NNU · 南京师范大学
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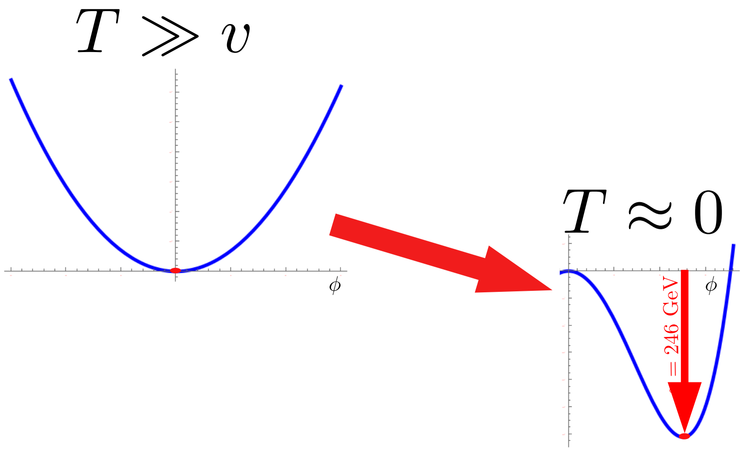
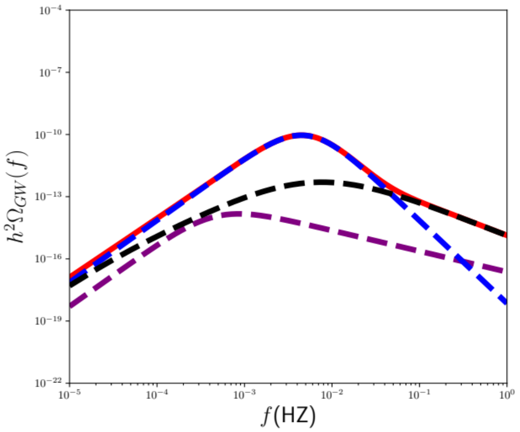


Gravitational waves from first order cosmological phase transitions

Peter Athron
(Nanjing Normal University)



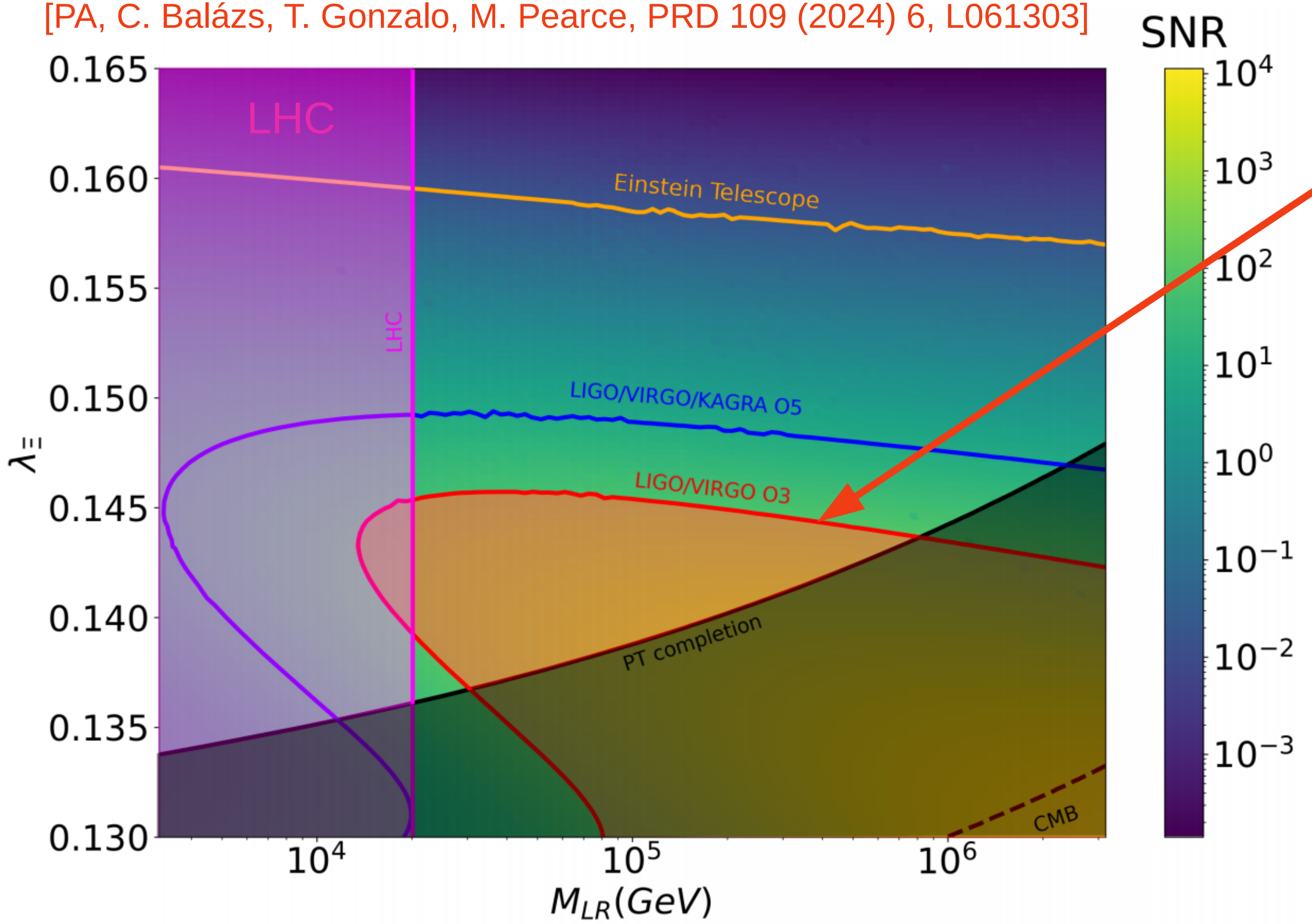
Prague: ICHEP

We are entering an era
where
robust GWs predictions matter

Precise GWs predictions matter

LIGO data already constrains well motivated Pati-Salam GUT models

[PA, C. Balázs, T. Gonzalo, M. Pearce, PRD 109 (2024) 6, L061303]



Constraint from LIGO data on a realistic model

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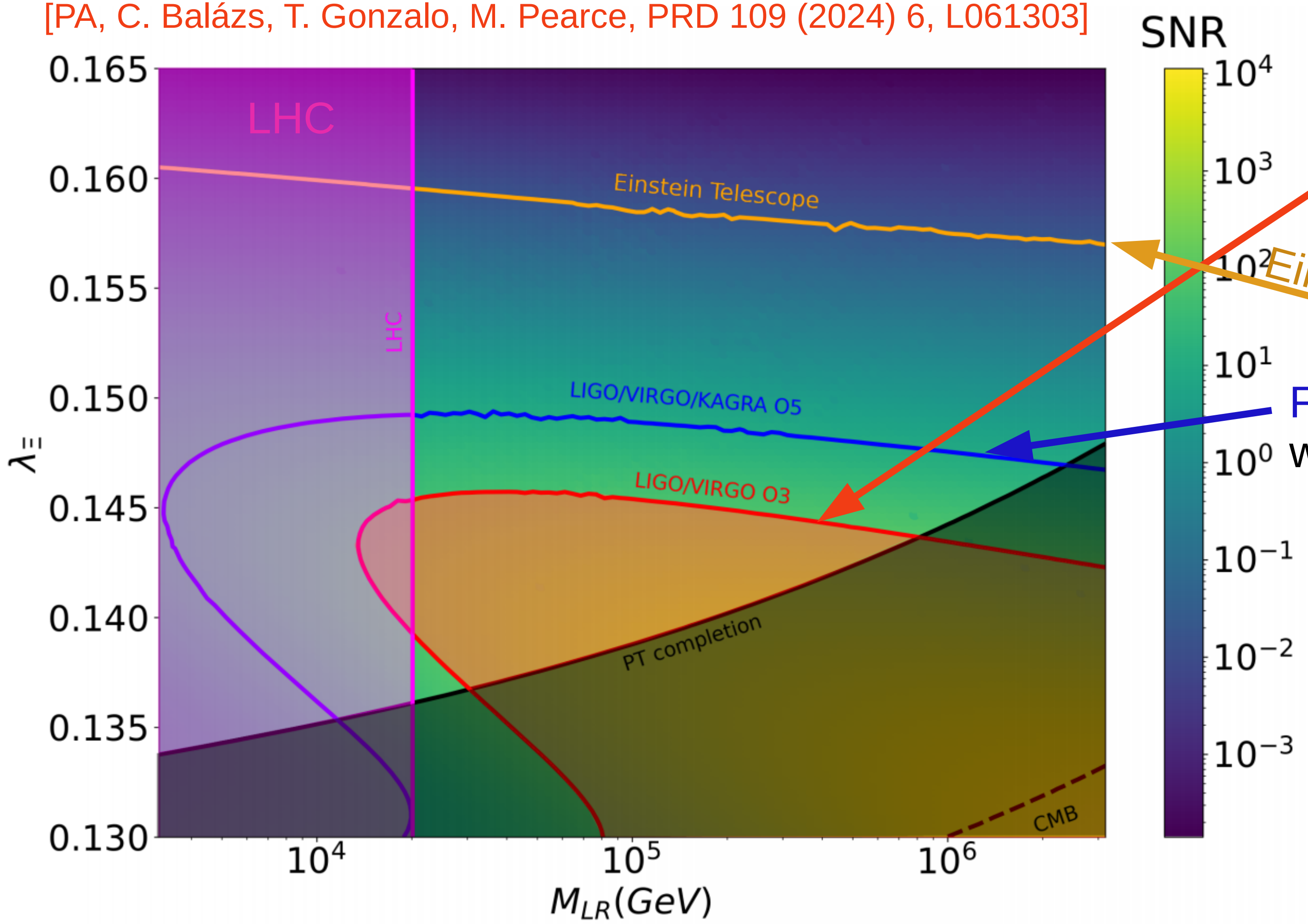


Constraint from LIGO data on a realistic model

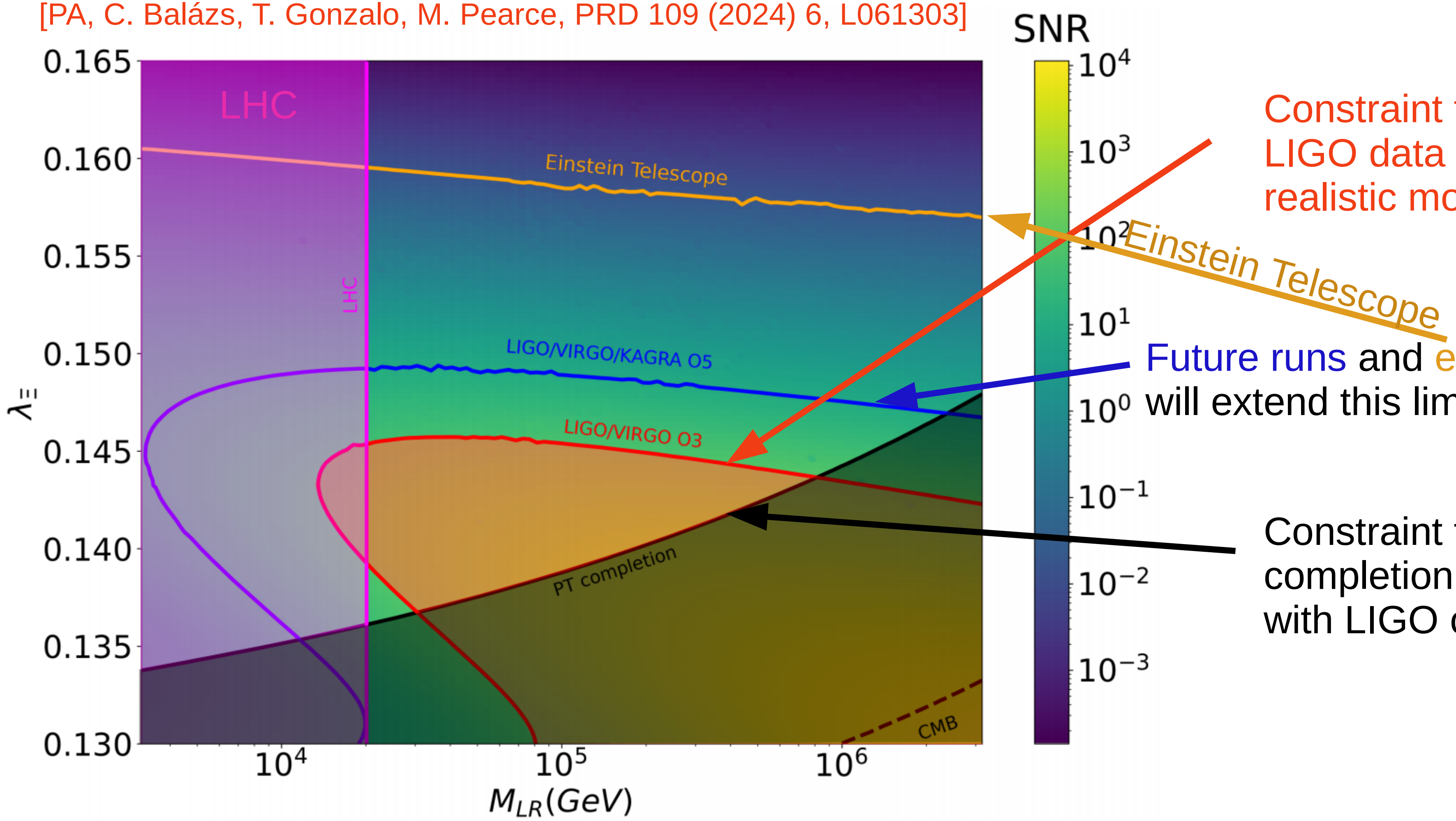
Einstein Telescope

Future runs and experiments will extend this limit

Precise GWs predictions matter

LIGO data already constrains well motivated Pati-Salam GUT models

[PA, C. Balázs, T. Gonzalo, M. Pearce, PRD 109 (2024) 6, L061303]



Constraint from LIGO data on a realistic model

Future runs and experiments will extend this limit

Constraint from completion competes with LIGO constraint

Big news last year:
A stochastic gravitational wave background has been observed
by multiple Pulsar Timing Arrays experiments

