Measurement of the anomalous spin precession frequency ω_a in the Muon $g-2$ experiment at Fermilab

Lorenzo Cotrozzi, on behalf of the Muon $g-2$ collaboration

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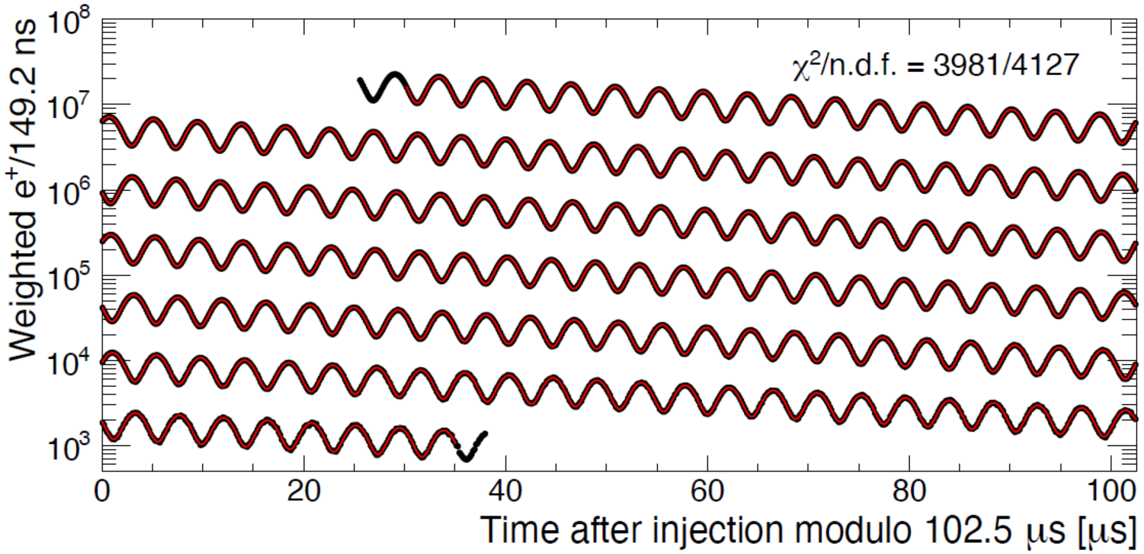
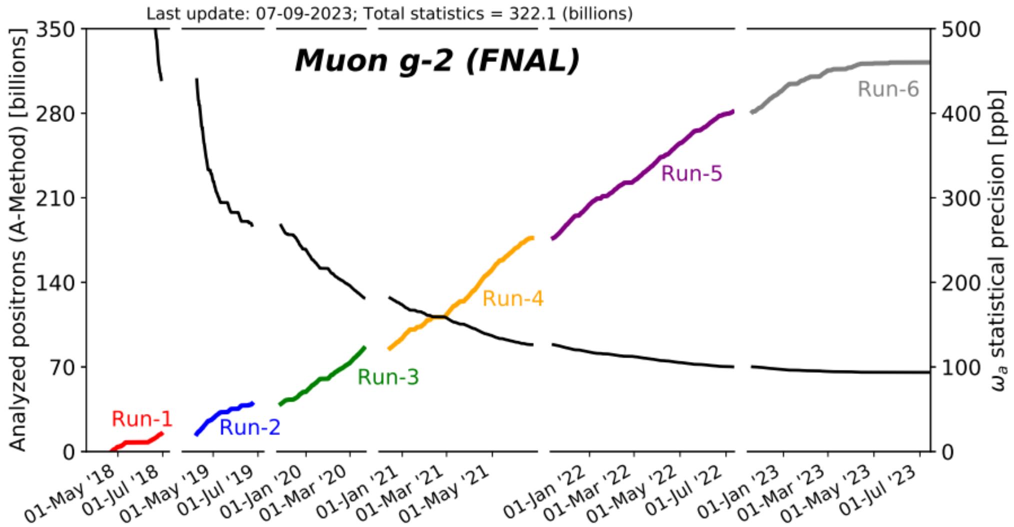
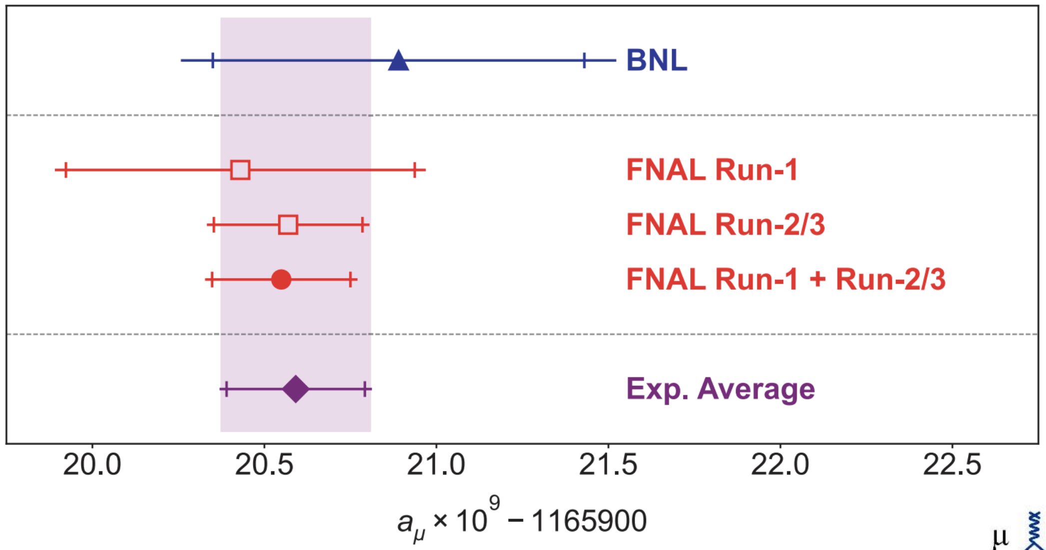
FERMILAB-SLIDES-24-0182-V

Outline

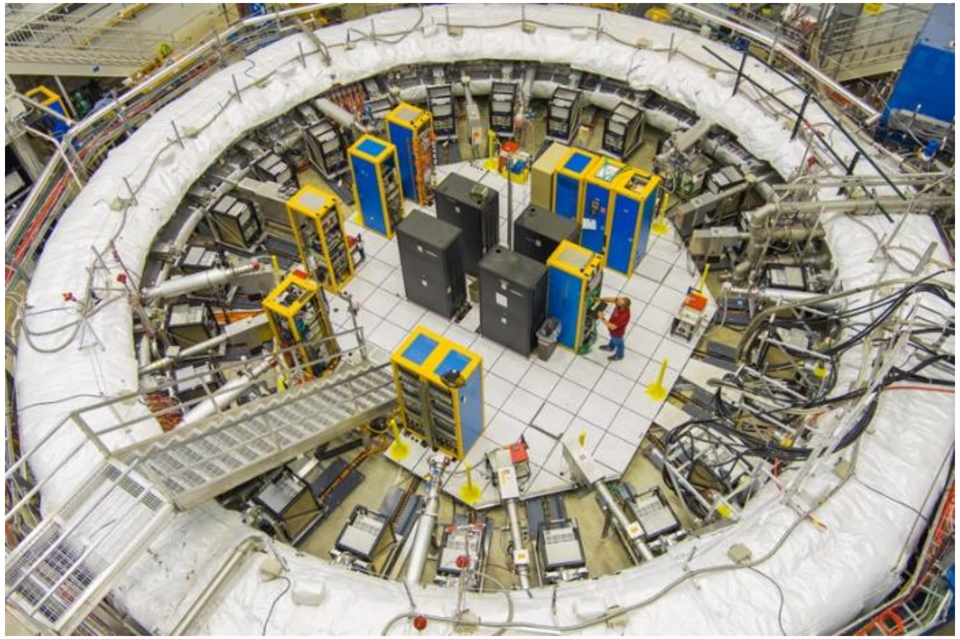
- Anomalous precession frequency ω_a in the Muon g-2 experiment at FNAL
- Run-2/3 result (2023) vs Run-1 (2021)
- Projections for Run-4/5/6 result







Experiment at Fermilab Muon Campus



Presented in the previous talk by **Kim Siang**

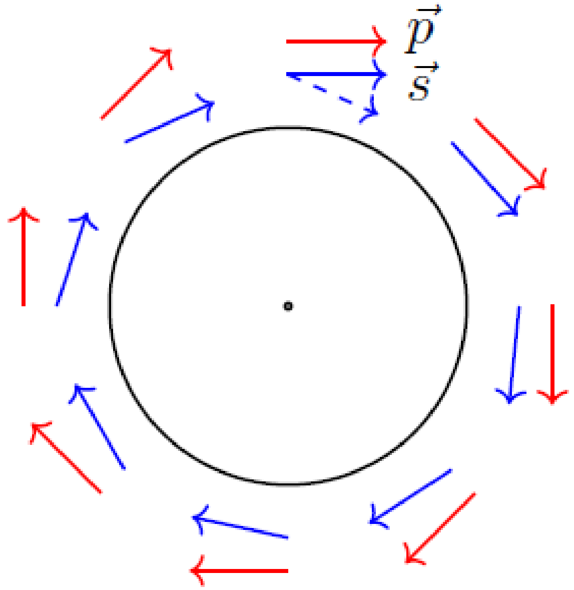
Anomalous spin precession in B-field

$g-2 \neq 0$
$a_\mu \neq 0$ } $\rightarrow$ spin precesses with anomalous frequency $\vec{\omega}_a = \vec{\omega}_{\text{spin}} - \vec{\omega}_{\text{c}}$

$$\vec{\omega}_a = -\frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E} - a_\mu \frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta}\right]$$

$\gamma = 29.3 \rightarrow p = 3.094\ \text{GeV/c}$
"magic momentum"

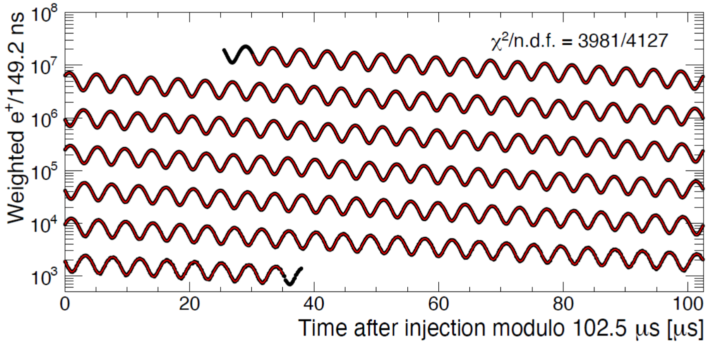
$\vec{\beta}\cdot\vec{B} = 0$

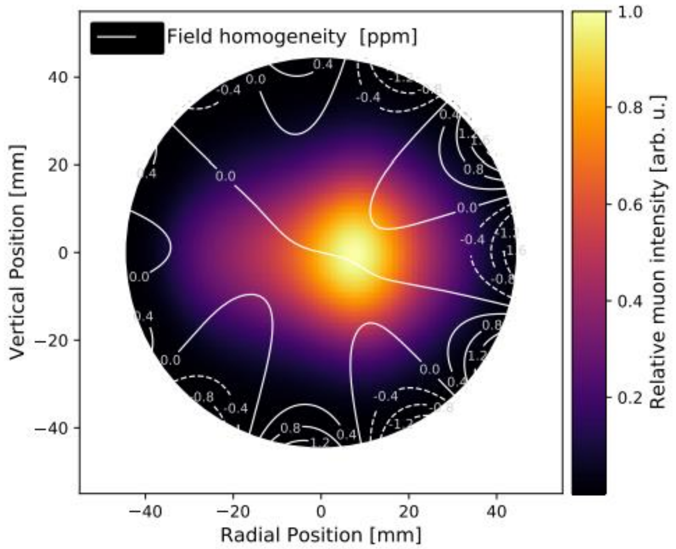


$\omega_c \sim 42.1$ rad/μs

$\omega_a \sim 1.439$ rad/μs ~
~ 12.4° per turn

Master formula for a_μ





$$a_\mu = \frac{\omega_a}{\omega_p} \times \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

External factors, known to 25 ppb

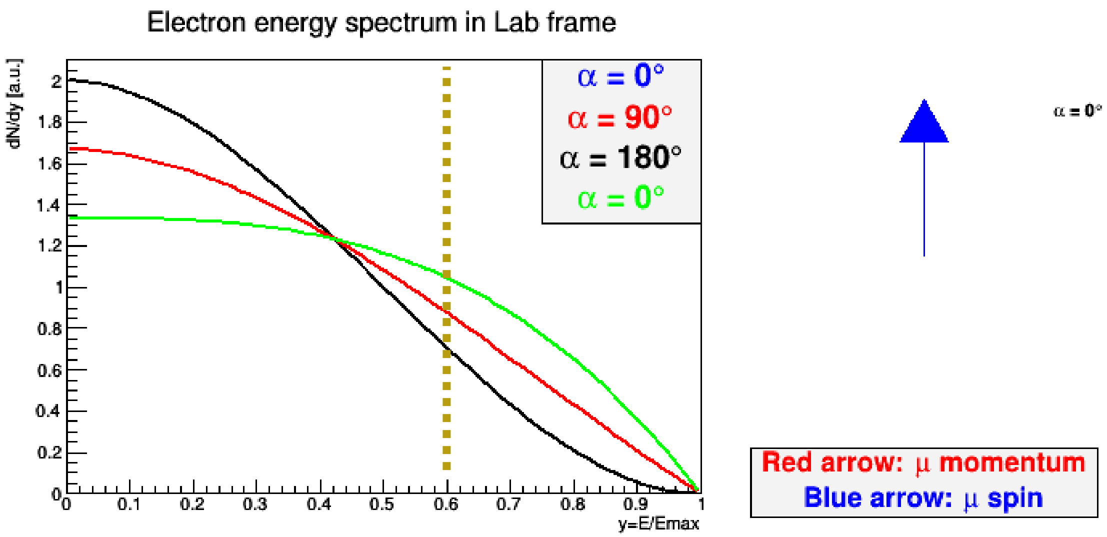
$$\frac{\omega_a}{\omega_p} = \frac{\omega_a^m}{\omega_p^m} \times$$ corrections for effects that…

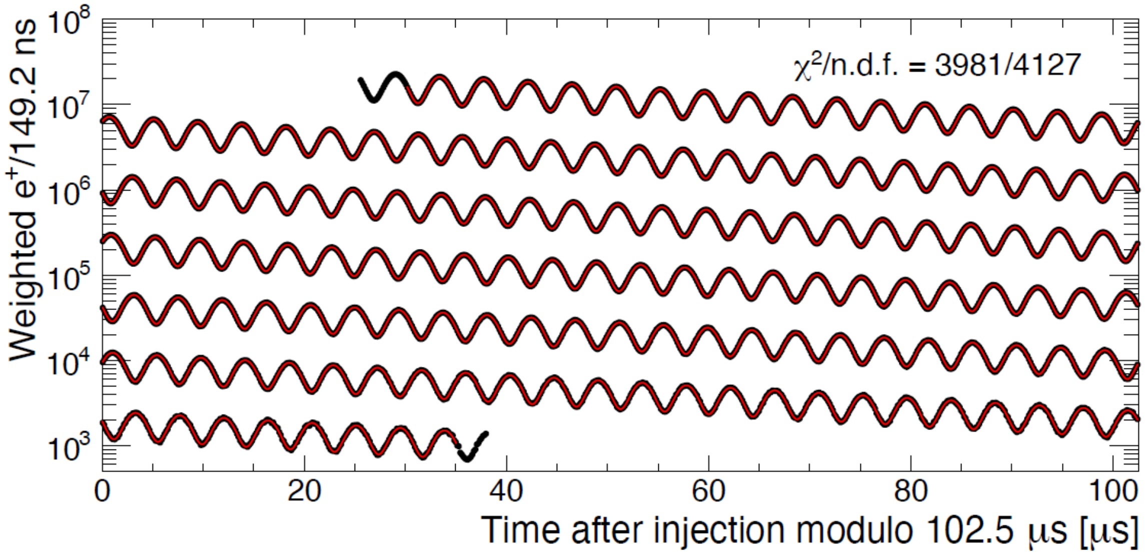
- Make spin precess slower (E-field, vertical motion)
- Make phase change within 700 μs
- Induce transient magnetic fields

