Thermal effects in CPT and unitarity constraints for higher-order CP asymmetries

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Tomáš Blažek, Peter Maták & **Viktor Zaujec**

Department of Theoretical Physics, Comenius University, Bratislava, Slovak Republic

peter.matak@fmph.uniba.sk, viktor.zaujec@fmph.uniba.sk

Particle density evolution in the expanding universe

At classical level, number density n_{i_k} of any particle species included in $\{i\}$ is affected by $\{i\} \leftrightarrow \{f\}$ reactions,

$$\dot{n}_{i_k} + 3\mathcal{H} n_{i_k} = -\mathring{\gamma}_{fi} + \mathring{\gamma}_{if} + \dots \tag{1}$$

Reaction rates

$$\mathring{\gamma}_{fi} = \int \prod_{\{i\}} [d\mathbf{p}_i] \mathring{f}_i(p_i) \int \prod_{\{f\}} [d\mathbf{p}_f] (2\pi)^4 \delta^{(4)}(p_f - p_i) |\mathring{M}_{fi}|^2 \equiv \int_{i\to f} \mathring{f}_i |\mathring{M}_{fi}|^2 \tag{2}$$

- ← classical phase-space densities $\mathring{f}_i$ (Maxwell-Boltzmann densities in equilibrium),
- ← zero-temperature Feynman rules entering the calculation of $\mathring{M}_{fi}$.

CP asymmetries at zero-temperature

The S-matrix unitarity $S^\dagger = S^{-1}$, or $1 - \mathrm{i}T^\dagger = (1 + \mathrm{i}T)^{-1}$, for the squared amplitude leads to [1–4]

$$\mathrm{i}T^\dagger = \mathrm{i}T - (\mathrm{i}T)^2 + (\mathrm{i}T)^3 - \dots \quad \Rightarrow \quad |T_{fi}|^2 = -\mathrm{i}T_{if}\mathrm{i}T_{fi} + \sum_n \mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \dots \tag{3}$$

Divide by $V_4 = (2\pi)^4\delta^{(4)}(0)$ to obtain $(2\pi)^4\delta^{(4)}(p_f - p_i)|\mathring{M}_{fi}|^2$. In CPT symmetric theory, the CP asymmetry $\Delta|T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2$ can be expressed as [4]

$$\begin{aligned}\Delta|T_{fi}|^2 = &\sum_n (\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni}) \\ &- \sum_{n,m} (\mathrm{i}T_{in}\mathrm{i}T_{nm}\mathrm{i}T_{mf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fm}\mathrm{i}T_{mn}\mathrm{i}T_{ni}) \\ &+ \dots\end{aligned} \tag{4}$$

From Eq. (4) the CPT and unitarity relations [5–7]

$$\sum_f \Delta|T_{fi}|^2 = 0 \tag{5}$$

are manifest at any perturbative order [4].

Thermal effects at the lowest order

As an example we consider $N_i \to lH$ decay within the seesaw type-I model. Right-handed neutrinos N_i, standard model lepton l and Higgs doublets H interact via Yukawa interactions

$$\mathcal{L} \supset -\frac{1}{2} M_i \bar{N}_i N_i - \left(\mathcal{Y}_{\alpha i} \bar{N}_i P_L l_\alpha H + \text{H.c.}\right). \tag{6}$$

To the lowest $\mathcal{O}(\mathcal{Y}^2)$ order,

$$\mathring{\gamma}_{N_i \to lH} = \int_{N_i \to lH} \mathring{f}_{N_i} |\mathring{M}|^2, \quad |\mathring{M}|^2 = 4 p_{N_i} \cdot p_l \sum_\alpha |\mathcal{Y}_{\alpha i}|^2 \tag{7}$$

that can be represented by the cut of the forward diagram Fig. 1a. Diagram in Fig. 1b leads to spurious contribution, equivalent to $\mathring{\gamma}_{N_i \to lH}$ times the total number of Higgs particles in the universe. Clearly, such contributions have to be omitted. Diagram in Fig. 1c corresponds to