ASTRONOMY AND SPACE | RESEARCH UPDATE

Pulsar timing irregularities reveals hidden gravitational-wave background

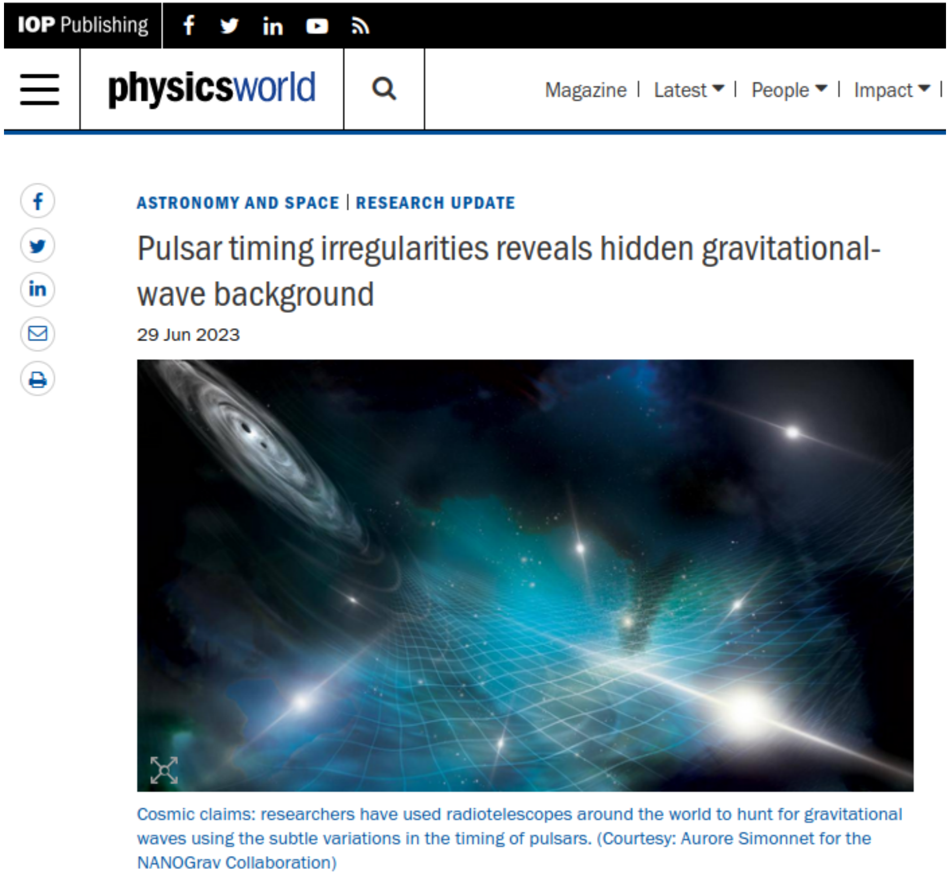
29 Jun 2023

Cosmic claims: researchers have used radiotelescopes around the world to hunt for gravitational waves using the subtle variations in the timing of pulsars. (Courtesy: Aurore Simonnet for the NANOGrav Collaboration)

Big news last year:
A stochastic gravitational wave background has been observed
by multiple Pulsar Timing Arrays experiments

Conservative interpretation:
a population of
supermassive black holes binaries

But more exotic interpretations are possible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DOUBLE WARNING

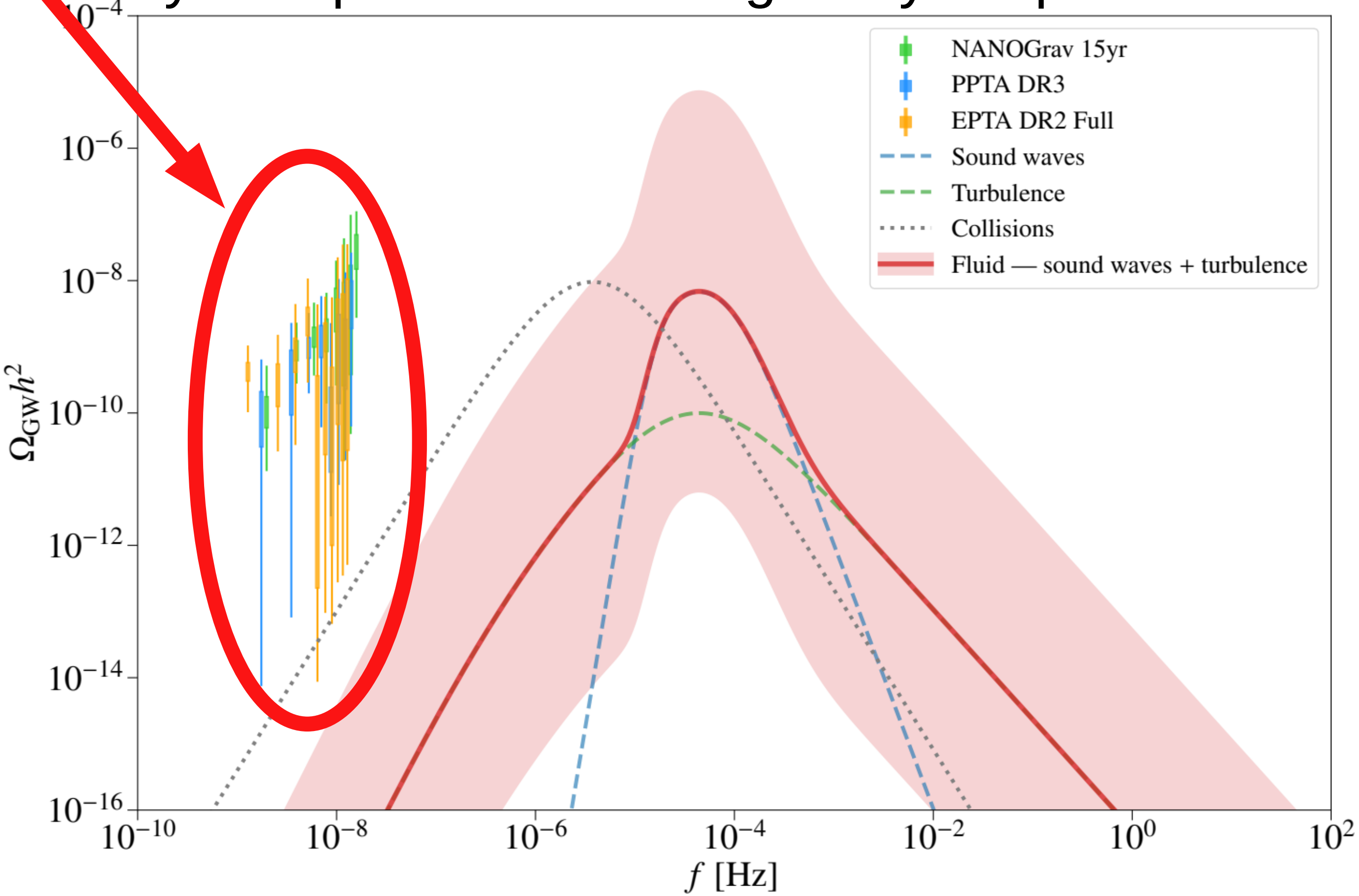
For specific models these predictions require great care!

We looked at one model

prominantly cited by NANOGRAV

as able to explain nHz signals from PTAs...

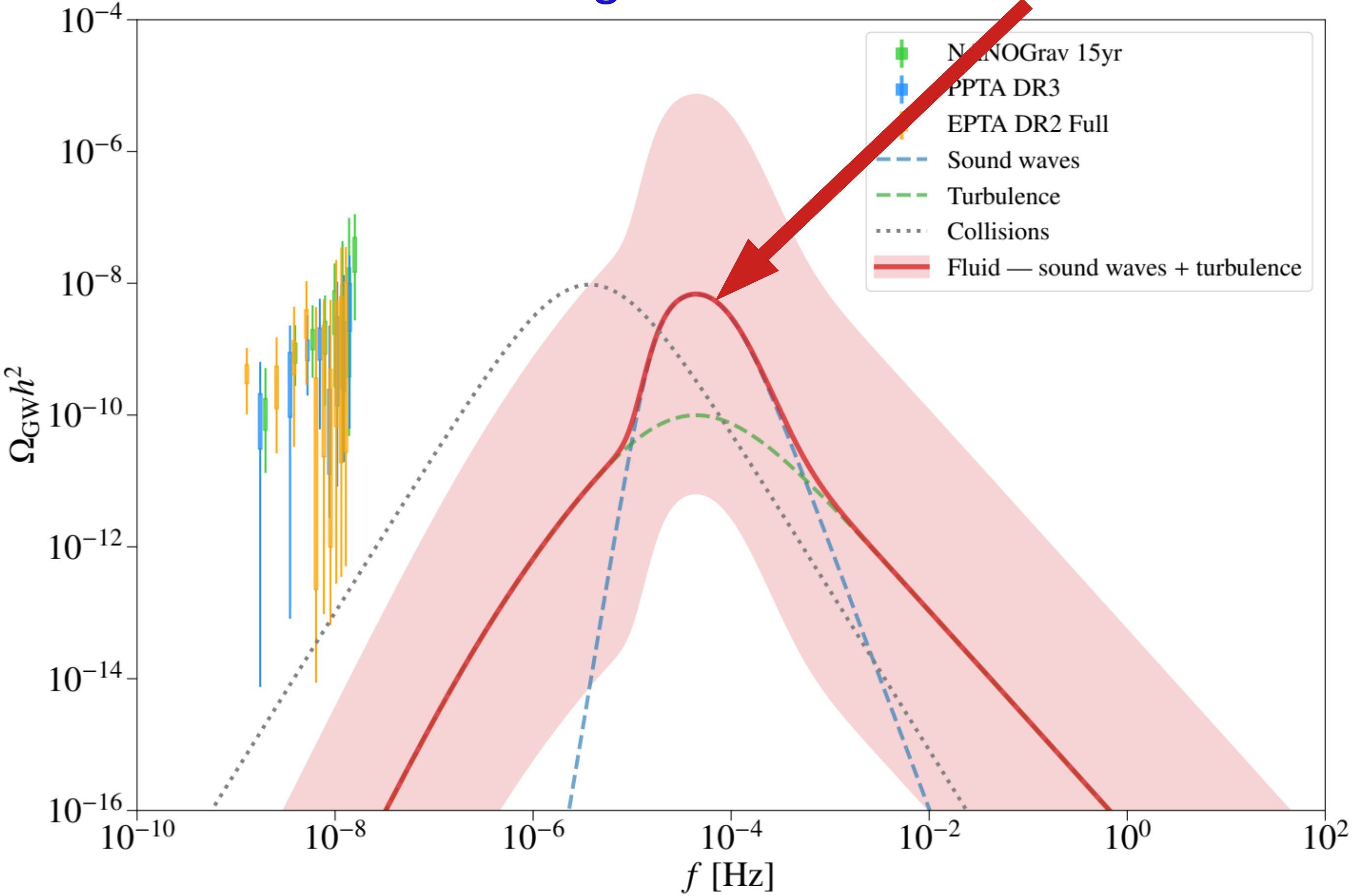
Big news last month:
A stochastic gravitational wave background has been observed
by multiple Pulsar Timing Arrays experiments



[PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Yongcheng Wu, Zhongxiu Xu, PRL 132 (2024) 22, 221001]

But for the protypical model of supercooled PTs cited by NANOgrav as a possible explanation:

GWs can't fit the signal with careful calculation



[PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Yongcheng Wu, Zhongxiu Xu, PRL 132 (2024) 22, 221001]

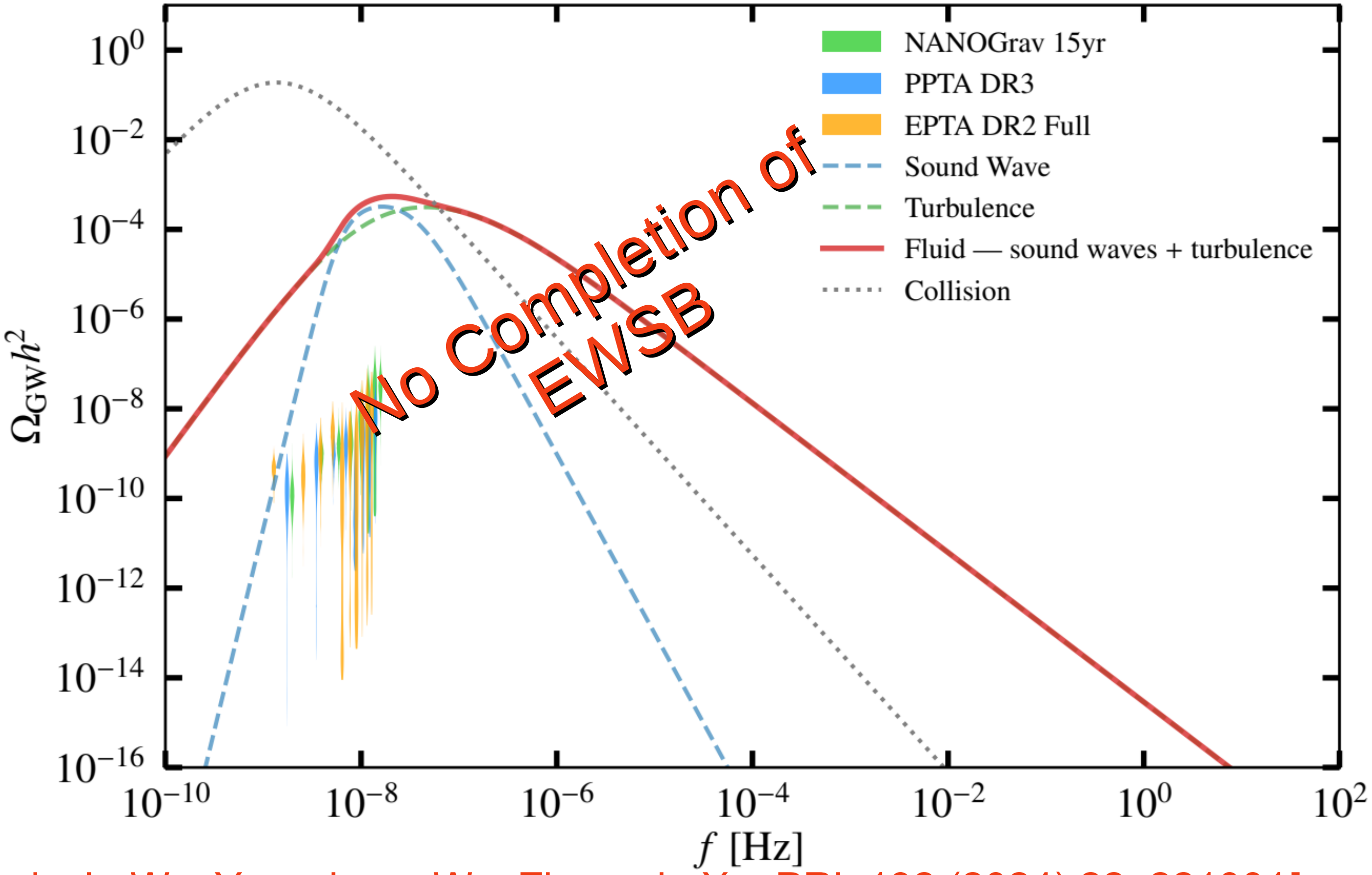
Big news last month:

A stochastic gravitational wave background has been observed

by multiple Pulsar Timing Arrays experiments

Larger signals are ruled out in this model because the PT does not complete

This is one of the subtle effects I will discuss today!

