Effect of thermal fluctuations on dark matter annihilation cross section

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1. Introduction

• **Motivation** : Relic abundance of dark matter in our universe is increasing accurately measured ($\Omega_c h^2 = 0.1200 \pm 0.0012$) by the successive generation of experiments. The Boltzmann equation determines the yields using the dark matter annihilation cross section as one of the input; the accurate computation of the latter including thermal contribution thus assumes importance.

• **Background** : We are interested in obtaining thermal correction to relic abundance of dark matter(DM). We present effect of thermal fluctuation on dark matter annihilation cross section at NLO, utilizing advanced techniques of thermal field theory(TFT), which is used as input in Boltzmann equation. We use generalized Grammer and Yennie (GY) technique, in order to deal with IR divergences encountered in annihilation cross section calculations .

2. Model and Feynman Rules

• **Model** : extension of SM by an $SU(2) \times U(1)$ singlet Majorana fermion χ and scalar doublet $\phi = (\phi^+, \phi^0)^T$. f is SM fermion doublet [2].

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{f}(i\not{D} - m_f) f + \frac{1}{2}\bar{\chi}(i\not{\partial} - m_\chi)\chi + (D_\mu \phi)^\dagger (D_\mu \phi) - m_\phi^2 \phi^\dagger \phi + (\lambda \bar{\chi} P_L f^- \phi^+ + h.c.)$$

• Feynman Rules for photon propagator in TFT

$$iD^{t_a,t_b}_{\mu\nu,k} = -g_{\mu\nu} \left\{ \begin{bmatrix} i\Delta_k & 0 \\ 0 & i\Delta_k^* \end{bmatrix} + 2\pi\delta(k^2) N_B(|k^0|) \begin{bmatrix} 1 & e^{\frac{k^0}{2T}} \\ e^{\frac{k^0}{2T}} & 1 \end{bmatrix} \right\}$$

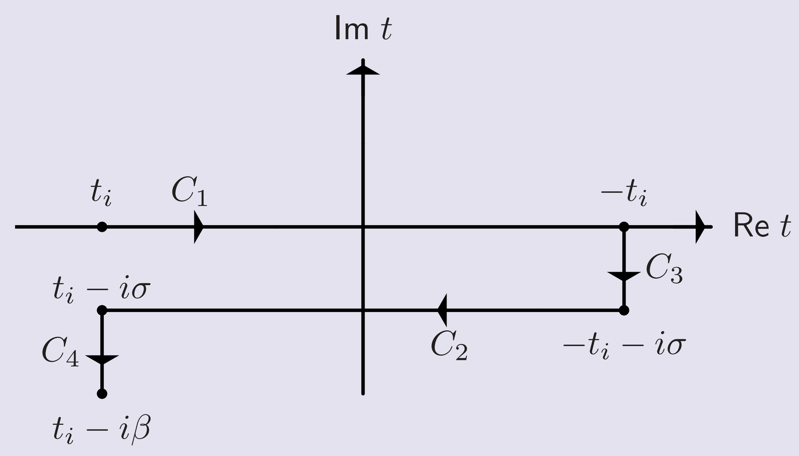
where $i\Delta_k = i/(k^2 + i\epsilon)$ and $N_B|k^0| = 1/(e^{|k^0|/T} - 1)$. Here $t_a, t_b (= 1, 2)$ refers to fields thermal type.

• Feynman Rules for fermion propagator in TFT

$$i\mathcal{S}^{t_a,t_b}_{\mathrm{f(p,m)}} = \begin{bmatrix} S & 0 \\ 0 & S^* \end{bmatrix} - 2\pi S' \delta(p^2 - m^2) N_{\mathrm{F}}(|p^0|) \begin{bmatrix} 1 & \epsilon_{p_0} e^{\frac{|p^0|}{(2T)}} \\ -\epsilon_{p_0} e^{\frac{|p^0|}{(2T)}} & 1 \end{bmatrix} ,$$

where $S = i/(\not{p} - m + i\epsilon)$, $S' = (\not{p} + m)$ & $N_F(p^0) = 1/(e^{|p^0|/T} + 1)$

3. Time Path in RTF in TFT



• Fig1 : The time path for real time formulation (RTF) of thermal field theories(TFT) in the complex t plane, where the y axis corresponds to '$Im\ t = \beta$', the inverse temperature.