$$\mathring{\gamma}_{N_i H \to lHH} = \int_{N_i \to lH} \mathring{f}_{N_i} \mathring{f}_H |\mathring{M}|^2 \tag{8}$$

with the same $|\mathring{M}|^2$ as in Eq. (7). To sum up the contributions such as in Fig. 1c avoiding those analogous to Fig. 1b, we shall draw the forward diagrams on a cylindrical surface [8]. In thermal equilibrium $\mathring{f}_H = \mathrm{e}^{-E/T}$ and the summation leads to

$$\sum_{w=0}^{\infty} \left(\mathring{f}_H^{\mathrm{eq}}\right)^w = \frac{1}{1 - \exp\{-E_H/T\}} = \boxed{1 + f_H^{\mathrm{eq}}} \tag{9}$$

$$\gamma_{N_i \to lH} = \int \dots \boxed{f_{N_i}(1 - f_l^{\mathrm{eq}})(1 + f_H^{\mathrm{eq}})} \tag{10}$$

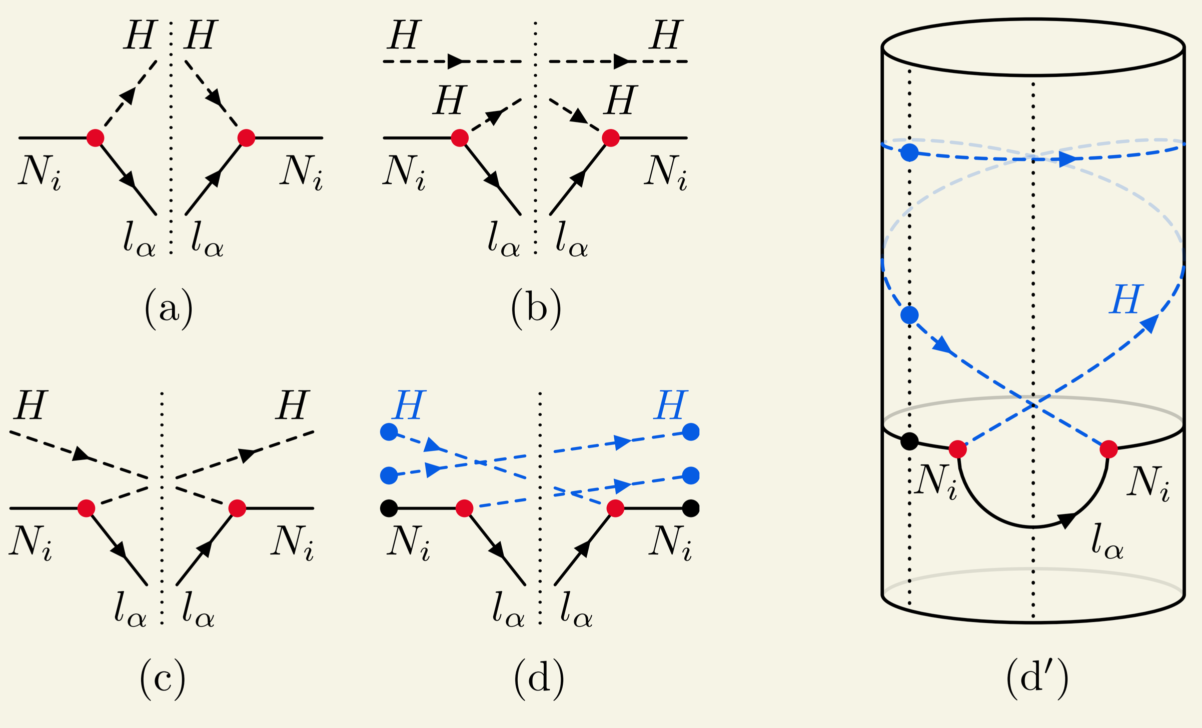


Figure 1: Planar and cylindrical diagrams contributing to the $N_i \to lH$ zero-temperature reaction rate and its thermal corrections.

Higher-order N_iQ scattering

As an example, we use the seesaw type-I model to show the emergence of thermal effects in zero-temperature calculations, while accounting for all contributing diagrams at the given order.