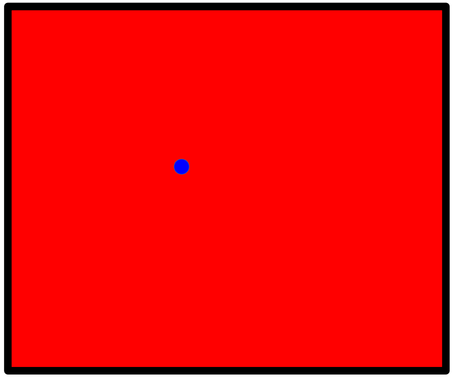


[PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Yongcheng Wu, Zhongxiu Xu, PRL 132 (2024) 22, 221001]

Does the Phase transiton complete?

Many studies only check nucleation

Nucleation: one bubble per Hubble volume



Hubble volume

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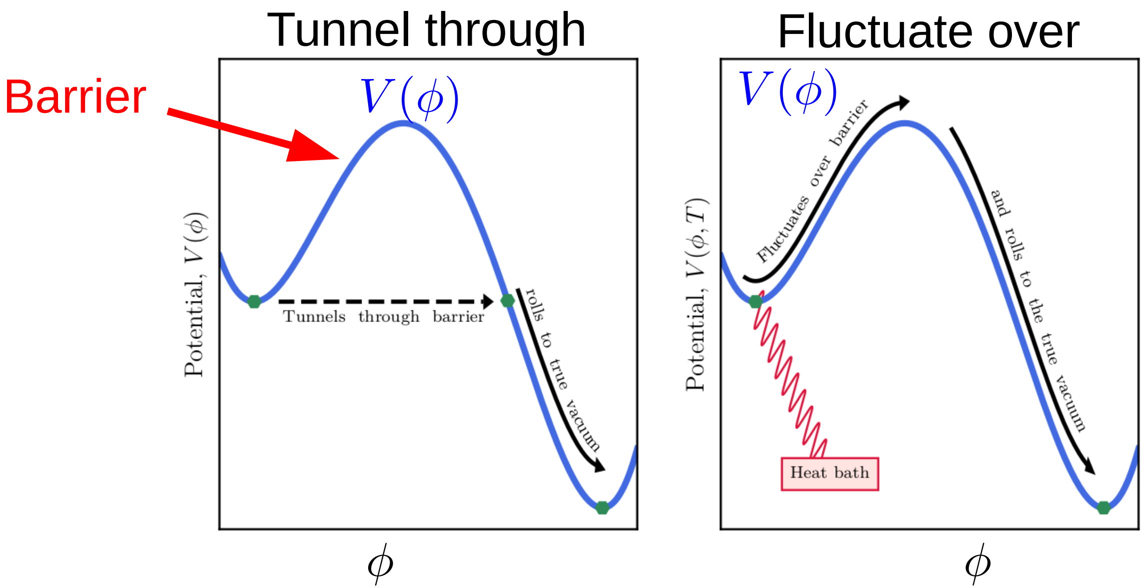
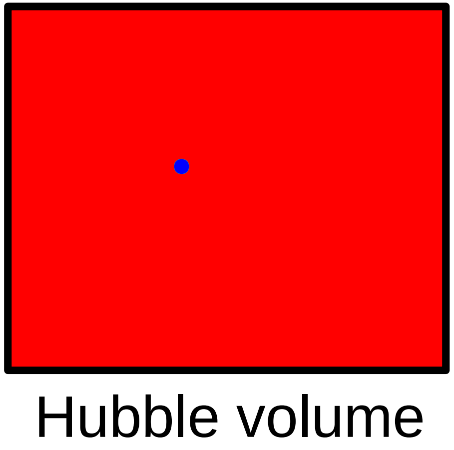




If the barrier disolves quickly with temperature

$\longrightarrow$ Exponential nucleation rate $\longrightarrow$ Bubbles rapidly fill space

"Fast transition" or "low supercooling"

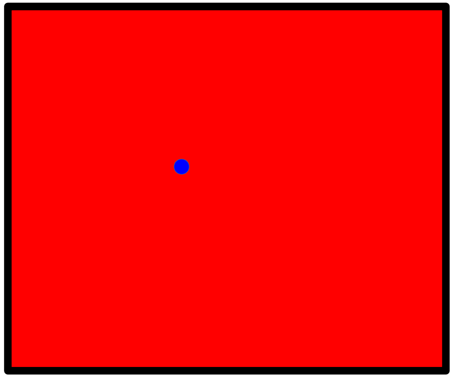
Does the Phase transiton complete?

Many studies only check nucleation

Nucleation: one bubble per Hubble volume

Not sufficient for scenarios with a lot of supercooling,

If the barrier persists to low temperatures,
$\longrightarrow$ nucleation rate can reach a maximum



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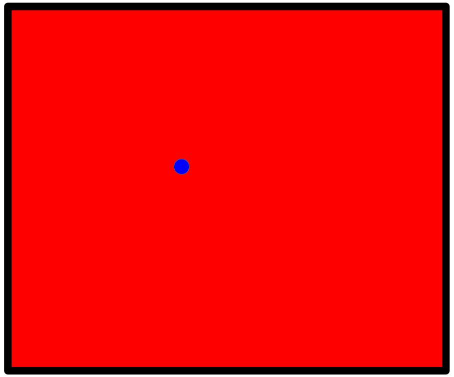

Hubble volume

For such slow transitions we need the false vacuum fraction $P_f \to 0$

$$P_f(T) = \exp\left[-\frac{4\pi}{3}\int_T^{T_c}\frac{dT'}{T'^4}\frac{\Gamma(T')}{H(T')}\left(\int_T^{T'} dT''\frac{v_w(T'')}{H(T'')}\right)^3\right]$$

Stochastic so actually check: $P_f < \epsilon$

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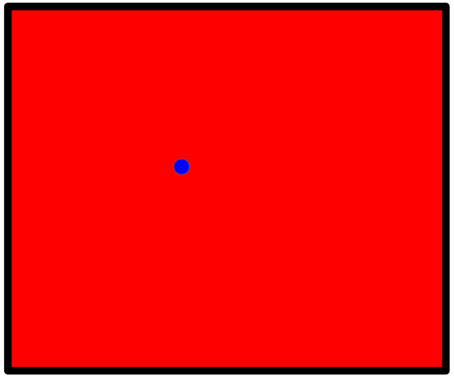
Nucleation: one bubble per Hubble volume



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Stochastic so actually check:
$P_f < \epsilon$

Warning: even this is not enough because space is expanding

Does the Phase transiton complete?

Many studies only check nucleation

Nucleation: one bubble per Hubble volume

Not sufficient for scenarios with a lot of supercooling,

If the barrier persists to low temperatures,
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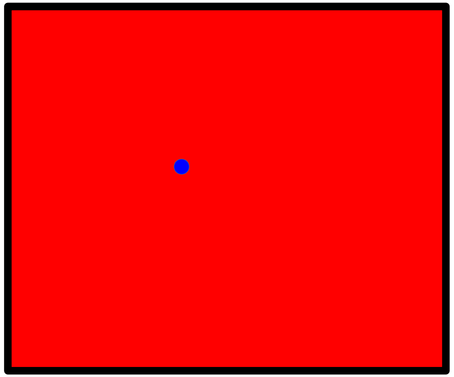

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Stochastic so actually check: $P_f < \epsilon$

Account for expansion of space-time and check $\dfrac{\mathrm{d}\mathcal{V}_f^{\mathrm{phys}}}{\mathrm{d}T} < 0$

Gravitational wave amplitude and frequency

Each component of the amplitude $h^2\Omega_{\text{GW-tot}} = h^2\Omega_{\text{coll}} + h^2\Omega_{\text{sw}} + h^2\Omega_{\text{turb}}$

is defined in terms of the energy density ρ via $\Omega_{\text{GW}}(f) \equiv \frac{1}{\rho_{\text{tot}}} \frac{d\rho_{\text{GW}}}{d\ln f}$

$$\Omega_{\text{GW}}(f) \propto R_{\Omega}\, K^n L^m$$

Length scale related to duration

redshift factor

Kinetic energy fraction

Redshift factor to account for redshifting from the transition time to today

Kinetic energy fraction is the energy that can be available to source GWs

Length scale that is sensitive to the lifetime of the source

Compute for specific model

Implicit dependence of the transition temperature and the velocity the bubble walls expand also influences things

Powers depend on the source and the modelling, coefficients found in simulation/calculations

The temperature choice really matters for gravitational wave signatures

Percolation tempearture

T_p: $P_f(T_p) = 0.71$

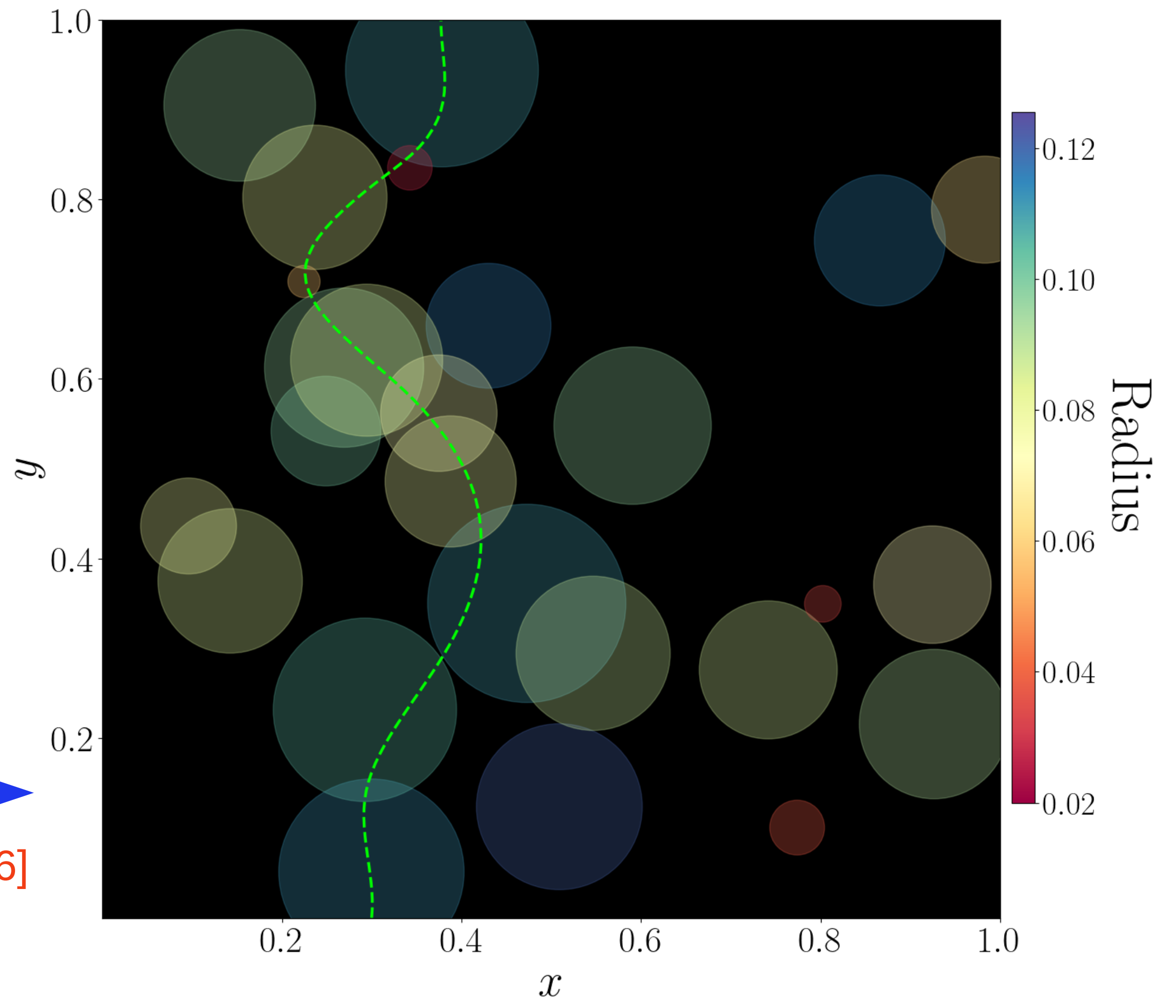
- Percolation is when there is a connected path between bubbles across the space

- Strongly linked to bubble collisions

- Good choice for a temperature at which to evaluate the GWs spectrum

Example from simple simulation →

[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]