4. GY technique in TFT

• Grammer and Yennie(GY) technique for Rearrangement of $-ig_{\mu\nu}$ [Virtual Photon] [3–5]

$$-ig_{\mu\nu} \to -i\{g_{\mu\nu} - b_{k,p_i,p_f} k_\mu k_\nu + b_{k,p_i,p_f} k_\mu k_\nu\}$$
$$G_{\mu\nu} = g_{\mu\nu} - b_{k,p_i,p_f} k_\mu k_\nu \ \&\ K_{\mu\nu} = b_{k,p_i,p_f} k_\mu k_\nu$$

• Rearrangement of Photon polarization sum [Real Photon]

$$\sum_{\mathrm{pol}} \epsilon^{*,\mu}(k)\, \epsilon^{\nu}(k) \to -g^{\mu\nu} \to -[\widetilde{G}^{\mu\nu}_k + \widetilde{K}^{\mu\nu}_k]$$

• Structure of $b[p_i, p_f.k]$

$$b_{p_i,p_f,k} = \frac{1}{2}\left[\frac{(2p_f - k).(2p_i - k)}{((p_f - k)^2 - m^2).((p_i - k)^2 - m^2)} + (k \leftrightarrow -k)\right]$$

5. Details and Approximations

• In GY technique, IR divergence cancels between the K-*photon* virtual correction and real $\tilde{K}$-*photon* contribution order by order [3–5].

• We calculate "*G-photon*" contribution to NLO-virtual correction in order to obtain thermal correction to DM annihilation cross section [1].

• Calculation was first performed in Ref. [2] within alternate approach. Our approach is manifestly IR finite.

• In our calculations, we take scalar mass to be heavy compared to dark matter mass, $m_\phi \gg m_\chi$. We calculate DM annihilation cross section in following approximations on non-thermal scalar propagator [1]

• Non-dynamical scalar (Heavy scalar) approximation

$$i\Delta_{l+k} := \frac{i}{(l+k)^2 - m_\phi^2}) \to \frac{i}{(-m_\phi^2)}$$

• Dynamical scalar approximation

$$i\Delta_{l+k} := \frac{i}{((l+k)^2 - m_\phi^2)} \to \frac{i}{(l^2 - m_\phi^2)}\left[1 - \frac{(2l\cdot k + k^2)}{(l^2 - m_\phi^2)}\right] .$$

6. DM annihilation cross section at LO (σ_{LO}) for the process $\chi\chi \to f\bar{f}$

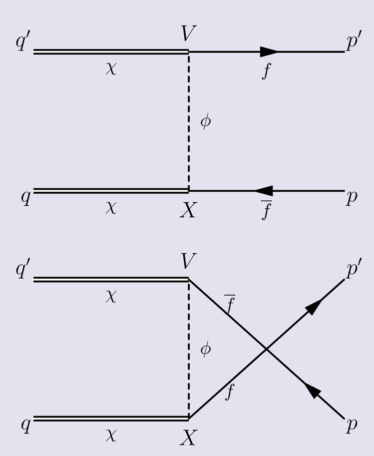


Fig.2 : tree level Feynman diagrams

• Momenta in CM frame

$q'^\mu = (H, 0, 0, P)$; $p'^\mu = (H, P'Sin(\theta), 0, P'Cos(\theta))$

$q^\mu = (H, 0, 0, -P); p^\mu = (H, -P'Sin(\theta), 0, -P'Cos(\theta))$

•Tree level DM annihilation cross section σ_{LO} in two approximations

$$\sigma_{LO}^{\text{heavy scalar}} = \frac{1}{12\pi s}\frac{P'}{P}\frac{\lambda^4}{m_\phi^4}\left[8H^2(H^2 - m_\chi^2) + m_f^2(5m_\chi^2 - 2H^2)\right] .$$

