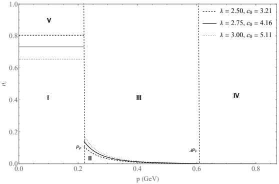

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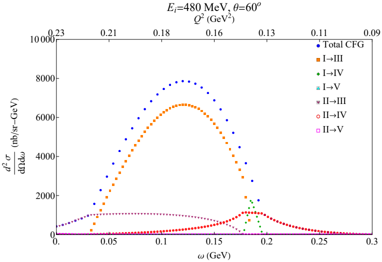
- Final nucleon can be in regions III, IV, or V
- Combining all regions we can compare CFG model to data



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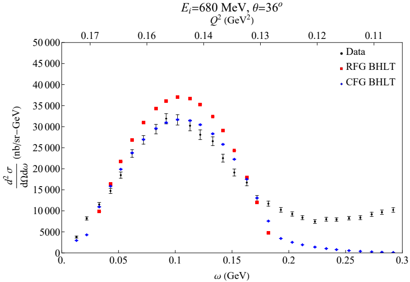

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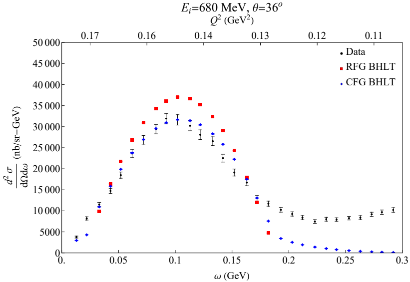

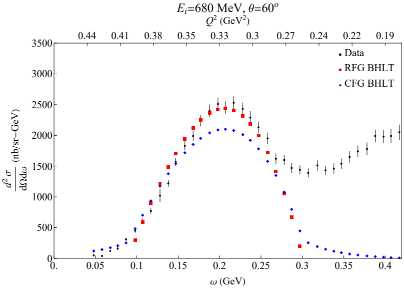

- We observe clear differences between the two nuclear models

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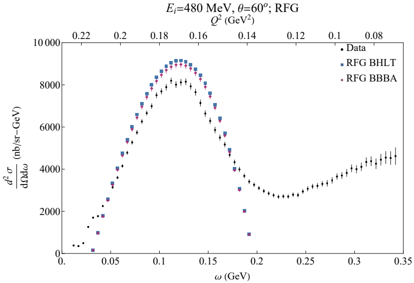

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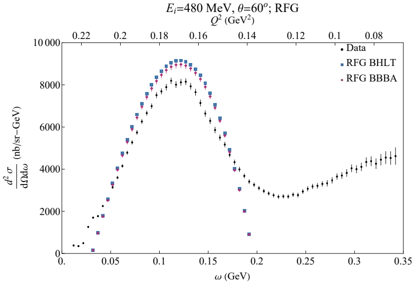

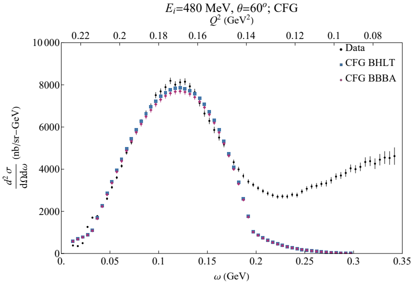

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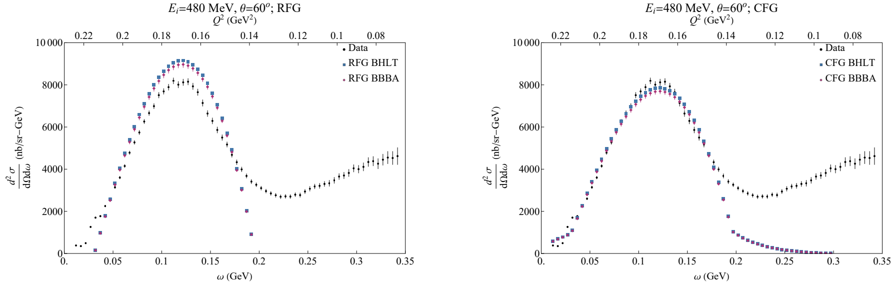