See poster tonight by Kim Siang

m=Measured values

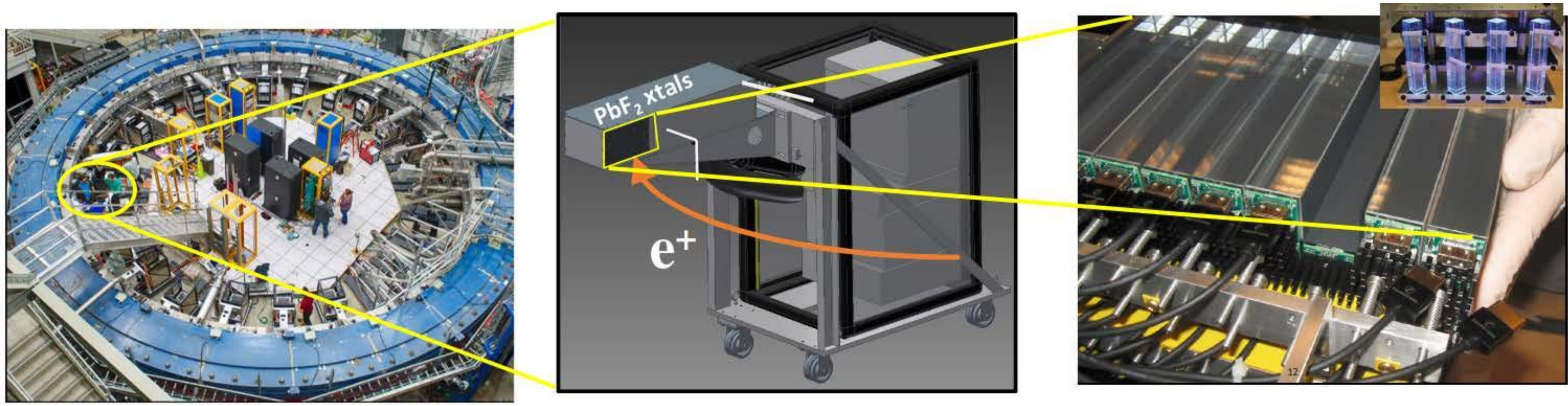
Principle of ω_a measurement





1. Weak decays violate parity:
   - polarized muon beam
   - preferred high-energy e^+ direction
2. Correlation in the lab frame between e^+ energy spectrum and ω_a phase
3. «Wiggle plot»: count high-energy e^+ over time, for about 700 μs (muon lifetime is ~ 64 μs in the lab)

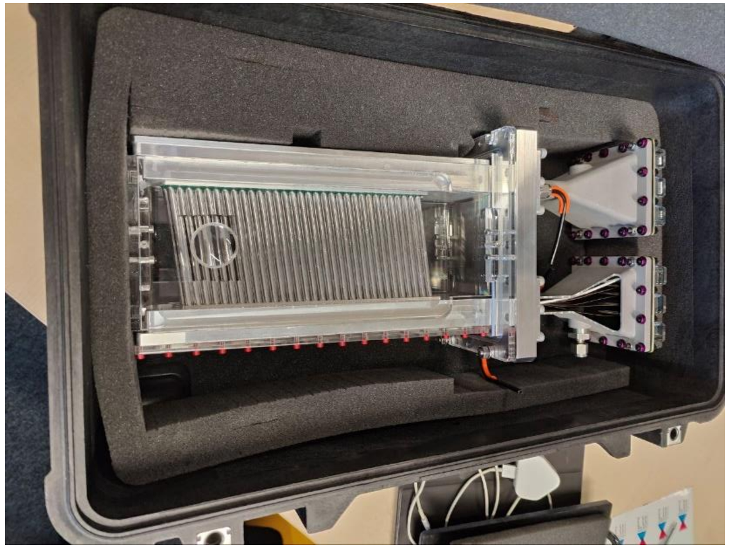
Detectors



24 e.m. calorimeters

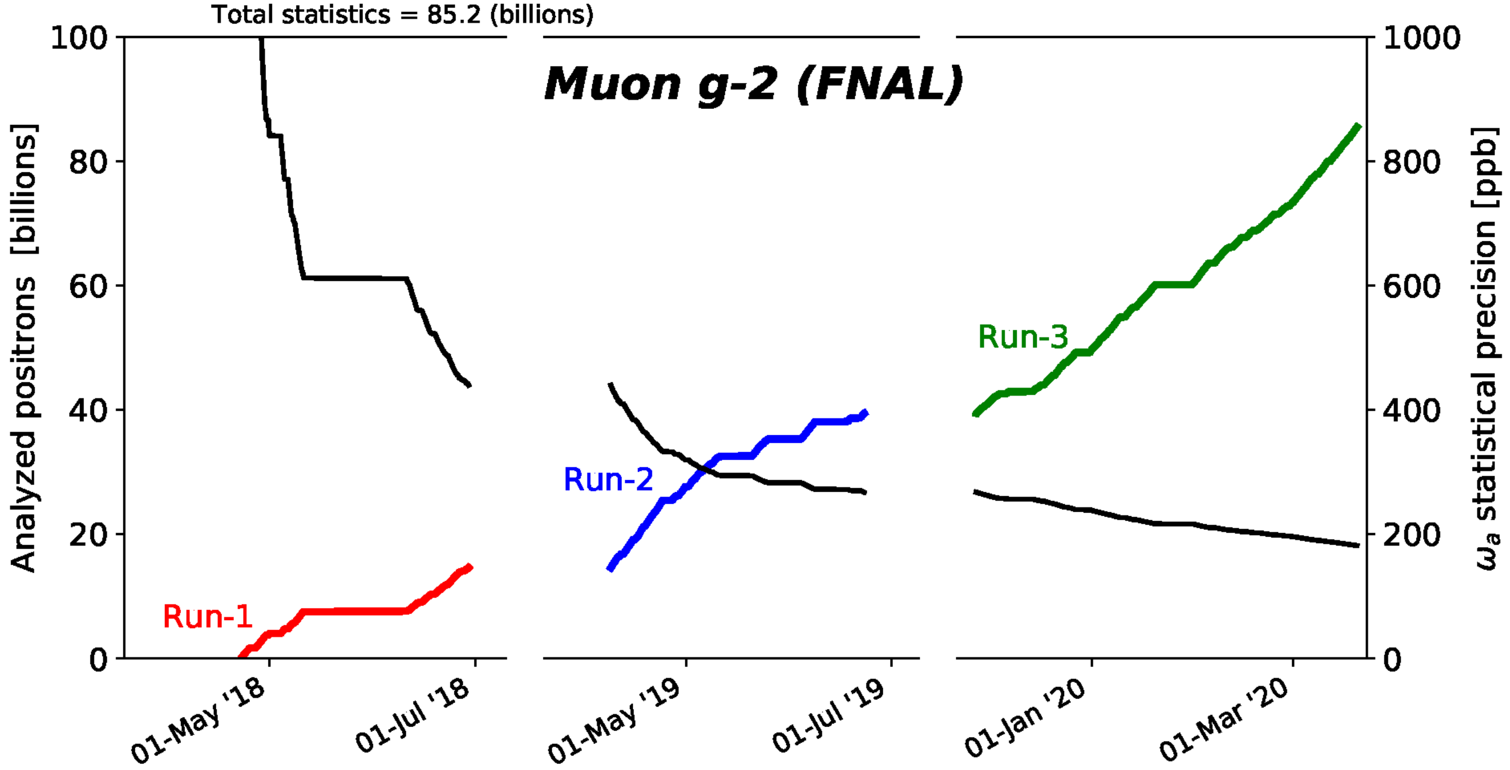
- Measure (E,t) of e^+
- Each made of 6×9 PbF_2 crystals, $15X_0$, read out by large-area SiPMs
- e^+ generate electromagnetic shower, SiPMs detect Cherenkov light ($n = 1.8$)

2 straw tube trackers



- Each has 8 modules and 32 planes
- 50:50 Argon:Ethane at 1 atm pressure
- Extrapolate decay vertex location to measure beam distribution

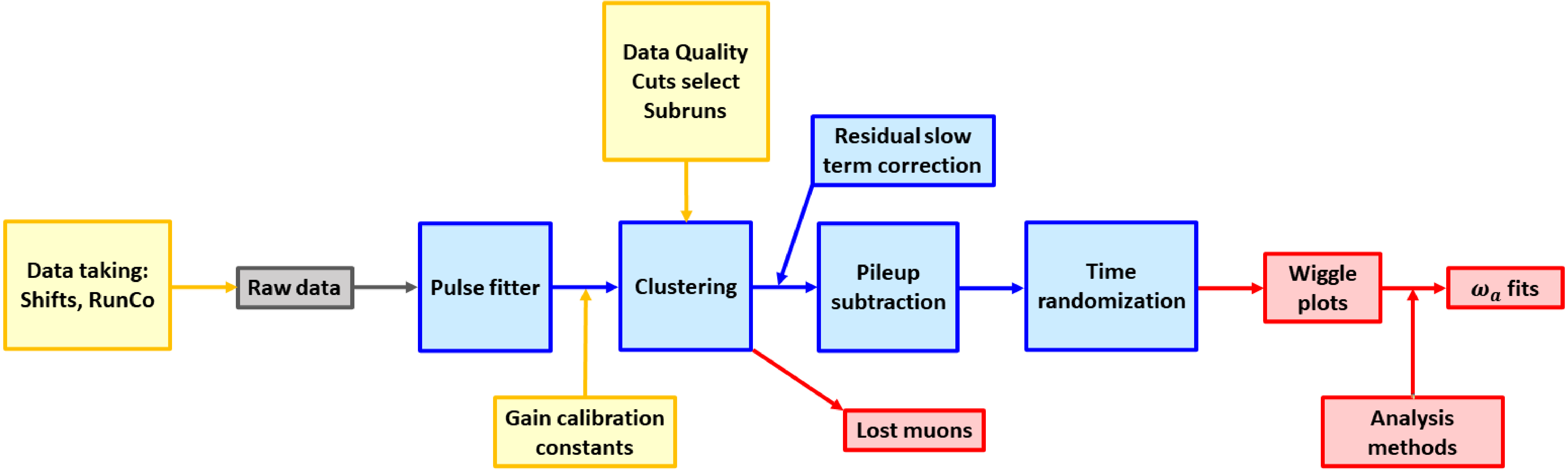
Run-2/3 (2019-2020 campaign)

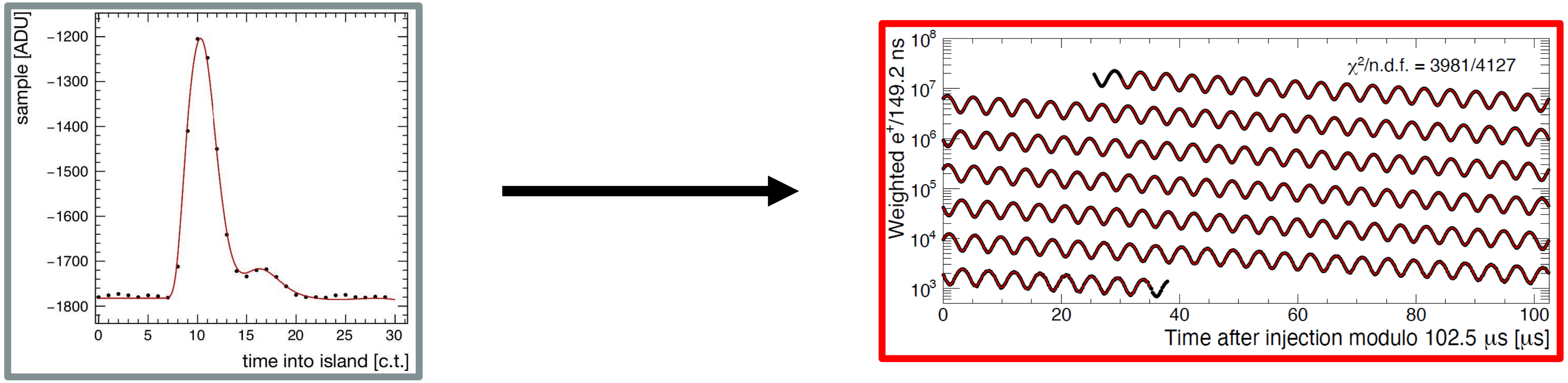


Dataset	Stat. unc. [ppb]
Run-1	434
Run-2/3	201
Combined Run-1+Run-2/3	185
FNAL design goal	100

Statistics: ~ 5 more muon decays than Run-1. Systematic limitations in Run-1 were fixed: reached 25 ppb on ω_a, 70 ppb on a_μ, surpassed design goal of 100 ppb

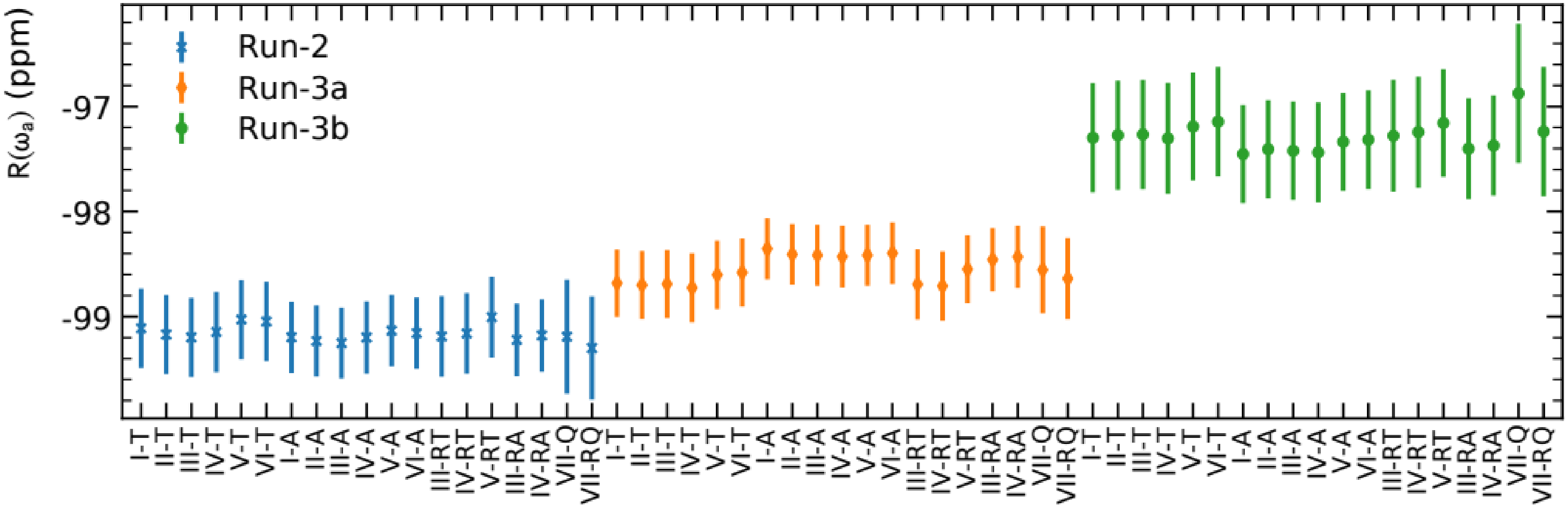
ω_a analysis flowchart





Run-2/3 ω_a analysis teams

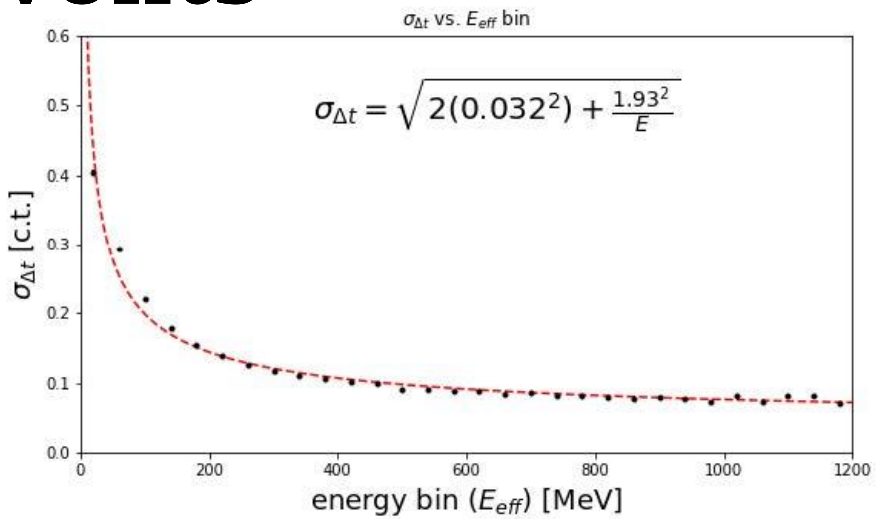
Group	I	II	III	IV	V	VI	VII
Pulse fitting & clustering	Local	Local $\Delta t'$	Local «ITA»	Local $\Delta t'$	Global	Global	Q
Pileup subtraction	Shadow	Empirical	«Semi» empirical	Empirical	Empirical	Empirical	-
Analysis methods	T, A	T, A	T, A, RT, RA	T, A, RT, RA	T, A, RT	T, A	Q, RQ