$$\sigma_{LO}(s) \overset{v\ small}{\longrightarrow} \frac{\lambda^4}{4\pi s}\frac{P'}{P}\left[\frac{m_\chi^2 m_f^2}{(m_\chi^2 + m_\phi^2 - m_f^2)^2} + \mathcal{O}(v^2)\right] .$$

7. t-channel Feynman diagrams (NLO)

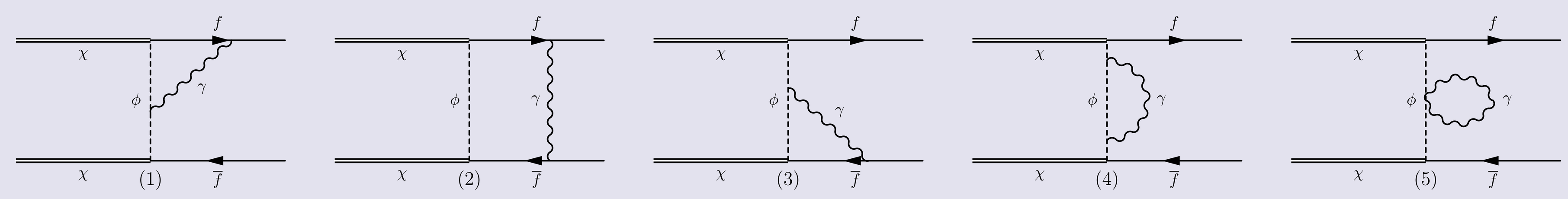


Fig.3 : The t-channel virtual photon corrections to the dark matter annihilation process at next to leading order (NLO). Diagrams are labeled from 1–5. Analogous contributions from the u-channel diagrams exist.

8. σ_{NLO} for sample case (non-dynamical scalar case)

• Useful formula for obtaining thermal correction

$$\int_0^\infty \omega \mathrm{d}\omega\, n_B(\omega) = \frac{\pi^2 T^2}{6} .$$

• We take Fig.3, diagram 1 for sample case, and obtain quadratic dependence of DM annihilation cross section in non-dynamical scalar approximation. We consider photon propagator to be thermal here,

$$\sigma^t_{NLO}(1,\gamma) = \frac{1}{32s(2\pi)^4}\frac{|\vec{p'}|}{|\vec{q'}|}\int \omega \mathrm{d}\omega\, n_B(\omega) I^t_{1,\gamma} .$$

$$I^t_{1,\gamma} = \frac{64\pi e^2\lambda^4}{3{m_\phi}^6 P'}\left[4P'\left(3H^4 + P^2\ P'^2\right) + \log\frac{H - P'}{H + P'} 3H\left(H^2 + P^2\right)\left(H^2 + P'^2\right)\right] .$$

$$\sigma^t_{NLO}(1,\gamma) = \frac{1}{32s(2\pi)^4}\frac{|\vec{p'}|}{|\vec{q'}|}\frac{\pi^2 T^2}{6} \times I^t_{1,\gamma}$$

9. Comparison of σ_{LO} and σ_{NLO}

•The relative size of the NLO contribution *for each* flavor of fermion pair for s-wave contribution is given by

$$\frac{\sigma^a_{NLO}}{\sigma^a_{LO}} = \frac{\pi\alpha T^2}{6m_\phi^2}\frac{m_f^2(22m_\chi^2 + 3m_f^2)}{m_f^2 m_\chi^2} ,$$
$$\approx \frac{11\pi\alpha}{3}\frac{T^2}{m_\phi^2} ,$$