Temperature dependence

Non-vacuum transition

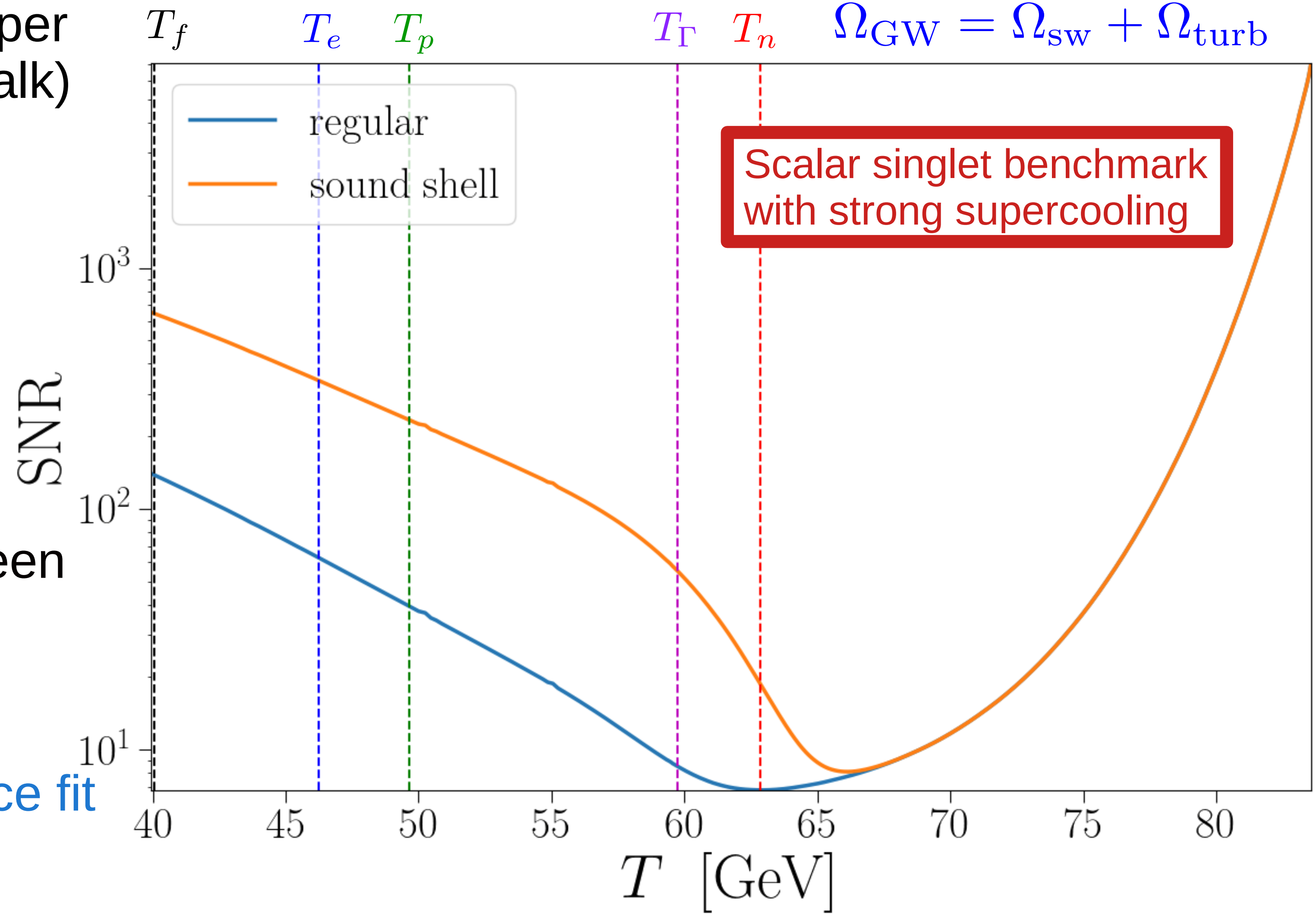
$\Omega_{\rm GW} = \Omega_{\rm sw} + \Omega_{\rm turb}$

Point from same paper (plot made for this talk)

Slow transition, nucleation far earlier than percolation

Detectability (SNR for LISA) very different between percolation vs nucleation!

Sound shell and lattice fit also very different



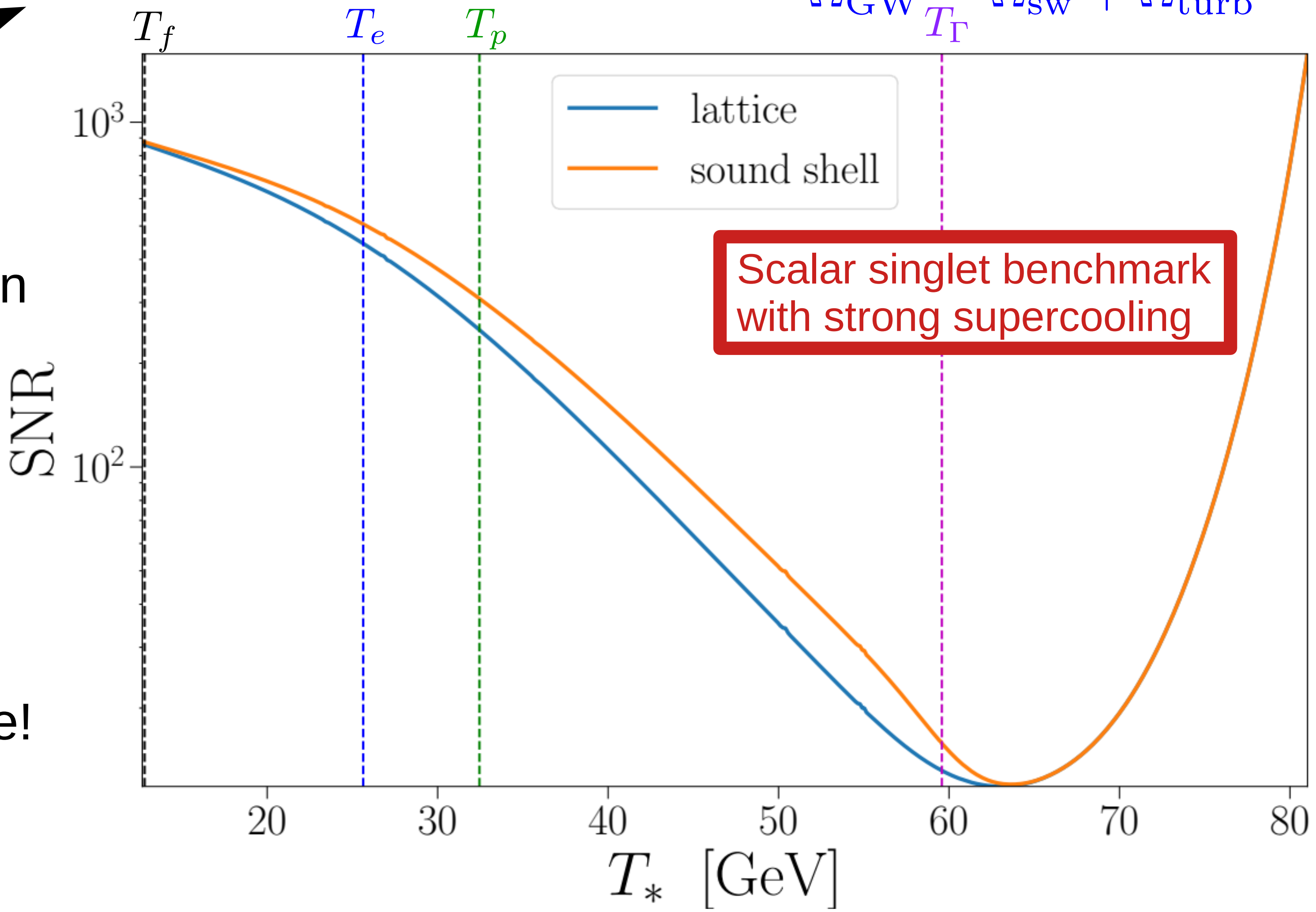
Temperature dependence

[PA, L. Morris, Z. Xu, JCAP 05 (2024) 075]

Non-vacuum transition

$\Omega_{\rm GW} = \Omega_{\rm sw} + \Omega_{\rm turb}$

From here
(but plot simplified)



Another slow transition but percolates and completes *before* nucleation

LISA SNR varies more than an order of magnitude!

Temperature dependence

[PA, L. Morris, Z. Xu, JCAP 05 (2024) 075]

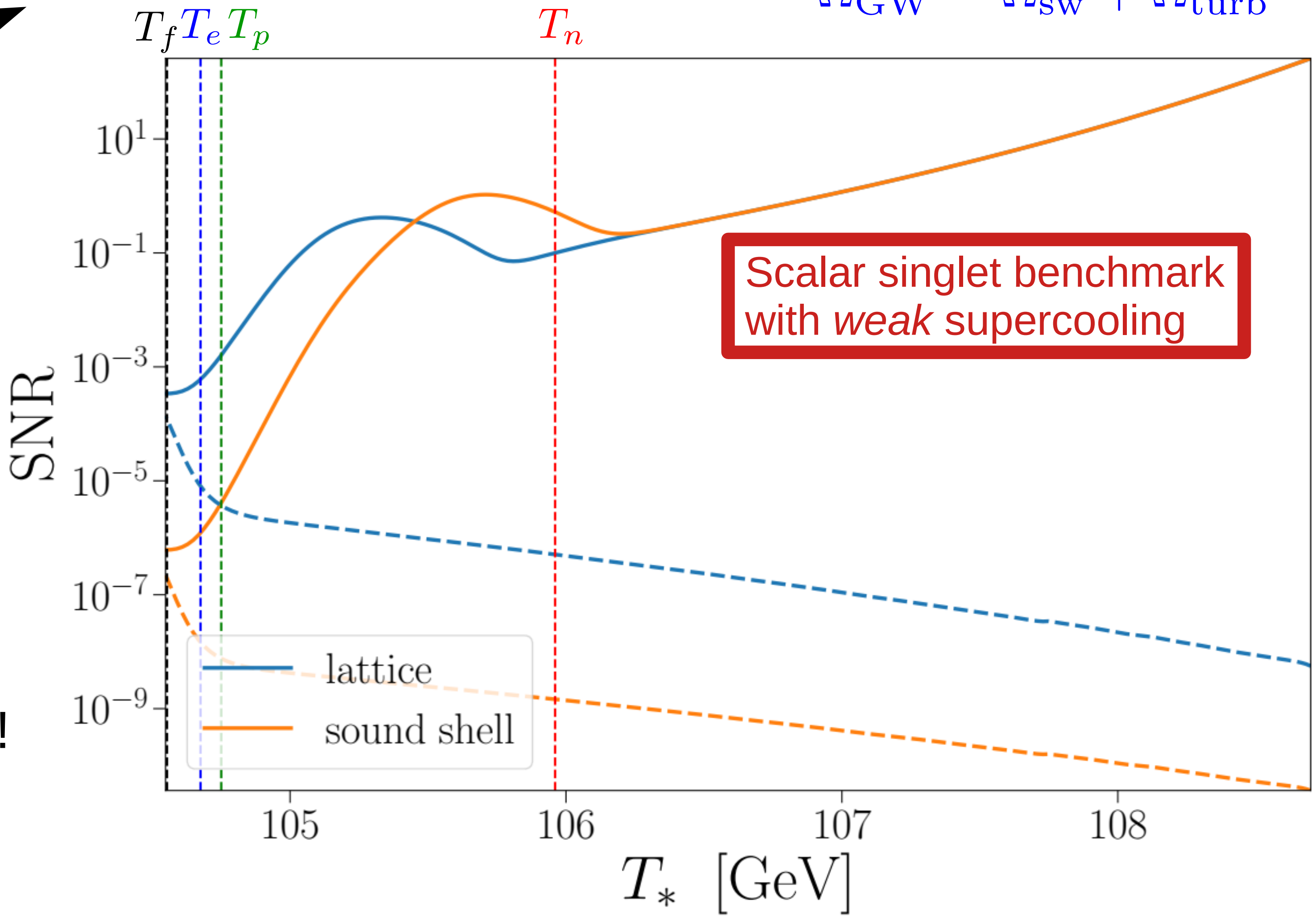
Non-vacuum transition

$\Omega_{\mathrm{GW}} = \Omega_{\mathrm{sw}} + \Omega_{\mathrm{turb}}$

Plot from here

A fast transition still has big variation between T_n & T_p

LISA SNR
Still varies more by orders of magnitude!



Temperature dependence

Nucleation temperature is a bad temperature to use

- not connceted to bubble collisions

Percolation is directly defined in terms of contact between bubbles

Percolation temperature is much better, but…

We still don’t know exactly correct temperature and...

Percolation criteria $P_f(T_p) = 0.71$ does not account for expanding space time

→ Temperature dependence represents a significant uncertainty

Time scales / length scales

Lattice simulations use the mean bubble separation

$$R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_B(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

bubble number density

Often estimated by taylor expanding the bounce action $\Gamma(t) = Ae^{-S(t)}$

$$S(t) \approx S(t_*) + \left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*}(t-t_*) + \frac{1}{2}\left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_*}(t-t_*)^2 + \cdots\,,$$

1st order $\longrightarrow$ explonential nucleation rate $\Gamma(t) = \Gamma(t_*)\exp(\beta(t-t_*)),$

$$\beta = -\left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*} = HT_*\left.\frac{\mathrm{d}S}{\mathrm{d}T}\right|_{T=T_*}$$

Widely used replacement $R_{\rm sep} = (8\pi)^{\frac{1}{3}}\frac{v_w}{\beta}$

Only valid for fast transitions (weak supercooling)

Even for fast transitions can give factor 2 or 3 error

Kinetic energy fraction

Common approach to generalise bag model

Efficiency coefficient

Kinetic energy fraction

$$K = \frac{\kappa \alpha_i}{1 + \alpha_i}$$

~ energy released by PT

overestimate

underestimate

$$\alpha_\rho = \frac{\Delta \rho}{\rho_R} > \alpha_\theta = \frac{\Delta\left(V - \frac{1}{4} T \frac{\partial V}{\partial T}\right)}{\rho_R} > \alpha_p = \frac{\Delta p}{\rho_R}$$

"Latent heat"

Trace anomaly $\theta = \frac{1}{4}(\rho - 3p)$

pressure

$\mathcal{O}(10)$

Important for fast transitions

[PA, L. Morris, Z. Xu , JCAP 05 (2024) 075]

$\mathcal{O}(1/10)$

Kinetic energy fraction

Common approach to generalise bag model

Efficiency coefficient

Kinetic energy fraction $K = \frac{\kappa \alpha_i}{1+\alpha_i}$ ~ energy released by PT

overestimate

underestimate

$$\alpha_\rho = \frac{\Delta\rho}{\rho_R} \quad > \quad \alpha_\theta = \frac{\Delta(V - \frac{1}{4}T\frac{\partial V}{\partial T})}{\rho_R} \quad > \quad \alpha_p = \frac{\Delta p}{\rho_R}$$

"Latent heat" Trace anomaly $\theta = \frac{1}{4}(\rho - 3p)$ pressure

New improvement $K = \frac{\bar\theta_f(T_*) - \bar\theta_t(T_*)}{\rho_{\rm tot}(T_*)}\kappa_{\bar\theta}(\alpha_{\bar\theta}(T_*), c_{s,t}(T_*))$ $\quad \bar\theta = \frac{1}{4}(\rho - \frac{p}{c_{s,t}^2})$

[F. Giese, T. Konstandin and J. van de Vis, JCAP 07 (2020) 057,(+K. Schmitz), JCAP 01 (2021) 072]

From particle physics theory to GWs

There is a long chain of steps needed to make GW predictions

Effective Potential

Transition rates

Thermal parameters that influence GW spectrum

$$\hat{V}_{\rm eff} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$$

PhaseTracing

Bubble dynamics, nucleation & growth

GW spectrum

At every step there are challenges :

- open questions & active investigation
- Tensions between rigour and feasibility,
- Subtle issues leading to common misunderstandings / mistakes

Numerical Packages

The good news is many of these issues can be avoided with careful numerical implementations

We are developing a set of numerical packages for PhaseTransitions: PhaseTracer, BubbleProfiler and...