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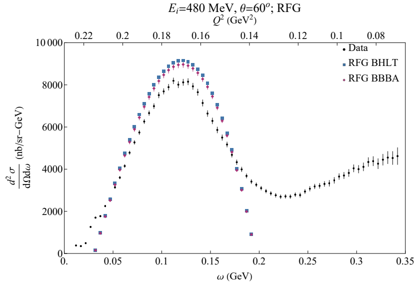

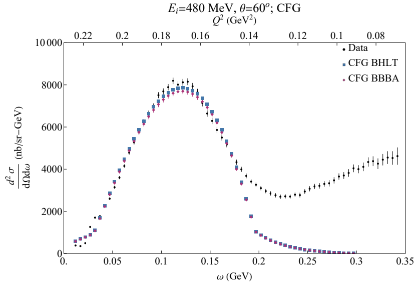

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- What happens for neutrino scattering?

Axial form factor

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- In the last few years many z-expansion based extractions of the axial form factor became available

Axial form factor

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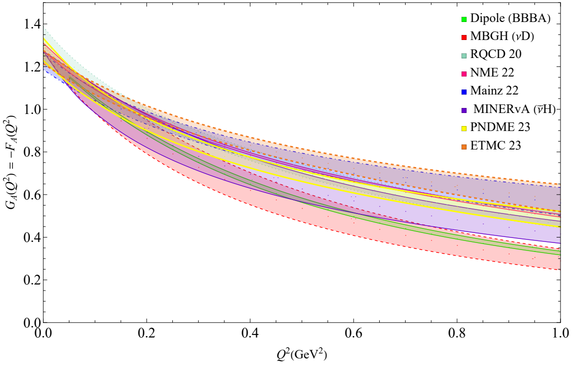
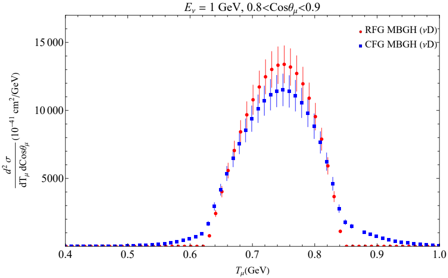


Neutrino scattering: hypothetical constant neutrino energy

- Consider the hypothetical scattering of a 1 GeV neutrino on Carbon



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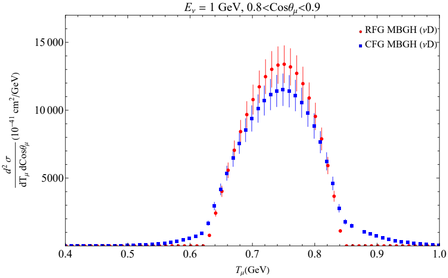

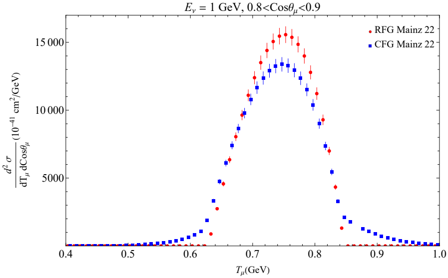

- $T_\mu = E_\nu - m_\mu - \omega$
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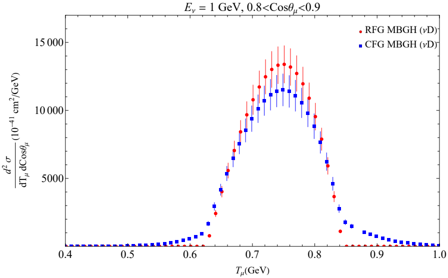

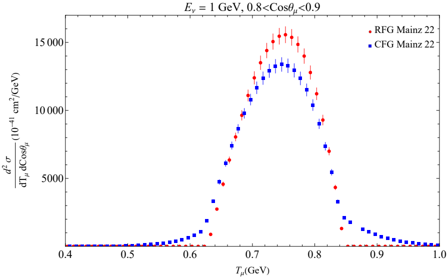

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- We observe clear differences between
- the two nuclear models and
- the two axial form factors extractions

MiniBooNE data: fix axial form factor, vary nuclear model

$$\frac{d\sigma_{\text{carbon,per nucleon,avg.}}}{dE_\ell d\cos\theta_\ell} = \int dE_\nu\, f(E_\nu)\, \frac{d\sigma_{\text{carbon,per nucleon}}}{dE_\ell d\cos\theta_\ell}$$

- Fix axial form factor, vary nuclear models, and compare to MiniBooNE data [Aguilar-Arevalo, et al., PRD **81** 092005, (2010)]