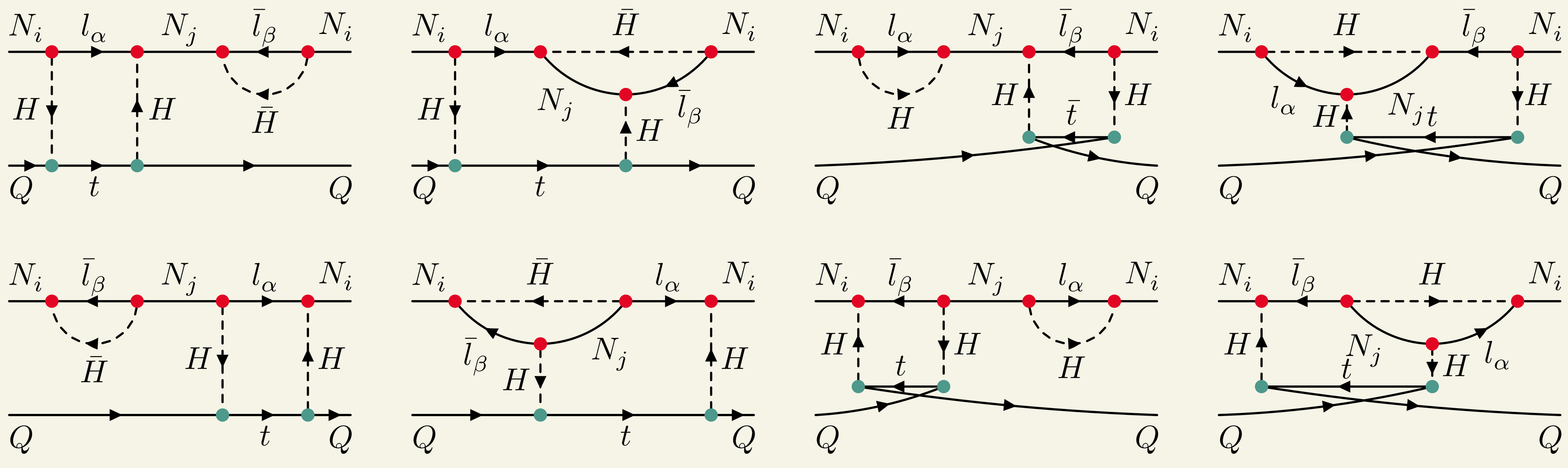


Figure 2: Forward-scattering diagrams contributing to N_iQ scattering at $\mathcal{O}(Y^4Y_t^2)$ order [4].

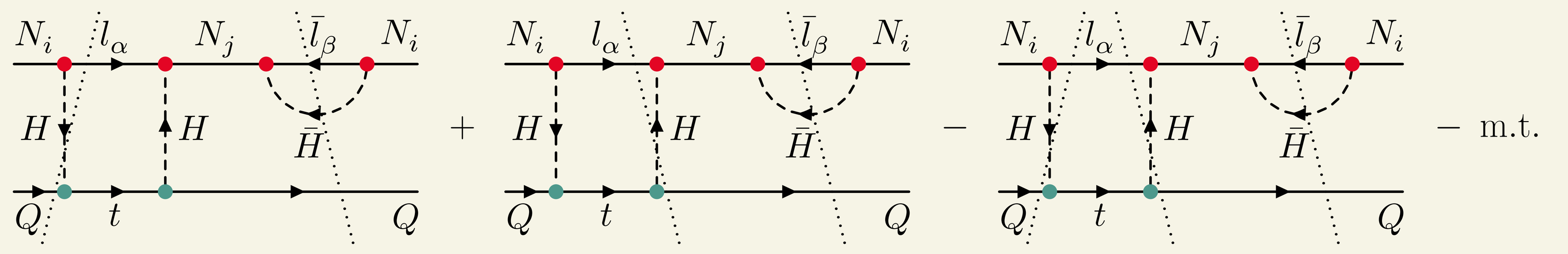
Accounting for all possible final states systematically, we obtain the unitarity relation [9–12]

$$\boxed{\Delta\mathring{\gamma}^{\mathrm{eq}}_{N_iQ\to lt} + \Delta\mathring{\gamma}^{\mathrm{eq}}_{N_iQ\to lHQ} + \Delta\mathring{\gamma}^{\mathrm{eq}}_{N_iQ\to \bar{l}\bar{H}Q}} + \boxed{\Delta\mathring{\gamma}^{\mathrm{eq}}_{N_iQ\to \bar{l}QQ\bar{t}}} = 0 \tag{11}$$