TransitionSolver is designed to treat nucleation and Gws as well as can feasiby be done in BSM studies

TransitionSolver finds possible FOPTs, checks they complete, computes thermal parameters and gravitational wave specra as well as we are able.

→ v1 Release is imminent, ETA by end of ~~summer winter 2023~~ soon

→ Future releases (v2) will automate effective potential,
Combine with PhaseTracer 2 / BubbleProfiler 2
link to DRalgo and BubbleDet for best feasible handing
of the effective potential and nucleation rate!

Conclusions

- Very exciting recent results indicate we have entered an era where GW experiments have sensitivity to SGBG from BSM physics
- Now it's very important to do calculations as carefully as possible – Many issues:
  - Effective potential – IR divergences, scale & gauge dependence
  - Vacuum Decay – bounce, double counting, and prefactor
  - Completion of the Phase Transition
  - Reference Temperature dependence of GW predictions.
  - Thermal parameters - kinetic energy & length scales (& bubble wall vlocity)
- It's very important that the theory community takes this seriously and BSM predictions are done as well as possible, as well as improving methods and understanding of uncertainties.
- We hope our review helps:

  PA, Csaba Balazs, Andrew Fowlie, Lachlan Morris, Lei Wu,
  Prog.Part.Nucl.Phys 135 (2024) 104094

The END

Thanks for listening!

Times scales for sources gravitational waves affect the GWs signal

Depends on the particle physics model

Can be related to a length scale, mean bubble separation used in hydrodynamical simulations of sound:

$$R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

bubble number density

Best treatment

Often estimated by taylor expanding the bounce action $\Gamma(t) = Ae^{-S(t)}$

$$S(t) \approx S(t_*) + \left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*}(t-t_*) + \frac{1}{2}\left.\frac{\mathrm{d}^2S}{\mathrm{d}t^2}\right|_{t=t_*}(t-t_*)^2 + \cdots ,$$

2nd order $\longrightarrow$ Gaussian nucleation rate $\Gamma(t) = \Gamma(t_*)\exp\left(-\frac{\beta_{\rm V}^2}{2}(t-t_*)^2\right),$

$$\beta_{\rm V} = \sqrt{\left.\frac{\mathrm{d}^2S}{\mathrm{d}t^2}\right|_{t=t_\Gamma}}$$

Can be used to replace mean bubble separation

$$R_{\rm sep} = \left(\sqrt{2\pi}\frac{\Gamma(T_\Gamma)}{\beta_{\rm V}}\right)^{-\frac{1}{3}}$$

Rough approximation

Times scales for sources gravitational waves affect the GWs signal

Depends on the particle physics model

Can be related to a length scale, mean bubble separation used in hydrodynamical simulations of sound:

$$R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3\int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

bubble number density

One more thing:

Alternative length scale - mean bubble radius

$$\bar{R}(T) = \frac{T^2}{n_B(T)} \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')} \int_T^{T'} dT'' \frac{v_w(T'')}{H(T'')}.$$

This has been proposed in the literature but not used in simulations

Gravitational waves and thermal parameters

Lattice fit to single broken power law for sound wave source :

[M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, PRD 96 (2017) 103520]

$$h^2\Omega^{\rm lat}_{\rm sw}(f) = 5.3\times 10^{-2}\, R_\Omega K^2 \left(\frac{HL_*}{c_{s,f}}\right)\Upsilon(\tau_{\rm sw}) S_{\rm sw}(f),$$

Shape

Speed of sound in false vacuum

Accounts for finite lifetime of source

Sound shell model:

[Hindmarsh PRL 120 (2018) 071301, (+Hijazi) JCAP 12 (2019) 062, + (C. Gowling, D.C. Hooper and J. Torrado), JCAP 04 (2023) 061]

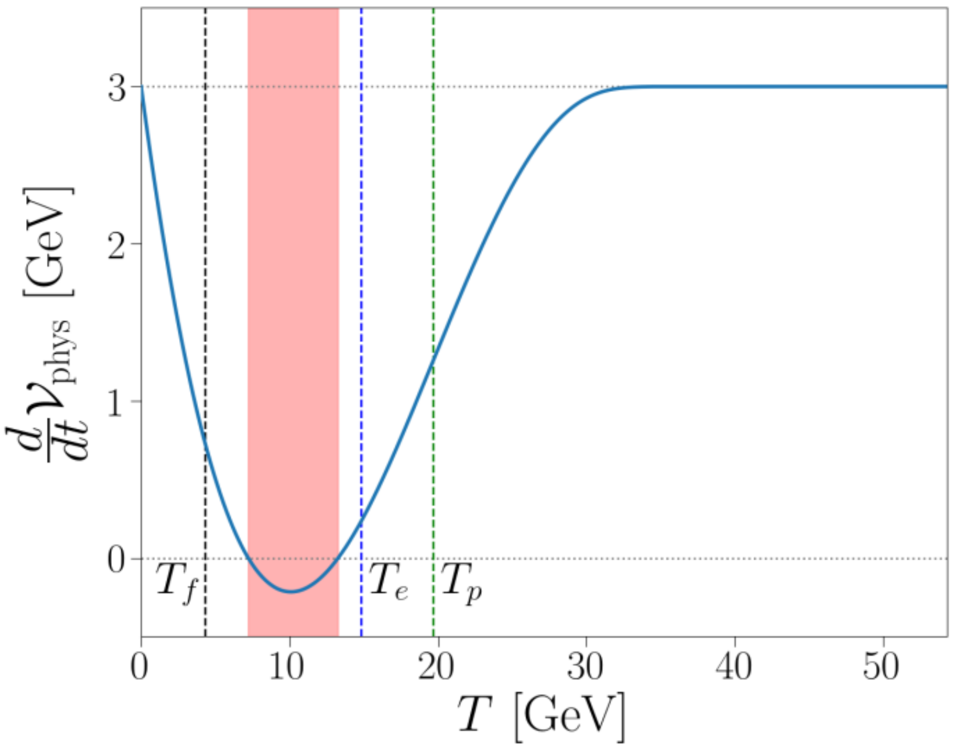
$$h^2\Omega_{\rm sw}(f) = 0.03 R_\Omega K^2 \left(\frac{H_* L_*}{c_{s,f}}\right)\Upsilon(\tau_{\rm sw})\frac{M(s,r_b,b)}{\mu_f(r_b)}$$

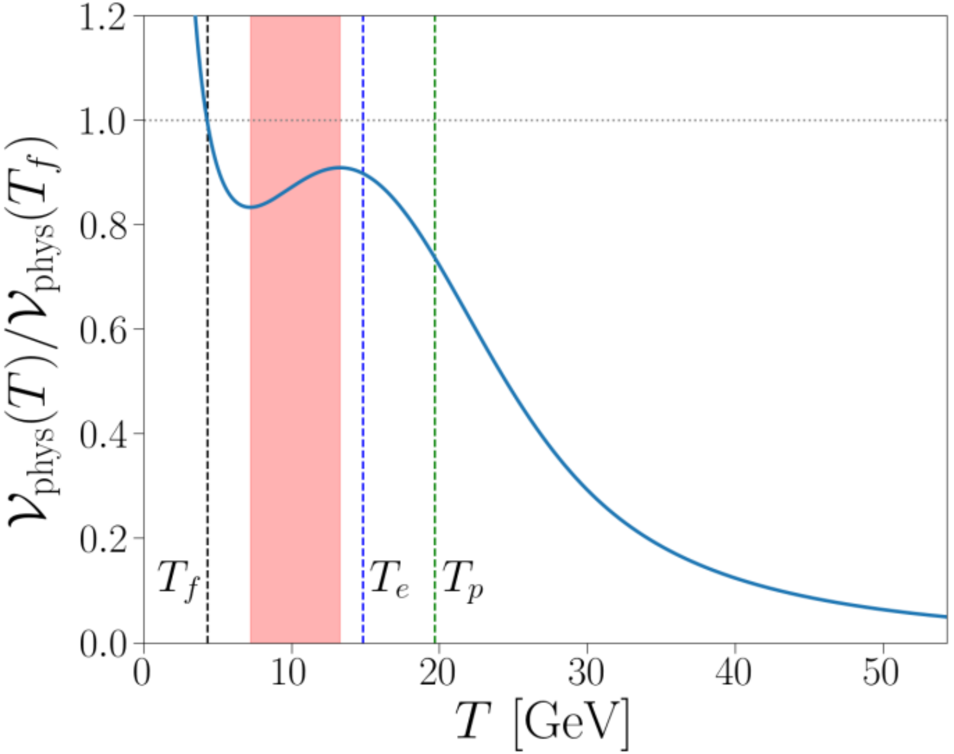
Shape

Sound shell model is new but very promising

Turbulence also contributes, but not well modeled $\longrightarrow$ Significant uncertainty!

Addional check for Percolation / completion





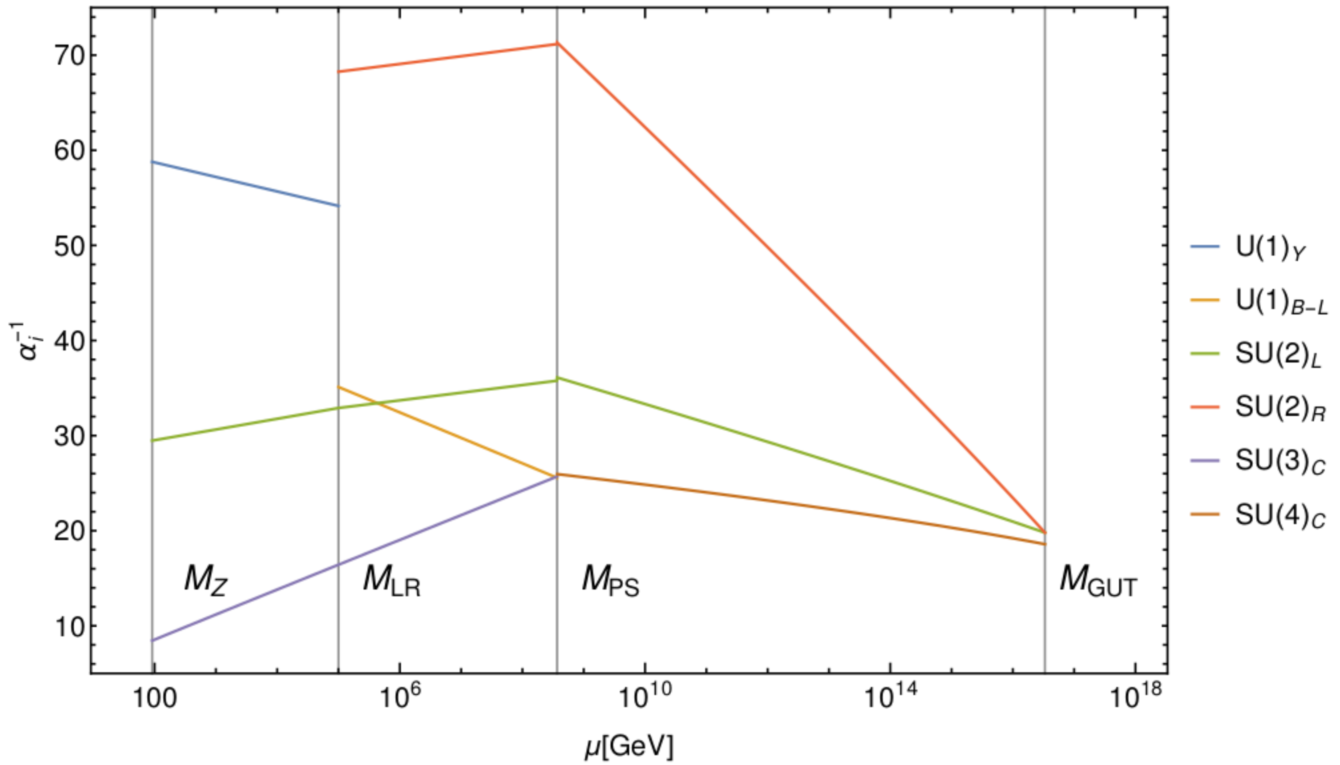
[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]

To ensure it really completes, also require: $\frac{\mathrm{d}\mathcal{V}_f^{\mathrm{phys}}}{\mathrm{d}T} < 0$

Non-trivial because whole volume is expanding

Pati-Salam two step grand unification

$$
\begin{aligned}
SO(10) &\to SU(4) \times SU(2)_L \times SU(2)_R \\
&\to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
&\to SU(3)_C \times SU(2)_L \times U(1)_Y
\end{aligned}
$$



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\end{aligned}
$$

Scalar fields at the Pati-Salam scale

Fields	$SU(4)_c$	$SU(2)_L$	$SU(2)_R$	Purpose
ϕ	1	2	2	Breaks SM
Δ_R	$\overline{10}$	1	3	Breaks LR
Δ_L	$\overline{10}$	3	1	Seesaw
Ξ	15	1	1	Breaks PS
Ω_R	15	1	3	Unification