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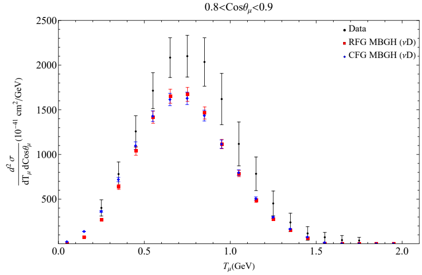

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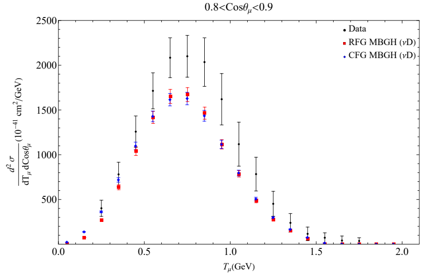

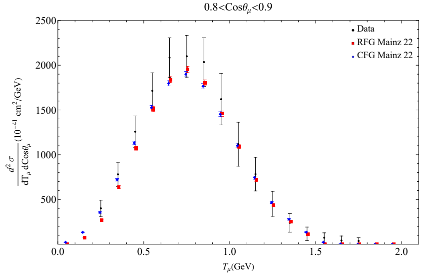

- With flux averaging, indistinguishable RFG and CFG models

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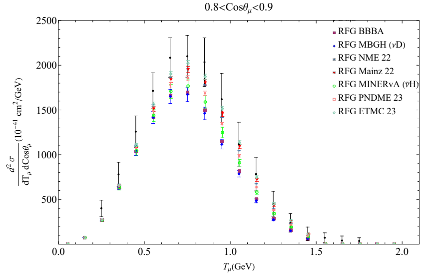

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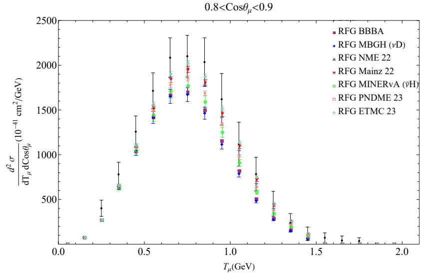

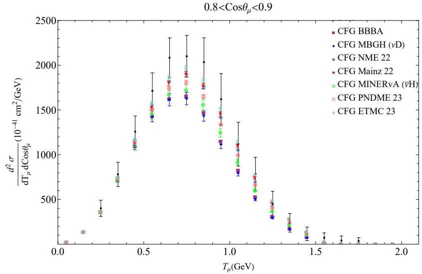

- Continuous spread from axial form factors for both nuclear models

Conclusions

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Thank you!

Backup slides

Cross section and nuclear tensor

- The (anti-)neutrino-nucleus cross section is

$$\frac{d\sigma^{\nu}_{\rm nuclear}}{dE_\ell\, d\cos\theta_\ell} = \frac{G_F^2|\vec{P}_\ell|}{16\pi^2\, m_T}\Bigg\{2(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)\, W_1 + (E_\ell + |\vec{P}_\ell|\cos\theta_\ell) W_2$$

$$\pm\frac{1}{m_T}\left[(E_\ell - |\vec{P}_\ell|\cos\theta_\ell)(E_\nu + E_\ell) - m_\ell^2\right] W_3 + \frac{m_\ell^2}{m_T^2}(E_\ell - |\vec{P}_\ell|\cos\theta_\ell) W_4 - \frac{m_\ell^2}{m_T}\, W_5\Bigg\}$$

where the upper (lower) sign is for neutrino (anti-neutrino) scattering.

- Neglecting the electron mass, the electron-nucleus cross section is

$$\frac{d\sigma^{e}_{\rm nuclear}}{dE_\ell\, d\cos\theta_\ell} = \frac{\alpha^2 E_\ell^2}{2q^4\, m_T}\left[2W_1(1-\cos\theta_\ell) + W_2(1+\cos\theta_\ell)\right]$$

- The hadronic tensor is

$$W_{\mu\nu} = -g_{\mu\nu} W_1 + \frac{p^T_\mu p^T_\nu}{m_T^2} W_2 - \frac{i\epsilon_{\mu\nu\rho\sigma} p_T^\rho p_T^\sigma}{2m_T^2} W_3 + \frac{q_\mu q_\nu}{m_T^2} W_4 + \frac{p^T_\mu q_\nu + q_\mu p^T_\nu}{2m_T^2} W_5$$