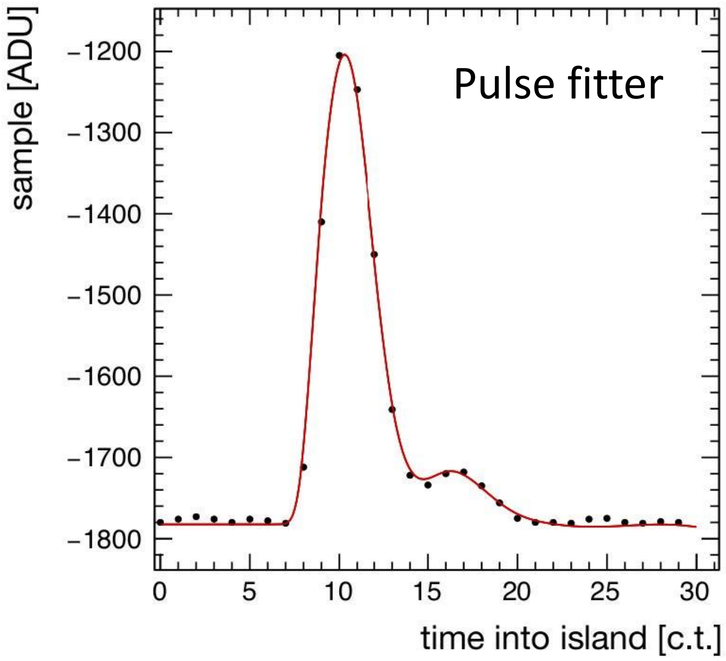


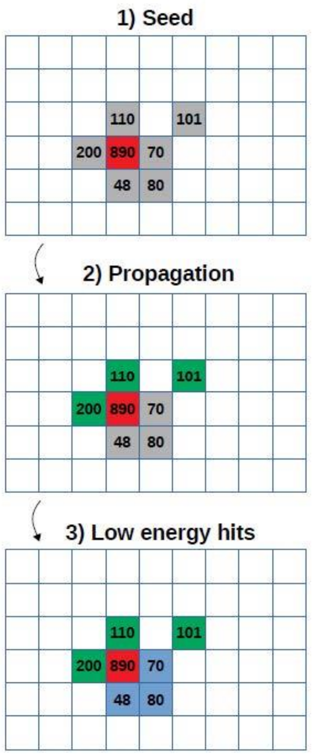
For details on combination: see next talk by Alberto Lusiani

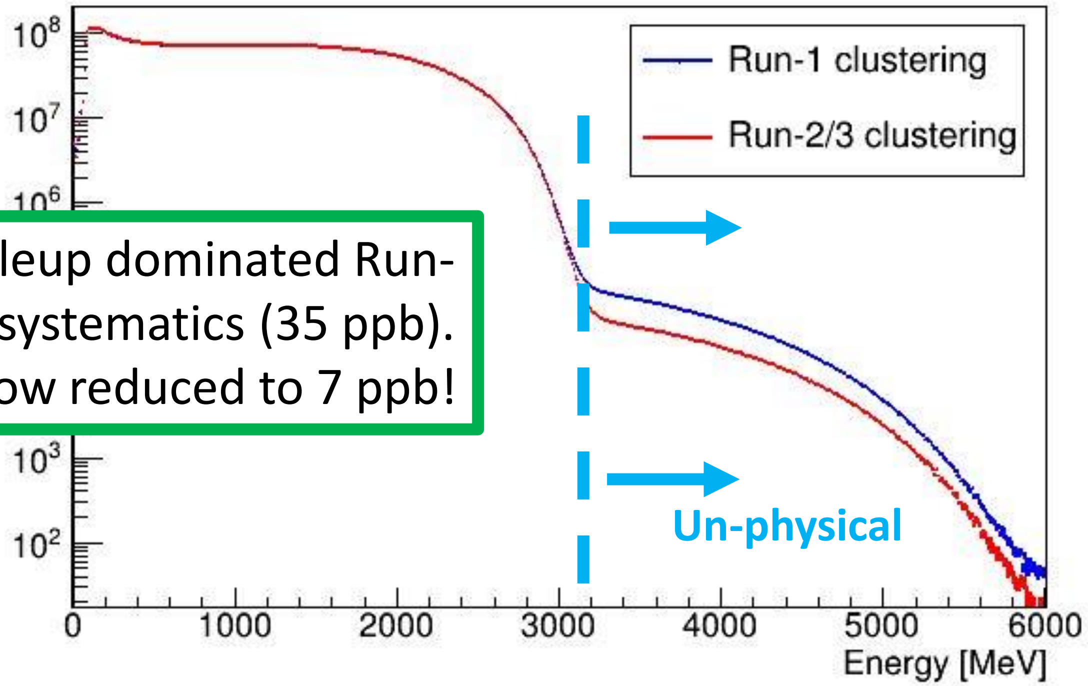
Reconstruct e^+ events

- Pulse fitter identifies traces on crystals
- New algorithms took into account detector's time and energy resolutions
- Reduced pileup in un-physical region



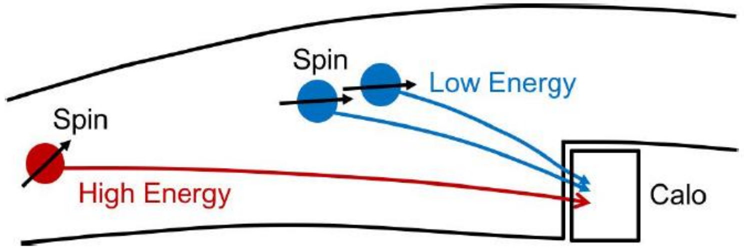


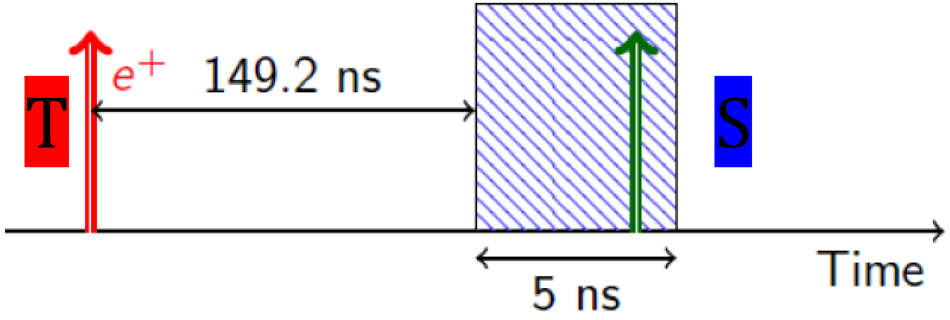




Pileup dominated Run-1 systematics (35 ppb). Now reduced to 7 ppb!

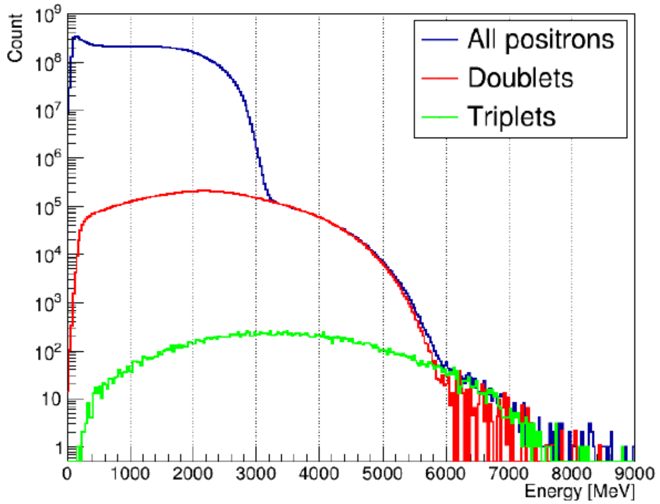
Improved pileup correction since Run-1
(example for «local» method)





For each T (Trigger) cluster that we find:

- Search for coincidence e^+ in S (Shadow) window, after 149.2 ns
- Superimpose the two clusters and pass to reconstruction algorithm



If not resolved: merge them and build pileup

In Run-2/3: also searched for triplets (2 shadows)

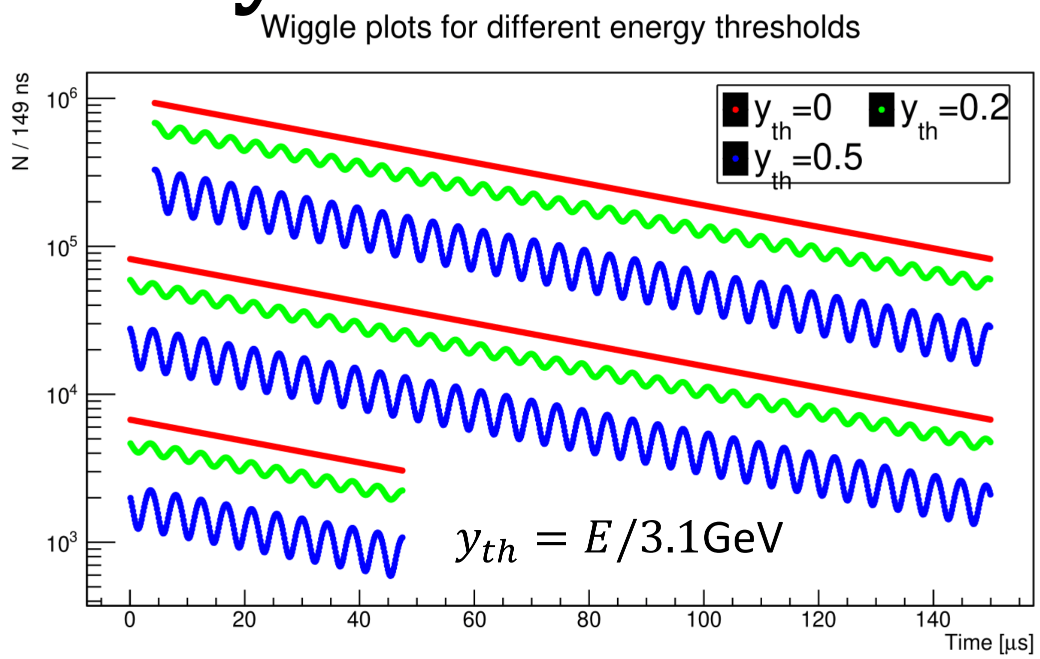
Methods for ω_a analysis

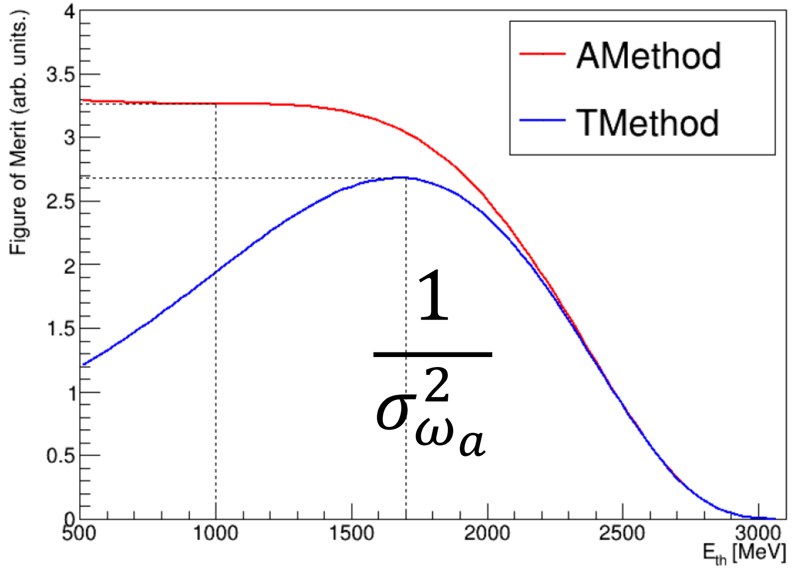
T-Method:

- Greater threshold: wider ω_a oscillations
- Lower threshold: more positrons
- Compromise: 1.7 GeV

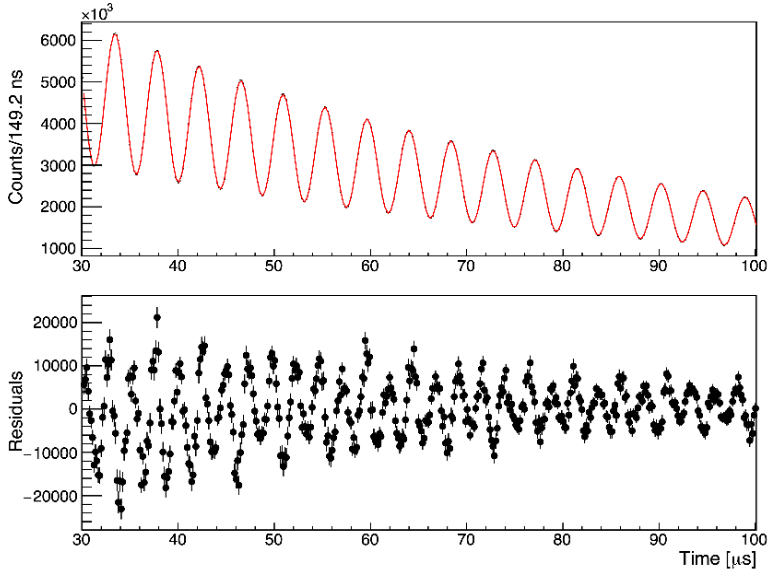
A-Method:

- Extract asymmetry (oscillation amplitude) as function of positron energy → $A(E)$
- Weight each positron event with $A(E)$
- σ_{ω_a}(A-Method)~ 90% σ_{ω_a}(T-Method)