Comparison of predictions for a weakly supercooled point

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Variation	$h^2\Omega^{\text{lat}}_{\text{sw}}$ $(\times 10^{-13})$	$h^2\Omega^{\text{ss}}_{\text{sw}}$ $(\times 10^{-14})$	$f^{\text{lat}}_{\text{sw}}$ $(\times 10^{-5})$	$f^{\text{ss}}_{\text{sw}}$ $(\times 10^{-4})$	$h^2\Omega_{\text{turb}}$ $(\times 10^{-16})$	f_{turb} $(\times 10^{-5})$	SNR_{lat}	SNR_{ss}	α $(\times 10^{-2})$	κ	K $(\times 10^{-3})$
None	3.590	5.673	129.6	60.20	3.898	287.0	10.08	2.031	5.450	0.2074	10.64
$T_* = T_e$	2.552	4.042	159.9	73.75	2.662	354.2	8.763	1.204	5.575	0.2096	10.99
$T_* = T_f$	2.146	3.410	181.7	82.91	2.187	402.5	8.110	0.8892	5.771	0.2129	11.54
$T_* = T_n$	676.5	1046	2.189	1.098	8968	4.849	1.310	5.142	4.297	0.1857	7.597
$R_{\text{sep}}(\beta)$	2.019	3.191	177.5	82.45	2.078	393.1	7.510	0.8449			
$K(\alpha(\theta))$	3.372	5.329			3.676		9.469	1.908	5.362	0.2011	10.23
$K(\alpha(p))$	1.428	2.256			1.649		4.010	0.8081	3.698	0.1682	5.997
$K(\alpha(\rho))$	14.45	22.84			14.61		40.59	8.172	10.35	0.2736	25.68
ϵ_2					11.03		10.11	2.064			
ϵ_3					290.2		11.21	3.406			
ϵ_4					301.7		11.26	3.462			

Comparison of predictions for a weakly supercooled point in SSM

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Differences in K: trace anomaly approximation is quite good in this case

Variation	$h^2\Omega_{\rm sw}^{\rm lat}$ $(\times 10^{-13})$	$h^2\Omega_{\rm sw}^{\rm ss}$ $(\times 10^{-14})$	$f_{\rm sw}^{\rm lat}$ $(\times 10^{-5})$	$f_{\rm sw}^{\rm ss}$ $(\times 10^{-4})$	$h^2\Omega_{\rm turb}$ $(\times 10^{-16})$	$f_{\rm turb}$ $(\times 10^{-5})$	$\mathrm{SNR}_{\rm lat}$	$\mathrm{SNR}_{\rm ss}$	α $(\times 10^{-2})$	κ	K $(\times 10^{-3})$
None	3.590	5.673	129.6	60.20	3.898	287.0	10.08	2.031	5.450	0.2074	10.64
$T_* = T_e$	2.552	4.042	159.9	73.75	2.662	354.2	8.763	1.204	5.575	0.2096	10.99
$T_* = T_f$	2.146	3.410	181.7	82.91	2.187	402.5	8.110	0.8892	5.771	0.2129	11.54
$T_* = T_n$	676.5	1046	2.189	1.098	8968	4.849	1.310	5.142	4.297	0.1857	7.597
$R_{\rm sep}(\beta)$	2.019	3.191	177.5	82.45	2.078	393.1	7.510	0.8449			
$K(\alpha(\theta))$	3.372	5.329			3.676		9.469	1.908	5.362	0.2011	10.23
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Comparison of predictions for a weakly supercooled point in SSM

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Differences in sound wave amplitude (sound shell): latent heat (and pressure) variants give substanial differences

Variation	$h^2\Omega_{\rm sw}^{\rm lat}$ ($\times 10^{-13}$)	$h^2\Omega_{\rm sw}^{\rm ss}$ ($\times 10^{-14}$)	$f_{\rm sw}^{\rm lat}$ ($\times 10^{-5}$)	$f_{\rm sw}^{\rm ss}$ ($\times 10^{-4}$)	$h^2\Omega_{\rm turb}$ ($\times 10^{-16}$)	$f_{\rm turb}$ ($\times 10^{-5}$)	$\mathrm{SNR}_{\rm lat}$	$\mathrm{SNR}_{\rm ss}$	α ($\times 10^{-2}$)	κ	K ($\times 10^{-3}$)
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Comparison of predictions for a weakly supercooled point in SSM

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Differences in sound wave SNR: latent heat (and pressure) variants give substanial differences

Variation	$h^2\Omega^{\rm lat}_{\rm sw}$ ($\times 10^{-13}$)	$h^2\Omega^{\rm ss}_{\rm sw}$ ($\times 10^{-14}$)	$f^{\rm lat}_{\rm sw}$ ($\times 10^{-5}$)	$f^{\rm ss}_{\rm sw}$ ($\times 10^{-4}$)	$h^2\Omega_{\rm turb}$ ($\times 10^{-16}$)	$f_{\rm turb}$ ($\times 10^{-5}$)	$\mathrm{SNR}_{\rm lat}$	$\mathrm{SNR}_{\rm ss}$	α ($\times 10^{-2}$)	κ	K ($\times 10^{-3}$)
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$T_* = T_e$	2.552	4.042	159.9	73.75	2.662	354.2	8.763	1.204	5.575	0.2096	10.99
$T_* = T_f$	2.146	3.410	181.7	82.91	2.187	402.5	8.110	0.8892	5.771	0.2129	11.54
$T_* = T_n$	676.5	1046	2.189	1.098	8968	4.849	1.310	5.142	4.297	0.1857	7.597
$R_{\rm sep}(\beta)$	2.019	3.191	177.5	82.45	2.078	393.1	7.510	0.8449			
$K(\alpha(\theta))$	3.372	5.329			3.676		9.469	1.908	5.362	0.2011	10.23
$K(\alpha(p))$	1.428	2.256			1.649		4.010	0.8081	3.698	0.1682	5.997
$K(\alpha(\rho))$	14.45	22.84			14.61		40.59	8.172	10.35	0.2736	25.68
ϵ_2					11.03		10.11	2.064			
ϵ_3					290.2		11.21	3.406			
ϵ_4					301.7		11.26	3.462			

Comparison of predictions for a strongly supercooled point

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

However the variation in K estimates is much smaller for strongly supercooled scenarios

Variation	$h^2\Omega_{\mathrm{sw}}^{\mathrm{lat}}$ $(\times 10^{-7})$	$h^2\Omega_{\mathrm{sw}}^{\mathrm{ss}}$ $(\times 10^{-8})$	$f_{\mathrm{sw}}^{\mathrm{lat}}$ $(\times 10^{-6})$	$f_{\mathrm{sw}}^{\mathrm{ss}}$ $(\times 10^{-6})$	$h^2\Omega_{\mathrm{turb}}$ $(\times 10^{-10})$	f_{turb} $(\times 10^{-6})$	$\mathrm{SNR}_{\mathrm{lat}}$	$\mathrm{SNR}_{\mathrm{ss}}$	α	κ	K
None	1.861	3.748	9.345	23.48	6.348	20.70	249.6	307.7	1.651	0.7175	0.4536
$T_* = T_e$	4.318	8.872	7.908	19.12	14.74	17.52	443.7	498.2	4.257	0.8422	0.6950
$T_* = T_f$	17.04	35.42	4.111	9.722	81.84	9.106	864.5	876.4	71.06	0.9831	0.9803
$R_{\mathrm{sep}}(\beta_V)$	1.193	2.402	12.80	32.17	3.394	28.36	222.6	356.9			
$K(\alpha(\theta))$	1.819	3.663			6.227		244.9	301.5	1.605	0.7269	0.4478
$K(\alpha(p))$	1.768	3.560			6.083		239.2	294.2	1.564	0.7269	0.4409
$K(\alpha(\rho))$	1.967	3.962			6.646		261.4	323.0	1.728	0.7383	0.4677
ϵ_2					17.95		700.0	742.2			
ϵ_3					0		18.36	130.9			
ϵ_4					288.4		11210	11230			

The duration affects the of the source of gravitational waves affects the GW signal a lot

This depends on the particle physics model

The duration can be related to a length scale and in hydrodynamical simulations of sound waves contributions the mean bubble separation is used:

$$R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

bubble number density

Best treatement

This can also be estimated by taylor expanding the bounce action

$$S(t) \approx S(t_*) + \left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*}(t-t_*) + \frac{1}{2}\left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_*}(t-t_*)^2 + \cdots\,,$$

2nd order $\longrightarrow$ Gaussian nucleation rate $\Gamma(t) = \Gamma(t_*)\exp\left(\frac{\beta_{\rm V}^2}{2}(t-t_*)^2\right),$

$$\beta_{\rm V} = \sqrt{\left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_\Gamma}}$$

Can be used to replace mean bubble separation

$$R_{\rm sep} = \left(\sqrt{2\pi}\frac{\Gamma(T_\Gamma)}{\beta_{\rm V}}\right)^{-\frac{1}{3}}$$

Rough approximation

The mean bubble separation varies a lot with temperature

Should not be used until $T \approx T_p$

For fast transitions

Estimating this with $\beta(T_p)$ GW amp. falls by factor 2 (larger variation in SNR)
Worse if using $\beta(T_n)$ as is standard practise

Mean bubble radius is more stable and $\beta(T)$ tracks this better.

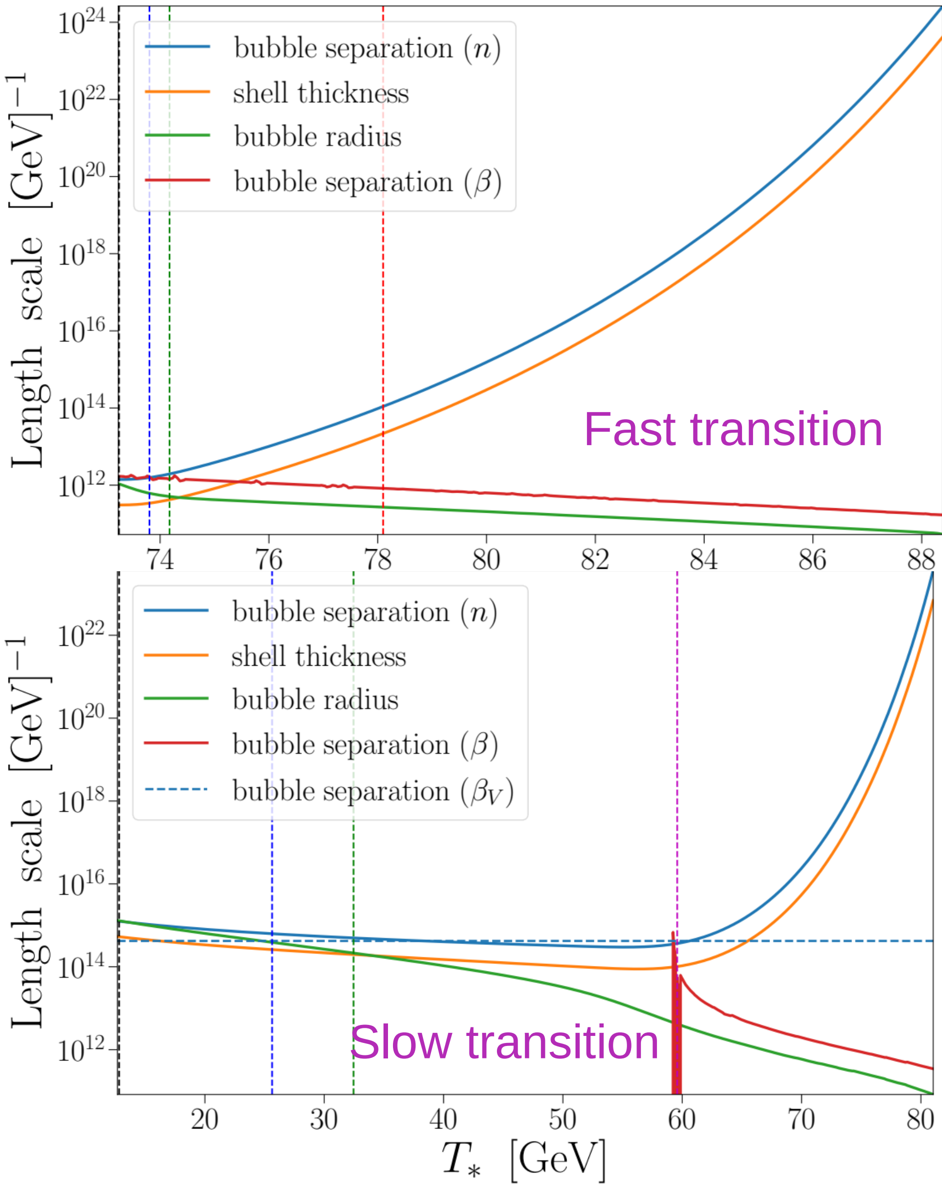
For slow transitions

Mean bubble radius varies more as bubbles have longer to grow.