Nuclear and hadronic tensor

- The nuclear tensor $W_{\mu\nu}$ can be expressed by the nucleon tensor $H_{\mu\nu}$

$$W_{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{m_T}{E_p} 2Vn_i(\boldsymbol{p}) \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} (2\pi)^4 \delta^4(p-p'+q) H_{\mu\nu} \left[1 - n_f(\boldsymbol{p'})\right]$$

- Performing some of the integrals

$$W_{\mu\nu} \equiv \int d^3p\, f(\boldsymbol{p}, q^0, \boldsymbol{q}) H_{\mu\nu}(\epsilon_{\boldsymbol{p}}, \boldsymbol{p}; q^0, \boldsymbol{q}) \,,$$

$$f(\boldsymbol{p}, q^0, \boldsymbol{q}) = \frac{m_T V}{4\pi^2}\, n_i(\boldsymbol{p})[1 - n_f(\boldsymbol{p} + \boldsymbol{q})] \frac{\delta(\epsilon_{\boldsymbol{p}} - \epsilon'_{\boldsymbol{p}+\boldsymbol{q}} + q^0)}{\epsilon_{\boldsymbol{p}} \epsilon'_{\boldsymbol{p}+\boldsymbol{q}}} \,.$$

Phase space integrals

$$W_1 = a_1 H_1 + \frac{1}{2}(a_2 - a_3) H_2 \,,$$

$$W_2 = \left[a_4 + \frac{\omega^2}{|\boldsymbol{q}|^2} a_3 - 2\frac{\omega}{|\boldsymbol{q}|} a_5 + \frac{1}{2}\left(1 - \frac{\omega^2}{|\boldsymbol{q}|^2}\right)(a_2 - a_3) \right] H_2 \,,$$

$$W_3 = \frac{m_T}{m_N}\left(a_7 - \frac{\omega}{|\boldsymbol{q}|} a_6\right) H_3 \,,$$

$$W_4 = \frac{m_T^2}{m_N^2}\left[a_1 H_4 + \frac{m_N}{|\boldsymbol{q}|} a_6 H_5 + \frac{m_N^2}{2|\boldsymbol{q}|^2}(3a_3 - a_2) H_2 \right] \,,$$

$$W_5 = \frac{m_T}{m_N}\left(a_7 - \frac{\omega}{|\boldsymbol{q}|} a_6\right) H_5 + \frac{m_T}{|\boldsymbol{q}|}\left[2a_5 + \frac{\omega}{|\boldsymbol{q}|}(a_2 - 3a_3) \right] H_2 \,,$$

$$a_1 = \int d^3p\, f(\boldsymbol{p}, q) \,, \qquad a_2 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{|\boldsymbol{p}|^2}{m_N^2} \,,$$

$$a_3 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{(p^z)^2}{m_N^2} \,, \qquad a_4 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{\epsilon_{\boldsymbol{p}}^2}{m_N^2} \,,$$

$$a_5 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{\epsilon_{\boldsymbol{p}} p^z}{m_N^2} \,, \qquad a_6 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{p^z}{m_N} \,,$$

$$a_7 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{\epsilon_{\boldsymbol{p}}}{m_N} \,.$$

Nucleon tensor

- For the RFG model

$$n_i(\boldsymbol{p}) = \theta(p_F - |\boldsymbol{p}|)\,, \quad n_f(\boldsymbol{p}') = \theta(p_F - |\boldsymbol{p}'|)$$

- For the CFG model

$$n_{\rm CFG}(\boldsymbol{p}) = \begin{cases} 1 - \left(1 - \dfrac{1}{\lambda}\right)\dfrac{c_0}{\pi^2} \equiv \alpha_0 & |\boldsymbol{p}| \le p_F \\ \dfrac{c_0}{3\pi^2}\left(\dfrac{p_F}{\boldsymbol{p}}\right)^4 \equiv \dfrac{\alpha_1}{|\boldsymbol{p}|^4} & p_F \le |\boldsymbol{p}| \le \lambda p_F \\ 0 & |\boldsymbol{p}| \ge \lambda p_F\,. \end{cases}$$

where $c_0 = 4.16 \pm 0.95$ and $\lambda \approx 2.75 \pm 0.25$