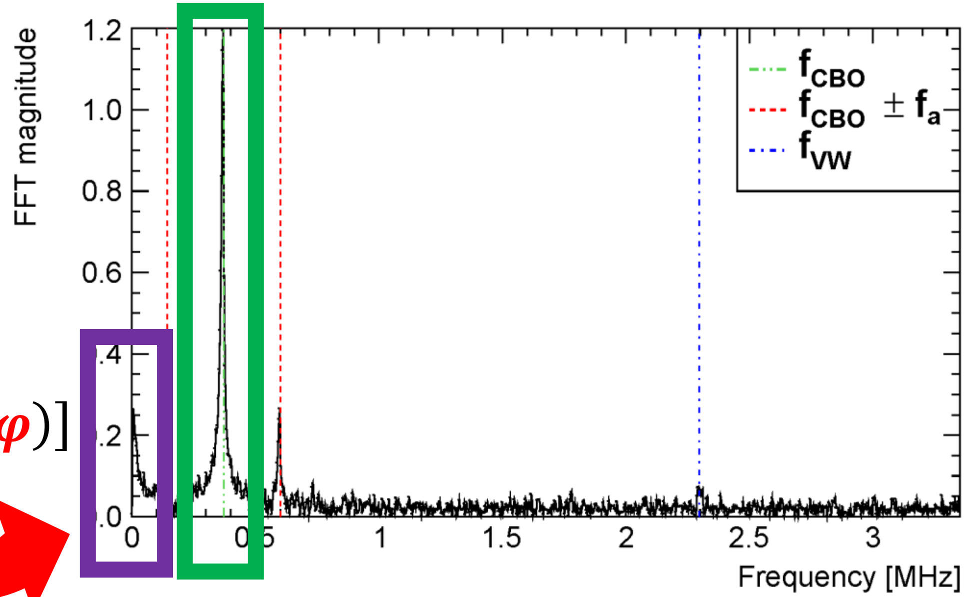
ω_a fit

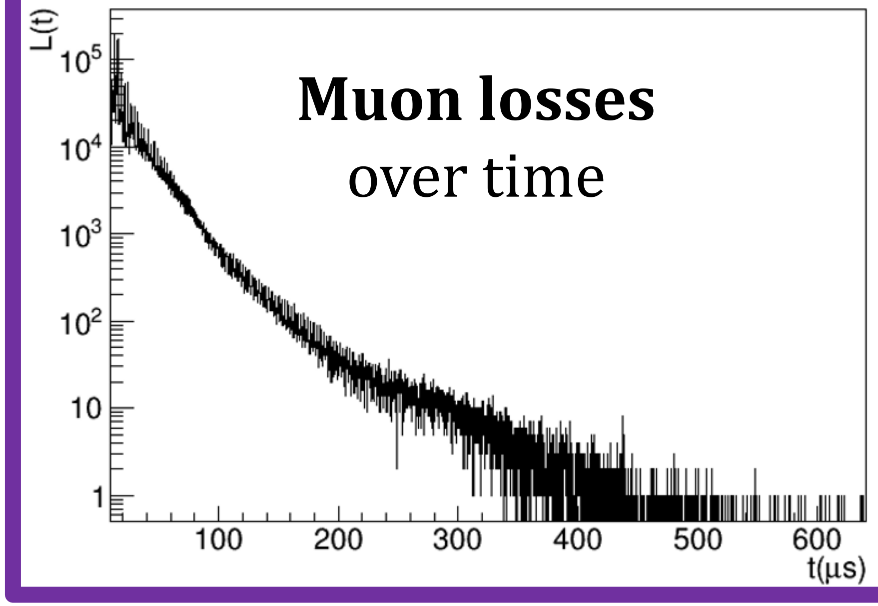


5-parameter fit

$$N(t) = N_0 e^{-\frac{t}{\gamma\tau_\mu}}[1 + A\cos(\omega_a t + \varphi)]$$

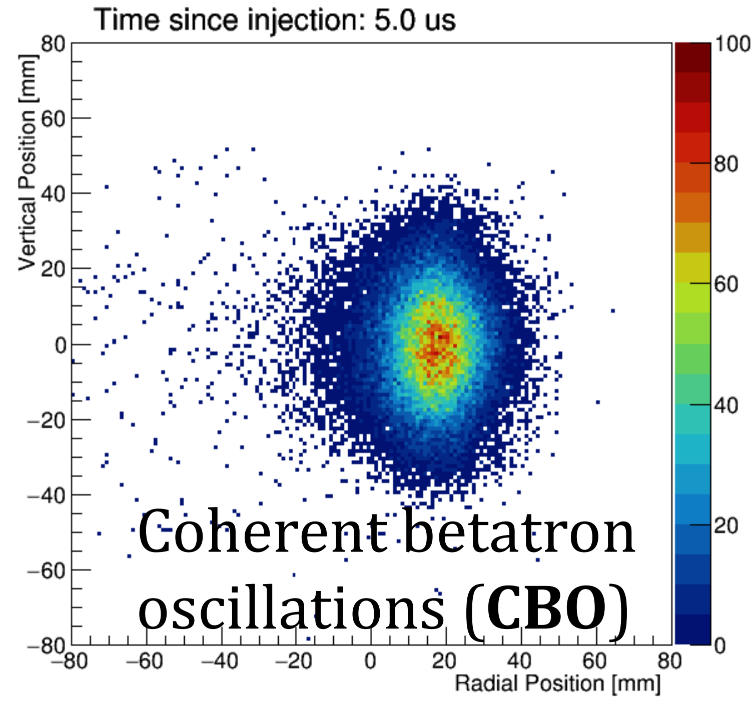
FFT of residuals





$$\Lambda(t) = 1 - k_{LM} \cdot J(t)$$

$$J(t) = \frac{\int_{t_0}^{t} L(t')e^{t'/\gamma\tau}dt'}{\int_{t_0}^{t_{end}} L(t)e^{t/\gamma\tau}dt}$$



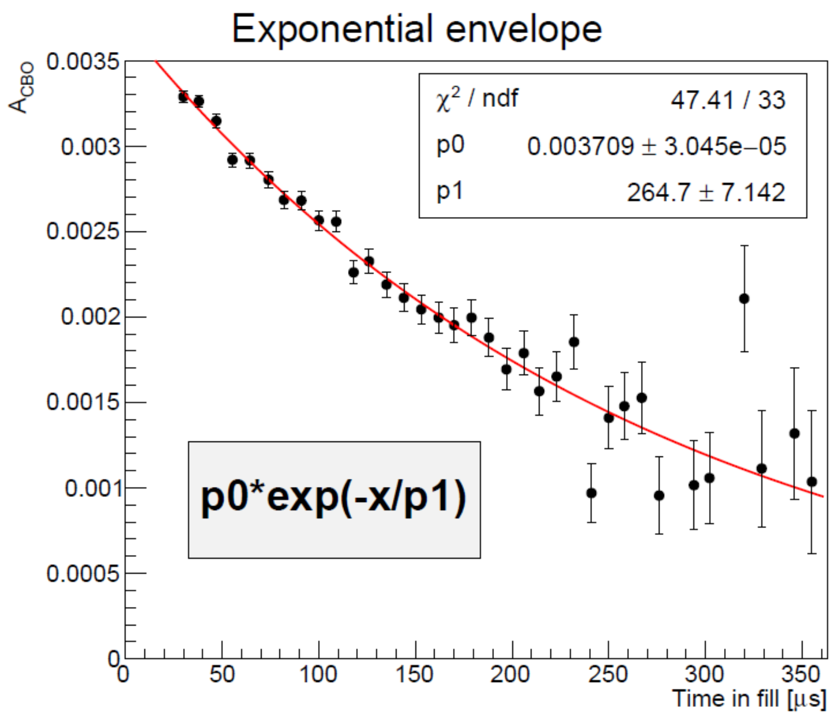
CBO model: amplitude vs time

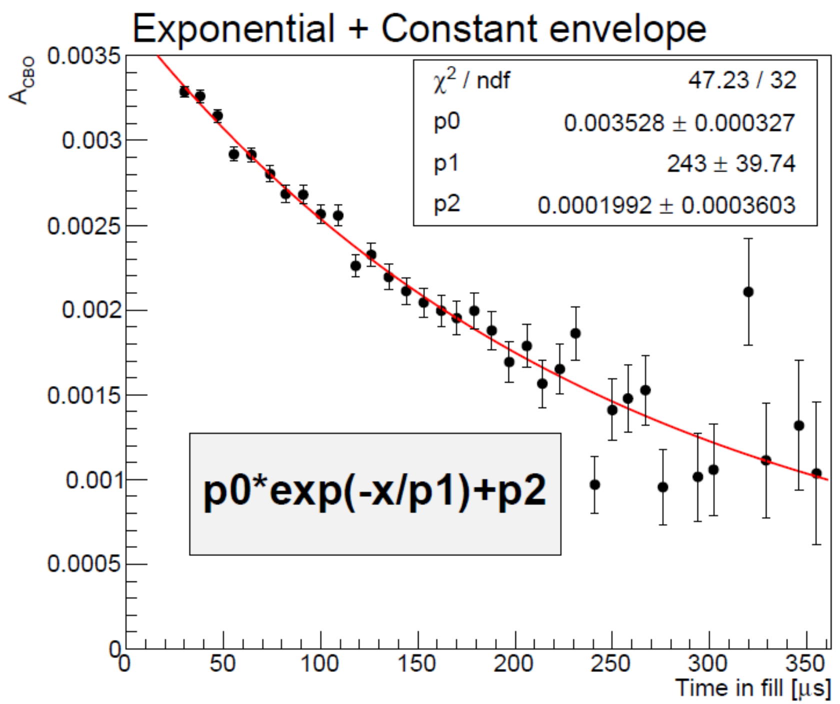
CBO dominated Run-1 systematics (38 ppb). Now reduced to 21 ppb!

$$CBO(t) = 1 + A_{CBO}\cos(\omega_{CBO}t + \varphi_{CBO}) \times e^{-t/\tau}$$

→ Decoherence

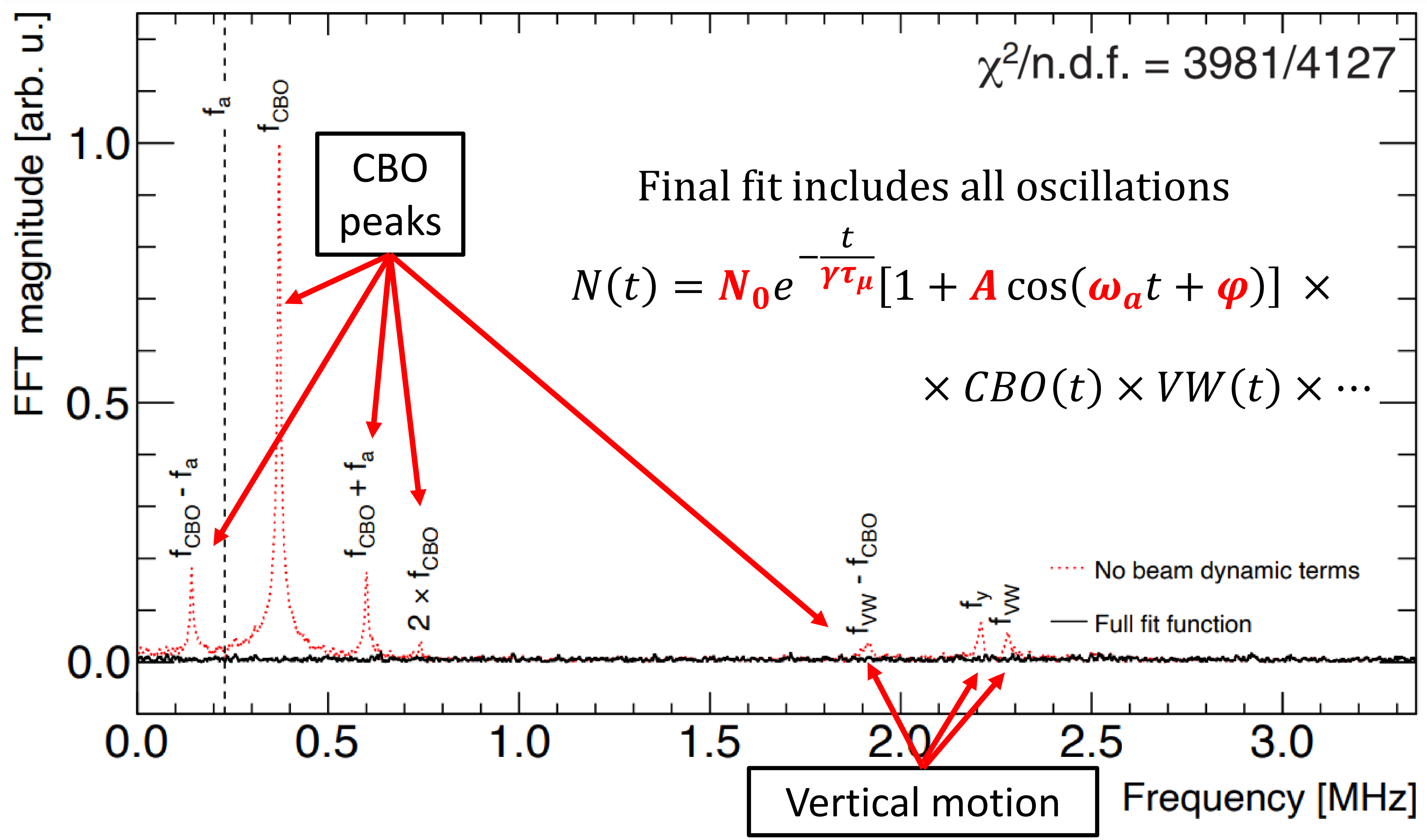
- Muons are an ensemble: betatron oscillations decohere over time
- Sliding window fits to determine good or bad envelopes: more statistics→more studies than Run-1; also input from tracker data



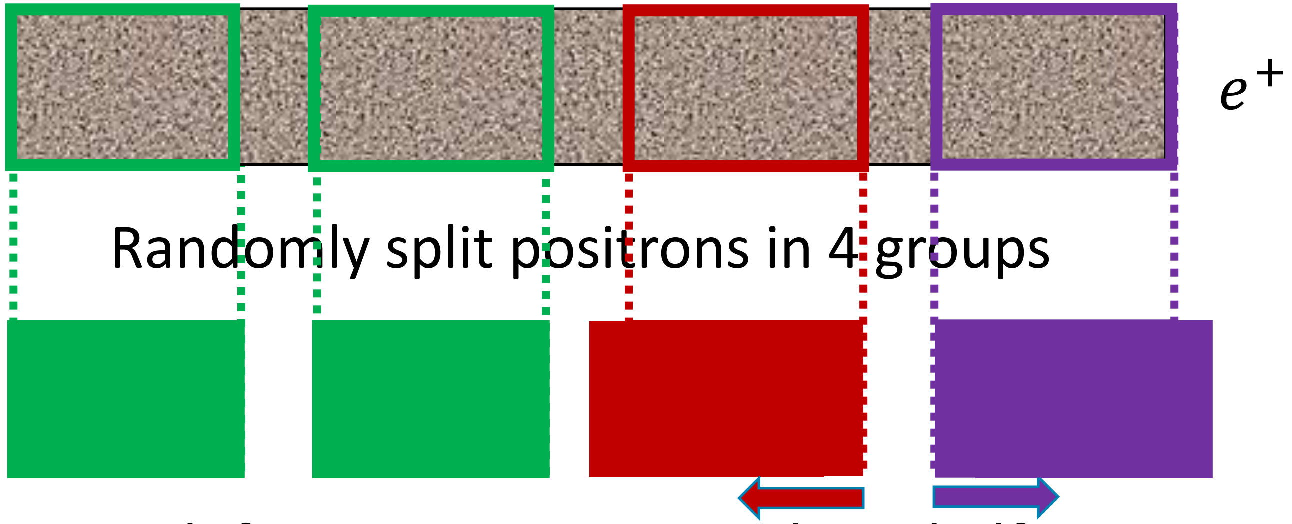


… and many other models tested

Run-2/3: ω_a fit and FFT of residuals



Ratio method wiggle plots

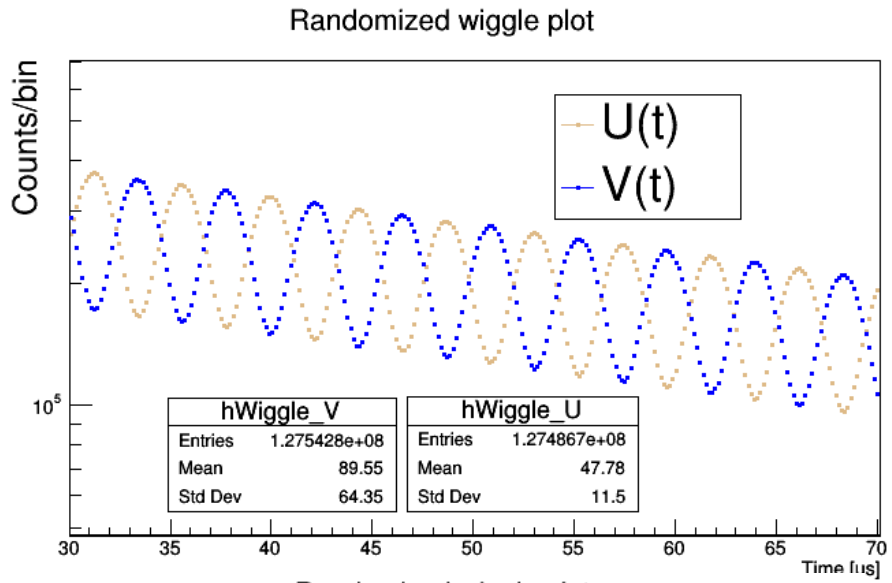


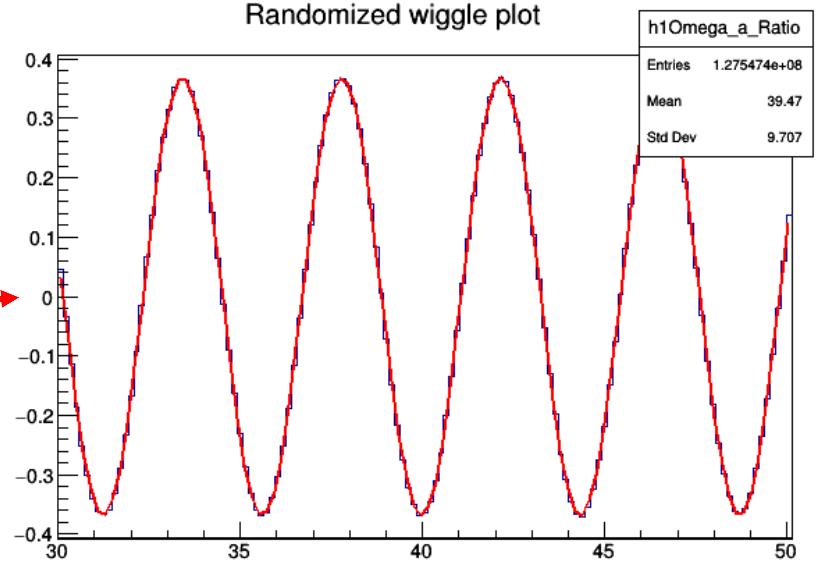
Randomly split positrons in 4 groups

Shift two groups in time by $\pm$ half precession period $T_a/2$. Recombine:

$\boldsymbol{V(t)}$	$\boldsymbol{U(t)}$

$$R(t) = [V(t) - U(t)]/[V(t) + U(t)]$$





It gets rid of muon lifetime and normalization N_0 in fit function. Any «slow» effect is highly reduced!