Using $\beta(T_p)$ makes no sense below T_Γ orders of magnitude errors above

β_V gives a factor 1.5 drop in GW amplitide

[PA, L. Morris, Z. Xu, arXiv:2309.05474]



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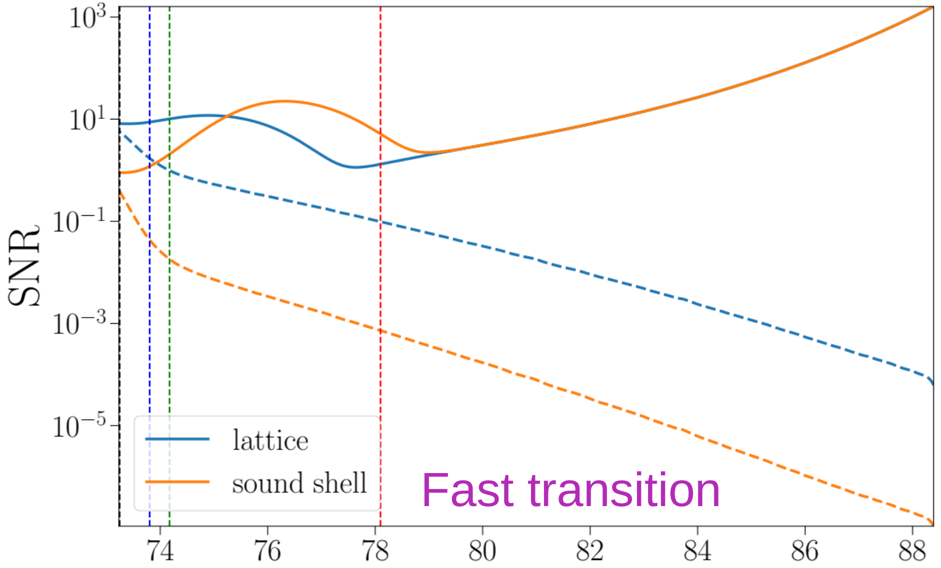
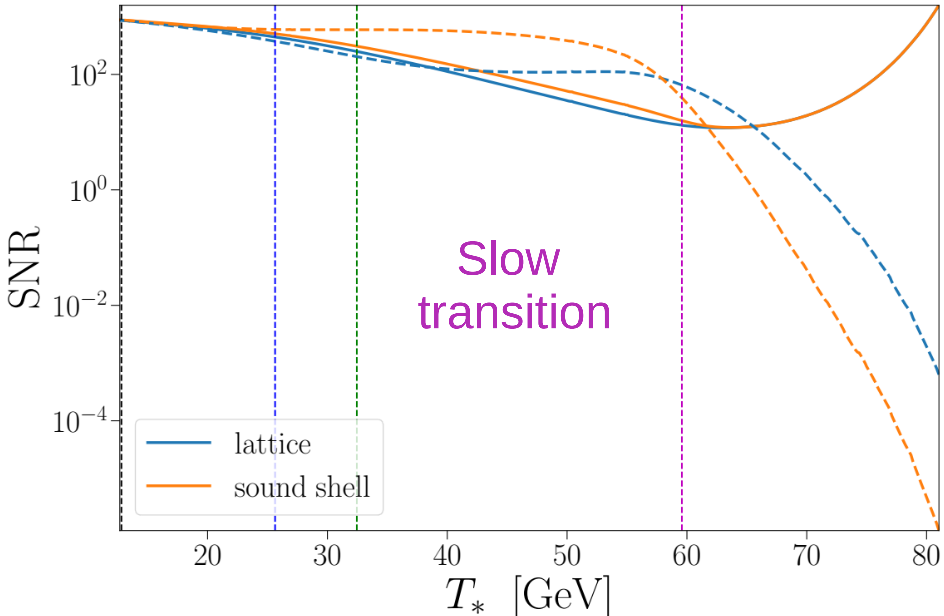




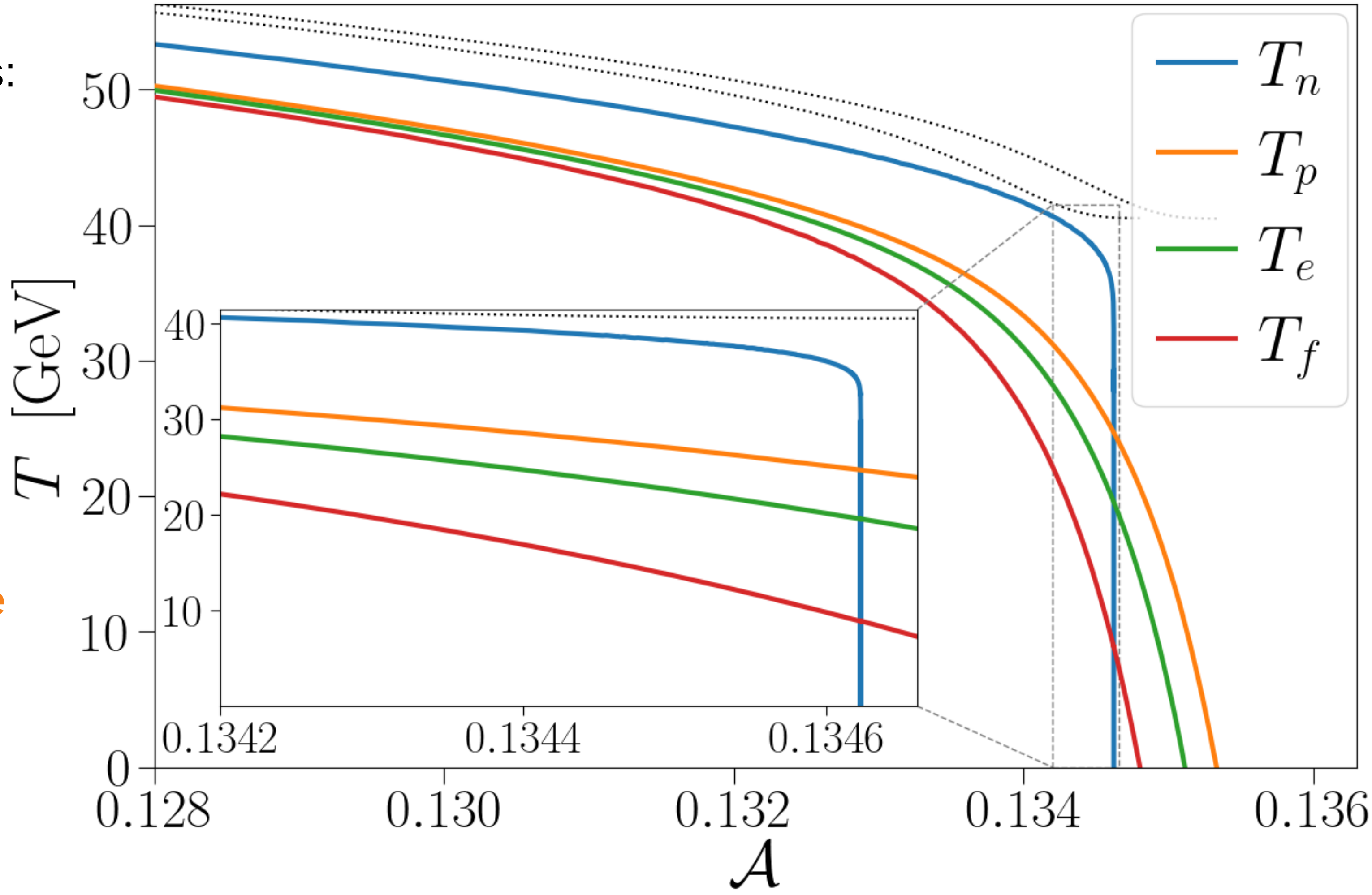
Milestone temperatures

[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]

Nucleation temperature is:

- Not related to bubble collisions
- Not related to other temperatures
- May not even exist

Percolation temperature is a better choice for GWs



From particle physics theory to GWs

$$\hat{V}_{\rm eff} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$$

Effective Potential: can be computed perturbatively with finite temperature quantum field theory

$$\hat{V}_{\rm eff} = V_0 + V_1^{T=0} + V_{1T}$$

Tree-level

Zero temperature Coleman-Weinberg corrections

Finite temperature corrections

Effective Potential: can be computed perturbatively with finite temperature quantum field theory

$$\hat{V}_{\rm eff} = V_0 + V_{1,T=0} + V_{1T}$$

$$V^{R_\xi}_{1,\,T=0} = \frac{1}{4(4\pi)^2}\left[\sum_\phi n_\phi m^4_\phi(\{\phi_j\},\xi)\left(\ln\left(\frac{m^2_\phi(\{\phi_j\},\xi)}{Q^2}\right) - k_s\right)\right.$$
$$+\sum_V n_V m^4_V(\{\phi_j\})\left(\ln\left(\frac{m^2_V(\{\phi_j\})}{Q^2}\right) - k_V\right) - \sum_V (\xi m^2_V(\{\phi_j\}))^2\left(\ln\left(\frac{\xi m^2_V(\{\phi_j\})}{Q^2}\right) - k_V\right)$$
$$\left.-\sum_f n_f m^4_f(\{\phi_j\})\left(\ln\left(\frac{m_f(\{\phi_j\})^2}{Q^2}\right) - k_f\right)\right],$$

$$V^{R_\xi}_{1T} = \frac{T^4}{2\pi^2}\left[\sum_i n_\phi J_B\left(\frac{m^2_{\phi_i}(\xi)}{T^2}\right) + \sum_j n_V J_B\left(\frac{m^2_{V_j}}{T^2}\right) - \frac{1}{3}\sum_j n_V J_B\left(\frac{\xi m^2_{V_j}}{T^2}\right) + \sum_l n_f J_F\left(\frac{m^2_{f_l}}{T^2}\right)\right]$$

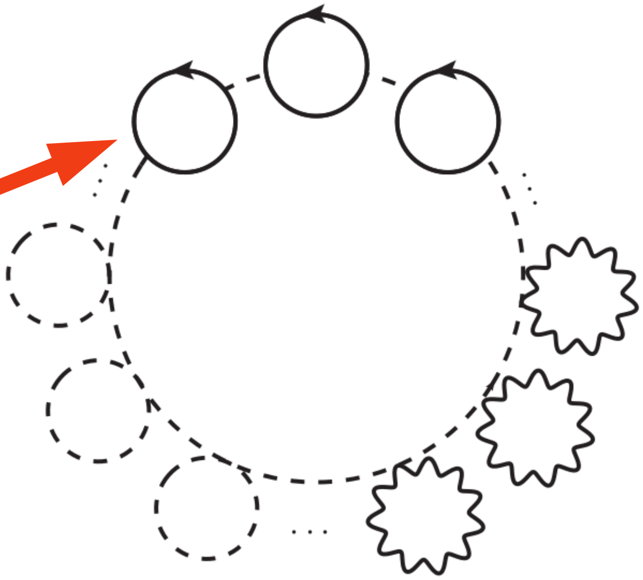
$$J_B(y^2) = \int_0^\infty dk\ k^2 \log\left[1 - e^{-\sqrt{k^2+y^2}}\right] \quad J_F(y^2) = \int_0^\infty dk\ k^2 \log\left[1 + e^{-\sqrt{k^2+y^2}}\right]$$

Effective Potential

Perturbative estimates of the effective potential can be tricky

Resummation needed to to deal with high temperatures spoiling perturbativity

Daisy diagram with N-loops:



Individual petals are inserted one-loop corrections

Resum daisy diagrams for leading order $\frac{T^2}{m^2}$

From particle physics theory to GWs

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Effective Potential: can be computed perturbatively with finite temperature quantum field theory

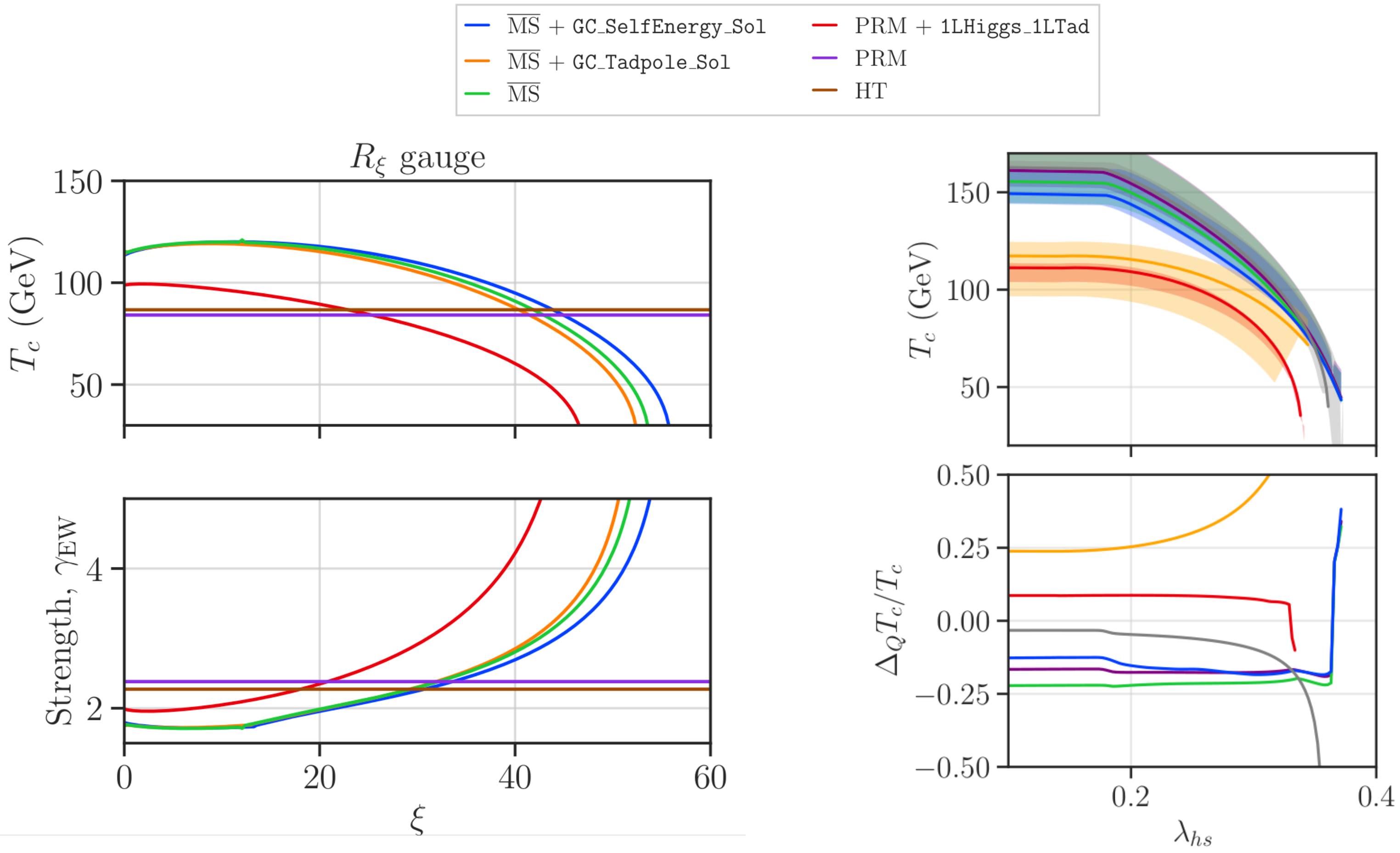
However there are problems appling this for phase transitons at finite temp

- Unphysical Gauge dependence
- Infrared divergences / problems with perturbativity for large T^2/m^2
- Many different scales in the problem
- thus large dependence on the renormalisation scale

Effective Potential

[PA, C. Balazs, A. Fowlie, L. Morris, G. White and Y.~Zhang, JHEP 01 (2023) 050]

Significant variance from gauge and renormalisation scale



Effective Potential

These issues have substantial impact on uncertainties in GW predictions

[Djuna Croon, Oliver Gould, Philipp Schicho, Tuomas V. I. Tenkanen, Graham White, JHEP 04 (2021) 055]

$\Delta\Omega_{\mathrm{GW}}/\Omega_{\mathrm{GW}}$	4d approach	3d approach
RG scale dependence	$\mathcal{O}(10^2 - 10^3)$	$\mathcal{O}(10^0 - 10^1)$
Gauge dependence	$\mathcal{O}(10^1)$	$\mathcal{O}(10^{-3})$
High-T approximation	$\mathcal{O}(10^{-1} - 10^0)$	$\mathcal{O}(10^0 - 10^2)$
Higher loop orders	unknown	$\mathcal{O}(10^0 - 10^1)$
Nucleation corrections	unknown	$\mathcal{O}(10^{-1} - 10^0)$
Nonperturbative corrections	unknown	unknown

High temperature effects can be resummed by effective field theory techniques

But non-perurbative effects may cause problems

Effective Potential

Most rigorous approach is to do this non-perturbatively on lattice

This is how we know SM EW and QCD transtions are smooth cross-overs

[K. Kajantie, M. Laine, J. Peisa, K. Rummukainen, M. Shaposhnikov, PRL 77 (1996) 2887-2890,
Y. Aoki, G. Endrodi*, Z. Fodor*, S. D. Katz*, and K. K. Szabo, Nature, 443:675–678, 2006]
[*Eötvös affiliation]

Downside: Very time consuming to do this on the lattice

Not feasible in general for new physics, we have:

- many models
- many transitions in specific models
- huge parameter spaces

$\longrightarrow$ Tension between rigour and feasability

Effective Potential

- Standard: 4D Perturbative approach with “Daisy resummation”

  Easy to implement
  Feasible for scans

- Better: 3D EFT Perturbative calculation

  Hard to implement*
  Feasible for scans

- Gold standard: non-perturbative lattice

  Hard to implement
  Not feasible for scans

* Very recently DRalgo code was developed to make this easier!