The last term – the one with two identical quarks in final state – is completely new [4]. This contribution might seem as nonsense, however, it is necessary for the completeness at the given perturbative order.

Thermal masses and quantum statistics

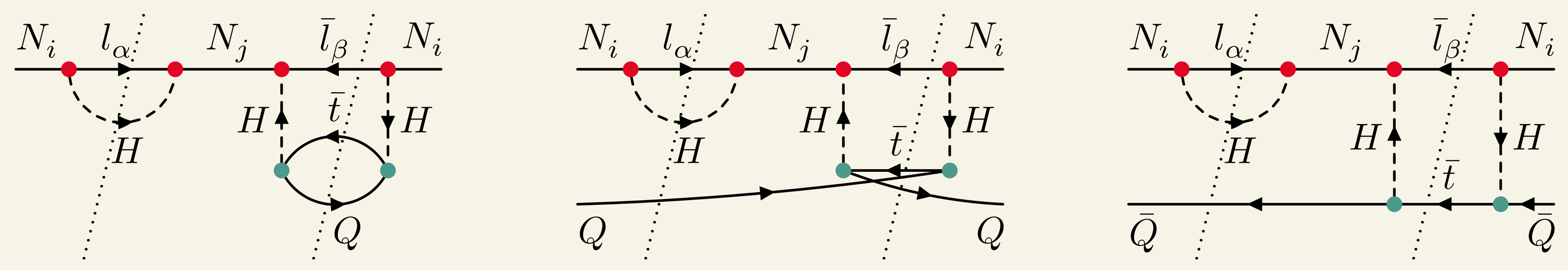
The anomalous-threshold cuts at $\mathcal{O}(Y^4Y_t^2)$ order lead to the Higgs thermal-mass effects, corrections for the leading order asymmetry ($N_i \to lH$). As an example we consider the cuts of one of the diagrams in Fig.2



leading to the mass-derivative term in the relation

$$\Delta\mathring{\gamma}^{\mathrm{eq}}_{N_iQ\to lHQ} = \boxed{\Delta\mathring{\gamma}^{\mathrm{eq}}_{N_i(Q)\to lH(Q)}} + \boxed{\frac{1}{4}\mathring{m}^2_{H,Y_t}(T)\frac{\partial}{\partial m_H^2}\bigg|_{m_H^2=0}\Delta\mathring{\gamma}^{\mathrm{eq}}_{N_i\to lH}} \tag{12}$$

while the first term represents the approximation of the quantum-statistical factor in the asymmetry-generating loop



Similarly, the last term in the unitarity relation in Eq. (11) contributes to the quantum statistical factor in the final state of the $N_i \to lQ\bar{t}$ decay. Including all windings for given initial state leads to the unitarity relations with quantum statistics

$$\boxed{\Delta\gamma^{\mathrm{eq}}_{N_iQ\to lt} + \Delta\gamma^{\mathrm{eq}}_{N_i(Q)\to lH(Q)} + \Delta\gamma^{\mathrm{eq}}_{N_i(Q)\to \bar{l}\bar{H}(Q)} + \Delta\gamma^{\mathrm{eq}}_{N_iQ\to \bar{l}QQ\bar{t}} = 0} \tag{13}$$

Here the mass-derivative part of the leading-order asymmetry has been taken out as it holds separately.

Conclusions

Anomalous thresholds → Thermal masses

Cylindrical diagrammatic representation → Quantum statistics

↓

CPT and unitarity constraints for equilibrium CP asymmetries at finite-temperature

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