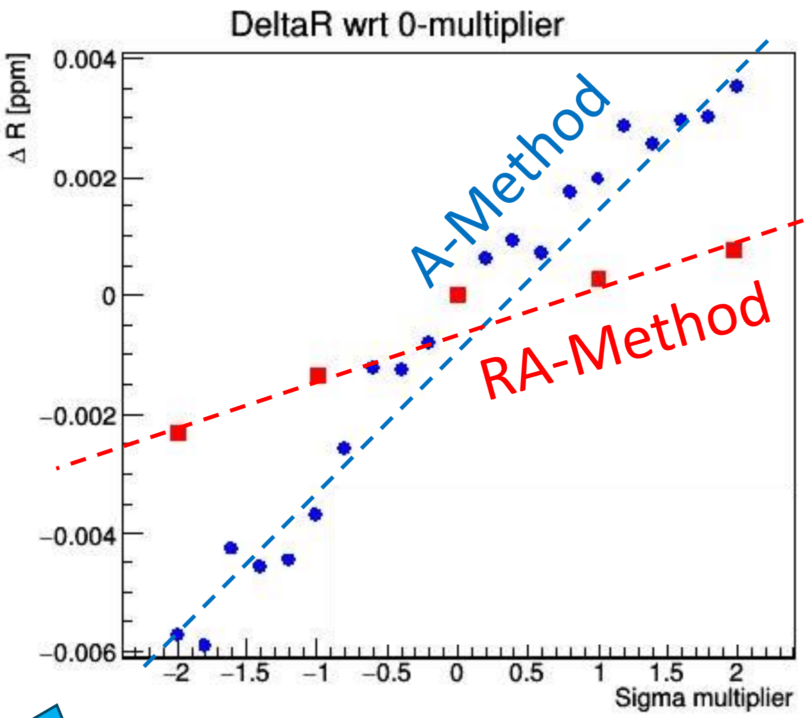
New "Ratio" A-Method

- Weight each positron with asymmetry function (like A-Method)

R: $\{V(t); U(t)\}$ → **RA**: $\{\bar{V}(t) = \sum_E A(E)\,V(E,t); \bar{U}(t) = \sum_E A(E)U(E,t)\}$

R: $\frac{V(t)-U(t)}{V(t)+U(t)}$ → **RA**: $\frac{\bar{V}(t)-\bar{U}(t)}{\bar{V}(t)+\bar{U}(t)}$

- Statistical uncertainty on ω_a is minimized
- Exponential due to muon lifetime is cancelled
- Reduces the sensitivity of ω_a to most «slow effects», such as **SiPM gain fluctuations**!



Prospects for ω_a analysis in Run-4/5/6

Statistical uncertainty:

- With Runs1-->6, we expect to surpass design uncertainty of 100 ppb
- 21.9 times the previous BNL experiment

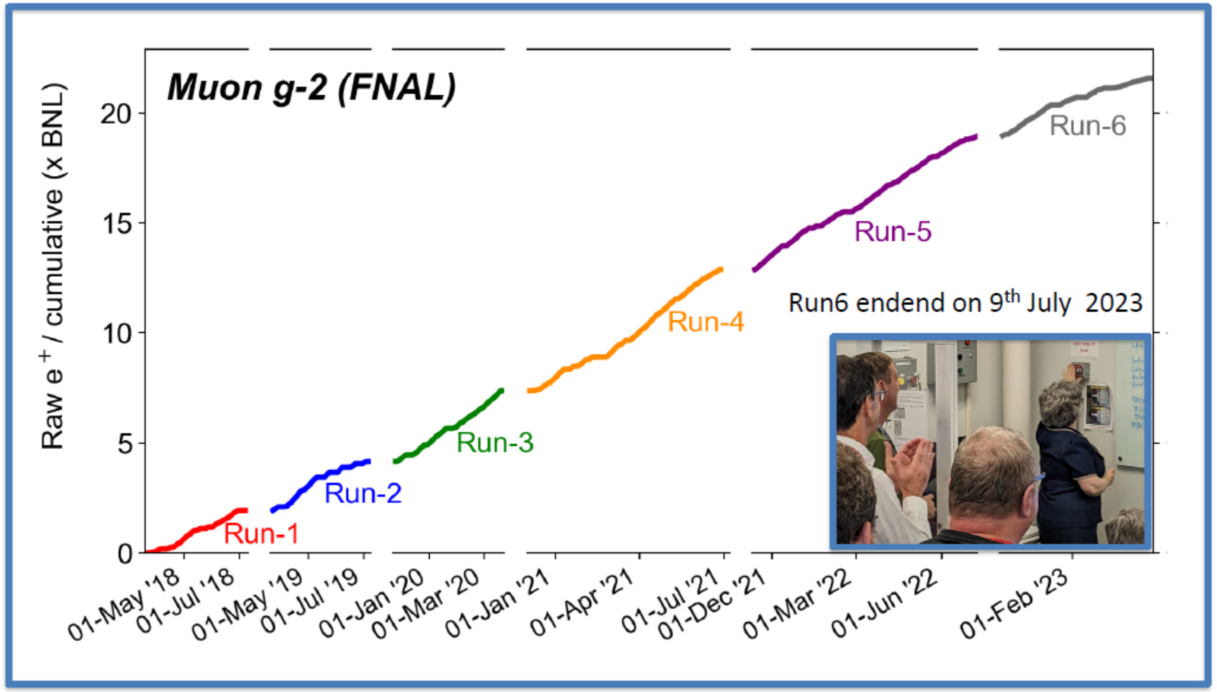
Running conditions in Run-4/5/6:

- Quadrupole Radio-Frequency switched on during Run-5 → reduced radial and vertical motion of muons, more stable beam and less muon losses

Systematic sources of uncertainty:

- CBO expected to be reduced, thanks to Quad RF and to dedicated studies in task forces

On 27 February 2023: proposal Goal of x21 BNL datasets!



Summary and conclusions

❖New muon a_μ experimental average has **unprecedented precision of 190 ppb**:

- Factor ~ 2 improvement in statistical and systematic uncertainties on ω_a
- Improved running conditions; upgraded reconstruction and pileup subtraction algorithms; more studies on beam dynamics effects

❖Future analysis is expected to meet desing goals:

- Surpassed goal statistics: 21+ times w.r.t. previous BNL experiment
- RF system ON: improved beam stability, ongoing evaluation of systematics

… and more details in recent 2024 paper

❖ «Detailed Report on the Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm» on [arXiv:2402.15410 [hep-ex]](arXiv:2402.15410)

Soon to be published on Phys. Rev. D

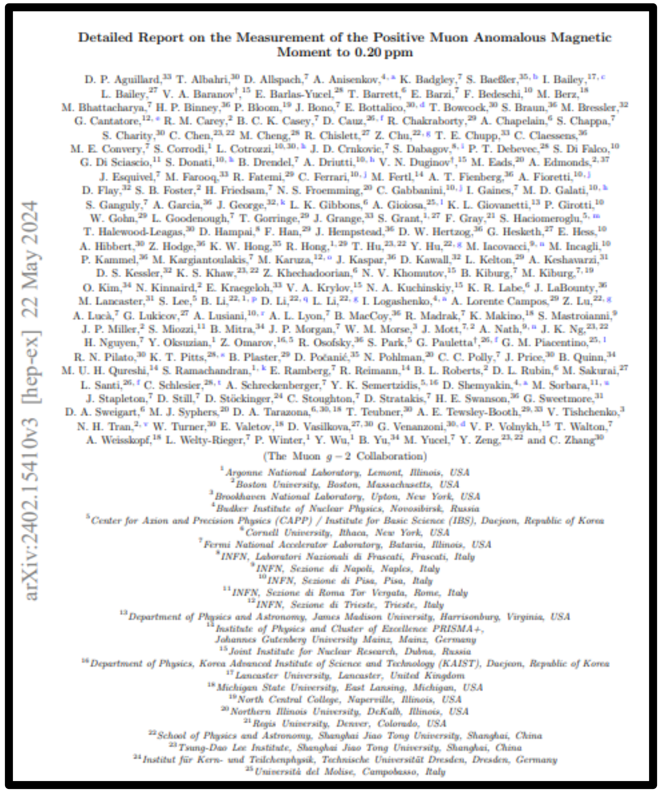
Accepted Paper

Detailed report on the measurement of the positive muon anomalous magnetic moment to 0.20 ppm

Phys. Rev. D

D. P. Aguillard et al.

Accepted 21 May 2024



arXiv:2402.15410v3 [hep-ex] 22 May 2024

Detailed Report on the Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm

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❖ A few more physics papers will come out in the future, e.g. Beyond Standard Model searches: see tomorrow’s talk on CPT/LIV by Baisakhi Mitra

THANK YOU FOR YOUR ATTENTION!

μ g-2

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LEVERHULME TRUST

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Fermilab

July 2023 collaboration meeting @ Liverpool, UK

ICHEP 2024 | PRAGUE

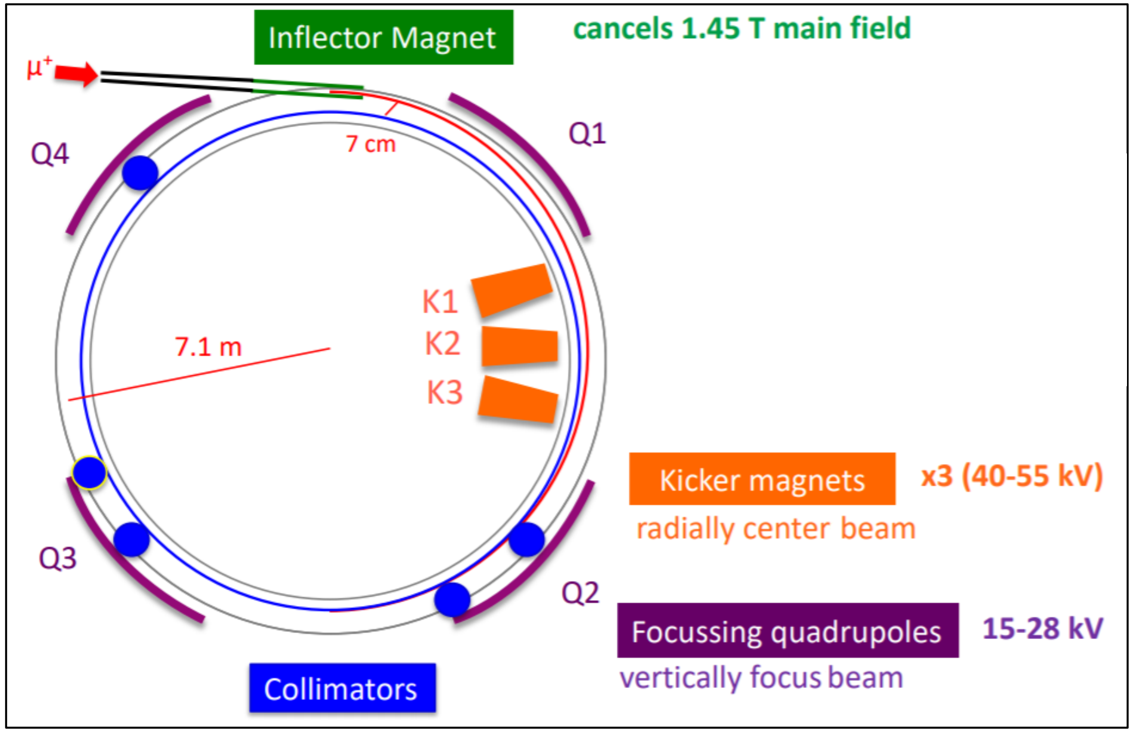
Extra: Bad resistors backup

Injection and muon storage

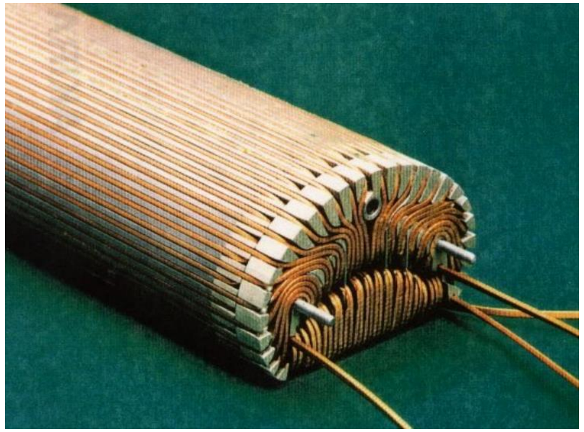
1. Inflector cancels main dipole field and injects at ~8 cm radially away from nominal orbit

2. 3 fast magnetic kickers provide 10 mrad kick and place muons in orbit

3. 8 Electrostatic Quadrupoles (ESQ) focus in the vertical direction



1.