[Andreas Ekstedt, Philipp Schicho, Tuomas V. I. Tenkanen, Comp.Phys.Comm. 288 (2023) 108725]

State of the art: match to 3DEFT models with lattice results where possible,
use 3DEFT where not available (or create new lattice results...)

See e.g. [PRD 100 (2019) 11, 115024, Phys.Rev.Lett. 126 (2021) 17, 171802]

From particle physics theory to GWs

$\hat{V}_{\text{eff}} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$

PhaseTracing

So cubic terms are generated at finite temperature

Tree-level cubic terms can also be introduced in SM extensions

These may or may not lead to first order phase transitions

Depends on detailed calculation, e.g. SM is a smooth cross-over for the measured Higgs mass..

...but could have been first order if the Higgs mass was much lighter.

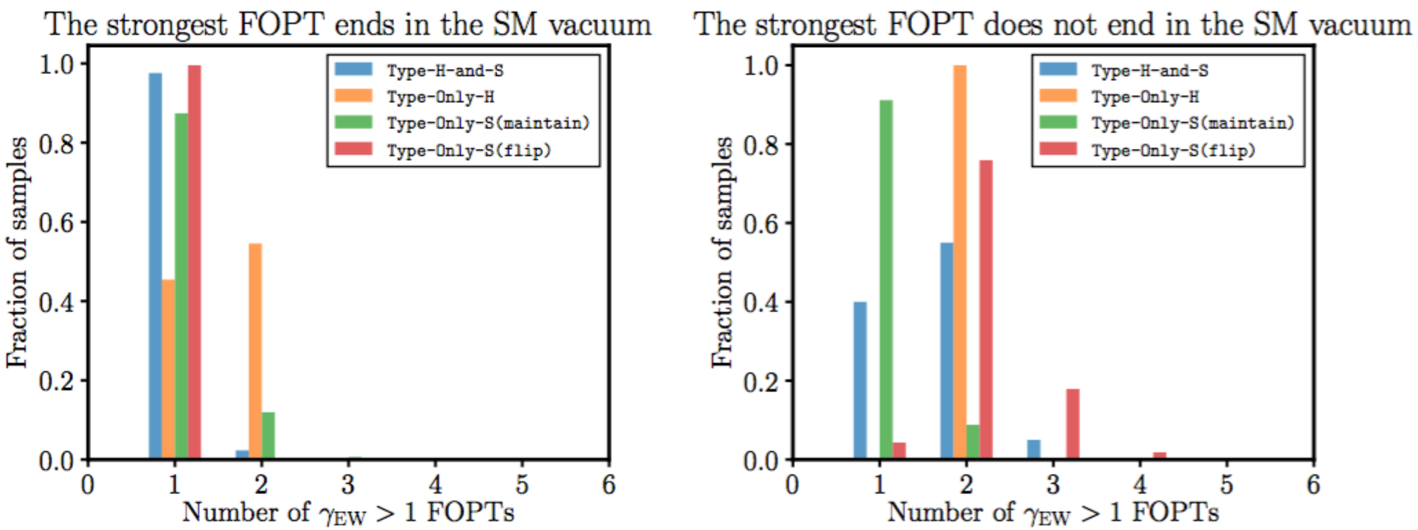
From particle physics theory to GWs

$$\hat{V}_{\rm eff} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$$

PhaseTracing

This is not straightforward:

multiple FOPTs and possible paths common in realistic models



[PA, Csaba Balazs, Andrew Fowlie, Giancarlo Pozzo, Graham White, Yang Zhang, JHEP 11 (2019) 151]

From particle physics theory to GWs

$$\hat{V}_{\rm eff} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$$

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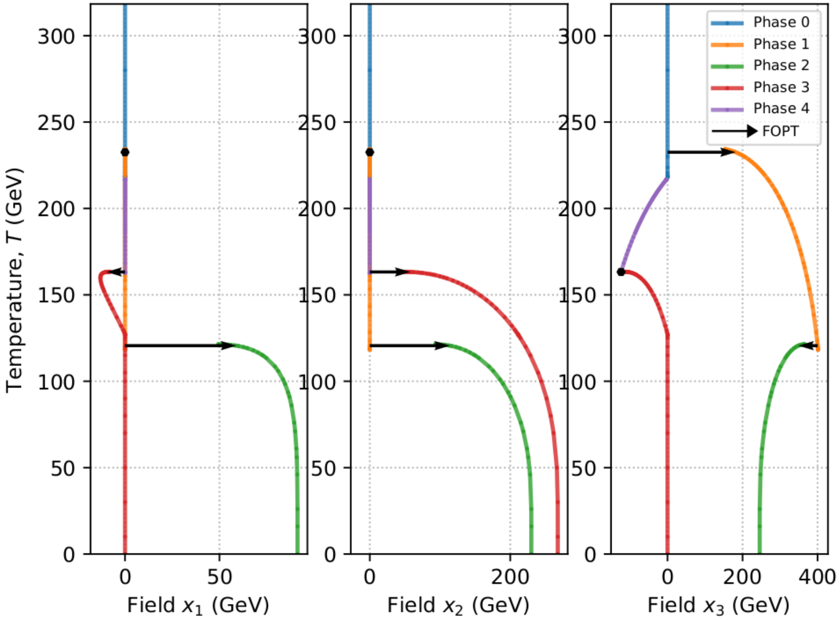


Careful algorithims needed to handle this, e.g.

- PhaseTracer ← *My own code, but I do recommend this one*
- Cosmotransitions ← *Tricky to use, often just hangs or exits*
- BSMPT ← *Simple and fast but won't get complicated patterns, mutiple PTs*

[PhaseTracer, PA, Csaba Balazs, Andrew Fowlie, Yang Zhang, Eur.Phys.J.C 80 (2020) 6, 567]

From particle physics theory to GWs

$\hat{V}_{\text{eff}} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \rightarrow \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$

Transition rates Semi-classical approx $\Gamma \approx Ae^{-B}$

Action at saddle point

Fluctuations around saddle point

B solved by finding a “bounce” instanton solution numerically

Tricky numerical problem, many public bounce solvers

CosmoTransitions [C. L. Wainwright, CPC 183 (2012) 2006–2013,],

AnyBubble [A. Masoumi, K. D. Olum and B. Shlaer, JCAP 1701 (2017) 051],

BubbleProfiler [PA, Balazs, Bardsley, Fowlie, Harries & White CPC 244 (2019) 448-468]

SimpleBounce [Ryosuke Sato, CPC 258 (2021) 107566]

All bounce solvers to date have some signifcant drawbacks

(numerical stability, reliability, noise/precision, speed, number of fields)

From particle physics theory to GWs

$$\hat{V}_{\text{eff}} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$$

Transition rates   Semi-classical approx $\Gamma \approx A e^{-B}$

Action at saddle point

Fluctuations around saddle point

A usually assumed less important,
Often estimated on dimensional grounds

$$A \approx T^4$$

$$A \approx T^4 \left(B/(2\pi T)^{3/2}\right)$$

Problem: what if A has exponential dependence?

$\longrightarrow$ Calculate it directly $\longrightarrow$ BubbleDet

[Ekstedt, Gould, and Hirvonen, arXiv:2308.15652]

Bubble nucleation

Bubbles of the new phase
form at random locations

The bubbles that already formed
grow in size

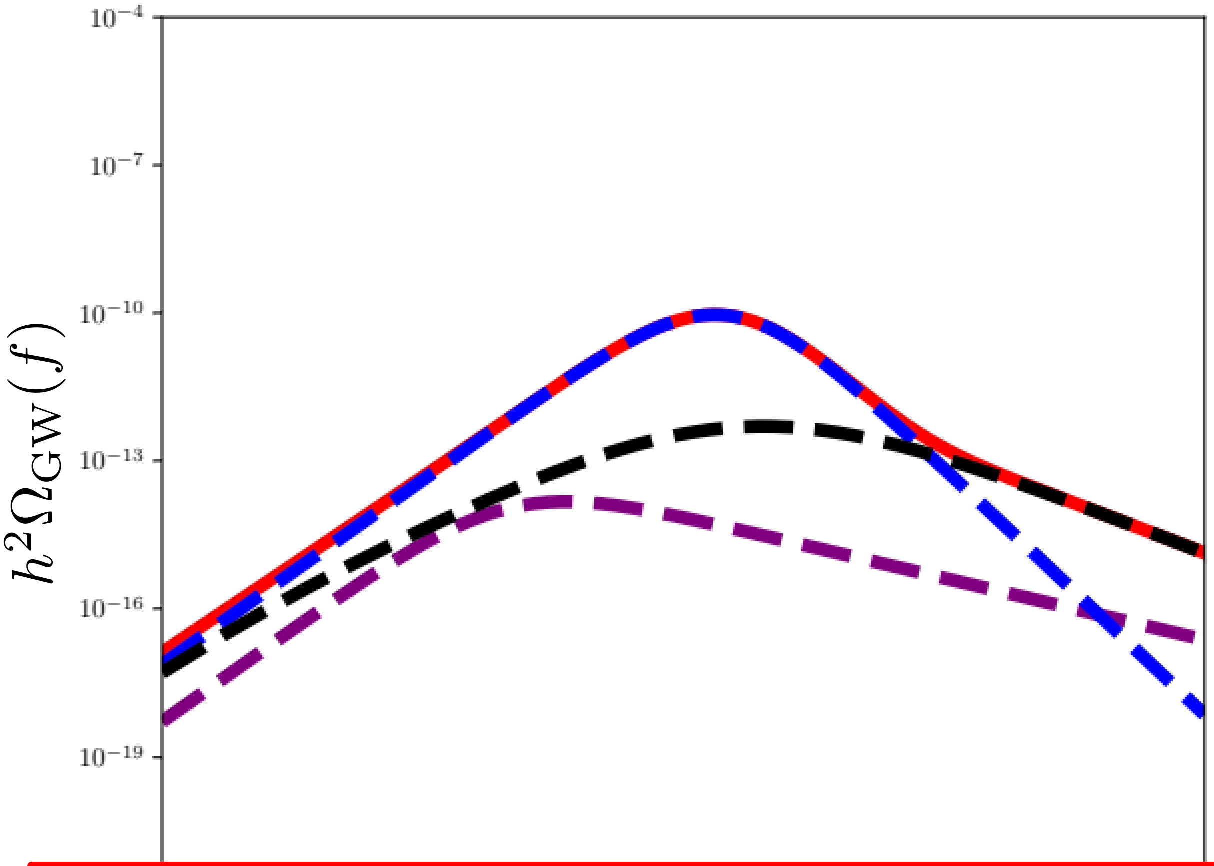
while more bubbles nucleate

As the bubbles grow,
and the number increases,
collisions become more likely

And more and more of the space is
converted to the true vacuum

[image: from Lachlan Morris]

$$\Omega_{\mathrm{GW}}(f) \equiv \frac{1}{\rho_{\mathrm{tot}}} \frac{d\rho_{\mathrm{GW}}}{d\ln f} \qquad h^2\Omega_{\mathrm{GW}} = h^2\Omega_{\mathrm{coll}} + h^2\Omega_{\mathrm{sw}} + h^2\Omega_{\mathrm{turb}}$$



The peak amplitide varies with the frequency

The signal has several contributions:

1) the collision of bubbles – which breaks their spherical symmetry.

2) waves of plasma accelerated. by the bubble wall.

3) shocks in the fluid leading to turbulence

Understanding this quantitatively requires hyrdodynamical simulations and/or clever modeling of how it happens

You will learn:

- Why its really important to have robust predictions for GWs now
- State of the art approaches
- Some big uncertainties in the predictions from first order phase transitions
- The cost of common approximations

Based on this review article:

- PA, C. Balázs, A. Fowlie, L. Morris, L. Wu, Prog.Part.Nucl.Phys 135 (2024) 104094 +
- PA, L. Morris, Z. Xu , JCAP 05 (2024) 075
- PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Y. Wu, Z. Xu, PRL 132 (2024) 22, 221001
- PA, C. Balázs, T. Gonzalo, M. Pearce, PRD 109 (2024) 6, L061303,
- PA, C. Balázs, L. Morris, JCAP 03 (2023), 006,
- PA, C. Balázs, A. Fowlie, L. Morris, G. White, Y. Zhang, JHEP 01 (2023) 050,