10. σ_{NLO} in dynamical scalar approximation

• s-wave contribution to NLO thermal correction considering scalar to be dynamic, $\langle\sigma v_{rel}\rangle \sim a + bv_{rel}^2$

Diagram	γ/f	Int^a_{NLO} (T^2 contribution)	Int^a_{NLO} (T^4 contribution)
1	γ	$-8m_\chi^2 m_f^2(m_f^2 - m_\phi^2)/D^4$	0
	f	$4m_\chi^2 m_f^2(5m_\chi^2 - 5m_f^2 + m_\phi^2)/D^4$	0
	$\text{Total}_{\gamma+f}$	$2m_\chi^2 m_f^2(5m_\chi^2 - 9m_f^2 + 5m_\phi^2)/D^4$	0
2	γ	$-8m_\chi^2 m_f^2/D^3$	0
	f	$-6m_f^2(2m_\chi^2 - m_f^2)/D^3$	$-\frac{21\pi^2T^2}{10m_\chi^2 D^3} m_f^2(2m_\chi^2 - m_f^2)$
	$\text{Total}_{\gamma+f}$	$-m_f^2(14m_\chi^2 - 3m_f^2)/D^3$	$-\frac{21\pi^2T^2}{10m_\chi^2 D^3} m_f^2(2m_\chi^2 - m_f^2)$
3	γ	$-8m_\chi^2 m_f^2(m_f^2 - m_\phi^2)/D^4$	0
	f	$4m_\chi^2 m_f^2(3m_\chi^2 - 2m_f^2 + m_\phi^2)/D^4$	0
	$\text{Total}_{\gamma+f}$	$2m_\chi^2 m_f^2(3m_\chi^2 - 6m_f^2 + 5m_\phi^2)/D^4$	0
4	γ	$32m_\chi^4 m_f^2/D^4$	$-\frac{56\pi^2T^2}{15D^5} m_\chi^2 m_f^2(m_\chi^2 - m_f^2)$
5	γ	$-16m_\chi^2 m_f^2/D^3$	0
All	$\text{Total}_{\gamma+f}$	$\frac{1}{D^3} m_f^2(2m_\chi^2 + 3m_f^2)+$ $\frac{2}{D^4} m_f^2 m_\chi^2(10m_\phi^2 + 24m_\chi^2 - 15m_f^2)$	$-\frac{21\pi^2T^2}{10m_\chi^2 D^3} m_f^2(2m_\chi^2 - m_f^2) +$ $-\frac{56\pi^2T^2}{15D^5} m_\chi^2 m_f^2(m_\chi^2 - m_f^2)$

• where $D = (m_\chi^2 - m_f^2 + m_\phi^2)$

11. Conclusion

• We presented thermal correction to annihilation cross section of DM due to pure thermal fluctuations at NLO for virtual corrections utilizing thermal field theory.
• We utilized generalized Grammer and Yennie technique in order to deal with IR divergence encountered and get thermal correction to IR finite G-photon contribution.
• Our calculations are performed in two approximations on scalar propagator, first considering scalar to be heavy and taking scalar mass term in the propagator and further in Dynamical scalar approximation.
• We took thermal part of the propagator for photon, fermion or anti-fermion for obtaining thermal correction due to thermal fluctuations. Only one of the propagator among them can be taken as thermal at a time.
• We find NLO thermal correction terms have a T^2 dependence, but are suppressed by additional powers of (heavy scalar) propagator, $D^n \sim m_\phi^{2n}$ and square of the fermion mass, m_f^2, both in LO and NLO.
• We calculate s-wave contribution to NLO thermal correction in dynamic scalar approximation and find $\mathcal{O}(T^2)$ & $\mathcal{O}(T^4)$ terms, which are suppressed by square of fermion mass, m_f^2.
• Thermal correction to finite remainder due to real-correction, i.e. $\tilde{G} - photon$ contribution is ongoing part of the work, which is necessary for relic abundance calculations of DM utilizing Boltzmann equation.

12. References

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