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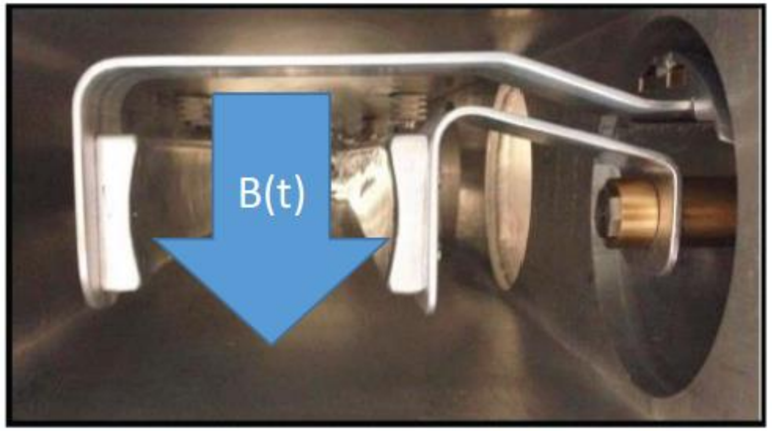

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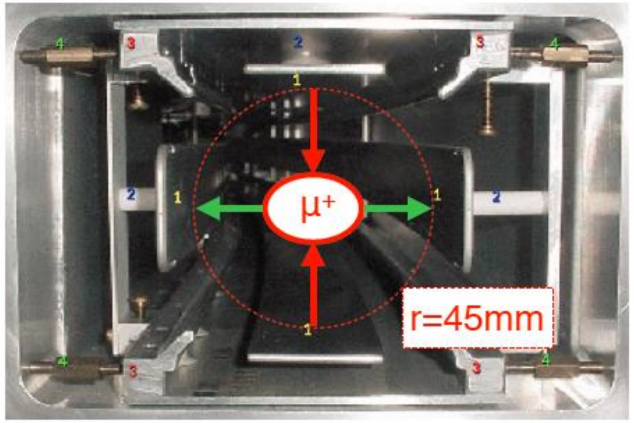
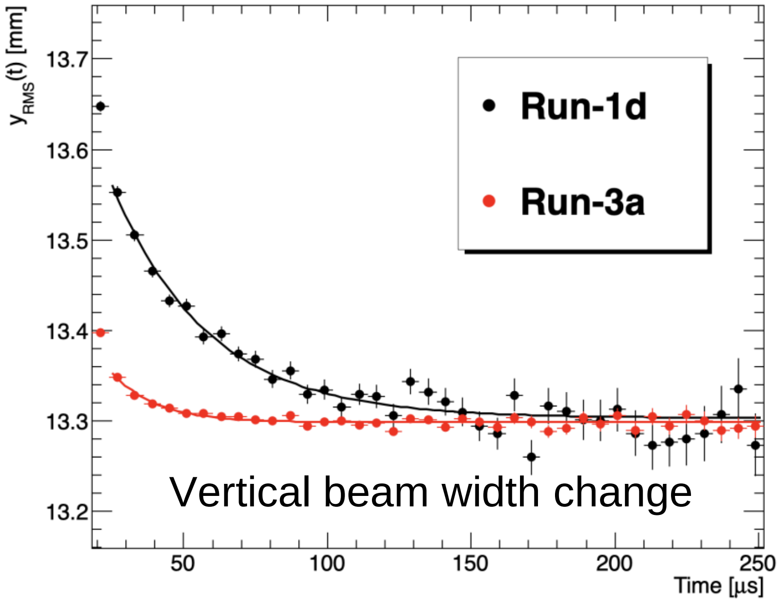
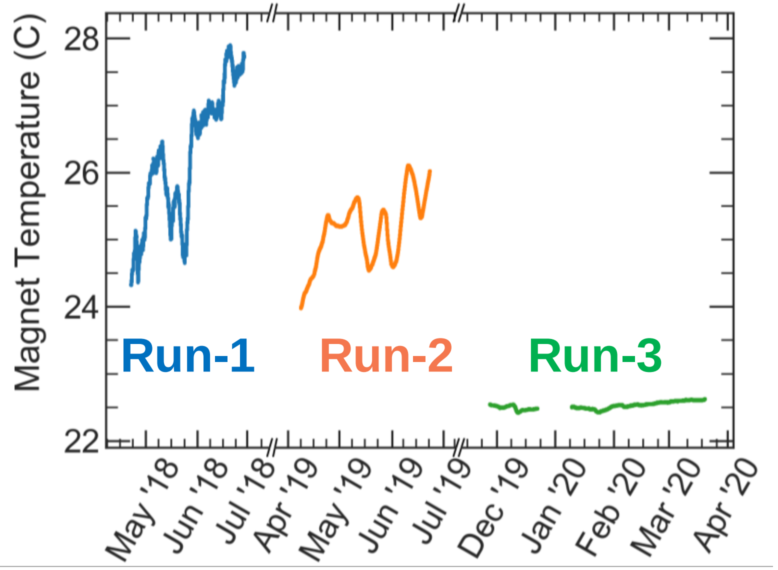
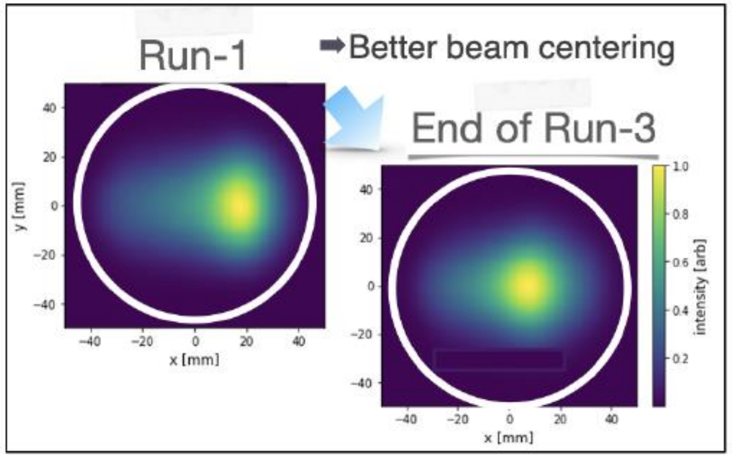


Run-2/3 improved running conditions

- Before Run-2: **fixed faulty resistors** in 2/32 quadrupole plates → better storage, more stable beam oscillations and reduced systematics
- After Run-2: added thermal insulation to ring → less variable magnetic field
- Mid Run-3: **upgraded kicker** cables for optimal kick → more centered beam







Extra: ω_a backup

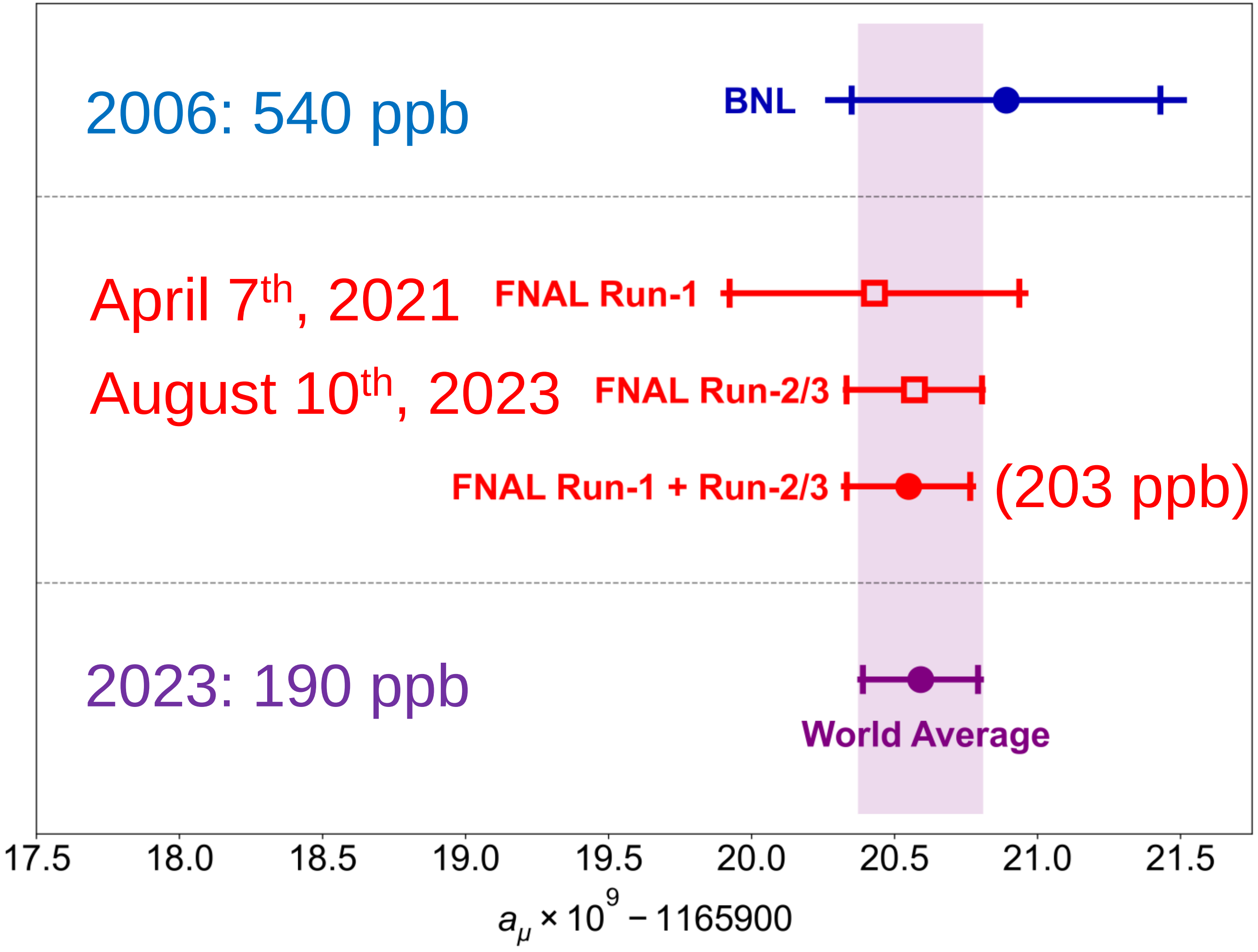
Run-1 vs Run-2/3 systematics

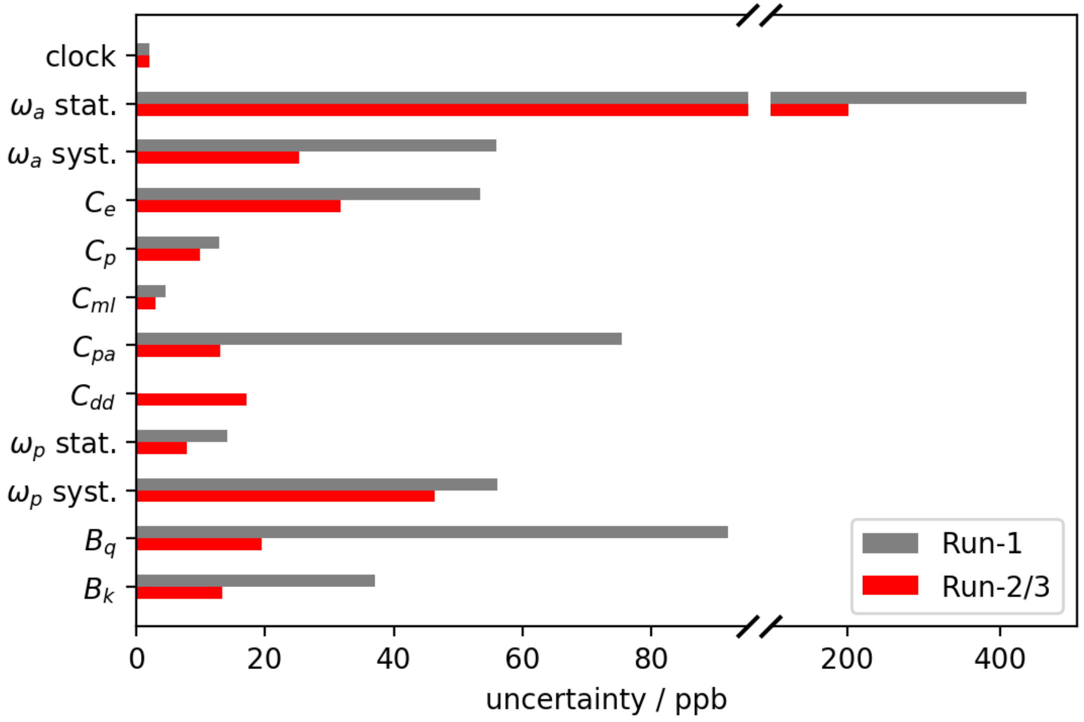
Quantity	Correction terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	···	434
ω_a^m (systematic)	···	56
C_e	489	53
C_p	180	13
C_{ml}	−11	5
C_{pa}	−158	75
$f_{\text{calib}}\langle\omega_p(x,y,\phi)\times M(x,y,\phi)\rangle$	···	56
B_k	−27	37
B_q	−17	92
$\mu'_p(34.7°)/\mu_e$	···	10
m_μ/m_e	···	22
$g_e/2$	···	0
Total systematic	···	157
Total fundamental factors	···	25
Totals	544	462

Quantity	Correction (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	···	201
ω_a^m (systematic)	···	25
C_e	451	32
C_p	170	10
C_{pa}	−27	13
C_{dd}	−15	17
C_{ml}	0	3
$f_{\text{calib}}\cdot\langle\omega'_p(\vec{r})\times M(\vec{r})\rangle$	···	46
B_k	−21	13
B_q	−21	20
$\mu'_p(34.7°)/\mu_e$	···	11
m_μ/m_e	···	22
$g_e/2$	···	0
Total systematic for $\mathcal{R}'_\mu$	···	70
Total external parameters	···	25
Total for a_μ	622	215

Run-2/3 Result: FNAL + BNL Combination

D. P. Aguillard et al, Phys. Rev. Lett. 131.161802 (2023)
D. P. Aguillard et al, arxiv:2402.15410 (2024)





- Running improvements
- ~ 5x statistics
- Analysis improvements (CBO, pileup, reconstruction, …)
- 70 ppb syst. → exceeded goal!

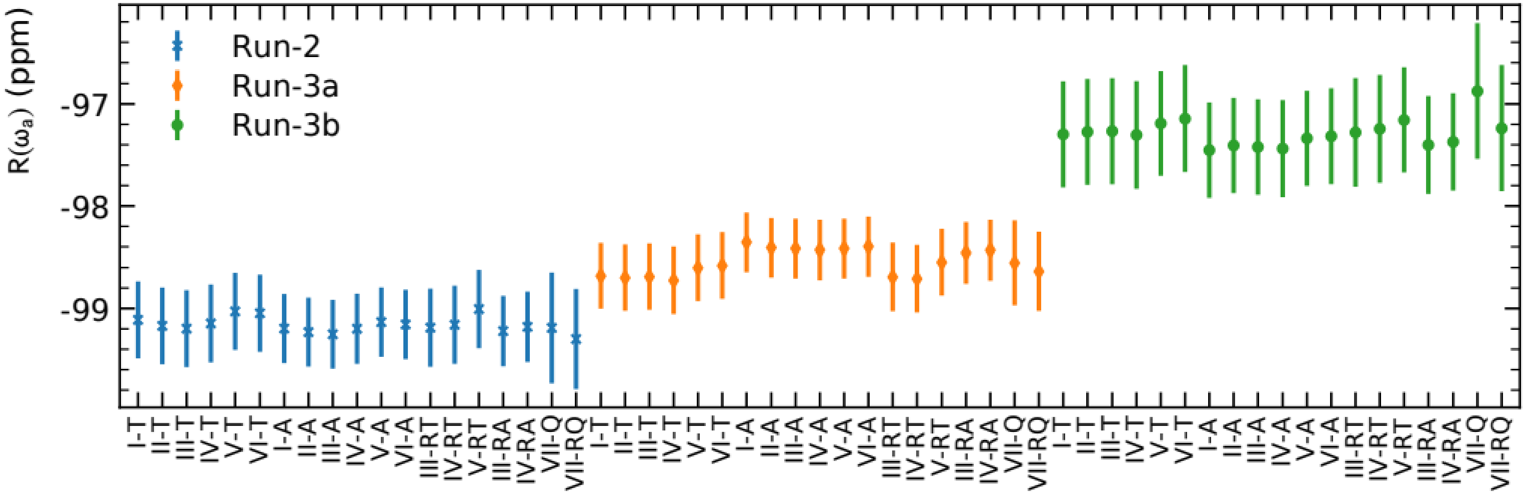
Blinded analysis

- **Hardware**: main clock is tuned at $(40 - \varepsilon)$ MHz
  Offset only known to two scientists external to the collaboration

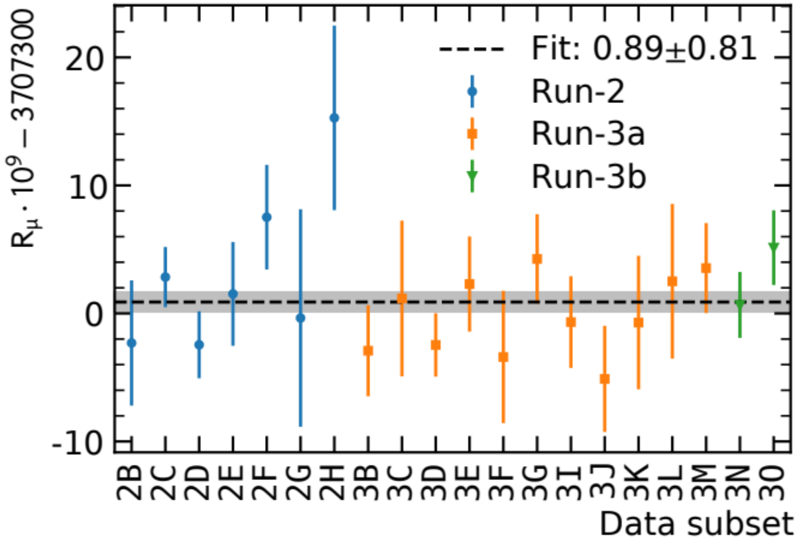
- **Software**: each ω_a analyzer applies their own, secret offset to their results

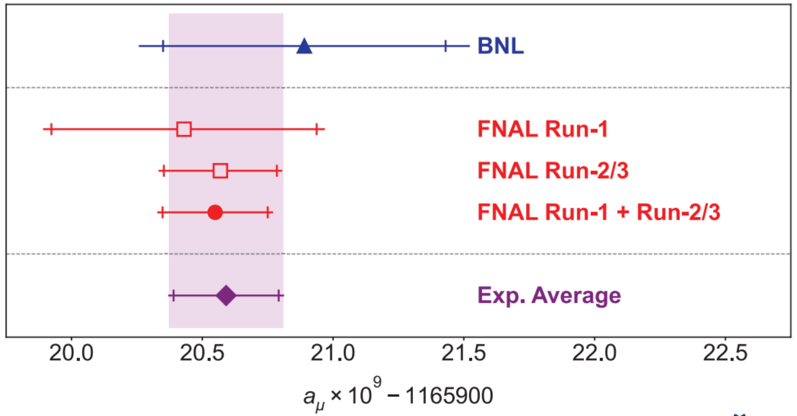
Run-2/3 unblinding

- **Software unblinding**

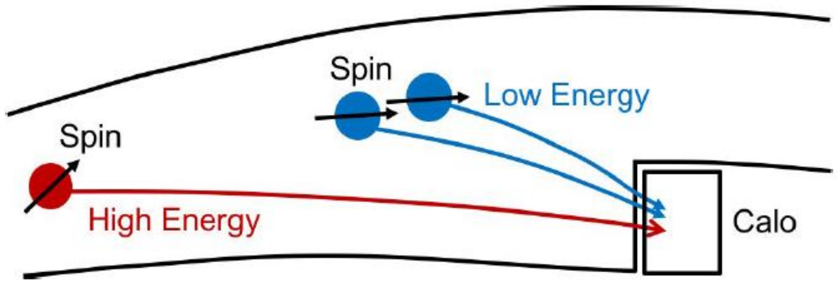


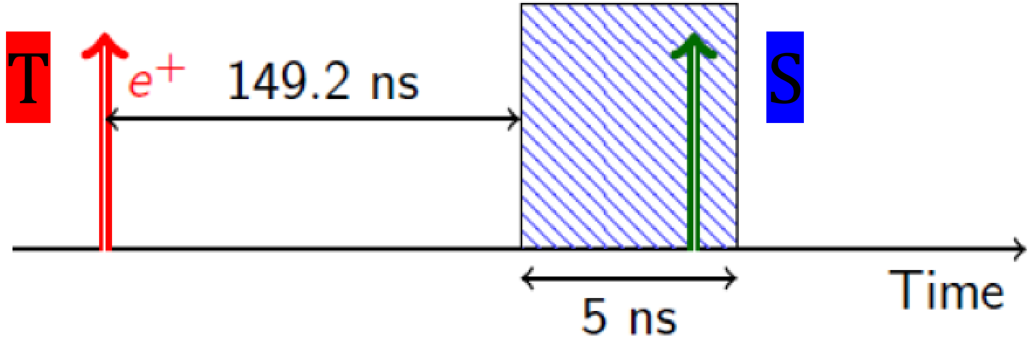
- **Hardware unblinding**





Example of new method to subtract pileup



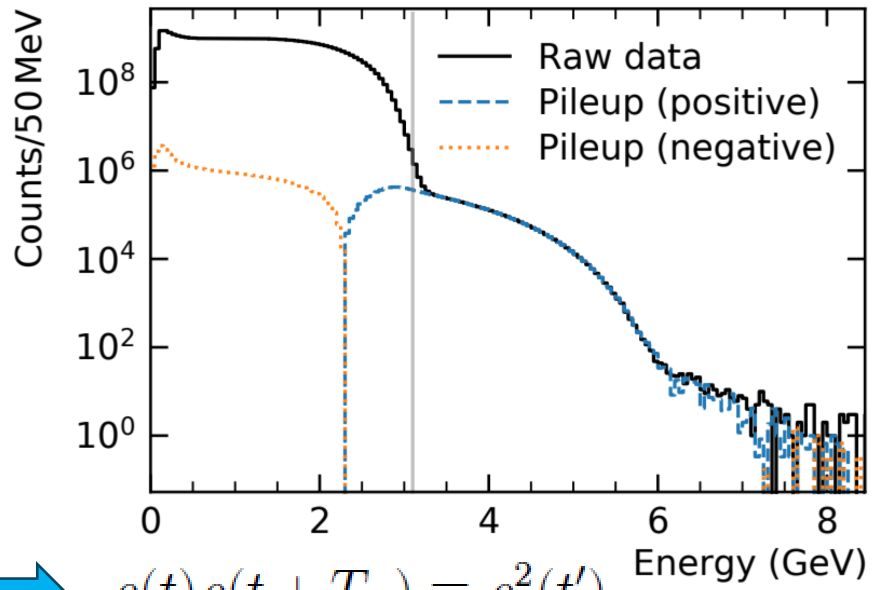


For each T (Trigger) cluster that we find:

- Search for coincidence e^+ in S (Shadow) window, after 149.2 ns
- Superimpose the two clusters and pass to reconstruction algorithm
  →If not resolved: merge them and build pileup

$$E_2 = (E_T + E_{S_1}) \qquad t_2 = \frac{(t_T + T_G/2)E_T + (t_{S_1} - T_G/2)E_{S_1}}{E_T + E_{S_1}}$$

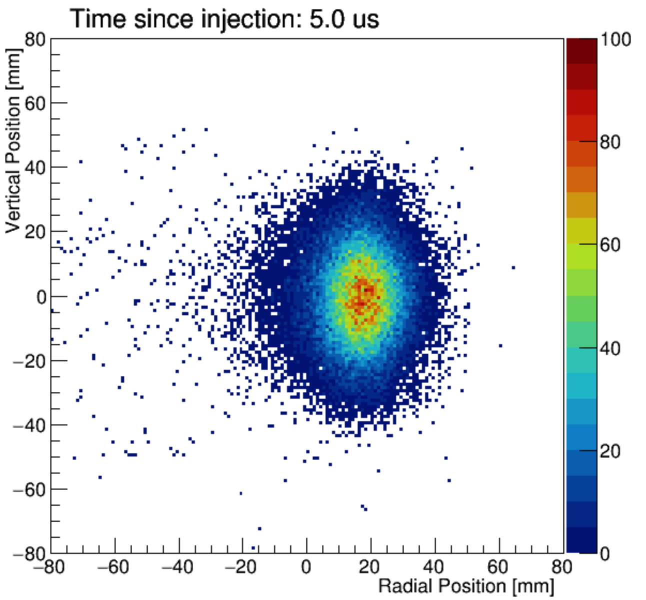
$$\rho(t)\rho(t + T_G) \equiv \rho^2(t')$$

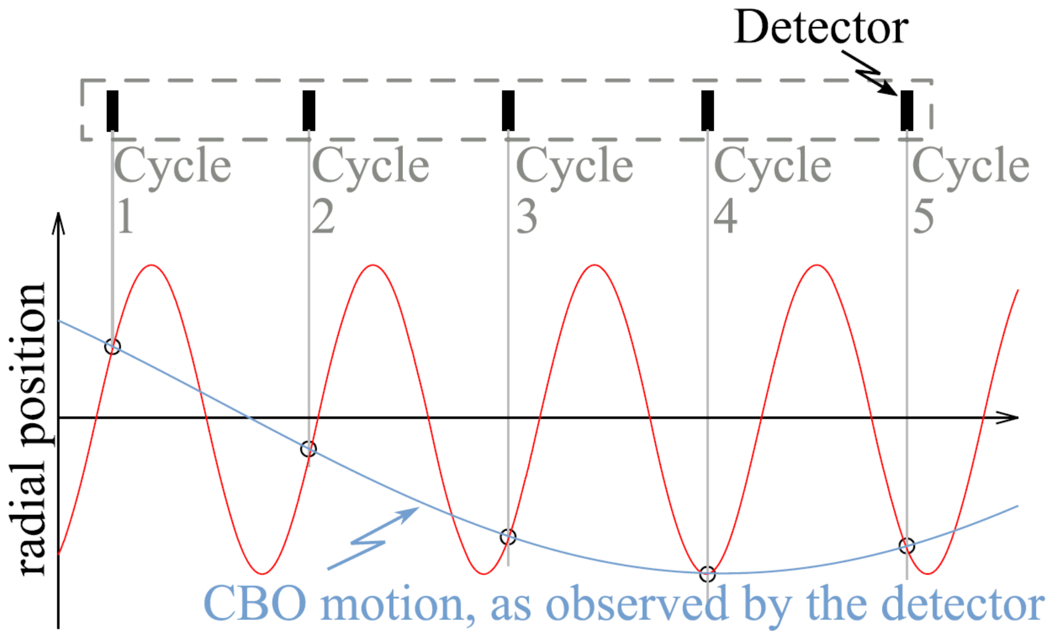


Finally: subtract merged event and add single events

Radial and vertical motion of the beam

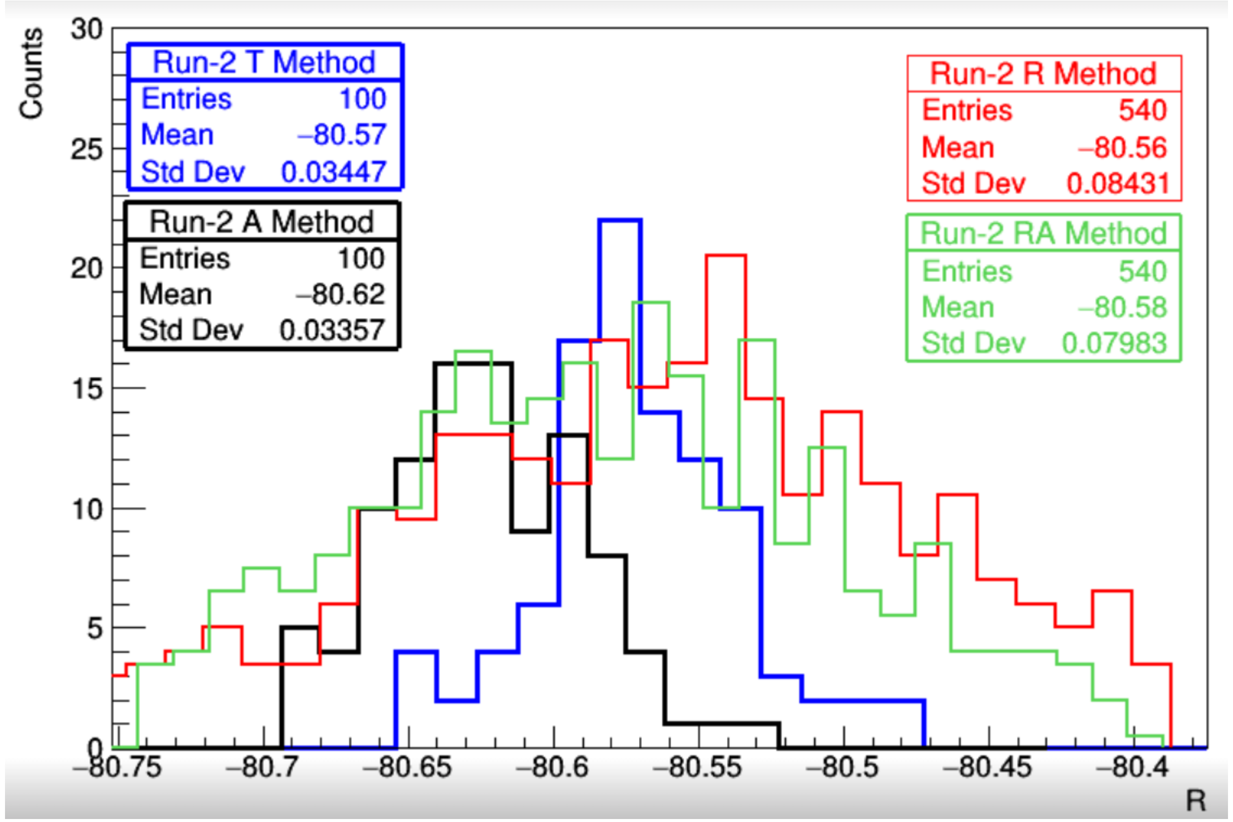
- Field index: n (quad voltages)
- Radial motion of the beam: $\omega_x = \omega_c\sqrt{1-n}$
- CBO is the aliased frequency $\omega_{CBO} = \omega_c - \omega_x$
- CBO period of about 2.7 μs

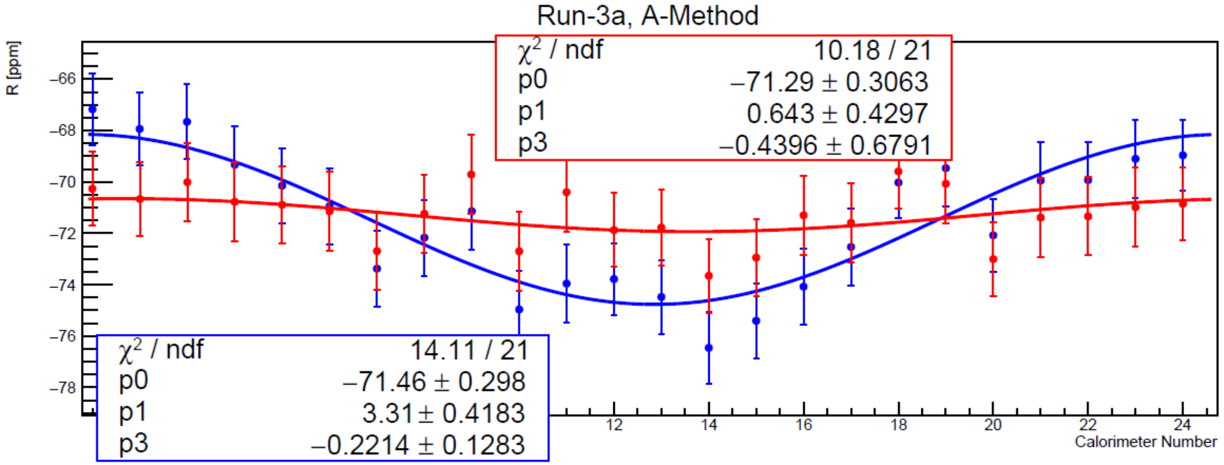




Quantity	Expression	Frequency [MHz]	Frequency [rad/μs]	Period [ns]
ω_a	$ea_\mu B/m$	0.23	1.439	4365
ω_C	v/R_0	6.7	42.0	149.2
ω_x	$\omega_C\sqrt{1-n}$	6.3	39.7	158.0
ω_y	$\omega_C\sqrt{n}$	2.2	13.8	454.2
ω_{CBO}	$\omega_C - \omega_x$	0.37	2.33	2686
ω_{VW}	$\omega_C - 2\omega_y$	2.3	14.4	435.3

Randomization



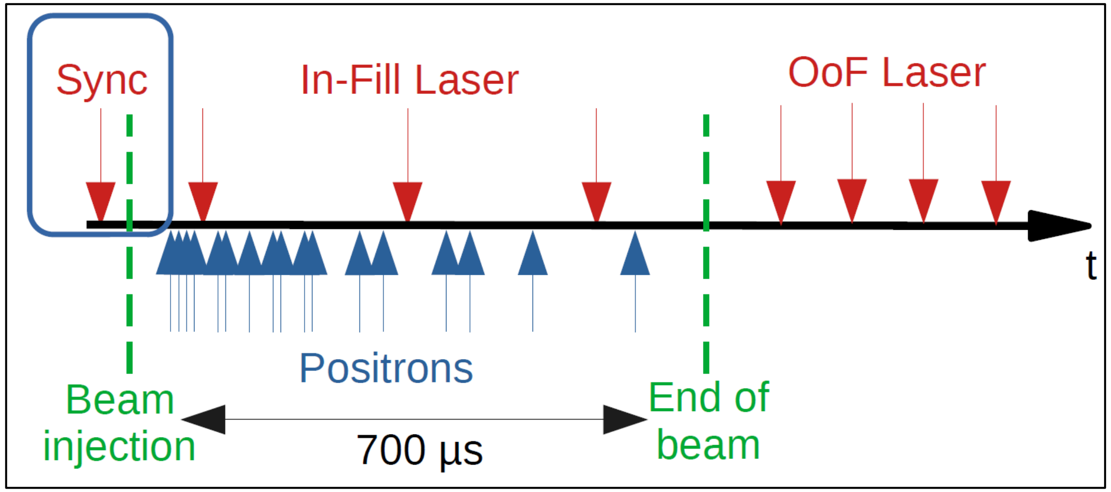


Laser-based gain monitoring system

Built by INFN/CNR-INO: time synchronization and calibration of 1296 SiPMs on timescales from ns to days/weeks. Gain changes dominated ω_a systematics at BNL: exceeded goal of 20 ppb at FNAL.

Standard operating mode:

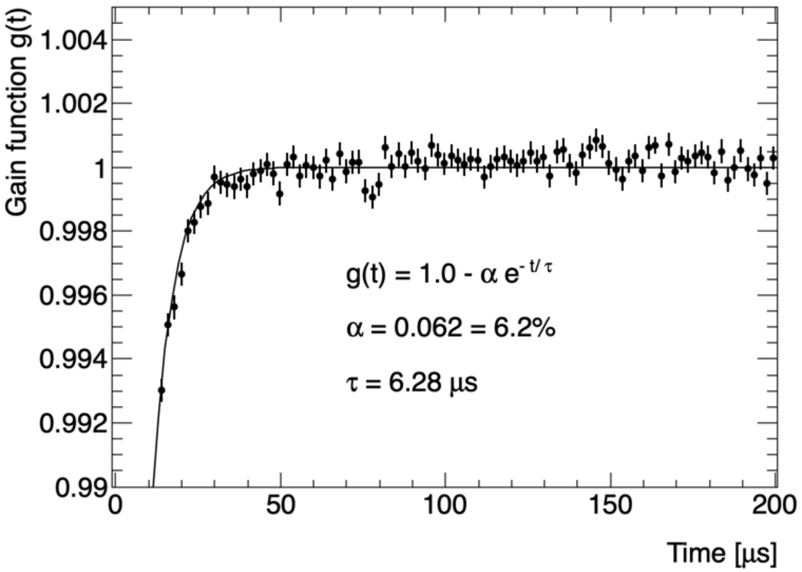
- **Sync pulse**: time synchronization at $\sim$ 50 ps
- **In-Fill pulses**: monitor rate-dependent gain changes at 10^{-4} during 700 μs of μ^+ beam
- **Out-of-Fill pulses**: monitor stability over days



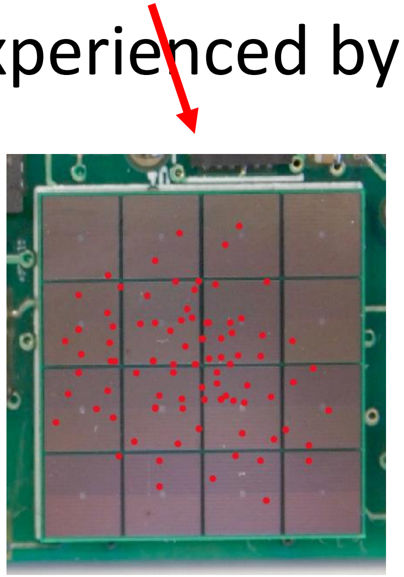
SiPM gain calibration

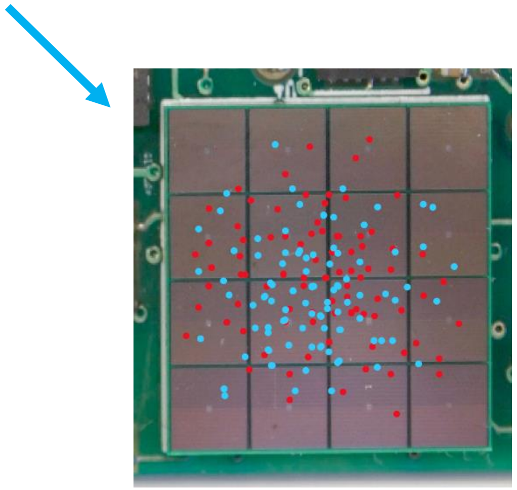
In-Fill: sag in power supply due to initial injection splash.

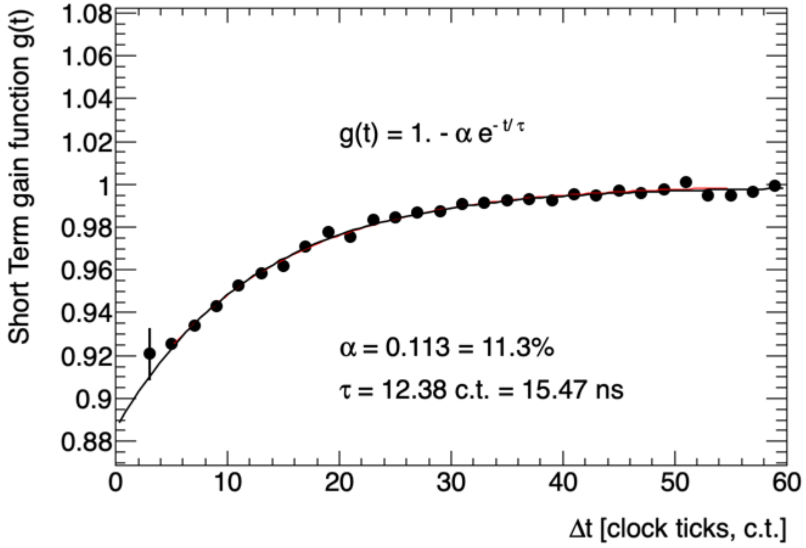
Recovery timescale of front-end electronics: $\mathcal{O}(10\ \mu s)$.



Short-term: consecutive positron hits within $\mathcal{O}(100\ ns)$. After the first hit, the recovery time of pixels reduce the gain experienced by the second hit.



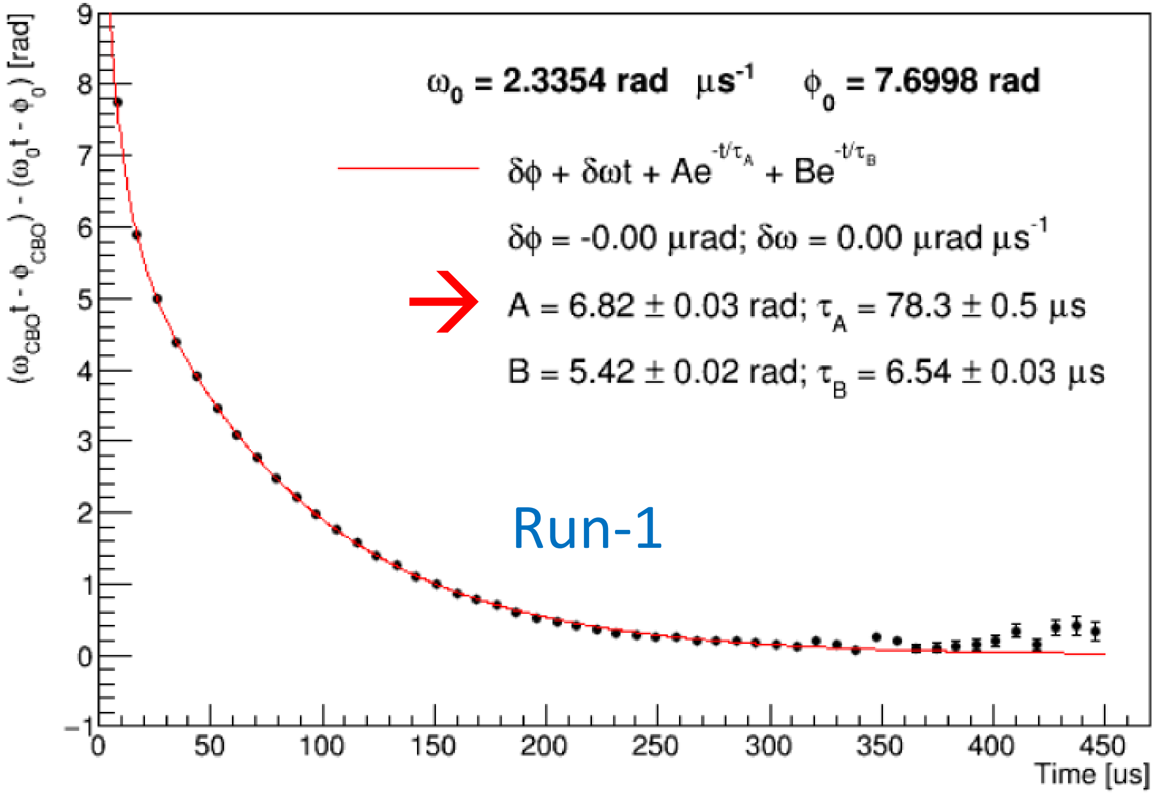


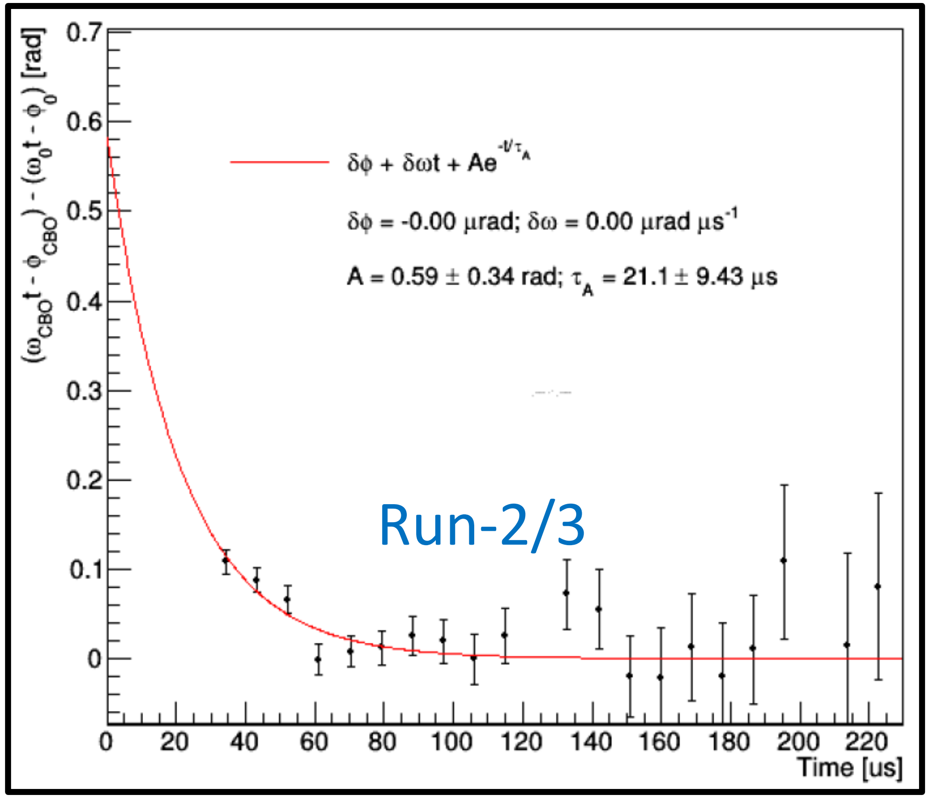


CBO model: frequency vs time

CBO dominated Run-1 systematics (38 ppb). Now reduced to 21 ppb!

- Exponential relaxation of CBO frequency
- Run-1: faulty ESQ resistors enhanced this effect 10 times!
- Sliding window fits to determine lifetime and constrain it in ω_a fits





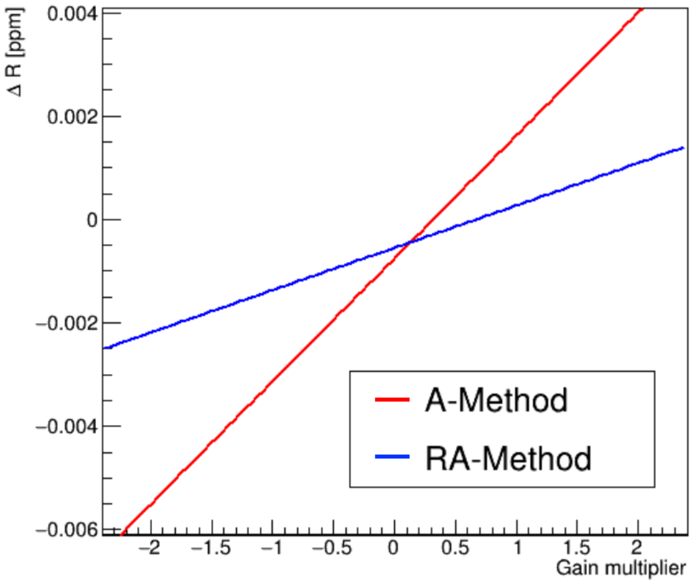
Run-4/5/6: current status and puzzles

- With much more statistics, we can investigate the residual slow term
  → energy leakage in calorimeters?
  → reconstruction effect?
- Further improved reconstruction with new pulse fitting technique
- Task forces in place to address dominating Run-2/3 systematics
- Quadrupole Radio-Frequency switched on during Run-5, to highly reduce radial and vertical motion of muons → more stable beam dynamics and much fewer lost muons!

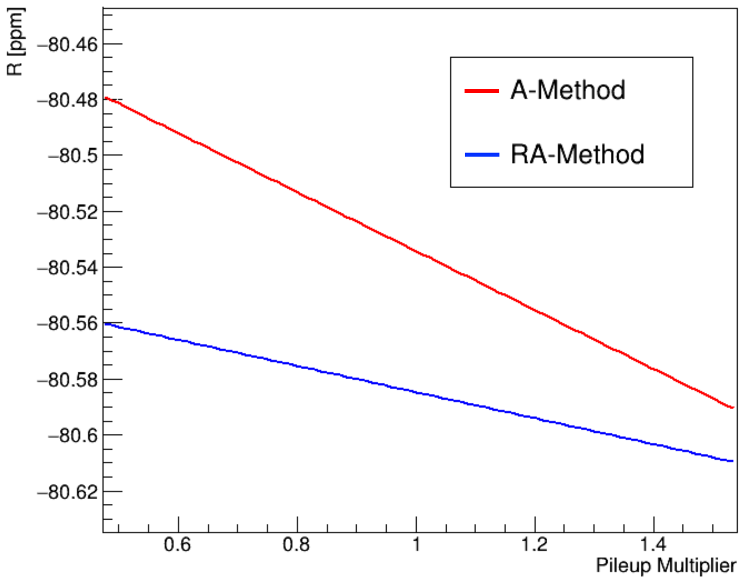
«Slow» effects: ratio vs non-ratio

$$\frac{d\langle p\rangle}{dt} \neq 0 \rightarrow \frac{\Delta\omega_a}{\omega_a} = \frac{1}{\omega_a}\cdot\frac{d\langle\varphi\rangle}{dt} = \frac{1}{\omega_a}\cdot\frac{d\langle\varphi\rangle}{d\langle p\rangle}\cdot\frac{d\langle p\rangle}{dt} \neq 0$$

$$\varphi(t) = \varphi(0) + \dot{\varphi}t + \cdots \rightarrow \boldsymbol{\omega_a} t + \varphi(t) = (\boldsymbol{\omega_a} + \dot{\boldsymbol{\varphi}})t + \cdots$$



Gain calibration



Pileup